THREE DIMENSIONAL ANALYSIS OF THE VIBRATION CHARACTERISTICS OF PIEZOELECTRIC DISCS

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INTRODUCTION

The piezoelectric transducer plays a paramount role in the ultrasonic non-destructive testing systems which are increasingly employed in industry. The active element in these transducers is the piezoelectric disc and its vibration characteristics govern the performance of the transducer. One dimensional theory (1D) [1], which assumes that the piezoelectric disc vibrates in the thickness direction only as shown in Fig 1, has been used for many years. However, the 1D theory cannot predict other vibration modes, which may affect the transducer behaviour in the frequency range of interest, especially for those transducers with a small diameter to thickness (D/T) ratio.

There have been many experimental reports which show that a variety of vibration modes exist in piezoelectric discs [2-4]. It has been found that there are five types of mode in the frequency range of interest; these are radial (R), edge (E), thickness shear (TS), thickness extensional (TE) and high frequency radial (A) modes. The vibration characteristics of piezoelectric discs have been analysed by three dimensional analytical theory [5], and two dimensional plate theory [6,7]. Although there is generally good agreement between the predictions by the plate theory and the measurements [3,6], some modes are not predicted accurately and the strength of excitation of the different modes has not been predicted. Finite element analysis, which was first developed for piezoelectric materials by Allik [8], has also been applied to piezoelectric discs [9,10]. However, the piezoelectric discs studied were of relatively small D/T ratio, and the frequency response functions, which enable the strength of excitation of the different modes to be determined, were not predicted.

In this paper the vibration characteristics of piezoelectric discs are analysed by a three dimensional finite element model, and the electrical impedance functions of PZT5A discs with D/T ratios of 20 and 10 are calculated. The calculations are checked by experimental measurements.

ANALYSIS

The general finite element formulation for piezoelectric materials has been given by Allik [8]. Since then it has been used in analyses of many piezoelectric devices. However, those finite element models usually used a mass condensation scheme to eliminate the static electrical degree of freedom. This procedure is not desirable since the resulting matrices are no longer banded. It has been shown that a direct scheme, which treats the electrical potential of each node as an extra degree of freedom, can be more effective, and details of this may be found elsewhere [11]. After obtaining the resonant frequencies and mode shapes, the mechanical frequency response function and electrical impedance response can be formed by the modal analysis technique [12]. This is described in more detail in references 11 and 13.

The resonant frequencies of piezoelectric discs were predicted for constant voltage excitation. These correspond to the short circuit resonant frequencies, or the frequencies at which the electrical impedances of the disc are zero if energy dissipation is not considered [8]. The mode shapes at these resonant frequencies were also computed. The disc was assumed to be axisymmetric with full electrodes on the top and bottom surfaces, and an 8 node quadrilateral axisymmetric piezoelectric element was used [11]. Since the frequency range around the first through thickness mode by the 1D model is of most interest, only the modes with frequencies up to approximately one and a half times the first through thickness frequency were predicted.

EXPERIMENTAL SET UP

The electrical impedances of piezoelectric discs were measured by the set up shown in Fig 2. The piezoelectric disc is in series with a reference resistor which is used to measure the current through the circuit, and excitation is provided by the function generator in the two channel frequency response analyser (Solartron type 1255). The voltages across the electrodes of the piezoelectric disc and the resistor are measured simultaneously by the FRA, which can analyse signals up to a frequency of 20 MHz. The system is controlled by a micro-computer to make a sweep measurement in the frequency range of interest by varying the frequency of the output voltage from the generator in the FRA. The electrical impedance can then be obtained directly from the FRA output as,

$$Z(\omega) = R\frac{V_1}{V_2}$$

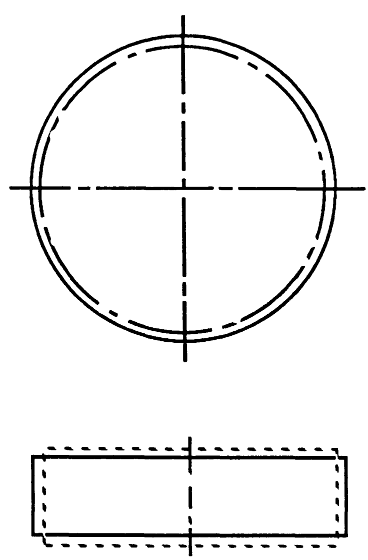

Fig 1. The mode shape of the first through thickness mode of piezoelectric discs assumed by one dimensional analysis.

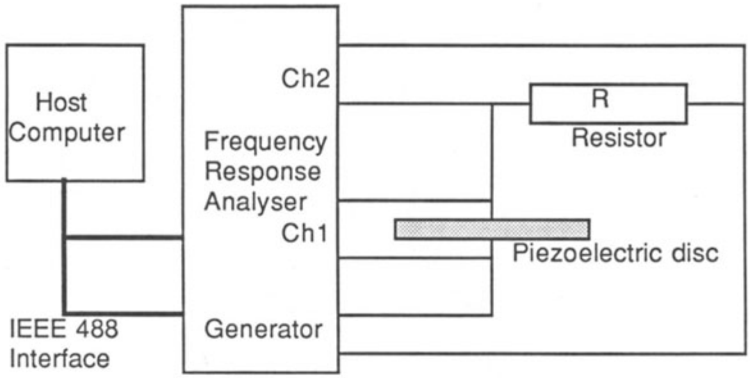

Fig 2. Apparatus used for measurement of the electrical impedance of piezoelectric discs.

where R is the resistance of the reference resistor, and V_1 and V_2 are the voltages across the piezoelectric disc and the resistor respectively.

RESULTS

Disc with D/T = 20

The disc analysed was 40.10 mm in diameter and 2.03 mm thick, giving a D/T ratio of approximately 20, which is the same order as the D/T ratios used in many NDT transducers. Since the disc is fairly thin the 1D analysis is also applicable, which gives the first thickness mode at 962 kHz. Therefore, all the modes in the frequency range from 0 to 1500 kHz were predicted. A total of 130 modes in this range were found. Of these, 66 were flexural modes which cannot be excited by a voltage applied to electrodes on the top and bottom surfaces of the disc, and so do not appear in the computed response functions.

The analysis predicted all the five types of modes described by previous workers and typical examples of the different types are shown in Fig 3(a) to 3(e). A detailed description of these mode shapes can be found in Reference 11. Since the disc is axisymmetric, only half of the cross section of the disc is plotted; the left edge is the central axis, and the right edge is the cylindrical surface. The broken line shows the finite element mesh itself, and the solid line represents the deformed shape of the disc at the resonant frequency.

Among these predicted modes, there is one thickness extensional mode, whose frequency of 964.9 kHz is very close to the first through thickness frequency of 962 kHz predicted by the 1D model. The corresponding mode shape is shown in Fig 3(d). The axial displacement of the surface is similar to that in the radial modes in that it varies with radial position about a mean value. In the case of the radial modes, this mean value is zero and therefore the points at which the displacement has its mean value form nodal circles. However, in this case, the mean value is non zero, so the axial displacement pattern is similar to that of the radial modes, but superimposed on a constant shift. This shift is analogous to the piston-like motion assumed in the one dimensional analysis as shown in Fig 1. The thickness extensional mode therefore has characteristics of the high order radial modes and one dimensional through thickness motion.

Fig 4 shows the predicted electrical impedance responses assuming the structural damping factor of 0.013 giving by the material manufacturers [14], together with the measured electrical impedance, and the response predicted by the 1D analysis [1]. It can be seen from Fig 4 that there is excellent agreement between the measured and the predicted electrical impedance responses. The predicted electrical impedance response in the thickness frequency range agrees well with the measured response both in the amplitudes of the response and in

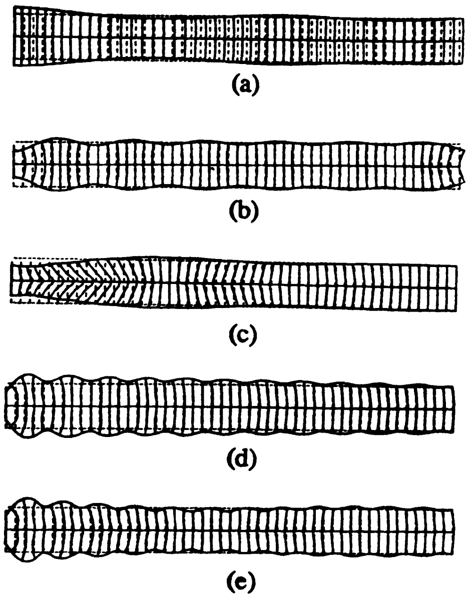

Fig 3 The five types of mode shape of a piezoelectric disc with a D/T ratio of 20.
(a) Radial mode (mode 4, 272 kHz);
(b) Edge mode (mode 13, 658 kHz);
(c) Thickness shear mode (mode 22, 873 kHz);
(d) Thickness extensional mode (mode 32, 965 kHz);
(e) High frequency radial mode (mode 37, 1026 kHz).

the values of the resonant frequencies. The measured resonant mode at a frequency of 951 kHz, which corresponds to a predicted frequency of 964.9 kHz, is the most strongly excited mode for this disc. The measured and predicted responses over the thickness shear modes range and the frequency range above the thickness modes show very weak excitation.

Disc with D/T = 10

Another PZT5A disc which had a diameter of 19.96 mm and thickness of 2.01 mm, giving a D/T ratio around 10, was also considered. The measured and predicted electrical impedances are shown in Fig 5.

It can be seen from both the prediction and measurement that the overall response is no longer dominated by a single mode, which was the case for the previous disc. Two strongly excited thickness modes at measured frequencies of 895.9 kHz and 933.7 kHz are also seen in the predicted curve at 964.7 kHz and 988.0 kHz. The predicted mode shapes of these two modes are similar to that shown in Fig 3(d).

Good agreement is shown between the form of the electrical impedance predicted by the FE model and the measurement. However, some discrepancy occurs in the resonant frequencies over the thickness modes frequency range. The discrepancy may come from a number of sources such as variation in the measurement of dimensions of disc (up to 2%), imperfection of the disc during manufacture, and inaccuracy in the material constants used in prediction. A check of the longitudinal velocity in the thickness direction of the piezoelectric disc showed a velocity of 4190 m/s which is 4% smaller than the value of 4350 m/s obtained from the data sheet [14] which was used in the analysis.

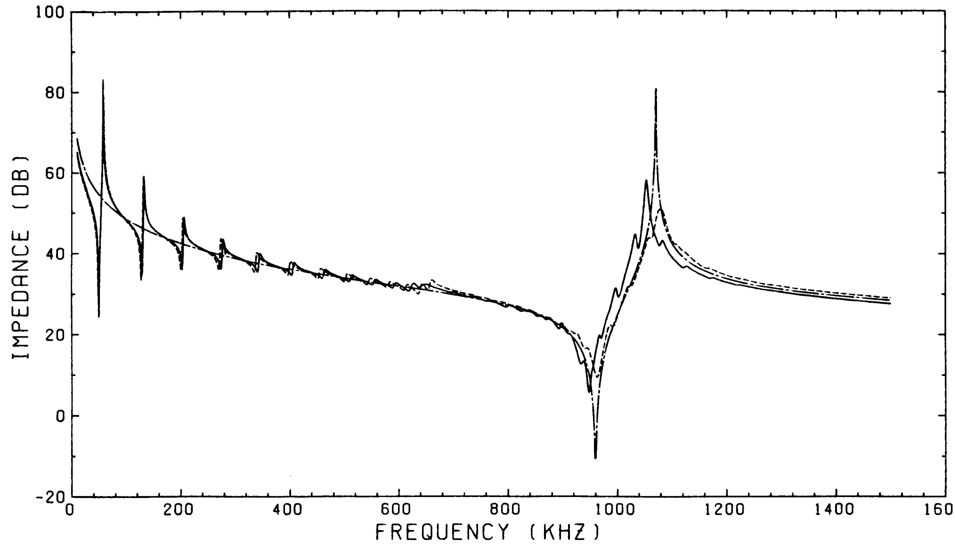

Fig 4 The measured and predicted electrical impedances of a PZT5A disc with a D/T ratio of 20. (_______ measurement; _ _ _ _ _ _ finite element prediction; _____ _ _____ 1D prediction)

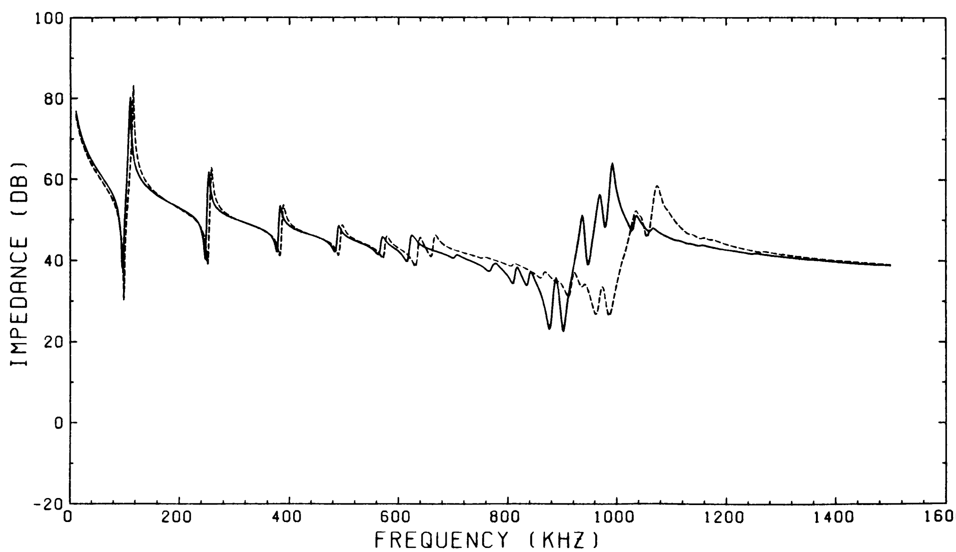

Fig 5 The measured and predicted electrical impedances of a PZT5A disc with a D/T ratio of 10. (_______ measurement; _ _ _ _ _ _ finite element prediction)

CONCLUSIONS

The vibration characteristics of piezoelectric discs with different D/T ratios have been studied by finite element analysis and experimental measurement. It has been shown that the resonant frequencies and their corresponding mode shapes are much more complicated than is assumed in one dimensional theory. There are a large number of resonant modes in the frequency range of interest, which have been classified into five groups according to their mode shapes: the radial modes (R), the edge mode (E), the thickness shear modes (TS), the thickness extensional modes (TE) and the high frequency radial modes (A). The thickness modes have characteristics seen both in high order radial modes and in the through thickness motion (piston motion) assumed by the one dimensional theory. The finite element predictions of the electrical impedance characteristics agreed well with the experimental measurements.

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