PREDICTION AND ANALYSIS OF TRANSIENT

EDDY-CURRENT PROBE SIGNALS

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PULSED EDDY-CURRENTS

In eddy-current nondestructive evaluation, the electromagnetic field is usually excited by a probe carrying a time-harmonic current and flaw information inferred from the amplitude and phase of the probe signal. In principle, transient excitation of eddy-currents would seem to offer great advantages since the probe response contains the equivalent information of a spectrum of frequencies. This paper explores a number of basic transient solutions due to normal air-cored coils and shows how the induced emf in a coil is related to its coupling coefficient.

FOURIER LAPLACE TRANSFORMS

We can make use of some standard time-harmonic results to calculate transient fields since the frequency and time-domain solutions are related through the Fourier transform. Thus we define

$$\begin{aligned} F(t) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} f(i\omega)e^{i\omega t}\ d\omega \\ &= \frac{1}{2\pi i}\int_{Br} f(s)e^{st}\ ds, \end{aligned} \tag{1}$$

where $F(t)$ represents the time-domain solution and $f(i\omega)$ its Fourier transform. Br denotes the Bromwitch contour taking the path of integration with respect to s in the complex plane to the right of any poles. (1) enables us to derive a transient solution from a corresponding time-harmonic solution.

Perhaps the most well known time harmonic fields in the theory of eddy-current NDE are those determined by Dodd and Deeds for normal rectangular section coils above multi-layered infinite conducting slabs [1]. To take initially a slightly more general case, suppose we consider a normal axially symmetric coil of arbitrary cross section carrying a current whose density has the form

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$$\bar{J}(\rho, z; s) = I(s) \overset{\circ}{J}(\rho, z)\hat{\phi}$$

in cylindrical polar coordinates. $I(s)$ is the coil current $\overset{\circ}{J}(\rho, z)$ is the turns density function and $\hat{\phi}$ is the azimuthal unit vector. Then the electric field $\Delta\bar{E}(\bar{r}, s)$, reflected at the plane surface of a conductor normal to the axis of the coil may be written as an inverse Fourier-Bessel transform,

$$\Delta\bar{E}(\bar{r}; s) = s\mu_0 I(s)\hat{\phi} \int_0^\infty \frac{1}{2\kappa}\Gamma(\kappa; s)e^{-\kappa z}\mathcal{J}(\kappa)J_1(\kappa\rho)\kappa \; d\kappa, \tag{2}$$

where μ_0 is the permeability of free-space and $\Gamma(\kappa; s)$ is the transverse electric reflection coefficient. Here we are assuming an isotropic conductor whose material properties vary only in the z-direction. $\mathcal{J}(\kappa)$ is defined through integrals over the coil turns density function. We shall write it as the Fourier-Bessel transform of a function $j(\kappa, \rho)$. That is

$$\mathcal{J}(\kappa) = \int_0^\infty j(\kappa, \rho)J_1(\kappa, \rho)\rho d\rho, \tag{3}$$

where $j(\kappa, \rho)$ is the Laplace transform of the turns density function. Thus

$$j(\kappa, \rho) = \int_0^\infty \overset{\circ}{J}(\rho, z)e^{-\kappa z} \; dz. \tag{4}$$

Although we can make further progress in some cases using analytical techniques [2] [3], the integration with respect to κ generally must be done numerically and it seems best to leave it until last. For a cylindrical coil of rectangular cross-section and with the coil turns density n, constant over the cross-section, we have

$$\overset{\circ}{J}(\rho, z) = n \qquad a_2 < \rho < a_1, \qquad -b < z - h < b \tag{5}$$

and zero otherwise. Here a_1 is the outer radius of the coil and a_2 the inner radius. $2b$ is the axial length and h the height of the coil center above the surface of the conductor. From (3) and (4) we find that

$$\mathcal{J}(\kappa) = \frac{2n}{\kappa}e^{-\kappa h}sinh(b\kappa)[a_1^2\chi(a_1\kappa) - a_2^2\chi(a_2\kappa)] \tag{6}$$

where χ arises from the radial integral of (3) and is defined in terms of a standard integral by

$$\begin{aligned}\chi(\alpha) &= \int_0^1 J_1(\alpha\rho)\rho d\rho \\ &= \frac{2\pi}{\alpha}[J_1(\alpha)\mathcal{H}_0(\alpha) - J_0(\alpha)\mathcal{H}_1(\alpha)]\end{aligned} \tag{7}$$

$\mathcal{H}_0$ and $\mathcal{H}_1$ being Struve functions.

TRANSIENT FIELDS

Evidently from (2) the frequency/time dependence of an arbitrary normal coil is determined by the function

$$\psi(\kappa, s) = sI(s)\Gamma(\kappa, s) \tag{8}$$

Carrying out the inverse Laplace transform of this function, according to (1), we define

$$\Psi(\kappa,t) = \frac{1}{2\pi i}\int_{Br} \psi(\kappa,s)e^{st}\ ds \tag{9}$$

Hence from the inverse Laplace transform of equation (2), the time dependent reflected electric field is given by

$$\Delta\bar{E}(\bar{r};t) = \mu_0\hat{\phi}\int_0^\infty \frac{1}{2\kappa}\Psi(\kappa,t)e^{-\kappa z}\mathcal{J}(\kappa)J_1(\kappa\rho)\kappa\ d\kappa \tag{10}$$

In some special cases where the reflection coefficient in (8) has a simple form, the inverse Laplace transform, equation (9), can be carried out analytically; the reflection coefficient for a half-space conductor allows such possibilities. The electric field may then be evaluated from (10) by numerical integration or perhaps by using a fast Fourier-Bessel transform. If both of the inverse Laplace and the Bessel transform must be carried out numerically, it would probably be best not to use (9) but instead use the fast Fourier transform, integrating numerically with respect to ω rather than s.

TRANSIENT PROBE RESPONSE

The emf induced in a probe coil due to the field reflected by a conductor is given by

$$v(s) = \frac{1}{I(s)}\int_\Omega \Delta\bar{E}(\bar{r};s)\cdot\bar{J}(\bar{r};s)d\bar{r}, \tag{11}$$

where Ω is the coil region. For an axially symmetric system, with the electric field given by (2), this becomes

$$v(s) = 2\pi s\mu_0 I(s)\int_0^\infty \frac{1}{2\kappa}\Gamma(\kappa,s)[\mathcal{J}(\kappa)]^2\kappa\ d\kappa \tag{12}$$

hence, the time domain probe response is given by

$$V(t) = 2\pi\mu_0\int_0^\infty \frac{1}{2\kappa}\Psi(\kappa,t)[\mathcal{J}(\kappa)]^2\kappa\ d\kappa, \tag{13}$$

Thus, given $\Psi(\kappa,t)$, the probe signal variation with time may be found. Let us find $\Psi(\kappa,t)$ for some cases of interest.

EVALUATION OF TRANSIENTS

Assuming we have a uniform half-space conductor of conductivity σ, the transverse electric reflection coefficient is given by [4]

$$\Gamma(\kappa;s) = \frac{\kappa-(\kappa^2+\mu_0\sigma s)^{1/2}}{\kappa+(\kappa^2+\mu_0\sigma s)^{1/2}}. \tag{14}$$

Suppose the coil carries a step current excitation defined by $\mathcal{I}(t) = I_0 u(t)$ with $u(t) = 1$, for $t > 0$ and $u(t) = 0$ otherwise. Then $I(s) = I_0/s$ and

$$\psi(\kappa,s) = I_0\left[\frac{2\kappa}{\kappa+(\kappa^2+\mu_0\sigma s)^{1/2}} - 1\right]. \tag{15}$$

Using standard Laplace transforms [5] we find ($t \geq 0$)

$$\Psi(\kappa,t) = \frac{2I_0}{\tau}\left[\sqrt{\frac{\tau}{\pi t}}e^{-t/\tau} - erfc\left(\sqrt{\frac{t}{\tau}}\right)\right] - I_0\delta(t) \tag{16}$$

where $\tau = \mu_0\sigma/\kappa^2$. Although this is a very simple example, it illustrates a number of issues raised in finding transient eddy-current solutions. The result may be derived by deforming the Bromwitch contour of integration to give a branch integral along the negative real axis to the point $-1/\tau$. The functional form of the result is found from the branch integral. Since the reflection coefficient is ubiquitous in the electromagnetic theory of penetrable bodies, consideration of its associated branch cuts is a recurring theme in working out time-dependent fields.

Another point to notice about the present example is that ψ varies as $s^{-1/2}$ for large s and as such, it would not be easy to transform numerically via the inverse FFT. Where we are forced to use numerical methods, it may be best to separate out any high frequencies dependence of this form and deal with it analytically, if possible. One can anticipate that the problem of a slowly converging integral will arise with surface breaking cracks since thin-skin perturbation theory gives rise to first order terms varying as $s^{-1/2}$. For subsuface cracks in contrast, the probe response dies away rapidly with frequency, therefore no such problem arises.

It is of interest to determine the integrated probe response as well as the time dependent probe signal. The former can be found in this case by first integrating (16) directly to give $(t \geq 0)$

$$\Psi_{int}(\kappa, t) = 2I_0\left[\sqrt{\frac{t}{\pi\tau}}e^{-t/\tau} + \frac{1}{2}(1+\frac{2t}{\tau})erf\left(\sqrt{\frac{t}{\tau}}\right) - \frac{t}{\tau}\right] - I_0u(t) \tag{17}$$

again $\tau = \mu_0\sigma/\kappa^2$. (13) with Ψ replaced by Ψ_{int} then gives the integrated probe response, $\mathcal{V}(t)$ say.

NUMERICAL RESULTS

As with the coil impedance characteristic, it proves useful to introduce a normalization convention with a free-space coil parameter as reference. The self-induced emf of the coil in free-space is given by an expression similar to (11) except that the electric field of the coil in an unbounded domain must replace $\Delta\bar{E}$. Assuming the excitation is due to a step current, the self-induced emf is theoretically a delta function, having the form $V_0\delta(t)$ for a transition at $t = 0$. The constant V_0 can be evaluated using the Fourier-Bessel representation of the free-space electric field in (11)-(13). For a normal coil of rectangular cross-section it is found that

$$V_0 = \pi\mu_0 \int_0^\infty [\mathcal{J}_0(\kappa)]^2 d\kappa \tag{18}$$

where

$$\mathcal{J}_0(\kappa) = \frac{4b}{\kappa}[1 - \frac{1}{b\kappa}e^{-\kappa b}sinh(\kappa b)][a_1^2\chi(a_1\kappa) - a_2^2\chi(a_2\kappa)] \tag{19}$$

The normalized coil emf and integrated response due to the reflected field are now defined as

$$V_n(t) = V(t)/V_0 \qquad \text{and} \qquad \mathcal{V}_n(t) = \mathcal{V}(t)/V_0 \tag{20}$$

In the limit as $t \to 0$ $(t > 0)$ the integrated signal becomes

$$\mathcal{V}_n(0) = -\frac{\int_0^\infty [\mathcal{J}(\kappa)]^2 d\kappa}{\int_0^\infty [\mathcal{J}_0(\kappa)]^2 d\kappa} = -k^2 \tag{21}$$

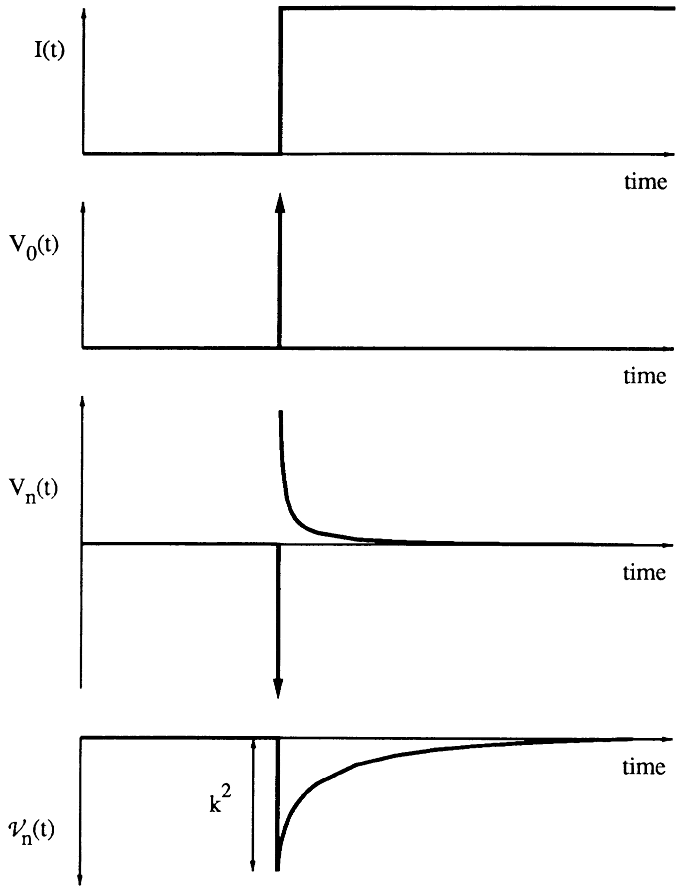

Fig. 1. Time dependent signal variation. Notation is defined in the text.

where k is the probe coupling coefficient; the same coupling parameter traditionally introduced in the theory of time harmonic coil impedance analysis. Fig. 1 compares the various time dependent signals for a step current excitation in a normal coil and shows the initial value of $\mathcal{V}_n$ as $-k^2$. Fig. 2 compares the induced emf as a function of time due to the reflected field for three coils of different cross-section, again assuming a step current gives rise to the electromagnetic field. The coils are all of 20 mm. O/D and are designated as pancake (2 mm. I/D 1 mm. axial length) square section (10 mm. I/D 5 mm. axial length) and solenoidal (16 mm. I/D 10 mm. axial length). The material conductivity was assumed to be 2.5×10^7 S/m. Naturally the coupling coefficient is greatest for the pancake coil. The corresponding integrated signals for the three coils are shown in Fig. 3.

CONCLUSION

Some simple time domain solutions for air-cored coils have been examined showing how the coil coupling coefficient is related to the induced emf. Clearly similar time dependent signals also arise in ferrite

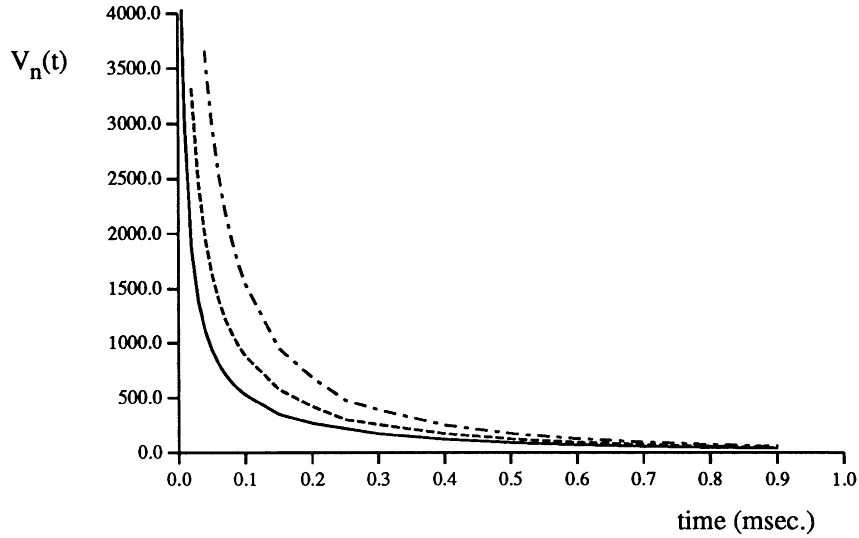

Fig. 2. Normalized induced emf due to the reflected field excited by a step current in the driving coil. Three coils are compared: pancake; dash-dot line, square section; dashed line, solenoidal coil; solid line. The instantaneous pulse reflection is not show

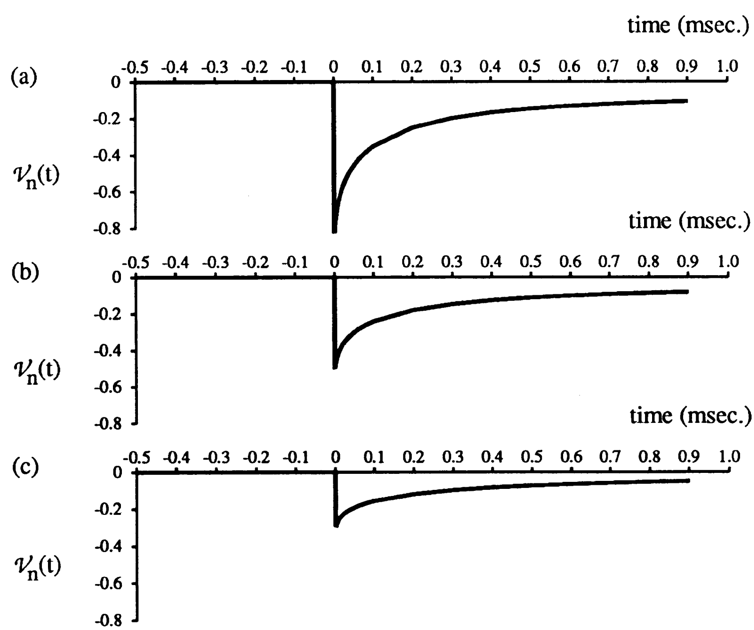

Fig. 3. Integrated probe signal due to the reflected field for (a) a pancake coil (b) square section coil (c) solenoidal coil.

cored probes. For cup cored-probes in particular it has been found that the frequency dependent impedance characteristic has the same form for a range of lift-off values [6] [7]. As a consequence, the time dependent self-induced emf in a cup-core probe has a shape that is insensitive to lift-off and depends only on a scaling factor. This behavior makes it easier to compensate for lift-off variations.

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