TRANSMISSION OF AN ULTRASONIC BEAM THROUGH A FLUID-SOLID INTERFACE

Alexander Sedov

Dept. of Mechanical Engineering
Lakehead University
Thunder Bay, Ontario
Canada P7B 5E1

Lester W. Schmerr, Jr.

Center for NDE
Dept. of Engineering Science and Mechanics
Engineering Research Institute
Iowa State University
Ames, Iowa 50011

INTRODUCTION

In immersion ultrasonic testing, a beam of sound must pass through a liquid-solid interface before it can interact with subsurface defects. In modern quantitative NDE studies, it is essential to know the beam properties in the solid so that flaw scattering variations, transducer diffraction corrections, etc. can be estimated. Using high frequency asymptotics and the method of stationary phase, we show here that analytical expressions can be derived for the wavefield radiated by a piston transducer, where the transducer is oriented normal to a plane liquid-solid interface (Fig. 1). In the main beam of the transducer these expressions will be shown to be equivalent to the solutions Schoch obtained for a single fluid medium [1].

EXACT SOLUTIONS

Schmerr and Sedov [2] have recently modeled the waves radiated by contact compressional and shear wave transducers. They showed that exact solutions for those cases could be obtained in terms of Hankel transform integral expressions. Following the same procedures for the geometry of Fig. 1, we may write, for example, the expression for the displacement, u_z, in the solid as

$$u_z(r,z,\omega) = -2i\omega\rho_o v_o a\int_o^\infty J_o(\xi r)J_1(\xi a)\alpha_1 \cdot\{(2\rho c_2^2\xi^2-\rho\omega^2)\ \exp(-\alpha_1(z-h)-\alpha_o h)-2\rho c_2^2\xi^2\exp(-\alpha_2(z-h) -\alpha_o h)\}\ d\xi/F(\xi) \quad (1)$$

with

Review of Progress in Quantitative Nondestructive Evaluation, Vol. 9
Edited by D.O. Thompson and D.E. Chimenti
Plenum Press, New York, 1990

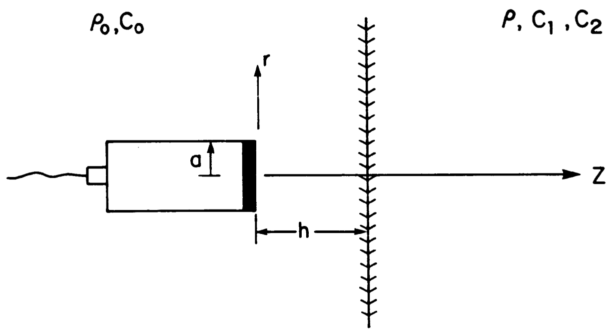


Fig. 1. Immersion transducer geometry.

$$F(\xi) = \alpha_o\left[(2\rho c_2^2\xi^2 - \rho\omega^2)^2 - 4\rho^2 c_2^4 \xi^2 \alpha_1 \alpha_2\right] + \rho\rho_o\omega^4\alpha_1 \quad (2)$$

and $\alpha_1 = (\xi^2 - k_i^2)^{1/2}$, $k_i = \omega/c_i$ (i - 1,2)

where ω is the angular frequency, c_i (i = 1,2) are wavespeeds for compressional and shear waves, respectively, a is the radius of the transducer and v_o the velocity on its surface, and ρ and ρ_o are the densities of the solid and liquid, respectively. Similar expressions, of course, can be written for the pressure in the liquid and the other velocity and stress components in the solid. Although Eq. (1) is exact, because of the infinite ξ-integration it is not in a form that is readily usable for analytical or numerical evaluation. However, for ka>>1 it is possible to extract relatively simple analytical expressions from these exact integrals that should prove valuable in a number of applications.

THE METHOD OF STATIONARY PHASE

To illustrate how one can go about evaluating integrals such as found in Eq. (1), consider the case when r<<a, i.e. we are near the central axis of transducer. In this case, and for ξa>>1 it is permissible to express the $J_1(\xi a)$ in its asymptotic form, i.e.

$$J_1(\xi a) = (2/\pi\xi a)^{1/2} \; [\exp(i\xi a - i\pi/4) \; - \; \exp(-i\xi a + i\pi/4)]/2i$$

Then Eq. (1) reduces to an expression of the form

$$u_z(r,z,\omega) = -i\omega\rho_o v_o a \int_0^\infty \sum_{m=1}^{4} g_m(\xi) \; \exp(i\psi_m + i\phi_m)d\xi \quad (3)$$

which can readily be evaluated by the method of stationary phase [3] where

$$\psi_m + \phi_m = \pm a\xi + i(\xi^2 - k_1^2)^{1/2}(z-h) + i(\xi^2 - k_o^2)^{1/2}h \;\; (m=1,2) \quad (4a)$$

are the phase terms for compressional waves and

$$\psi_m + \phi_m = \pm a\xi + i(\xi^2 - k_2^2)^{1/2}(z-h) + i(\xi^2 - k_o^2)^{1/2}h \quad (m = 3,4) \quad (4b)$$

are the corresponding phases for shear waves. We have shown these phase terms as split into two pieces, ψ_m and ϕ_m, because in the stationary phase evaluation process, it is necessary to decompose the integrands in Eq. (3) into a slowly varying term $g_m(\xi)$ exp $(i\psi_m)$ and a rapidly varying exponential, exp $(i\phi_m)$. Although this decomposition is necessary, it is not immediately obvious what the "correct" choice might be. Consider, therefore, several possible choices. First, let

$$\phi_m = \pm a\xi + i(\xi^2 - k_{1,2}^2)^{1/2}(z-h) \quad (5)$$

In this case, we can show that the stationary phase points of the integral occur at

$$\xi_s = \pm k_{1,2} \cos\theta_a$$

where $\cos\theta_a$ is shown in Fig. 2a. Physically, this choice can be shown, by examining the phase term ϕ_m at the stationary phase point, to represent waves traveling from a point (r = a, z = h) to the point (r = o, z) (Fig. 2a). In contrast, if we had chosen

$$\phi_m = \pm a\xi + i(\xi^2 - k_{1,2}^2)^{1/2} z \quad (6)$$

with stationary phase points

$$\xi_s = \pm k_{1,2} \cos\theta_b$$

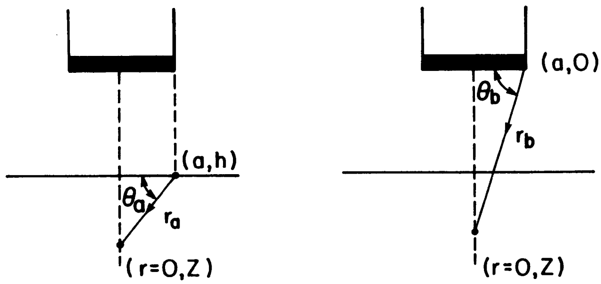
we would be representing waves travelling from point (r = a, z = 0) to point (r = 0, z) (Fig. 2b). A change in our choice of ϕ_m, as Figs. 2a and 2b demonstrate, is therefore, in effect a choice of where the waves appear to arise from in the fluid medium. As Candel and Crance [4] point out, the natural point of expansion is the virtual source location and this also leads to results which agree closely with exact values. For compressional wave terms this virtual source point is labelled as (r_v, z_v) (Fig 3a) corresponding to a choice of ϕ_m given by

$$\phi_m = \pm r_v\xi + i(\xi^2 - k_1^2)^{1/2}(z - z_v) \quad (7)$$

and stationary phase point

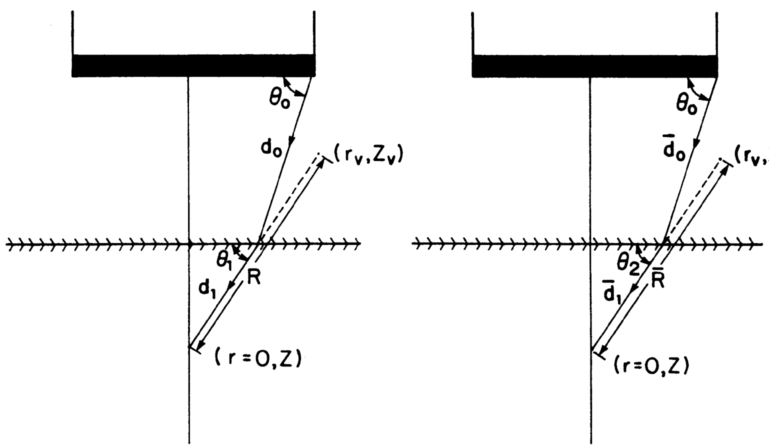
$$\xi_s = \pm k_1 \cos\theta_1$$

Similarly for shear wave terms we have a virtual source location $(\bar{r}_v, \bar{z}_v)$ (Fig. 3b) with



(a) (b)

Fig. 2. Ray paths corrresponding to two different choices for ϕ_m



(a) (b)

Fig. 3. Ray paths corresponding to (a) P-waves in the solid (b) shear waves in the solid.

$$\phi_m = \pm \bar{r}_v \xi + i(\xi^2 - k_2^2)^{1/2}(z - \bar{z}_v) \tag{8}$$

and stationary phase point

$$\xi_s = \pm k_2 \cos\theta_2$$

A straight forward but lengthy calculation shows that for the geometry of our problem we have

$$r_v = a - h[(c_1/c_o)^2 - 1] \cos^3\theta_o/\sin^3\theta_o \tag{9a}$$

and

$$z_v = h - h(c_o/c_1) \sin^3\theta_1/\sin^3\theta_o \tag{9b}$$

with similar expressions for $(\bar{r}_v, \bar{z}_v)$ if we make the replacements $c_1 \rightarrow c_2$, $\theta_1 \rightarrow \theta_2$.

With the proper choice of ϕ_m, the evaluation of the integrals of Eq. (3) then follows directly via the same procedures followed in the contact case [2]. For u_z, for example, we find

$$u_z(r,z,\omega) = (2i\ v_o/\omega)[(\rho_o c_o/(\rho_o c_o + \rho_1 c_1))$$

$$\cdot \exp i(k_1(z-h) + k_o h) - \{(a/R)^{1/2}(\sin^2\theta_1/|\cos\theta_1|)^{1/2}$$

$$\cdot (1 - 2\beta_2^2 \cos^2\theta_1/\beta_1^2) \sin\theta_1 J_o(k_1 r\cos\theta_1)\exp i(k_o d_o + k_i d_1)$$

$$+ (a/\bar{R})^{1/2}(\sin^2\theta_2/|\cos\theta_2|)^{1/2} 2\cos^2\theta_2 \ (1 - \beta_1^2 \cos^2\theta_2/\beta_2^2)^{1/2}$$

$$\cdot J_o(k_2 r\cos\theta_2) \exp i(k_o\bar{d}_o + k_2\bar{d}_1)\}/G(\theta_1)] \tag{10}$$

where $\beta_1 = c_1/c_o$, $\beta_2 = c_2/c_o$

and

$$G(\theta_1) = (\rho/\rho_o)(\beta_1^2 - \cos^2\theta_1)^{1/2} [(2\beta_2^2 \cos^2\theta_1/\beta_1^2 - 1)^2$$

$$+ 4(\beta_2^4/\beta_1^4) (\beta_1^2/\beta_2^2 - \cos^2\theta_1)^{1/2}\cos^2\theta_1 \sin\theta_1] + \sin\theta_1 \tag{11}$$

and R, $\bar{R}$, d_o, d_1, $\bar{d}_o$, $\bar{d}_1$ are all distances as defined in Figs. 3a, b.

Expressions such as Eq. (10) can be put into a more transparent form if we introduce plane wave transmission coefficients for the interface. For example, for the normal stress, σ_{zz}, we find

$$\sigma_{zz}(r,z,\omega) = v_o[T_p(\pi/2)\exp i(k_1(z-h) + k_o h)$$

$$-(a/R)^{1/2}(\sin^2\theta_1/|\cos\theta_1|)^{1/2} T_p(\theta_1) J_o(k_1 r\cos\theta_1)$$

$$\cdot \exp i(k_o d_o + k_1 d_1) - (a/\bar{R})^{1/2}(\sin^2\theta_2/|\cos\theta_2|)^{1/2} T_{sv}(\theta_2)$$

$$\cdot J_o(k_2 r\cos\theta_2) \exp i(k_o\bar{d}_o + k_2\bar{d}_1)] \tag{12}$$

where

$$T_p(\theta_1) = -2\rho c_1(1-2\beta_2^2\cos^2\theta_1/\beta_1^2)^2/G(\theta_1)$$

$$T_{SV}(\theta_2) = -8\rho c_1(1-\beta_1^2\cos^2\theta_2/\beta_2^2)^{1/2}\sin\theta_2\cos^2\theta_2/G(\theta_1)$$

are transmission coefficients relating the stress in the solid to an incident wave of unit velocity for P and SV waves, respectively.

We can easily see from Eq. (12) that the response of the transducer consists of three terms; a direct wave that travels normally from the face of the transducer and P and SV waves that travel from the edge of the transducer. If the same approximations were made for the transducer radiating into a single fluid medium, we would find for the radiated pressure, p:

$$p = \rho_o c_o v_o\left[\exp(ik_o z) - J_o(k_o r\cos\theta_o)\exp i(k(a^2+z^2)^{1/2})\right] \tag{13}$$

which is the solution Schoch [1] found over forty years ago. As Schmerr and Sedov have pointed out [2], although Schoch-like solutions such as Eq. (12) were derived under the assumption r<<a, in fact they can be used for r<a with little error if the distance from the transducer is sufficiently large. Thus, the range of validity of Eq. (12) is much larger than might be expected from the formal analysis.

For r>a, i.e. outside the transducer main beam, we can also use our modified stationary phase approach to obtain, for example, the stress σ_{zz} as

$$\begin{aligned}\sigma_{zz}(r,z,\omega) = & -ik_1a^2v_o\ \{(R|\cos\theta_1|\sin\theta_1/r)^{1/2}T_p(\theta_1)\\ & \cdot[J_1(k_1a\cos\theta_1)/k_1a|\cos\theta_1|]\exp i(k_od_o+k_1d_1)/R\}\\ & -ik_2a^2v_o\ \{(\bar{R}|\cos\theta_2|\sin\theta_2/r)^{1/2}T_{SV}(\theta_2)\\ & \cdot[J_1(k_2a\cos\theta_z)/k_2a|\cos\theta_2|]\exp i(k_o\bar{d}_o+k_2\bar{d}_1)/\bar{R}\}\end{aligned} \tag{14}$$

This may be compared to the single medium fluid problem where

$$p = -ika^2\rho_oc_ov_o[J_1(ka\sin\theta)/ka\sin\theta]\exp(ikR_o)/R_o \tag{15}$$

The major differences between Eqs. (14) and (15) are 1) the existence of the shear wave term in the solid solution of Eq. (14) and 2) the fact that the ordinary directivity function $J_1(u)/u$ of the fluid problem is modified in the case of the solid by the existence of the transmission coefficient and geometrical spreading terms.

CONCLUSIONS

We have shown that a modified version of the method of stationary phase can be used to extract analytical expressions for the waves radiated by a piston transducer normally through a fluid solid interface. These solutions are generalizations, for a solid medium, of the Schoch solutions for a fluid.

ACKNOWLEDGEMENT

Professor A. Sedov was supported in this work by the Natural Sciences and Engineering Research Council of Canada (Grant No. A4977). This research was supported (for L. W. Schmerr) by the Center for Nondestructive Evalution at Iowa State University and was performed at Ames Laboratory. Ames Laboratory is operated for the U.S. Department of Energy by Iowa State University under Contract No. W-7405-ENG-82.

REFERENCES

1. Schoch, A., "Betrachtungen uber das Schallfeld einer Kolbenmembran," Akust. Z, 6, 318-326, 1941.
2. Schmerr, L. W. and A. Sedov, "An Elastodynamic Model for Compressional and Shear Wave Transducers," J. Acoust. Soc. Am., 1989, (to appear).
3. Kino, G. S. Acoustic Waves, Prentice-Hall Inc., Englewood Cliffs, N. J., 1987.
4. Candel, S. M. and C. Crance, "Direct Fourier Synthesis of Waves in Layered Media and the Method of Stationary Phase," J. Sound Vib., 74, 1981.