AN ANALYSIS OF ULTRASONIC FLAW SCATTERING AMPLITUDE AS A RANDOM VARIABLE

Steven P. Neal

Department of Mechanical and Aerospace Engineering
University of Missouri-Columbia
Columbia, MO 65203

Donald O. Thompson

Center for NDE
Iowa State University
Ames, IA 50011

INTRODUCTION

The use of prior information is an important component in the ultrasonic detection, classification, and characterization of flaws. In order to take full advantage of advanced digitally based approaches to flaw detection, classification, and characterization, use of prior information will be critical. Some advanced techniques involve probabilistic approaches which start with a stochastic model for a flaw signal in which the flaw's scattering amplitude is assumed to be an uncorrelated, Gaussian random variable with zero mean and known variance [1-4]. The goal of the work presented here was to analyze scattering amplitude as a random variable with emphasis on evaluation of these assumptions.

The body of this paper is separated into a background section and an analysis section. The background section focuses on the nature of scattering amplitudes with emphasis on characteristics which influence scattering amplitude as a random variable. The analysis section concentrates on evaluating the assumptions that scattering amplitude is an uncorrelated, Gaussian random variable with zero mean and known variance. In this section it is shown that these assumptions are, in general, not valid. The paper is concluded with a summary section.

BACKGROUND

Consider an idealized scattering problem involving the interrogation of a flaw in an otherwise isotropic, homogeneous, and unbounded solid by a unit amplitude, longitudinally polarized delta function plane wave. In the far field, spherically spreading waves resulting from scattering at the flaw can be described in terms of the flaw's scattering amplitude. The scattered field can be written as [5]

$$u_i = \frac{A(\hat{e}_i,\hat{e}_s,\omega)}{r} e^{j(\omega t-k\hat{e}_i\cdot\vec{r})} \qquad (1)$$

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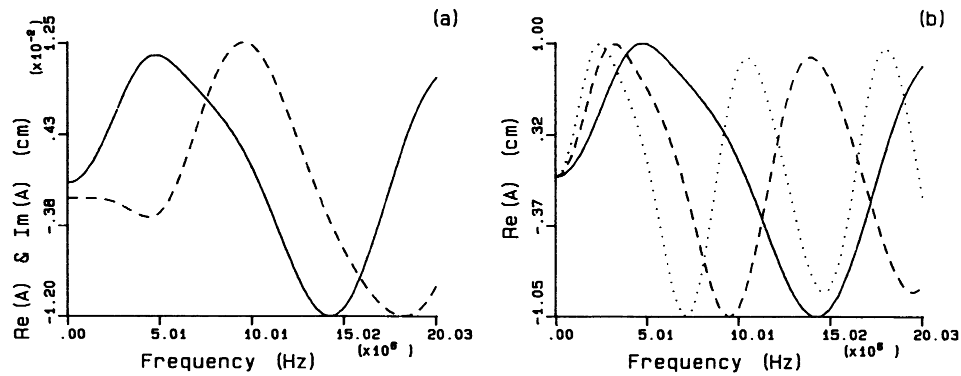

Fig. 1. Scattering amplitudes for spherical voids in stainless steel. a) $R[A(\omega)]$ (solid) and $Im[A(\omega)]$ (dashed) for a 200μm diameter void; b) $Re[A(\omega)]$ for three different diameter voids: 200μm (solid), 300μm (dashed), 400μm (dotted).

where $\hat{e}_i$ and $\hat{e}_s$ are unit vectors in the incident and scattering direction, respectively, ω equals the circular frequency ($2\pi f$), c equals the longitudinal wave speed, k is the magnitude of the wave vector (ω/c), and the far field scattering amplitude is given by $A(\hat{e}_i,\hat{e}_s,\omega)$.

The Born approximation is particularly useful as a vehicle for discussing scattering amplitude characteristics. In the Born approximation, the scattering amplitude is purely real and for a flaw with center of inversion symmetry can be written as [5]

$$A(\omega) \sim -R\, k^2a^3[\sin(2ka)-2ka(\cos(2ka))]/(2ka)^3 \qquad (2)$$

where R is the acoustic reflection coefficient and a is the effective flaw radius. In the long wavelength (low frequency) limit, Eq. 1 reduces to $A(\omega) \sim R\, k^2a^3$. This is a universal result for volumetric scatterers at long wavelengths [6]. In the long wavelength limit: 1) the sign of $A(\omega)$ is determined by R; 2) at a given frequency (i.e., at a given k), $A(\omega)$ is proportional to a^3; and 3) for a given flaw size, $A(\omega)$ is proportional to ω^2. At high frequencies, Eq. 1 reduces to $A(\omega) \sim R\, k^2\, a \cos(2ka)$. At high frequencies: 1) the sign of $A(\omega)$ is not controlled simply by R and 2) $A(\omega)$ oscillates about zero at a given frequency for varying flaw sizes.

These observations are demonstrated graphically for Born scatterers in Ref. 1. Here, this behavior is demonstrated for spherical voids in stainless steel (ρ=7.9g/cc, c_L=0.58cm/μs, c_S=0.31cm/μs) using calculated scattering amplitudes [7]. Figure 1(a) shows the real part (solid line) and imaginary part (dashed line) of the scattering amplitude for a 200μm diameter void. The oscillatory nature is characteristic of scattering amplitudes for both volumetric scatterers and cracks. Figure 1(b) shows the real part of the scattering amplitude for three different size voids as indicated in the figure caption.

Scattering amplitude characteristics for volumetric scatterers, which collaborate the observations made based on the Born approximation, can be summarized as follows: 1) $A(\omega)$ is dependent on ka; 2) the initial sign of $Re[A(\omega)]$ and $Im[A(\omega)]$ is controlled by the reflection coefficient between the flaw and the host; 3) the real and imaginary parts for volumetric scatterers start out from zero frequency with opposite sign; 4) for a given flaw/host combination, $A(\omega)$ for all flaw sizes start out from zero frequency "in phase"; and 5) at intermediate and high frequencies, scattering amplitudes for different flaw sizes are "out of phase".

Analysis

In this section, the notation approach used by Papoulis [8] is adopted. Using this notation, $A(\omega,\zeta)$ can represent four things: 1) for a particular flaw, ζ_k, $A(\omega,\zeta_k)$ represents the complex scattering amplitude as a function of frequency for the k^{th} flaw; 2) at a particular frequency, ω_i, $A(\omega_i,\zeta)$ is a complex random variable whose real and imaginary parts vary over the family of flaws, 3) at particular frequency and for a particular flaw, $A(\omega_i,\zeta_k)$ is a complex number which represents one outcome or sample of the random variable, $A(\omega_i,\zeta)$; and 4) the general notation $A(\omega,\zeta)$ represents a family or ensemble of scattering amplitudes. Further, if $A(\omega_i,\zeta)$ is Gaussian, it is a univariate complex random variable whose real and imaginary parts are bivariate Gaussian [9]. For $A(\omega_i,\zeta)$ to have zero mean, both $Re[A(\omega_i,\zeta)]$ and $Im[A(\omega_i,\zeta)]$ must have zero mean. For $A(\omega_i,\zeta)$ to be uncorrelated, $Re[A(\omega_i,\zeta)]$ can not be correlated with $Im[A(\omega_i,\zeta)]$ [8,9].

Preliminary observations

Unlike noise which can frequently be measured and then characterized, it is not possible to measure a family of scattering amplitudes. In addition, prior information which may be available is information relative to the physical characteristics of flaws (i.e., size, orientation, composition, etc.) rather than information about scattering amplitudes directly. A realistic procedure might be to 1) establish the flaw distribution characteristics (in practice, based on a combination of prior destructive evaluations and knowledge of the manufacturing processes); 2) generate a family of scattering amplitudes, $A(\omega,\zeta)$, corresponding to randomly chosen flaws out of the distribution; and 3) use the family of scattering amplitudes to analyze the random variables represented by $A(\omega_i,\zeta)$. An important restriction is that a forward scattering solution, either analytical or numerical, must exist in order to generate scattering amplitudes corresponding to flaws in the distribution. A forward scattering solution essentially involves determining a flaw's scattering amplitude given the flaw's physical characteristics and knowledge of the incident wave field.

In order to analyze scattering amplitude distributions in a research environment, a flaw distribution must be assumed. In certain cases, the size distribution associated with voids or inclusions is approximately lognormal [1,10]. The random variable analyses presented in this paper are based on lognormal distributions of spherical voids in stainless steel.

Random variables can be classified as either deterministic or nondeterministic [11]. A random variable which has no structure and simply wanders on in a "noise-like manner" can be classified as a nondeterministic random variable. Scattering amplitude, which does not have a random noise-like character, can be considered to be a deterministic random variable. For example, Eq. (2) shows that an ensemble of scattering amplitudes associated with an ensemble of Born scatterers would have a deterministic form where the randomness would be associated only with the random variable, a, which represents the radii of the flaws in the ensemble. A primary consequence of this deterministic nature is the ability to anticipate results associated with the analysis of scattering amplitude.

Scattering amplitude characteristics for volumetric scatterers were summarized in the background section. The implications of these characteristics in terms of the behavior of scattering amplitude as a random variable can be summarized as follows: 1) the scattering amplitude distribution will not be the same in different ka ranges; 2) at long wavelength for a distribution of flaws with a given flaw/host combination, the scattering amplitude distribution will not be Gaussian since $Re[A(\omega)]$ and $Im[A(\omega)]$ will either be positive for all flaws or negative for all flaws; 3) as a consequence of the previous point, the scattering amplitude mean, $m_A(\omega)$, will not be equal to zero at long wavelength; 4) also at long wavelength, since scattering amplitudes are "in phase", the scattering amplitude variance, $\sigma_A^2(\omega)$, will be relatively low; 5) at high frequencies, since scattering amplitudes oscillate about zero at a given frequency for different flaw sizes, $m_A(\omega)$ is expected to tend toward zero; 6) since scattering amplitudes are "out of phase" at high frequencies, $\sigma_A^2(\omega)$ is expected to be relatively large; 7) at low frequencies, the real and imaginary parts of the scattering amplitude are expected to be negatively correlated since they start out with opposite signs; and 8) at high frequency, the oscillatory nature of scattering amplitudes is expected to cause the real and imaginary parts to be uncorrelated.

Distribution

A flaw size distribution determines, at each frequency, a distribution associated with the real part and a distribution associated with the imaginary part of the scattering amplitude distribution. The probability density function (pdf) transformation problem is to determine the pdf for the real part, $f_{Re[A]}(Re[A(\omega_i,\zeta)])$, and the pdf for the imaginary part, $f_{Im[A]}(Im[A(\omega_i,\zeta)])$, given the pdf for the flaw distribution, $f_a(a)$ (a=radius). For the real part, the solution to this transformation problem can be stated as [1,11]

$$f_{Re[A]}(Re[A(\omega_i,\zeta)]) = \left| \frac{da}{dA} \right| f_a(h(A)) \tag{3}$$

In order to determine $f_{Re[A]}(Re[A(\omega_i,\zeta)])$, an analytical forward solution, $Re[A(\omega_i,\zeta)]=g(a)$, must exist, and it must be possible to "reverse" this relationship to find $a=h(Re[A(\omega_i,\zeta)])$ [1]. Analogous relationships can be stated for the imaginary part. Even if an analytical forward solution exists, it will generally not be possible to find the reverse relationship. Thus, in general, given $f_a(a)$, the scattering amplitude distribution cannot be determined analytically.

For a lognormal distribution of volumetric scatterers, the transformation represented by Eq. (3) is possible at long wavelength [1]. For a particular frequency at long wavelength, $Re[A(\omega_i,\zeta)]$ is proportional to a^3. In addition, $Im[A(\omega_i,\zeta)]$ is proportional to a^6 at long wavelength. Carrying out the transformation yields the results that for volumetric flaws at long wavelength, a lognormal distribution of flaws results in a lognormal distribution for $Re[A(\omega_i,\zeta)]$ and for $Im[A(\omega_i,\zeta)]$ [1].

Provided that a forward solution exists, the distribution can be considered numerically using a family of calculated scattering amplitudes. The long wavelength result just stated can be confirmed numerically. The following procedure was used to establish a family of scattering amplitudes:

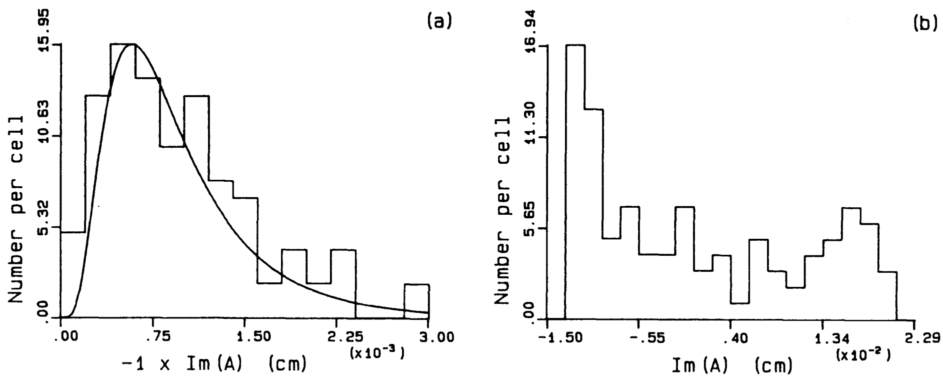

Fig. 2. Scattering amplitude distribution for m_a=300μm and σ_a=50μm. a) imaginary part at 2MHz; b) imaginary part at 15MHz.

1. Select the mean, m_a, and breadth (as represented by the standard deviation, σ_a) of the assumed lognormal flaw distribution;
2. Randomly generate N flaw sizes (ζ_k k=1,N) out of the assumed distribution;
3. Calculate the scattering amplitudes [7] ($A(\omega,\zeta_k)$ k=1,N) associated with the randomly chosen flaw sizes.

The histogram in Fig. 2(a) represent the distribution of the imaginary part ($Im[A(\omega_i,\zeta_k)]$ k=1,100) at 2MHz for a lognormal distribution of spherical voids in stainless steel with m_a=300μm and σ_a=50μm. All values of the imaginary part are negative; however, for plotting convenience, the distribution of (-1.0) times the imaginary part is shown. At 2MHz for flaw radii ranging from 150μm to 450μm, the ka range is from 0.16 to 0.49. Superimposed on the histogram is a lognormal pdf [11,1]. The lognormality of the scattering amplitude distribution is verified as the conformance between the histogram and the lognormal pdf is excellent. Similar results were obtained for the real part ($Re[A(\omega_i,\zeta_k)]$ k=1,100) at 2MHz [1].

The goal is to determine if the Gaussian assumption is reasonable, given that it is not strictly valid as stated on the previous page. The symmetry of a lognormal distribution increases as the coefficient of variation (σ_a/m_a) decreases. Thus, for a narrow flaw distribution, the scattering amplitude distribution is quite symmetric and is nearly Gaussian. As the flaw distribution breadth increases, the distribution becomes less symmetric (Fig 2(a)), and the scattering amplitude distribution becomes less Gaussian. Determining if a broad flaw distribution leads to a scattering amplitude distribution which is "reasonably Gaussian" is a function of the application. For example, Neal [1,2] showed that in utilizing an optimal Wiener filter to determine scattering amplitude estimates, the optimal Wiener filter performed well even for relatively broad flaw distributions.

At intermediate and high frequencies, scattering amplitudes have a more complicated and generally oscillatory nature as demonstrated in Fig. 1. In general, the scattering amplitude distribution cannot be determined analytically at these frequencies. At long wavelength, the scattering amplitude distribution is lognormal independent of the mean and breadth of the flaw distribution. At intermediate and high frequencies,

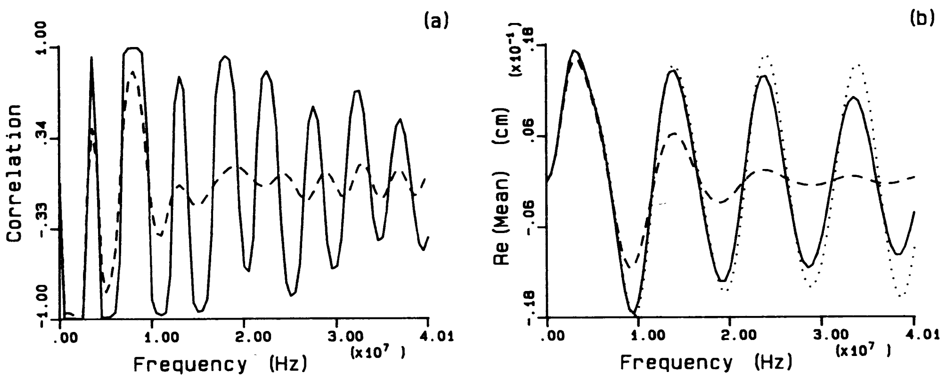

Fig. 3. Scattering amplitude correlation and mean. a) sample correlation coefficient for m_a=300μm and σ_a=10μm (solid) and σ_a=50μm (dashed); b) sample mean for real part with m_a=300μm and σ_a=10μm (solid) and σ_a=50μm (dashed) (the dotted line is A(ω) for a=300μm).

the sinusoidal nature of scattering amplitudes dictates that the characteristics of the scattering amplitude distribution will be a function of the mean and the breadth of the flaw distribution. This concept can be clarified by considering the Born approximation at high frequencies. If the flaw distribution is narrow, then cos(2ka) will vary over only a small portion of one period, resulting in a narrow, unimodal scattering amplitude distribution (i.e., a distribution whose pdf has only one peak). If the flaw distribution is broad enough so that cos(2ka) varies over more than one period, then a different (potentially bimodal) distribution will result.

The histogram in Fig. 2(b) represents the distribution for $Im[A(\omega_i,\zeta_k)]$ (k=1,100) at 15MHz for the flaw distribution considered in Fig. 2(a). At 15MHz, the ka range is from 2.4 to 7.35. The distribution is unsymmetric and shows some bimodal tendencies. For a narrow flaw distribution (not shown here), the distribution of the imaginary part appears to be unimodal and relatively symmetric. Similar behavior was observed for the real part [1]. Summarizing, intermediate and high frequencies: 1) the nature of the distribution is a function of the flaw distribution, 2) for a narrow flaw distribution the distribution may be somewhat Gaussian, and 3) in general, the distribution is not reasonably Gaussian.

Correlation, mean, and variance

Definitions for the correlation coefficient, mean, and variance for a random variable are based on an infinite number of observations [11]. Here, these parameters are estimated using a sample correlation coefficient, sample mean, and sample variance, respectively, based on a sample size of 100 (ζ_k, k=1,100). These estimates are defined explicitly in terms of scattering amplitudes in Ref. 1.

Sample correlation coefficients versus frequency are given in Fig. 3(a) for a narrow flaw distribution (m_a=300μm, σ_a=10μm) in dashed line and for the broader distribution (m_a=300μm, σ_a=50μm) in solid line. The plots are shown from 0-40MHz. Due to the deterministic nature of scattering amplitudes, the correlation is very structured. For the narrow distribution (dashed line), the general character of the plot is similar to a

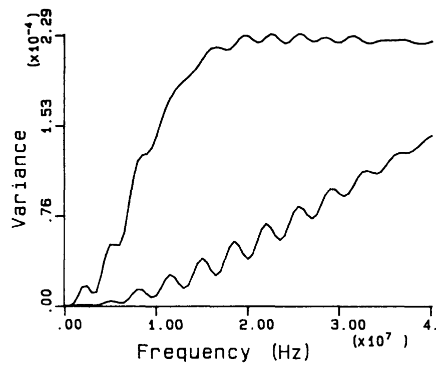

Fig. 4. Sample variance for m_a=300μm and σ_a=10μm (lower curve) and σ_a=50μm (upper curve).

plot of the product of the real and imaginary parts of the scattering amplitude for a 300μm radius spherical void in stainless steel. As expected, the correlation is negative at very low frequencies. For the broader flaw distribution (solid line), the real and imaginary parts are "out of phase" at intermediate and high frequencies (see Fig. 1(a)), and the correlation coefficient tends to oscillate about zero. The correlation is negative at very low frequencies even for the broader distribution.

Figure 3(b) shows the sample mean for the real part for the two flaw distributions. The solid line represents the sample mean for the narrow distribution (m_a=300μm, σ_a=10μm), and the dashed line corresponds to the broader distribution (m_a=300μm, σ_a=50μm). The dotted line is the calculated real part of the scattering amplitude for a 300μm radius spherical void in stainless steel [7]. For the narrow distribution, the scattering amplitude mean (solid line) approaches the scattering amplitude for a flaw with radius equal to the mean radius (dotted line). This is especially evident at low frequencies where scattering amplitudes for different flaw sizes are "in phase". At higher frequencies, scattering amplitudes become "out of phase" and cancelation begins to force the mean toward zero.

The sample variance for these two flaw distributions is shown in Fig. 4. The lower curve is for the narrow distribution (σ_a=10μm) and the upper curve is for the broader distribution (σ_a=50μm). This figure shows the anticipated results. That is, the variance increases with increasing frequency as the scattering amplitudes for different flaw sizes effectively get "out of phase". Also, at a particular frequency, the scattering amplitude variance increases as the flaw distribution breadth increases. It is to be reemphasized that in order to estimate the scattering amplitude variance associated with a given flaw distribution, a forward scattering solution must exist.

The effect of errors in estimating the mean and breadth of the flaw distribution was also investigated. Graphical results are given in Ref. 1. These results show that an error in estimating the flaw distribution mean or breadth will result in errors in both the scattering amplitude mean and variance. It was also observed that at low frequencies, both the scattering amplitude mean and variance are relatively insensitive to errors in the flaw distribution mean and breadth.

SUMMARY

In general, given a flaw distribution, the scattering amplitude distribution cannot be determined analytically. If a forward scattering solution exists, it may be possible to study the distribution numerically. At long wavelength for a narrow flaw distribution, the scattering amplitude distribution may be reasonably Gaussian. In general, it is not reasonable to assume that scattering amplitude is Gaussian. For a given flaw/host combination, the deterministic nature of scattering amplitudes dictates that scattering amplitude is not uncorrelated and does not have zero mean at all frequencies. Scattering amplitude variance increases with frequency and also increases with increasing flaw distribution breadth.

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