A MODEL-BASED RECONSTRUCTION METHOD FOR INCOMPLETE PROJECTION INDUSTRIAL COMPUTED TOMOGRAPHY IMAGING

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INTRODUCTION

In computerized tomography, the cross-sectioned image of an object can be reconstructed from a set of projection data. It provides the ability to image internal structure which can not be inspected effectively with alternate techniques. Based on the Fourier slice theorem[1], projections in a full angular range and with sufficiently fine angular spacing are required to reconstruct a unique image. In some situations, however, complete projections are not available due to physical limitations in the data acquisition process. Image quality is degraded by the absence of complete data. Because most manufactured parts were built from a designer's blueprint or solid modeling electronic database, a great deal is known about the physical structure of the part. Incorporating a priori information extracted from the CAD model has the potential to enhance incomplete projection CT image quality. In this paper, a model-based CT reconstruction method is presented. The a priori information used to enhance incomplete projection CT image quality is extracted from a 3-D solid modeling electronic database. Engineering database matching is conducted to extract the proper 2D cross-sectioned model image corresponding to the CT projection plane. A moment-based registration method is applied to ensure proper use of a priori information for model-based CT reconstruction. Furthermore, a projection substitution scheme, including projection alignment and automatic scaling method, is developed so that the projection data in the missing angular range calculated from a model image can be automatically rescaled to match the projection data in the available angular range. Experimental results of applying the model-based CT reconstruction method to an industrial part in both the limited-angle and the penetration-limited incomplete projection situations are presented and described. It is shown that the use of a priori information from solid models is a powerful technique for enhancing the quality of incomplete data CT images.

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In general, the incomplete projection situation can be divided into two categories: limited-angle and penetration-limited. In the limited-angle situation, incomplete data results when sections of a complete part obstruct the data acquisition scan over a portion of the angular range. Projection data are not available in some angular range. In the penetration-limited situation, the x-ray path length through certain sections of the part is too large to allow sufficient x-ray penetration for meaningful measurement. Certain portion of projection data are therefore saturated due to over-attenuation. Image quality is degraded by the absence of complete data in both situations. A model-based reconstruction method has been developed to incorporate a priori information extracted from a 3-D solid modeling electronic database to compensate incomplete projection data. The model-based reconstruction algorithm is schematically illustrated in Fig. 1. It consists of a priori information extraction and registration, projection scaling factor calculation, and projection substitution. Engineering database matching, which correlates the electronic model, blueprint specification and data acquisition setup is applied such that a 2D cross-sectioned image can be extracted from a solid model with the desired orientation and position of the CT projection plane. Accurate registration between the model cross-sectioned image and the CT image reconstructed from the incomplete projection data set is required to ensure proper utilization of a priori information for model-based CT reconstruction. Since the a priori information extracted from a 3D solid model does not contain the information of the CT penetration coefficient, the scaling factor has to be estimated such that the projection data calculated from the model image can be rescaled to match the acquired projection data. An automatic scaling method has been developed. It uses the averaged ratio of area covered by co-existing projection pairs over the available angular range as the scaling factor for projection alignment. This method ensures the proper alignment of the projection data set and establishes the success of the model-based CT reconstruction. Finally, projection data acquired over the available angular range and projection data calculated from the model image over the missing angular range are used for reconstruction in the limited-angle situation. For the penetration-limited situation, projection substitution is applied to the penetration-limited portion of the projection data to create a model-enhanced projection data set for reconstruction.

A PRIORI INFORMATION EXTRACTION AND REGISTRATION

A CAD electronic database can be created using a solid modeling development tool such as the TRUCE[2] solid modeller developed by the solid modeling program at General Electric Company Corporate Research and Development. In order to derive the correct cross-section from the 3-D solid model, engineering database matching has to be conducted. It correlates the electronic model, drawing specification and the data acquisition setup so that the two-dimensional cross section which corresponds to the CT projection plane can be successfully extracted from a 3-D solid model. The polygon shaded 3-D solid model of a GE Aircraft Engine blade is shown Fig. 2.(a). Figure 2.(b) shows a typical CT image reconstructed from a penetration-limited projection data set. The two-dimensional cross-sectioned model

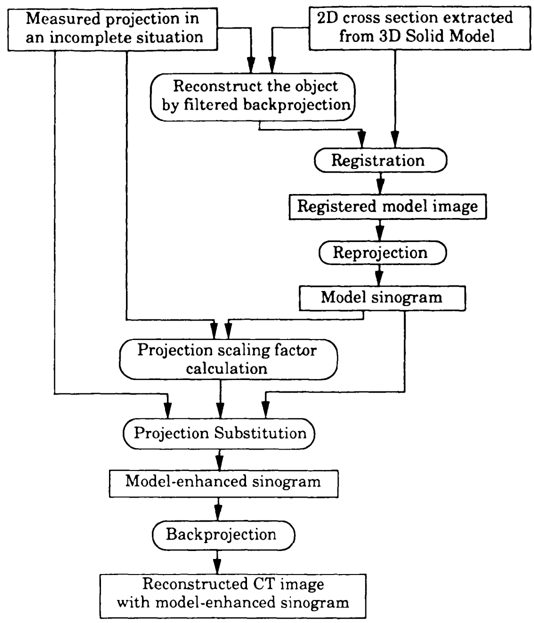


Figure 1. Model-Based CT Reconstruction Algorithm

image extracted from a 3-D solid model with the desired orientation and position of the CT projection plane is shown in Fig. 2.(c).

To properly utilize a priori information from a CAD model for incomplete CT reconstruction, accurate registration between the model image and the CT image is required. A two-dimensional moment-based registration method[3] has been developed and applied to ensure proper utilization of a priori information for model-based CT reconstruction. A geometric transformation is estimated based on the first and second moments of the model image and the reconstructed CT image with incomplete projections. The translation factor is calculated from the offset of the object's central gravity point, which can be computed as the first moment about the x and y axes. The axis of elongation, defined as the line for which the integral of the square of the distance to points in the object is a minimum, is used to represent the object orientation. The rotation is computed from the variations of the orientation angle. The orientation angle is defined as the angle between the axis of elongation and the x axis and can be computed as function of the second moments about the central gravity point. A transformation matrix, containing both the translation and rotation factors, is then applied to the model image extracted from a solid model such that the resultant image is geometrically correlated to the CT image. Figure 3.(a) shows the reconstructed CT image from incomplete projections and Figure 3.(b) shows its registered model image after applying the moment-based registration method.

PROJECTION SUBSTITUTION SCHEME

A projection substitution scheme has also been developed. It consists of projection

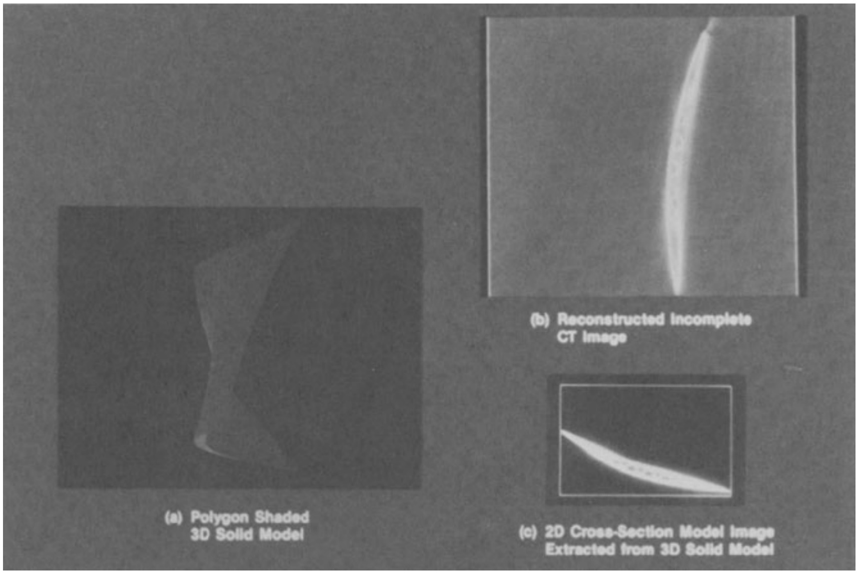
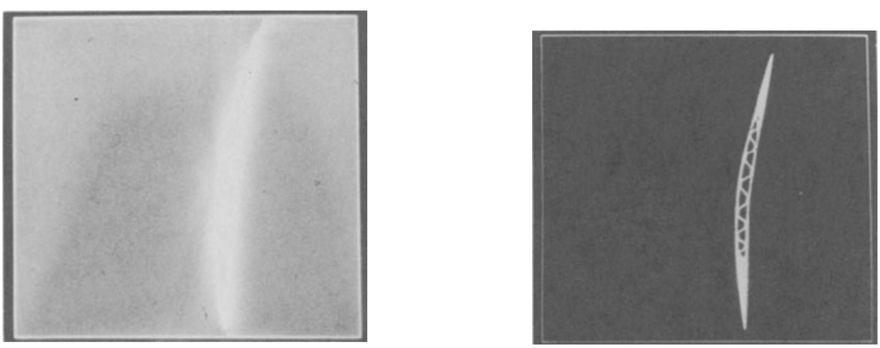


Figure 2. A Priori Information from 3D Solid Model



(a) Reconstructed CT image from incomplete projection

(b) Registered model image

Figure 3. Moment-Based Registration

alignment and automated scaling algorithms. Since the center detector number and the first viewing angle of the projection computed from the registered model image may be different from those used in the data acquisition, projection alignment must be conducted. In addition, the CT penetration coefficient is generally unknown. In order to ensure proper projection substitution, a scaling factor has to be estimated such that the projection calculated from the model image can be rescaled to match the experimentally acquired projection data. An automatic scaling method has been developed to automatically estimate the scaling factor from the co-existing projection pairs over the confident angular range. The co-existing projection pair are the projection data acquired from the available angular range and its corresponding projection data calculated from the registered model image. For the limited-angle situation, the co-existing projection pairs are selected from the measurable angular range. For the penetration-limited situation, the co-existing projection pairs are selected from the angular range without over-attenuation problems. Figure 4.(a) shows the profile of a co-existing projection data set at one particular view angle : one is extracted from the measured projection data set and onc is calculated from the model image. These two projection profiles have noticeably different scales.

The reconstruction procedure that is most commonly used on CT scanners is based on the filtered-backprojection algorithm[4]. The relationship between the reconstructed image $f(r,\phi)$, represented in a polar coordinate system, and the fan-beam projections $P_\theta(t)$ can be stated as follows :

$$f(r,\phi) = \frac{1}{2}\int_0^{2\pi} \frac{1}{U^2} \int_{-t_m}^{t_m} P_\theta(t) \ [\frac{D}{(D^2+t^2)^{1/2}}] \ h_1(t'-t) \ dtd\theta \qquad (1)$$

where

$$\begin{aligned} U &= \frac{(D + r\sin(\theta-\phi))}{D} \\ t' &= D[\frac{r\cos(\theta-\phi)}{D + r\sin(\theta-\phi)}] \end{aligned} \qquad (2)$$

D is the source-to-detector distance. The generalized distance function D between the reconstructed image $f(r,\phi)$ and the rescaled model image $\kappa f_{model}(r,\phi)$ can be presented as

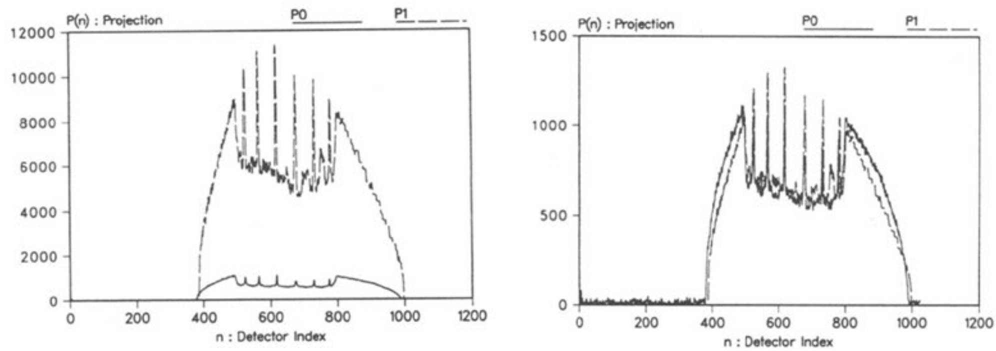
$$D = [f(r,\phi) - \kappa f_{model}(r,\phi)]^2 \qquad (3)$$

The scale factor κ can be estimated by solving $\frac{\partial D}{\partial \kappa} = 0$, i.e.

$$[f(r,\phi) - \kappa f_{model}(r,\phi)] = 0 \qquad (4)$$

Combining Eqs (1) and (4), one obtains

$$\frac{1}{2}\int_0^{2\pi} \frac{1}{U^2} \int_{-t_m}^{t_m} [P_\theta(t) - \kappa\hat{P}_\theta(t)] \ [\frac{D}{(D^2+t^2)^{1/2}}] \ h_1(t'-t) \ dtd\theta = 0 \qquad (5)$$



(a) Before scaling **(b) After scaling**

Figure 4. Profile of A Co-existing Projection Data Set at One View Angle

where $\hat{P}_\theta(t)$ represents the projection data calculated from the model image. Due to the incomplete projection situation, measured projection data at some view angle may not exist or may be inaccurate. In order to estimate the scaling factor κ, only those projection data over the confident angular range are used for calculation. The scaling factor κ is given by the averaged ratio of area covered by co-existing projection pair over the confident angular range. i.e.

$$\kappa = E[\ \frac{\int_{-t_m}^{t_m} P_\theta(t)dt}{\int_{-t_m}^{t_m} \hat{P}_\theta(t)dt}\ ] \tag{6}$$

where E[.] denotes the mathematical expectation over the available angular range. For the limited-angle situation, the projection data in the measurable angular range are used for the scaling factor calculation. For the penetration-limited situation, measured projection data without over-attenuation problems are used for estimation. Figure 4.(b) shows the profile of one co-existing projection data set after applying the estimated scaling factor to the projection data calculated from model image. This method ensures the proper alignment of the projection data set and established the success of the model-based CT reconstruction.

EXPERIMENTAL RESULTS

The experimental data set that was used for validation consists of a full 360° angular range projection data set for a pressure welded blade disk acquired on GE's new industrial X-ray CT inspection system. The limited-angle projection situation arises where the part is inaccessible at some view angles due to obstruction on the scanning path. Two missing cones each with 40° included angle, are placed in the most highly penetrating directions. Figure 5.(a) shows the limited-angle sinogram

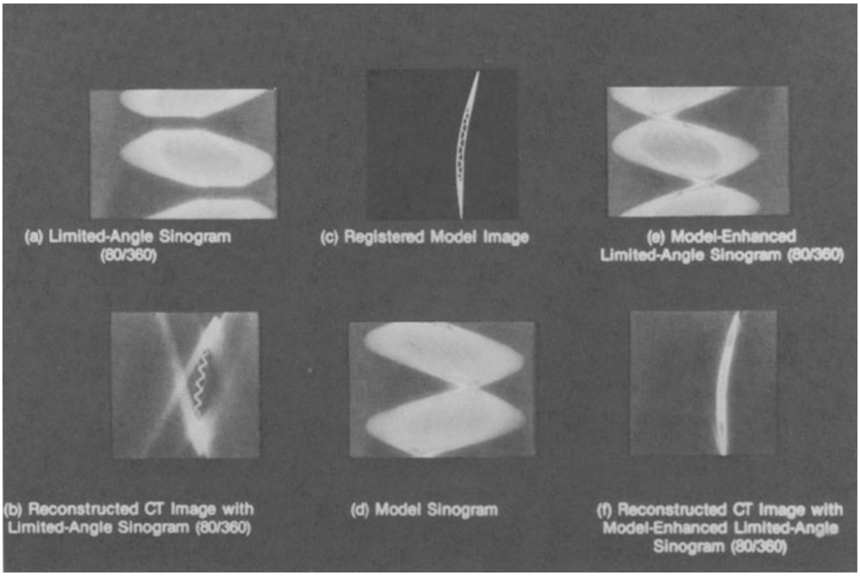


Figure 5. Model-Based Reconstruction with Limited-Angle Projections

data selected from the complete data set, with 80° missing angle out of 360° full scanning angular range. The reconstructed CT image with this limited-angle sinogram data is shown in Fig. 5.(b). The reconstructed image is seriously distorted. Major portions of the walls are missing and significant streak artifacts are present. The image is hardly useable for any quantitative measurement.

Using the model image shown in Fig. 5.(c), which is properly registered with the orientation and position of the reconstructed CT image with incomplete projection and preserves its internal structure, the model sinogram data can be calculated via the reprojection method and is shown in Fig. 5.(d). Applying the projection alignment, rescaling, and projection substitution schemes, the model-enhanced limited-angle sinogram is achieved by combining the limited-angle sinogram data and the model sinogram data and is shown in Fig. 5.(e). The reconstructed CT image with model-enhanced limited-angle sinogram data is produced and shown in Fig. 5.(f). The improvement in image quality is substantial.

In addition, applying the model-based CT reconstruction method to enhance penetration-limited CT image quality has also been studied and validated. Fig. 6.(a) shows the complete sinogram data of the pressure-welded blade disk. Due to the relatively high aspect ratio, the penetration-limited situation has occurred due to over-attenuation. The reconstructed CT image with the penetration-limited sinogram is shown in Fig. 6.(b). It is noticeable that those walls parallel to the most penetration limited direction are not as firm as the others. Using the model-based CT reconstruction technique, the model-enhanced sinogram has been generated and is shown in Fig. 6.(c) and the reconstructed image with model-enhanced sinogram data is presented in Fig. 6.(d). The result shows significant improvement and those less firm walls have been rebuild.

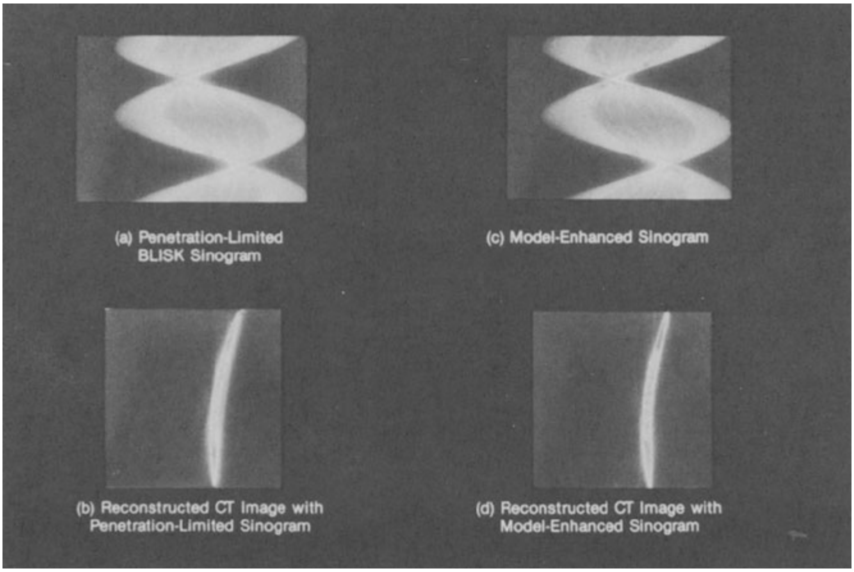


Figure 6. Model-Based Reconstruction with Penetration-Limited Projections

CONCLUSION

A model-based CT reconstruction method has been developed to incorporate a priori information extracted from a 3D solid model into incomplete data CT images. Significant improvement in image quality is demonstrated using this new technique. It requires detailed a priori information about the part. However, it provides excellent image quality and rapid calculation of the final image.

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