

FLAW DETECTION USING A PRIORI KNOWLEDGE WITH LIMITED VIEW APERTURE SYSTEM

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INTRODUCTION

In CT imaging with limited view-angle data, the image of the object's slice is usually distorted such that it is difficult to interpret the image. Usually in industrial applications, one deals with quality testing of products which are built from an original blueprint or model. The objective of this paper is to use the knowledge about the model and try to establish whether there is any significant difference between the object under test and the model object. We will first formulate the problem as a deconvolution problem. Then we will use the CLEAN deconvolution algorithm to restore the image.

PROBLEM STATEMENT

In x-ray CT, one measures the projections of the object at a certain slice. A projection at a given angle can be denoted by [1]

$$P_{\theta}(t) = \int f(s, t) ds \quad (1)$$

where $f(s, t)$ is related to the degree of transparency of the object at the slice where the plane of the ray passes. As shown in the diagram, $f(s, t)$ has nonzero values in the support of the slice in the (s, t) coordinate system. The relationship between (s, t) and (x, y) coordinates is given by a rotation matrix.

$$\begin{pmatrix} s \\ t \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \quad (2)$$

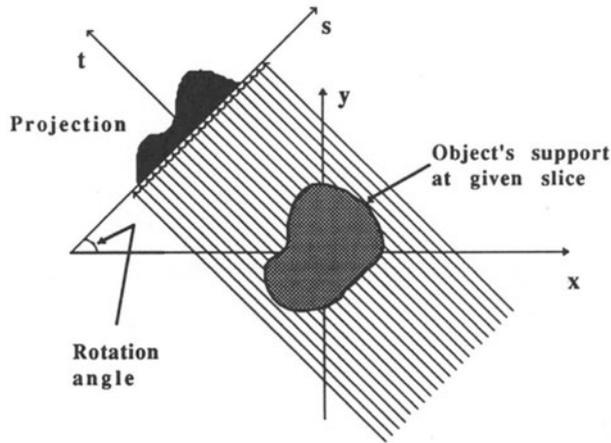


Figure 1. Diagram showing one projection of the slice.

If the Fourier transform of the object function $f(x, y)$ is defined by $F(u, v)$.

$$F(u, v) \doteq \iint_{-\infty}^{\infty} f(x, y) e^{-2\pi j(xu + yv)} dx dy \quad (3)$$

By appropriate change of variable and substitution, we obtain the Fourier slice theorem [1]. The significance of the Fourier slice theorem is in the fact it relates each projection to the values of the Fourier transform of the object in a line for the given direction. By accumulating more projections one is collecting information about $F(u, v)$ in the corresponding direction. The image of the object function can be obtained from a complete sample of the $F(u, v)$. However, if $F(u, v)$ is not sampled for particular angles, and then transformed by the usual signal processing steps, one obtains an image which can suffer severe distortions. The above distortion can be formulated as a convolution distortion. Let $\Psi(u, v)$ be the sampling function in the uv domain, then be shown below.

$$F^{obs}(u, v) = \Psi(u, v)F(u, v) \quad (4)$$

where

$$\Psi(u, v) = \begin{cases} 1 & \text{if } (u, v) \in W \\ 0 & \text{otherwise} \end{cases} \quad (5)$$

and $F^{obs}(u, v)$ is the observed function. Since the sampling function is multiplied in the Fourier domain, its Fourier transform is convolved with the true object image in the image domain.

CLEAN DECONVOLUTION TECHNIQUE

A technique we have used extensively in radio astronomy to reconstruct images with limited data is called CLEAN [2]. The CLEAN algorithm is used to deconvolve a point spread response from a reconstructed image obtained by the Fourier transformation of an incompletely sampled spatial frequency function [3, 4, 2]. It was developed by Högbom [3] for deconvolving the synthesized beam of an aperture synthesis antenna array from the intensity distribution of a radio source. It provides good estimates of the spatial frequency function at the unsampled frequencies and has yielded many significant results in radio astronomy applications.

DECONVOLUTION BY CLEAN

When a point source is observed by a system, the output of a mapping routine is the system response in the spatial domain. This response is termed the *dirty* beam in radio astronomy. It has very bad sidelobes due to missing data in the spatial frequency domain. If there are several point sources, then the output image of the mapping routine is a superposition of the several impulse responses. Any source can be assumed to be an equivalent collection of spatially separated point sources. The idea of the CLEAN algorithm is to find the positions and strengths of the point sources that comprise a given source and convolve these point sources with a new beam that does not have any corrupting sidelobes like the *dirty* beam. The new beam is called the *clean* beam. It is obtained by fitting a Gaussian function to the main lobe of the *dirty* beam. $\Psi(u, v)$ (Eqs. 4 and 5) can be regarded as the Fourier transform of the system response.

The CLEAN algorithm is implemented as follows:

1. Create an image by a reconstruction process.
2. Create a point spread function, or *dirty* beam, due to missing data in the spatial frequency domain.
3. Find the highest intensity peak in the image by a search routine.
4. Scale the beam to this peak intensity and center it on the peak.
5. Subtract the product of the beam, the peak strength, and a damping factor called the *clean* gain on a pixel-by-pixel basis from the image to create a file of residues. The peak is multiplied by the *clean* gain, which is less than unity, to ensure that the subtractions are done at a slower rate so as to represent the minute details of the source structure properly. Normally this value is set to 0.1.
6. Store the value of the peak intensity subtracted, and its location.
7. Go to step 3 and repeat by replacing the original image file by the file of residues.
8. Continue looking until only noise remains in the residue file.
9. Create a 'CLEAN' beam by fitting a Gaussian to the central portion of the beam.
10. Create an image by convolving the CLEAN beam with the file of subtracted intensities and their locations (see step 6). Convolution with the CLEAN beam is carried out to suppress the higher spatial frequencies which are spuriously extrapolated by CLEAN.
11. Add the noise in the residual file to the result of step 10, to give the CLEANed image. This is done since discarding the residuals might destroy some information that is present in the residual map. Also this gives a realistic representation of the information.

USING A PRIORI KNOWLEDGE

A priori models of sources can be used effectively in CLEAN to improve the accuracy of the algorithm. As mentioned above, the main assumption in CLEAN is that a source is a composition of point sources and hence, the algorithm performs best if the source has finite and isolated components. This condition is satisfied if we subtract the a priori known image from the measured image. The details for normalizations and related

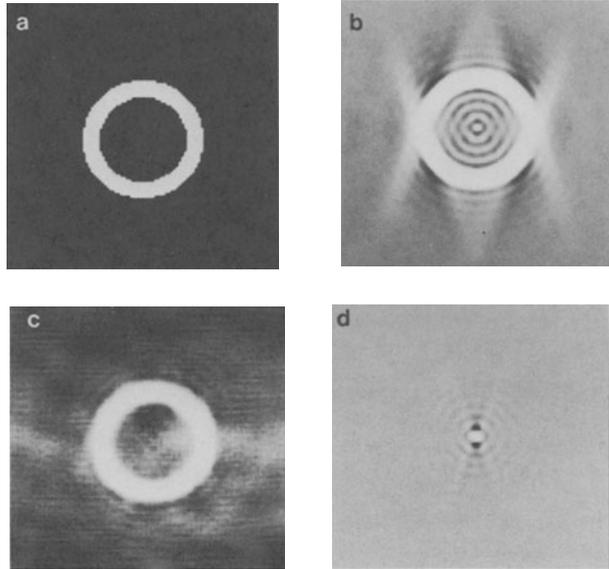


Figure 2. a) Original image b) Distorted image (flawless) c) CLEANed image d) Distorting function

procedures are given in [5]. CLEAN then needs only to find the deviations of the observed source from the the known model which improves both the image quality and speed of convergence is improved [2].

SIMULATION AND PROCESSING STEPS

We simulated a limited view tomographic scan by first creating a model object and then simulating Fourier slices of CT observations by analytically sampling along radial lines the known 2-D Fourier transform of the model object in the spatial frequency domain.

In this set of experiments a 5% flaw was used (i.e. the intensity of the flaw was 5% down from the surrounding material), no noise was added, and samples were taken radially every 0.0125 of a maximum spatial frequency of unity. The scans were between 20° to 160° , with samples being taken every two degrees. A simulation of the model with the flaw, and a simulation of the model without the flaw, were gridded into a 128×128 array. Data sets were then inverse Fourier transformed to yield the raw images. The raw image with the flaw was then subtracted from the raw image without the flaw to produce the raw image of the flaw. These raw images were then corrected using the CLEAN algorithm, giving the final results displayed in Figs. 2.c and 3.b. The CLEANed image of the model shown in Fig. 2.c is clearly circular and does not suffer from many of the distortions observed in the distorted image. For example, the bright rings and 'X-shape' distortions that are present in the distorted image has been removed. However, some loss of resolution and general blurring is observed. There is also a significant gain in the contrast of the image. This is achieved by collecting the power spread by the sidelobes of the distorting function. The enhanced image of the flaw shown in Fig. 3.b is similarly

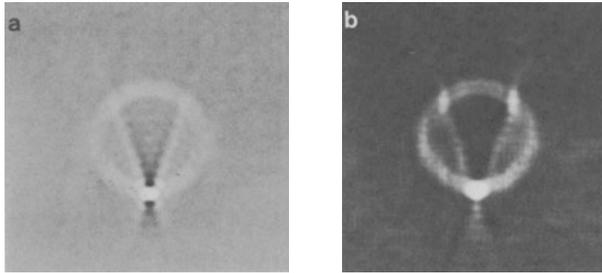


Figure 3. (a) Distorted image of flaw (b) CLEANed image of flaw

improved. CLEAN processing has improved the contrast significantly. In general, CLEAN has reduced the amount of distortion resulting from incomplete sampling.

All of the processing after the simulated observation was performed using Astronomical Image Processing System (AIPS), a set of routines created by the National Radio Astronomy Observatory (NRAO¹) for use in astronomy. These routines have also been applied in a number of areas of signal processing.

CONCLUSION

From the above examples we see that the CLEAN algorithm has worked very well in deconvolving the system response from the difference image. The corrected image agrees well with the flaw originally introduced in the simulation. Therefore CLEAN has the potential to detect unknown flaws to a high degree of accuracy given an a priori model of an object and the accurate observing conditions.

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