A liability management model for banks of the farm credit system

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The Federal Intermediate Credit Banks through local Production Credit Associations and other financial institutions have become significant suppliers of non-real-estate credit to agricultural producers. The increased volume has been accompanied by fluctuations in the funding volume and costs at which the funds are obtained. The liability management model developed and empirically tested in this study is an operational procedure that may be useful to the banks as an aid in selecting liability structures that minimize the average cost of credit that they provide to PCAs and OFIs.

The model requires an estimate of monthly expected debt costs for the Farm Credit System securities for a 18-month future period and quarterly estimates for an additional six quarters. It also requires an estimation of monthly and quarterly debt requirements for the 3-year future period. With these coefficients, the model generates an efficient frontier of debt portfolios for a 3-year period. That efficient frontier is structured so that a minimum expected-cost debt portfolio is derived for each level of cost variance. A movement to a lower expected-cost portfolio on the efficient frontier entails a higher level of cost variance. The model can be used to obtain a new efficient frontier when a change in projected expected debt costs (interest rate change) or debt requirements becomes evident. In most situations, the model would be used at least once a month when a decision to participate in a bond issue is made. The model also can be used to assess the impact of alternative debt policy constraints on cost and risk.

The model was empirically tested by using data from the FICB of Omaha. In the application, two projections of interest rates were converted into expected debt costs. The variance-covariance matrix of the debt costs was generated by using the expectations theory of the term structure of interest rates; interest rates from 1965-1977 were used in this estimation. Two forecasts of stochastic future debt needs were generated with linear regression equations, and the expected value and the standard error of the forecasts were used as the mean and standard deviation parameters of a normal density function. With the use of an inventory model, the stochastic debt needs were converted into optimal deterministic bond and note debt to be outstanding.

Two historical tests with data from the periods 1975-77 and 1976-78 indicate that debt-issuance strategies (debt portfolios) were available that would have reduced the expected cost of debt by 4 to 5 percent ($7 to $9 million) for the 3-year period. The reduction in expected cost, however, was accompanied by a large cost risk (standard deviation). In contrast, debt portfolios were available that had a lower cost risk but at higher expected costs. The test results also suggest that portfolio diversification occurs over time rather than between debt issues at a point in time.

With the most probable forecast of debt cost, expected cost is projected to steadily decrease during 1979, increase slightly during 1980, and then decrease slightly during 1981. The discounted expected cost of the efficient frontier generated for this forecast ranged from $227 to $237 million for the 3-year period; the discounted standard deviation ranged from $28 to $19 million. At low variance (high cost) levels on the efficient frontier, the first year of the portfolios contains discount notes and term bonds. The discount notes provide funds between term bond issues. At greater levels of variance (lower cost), discount notes and 6-month bonds substitute for some term bonds. Late the first year and early the second year in low-cost portfolios, a large volume of term bonds is used to lock in a low interest cost for the duration of the planning horizon. The usage of term bonds is greatest at the high variance levels. The third year of the portfolios involves extensive use of discount notes, especially at the high variance levels.

With the recession forecast of debt cost, expected debt cost is projected to increase sharply for the first half of 1979, peak during the middle of 1979, and then fall sharply through 1980. During 1981 the decrease continues but is less drastic. The discounted expected cost of the efficient frontier generated ranged from $237 to $275 million for the 3-year period; the discounted standard deviation ranged from $45 to $18 million. At the low variance (high cost) levels on the efficient frontier, the first year of the portfolios contains primarily term bonds and discount notes. At greater variance (lower cost) levels, 9-month and 6-month bonds are used the first few months of the first year to lock in a low cost while interest rates increase during the first half of the first year. As interest rates fall during the second and third years, short-term securities are used extensively, 9-month and 6-month bonds at the lower variance levels and discount notes at the higher variance levels.

In general, with all applications, a movement down the efficient frontier from low expected-cost and high cost-variance portfolios to higher expected-cost but lower cost-variance portfolios typically entails a shift from 1-month discount notes to 6-month bonds to 9-month bonds to term bonds. A projected decrease in expected interest rates over the planning horizon will cause shorter-term bonds and notes to be used to take advantage of the decrease. The specific terms used depend upon the duration of the movement and variance level on the efficient frontier. The long-term activities used at lower variances will be term bonds; long-term activities used at higher variances will be 9-month bonds. Short-term activities used at lower variances will be 9-month and 6-month bonds; short-term activities used at higher variances will be discount notes.

Fluctuations of debt needs over the planning horizon require, at all variance levels, the use of some short-term securities, which mature when debt needs decrease. A steady growth in debt needs permits the use of all terms to maturity, the selection of which depends upon the expected cost and variance of the portfolio.

Two versions of the model were constructed and tested, one version without debt policy constraints and another version with debt policy constraints. As would be expected, with the constraints, expected debt cost was higher at each level of variance. This resulted in a
shift to the right of the efficient frontier. With the most probable forecast of interest rates, expected cost was $5 to $8 million higher when constraints were included; with the recession forecast, expected cost was $2 to $3 million higher. The increase in expected cost was greater for the most probable forecast of rates because the debt policy constraints limited the extensive use of a low-cost term bond. With the recession forecast, the policy constraints generally limited the high levels of variance (cost risk) to which the bank could be exposed by truncating the upper section of the efficient frontier. Unfortunately, the policy constraints also truncated the lower section of the efficient frontier and limited the low-variance solutions as well.
The cooperative Farm Credit System has been established to improve the income and well-being of U.S. farmers and ranchers by furnishing sound, adequate, and constructive credit (U.S. Congress, 16). As a member of the System, a district Federal Intermediate Credit Bank (FICB) serves as the intermediary between national sources of money and local Production Credit Associations (PCAs) and other qualified financial institutions (OFIs) within the district. The FICB obtains most of its funds by participating in routinely issued System bonds and notes.

The FICBs, through local PCAs, have become significant suppliers of non-real-estate credit to farmers, increasing their market share from 16 percent of total non-real-estate farm debt in 1968 to 24 percent in 1978. The volume during the same period increased from $3.5 billion to $13.5 billion (Evans and Simunek, 5). This increase in volume has made cost control crucial to the System. An increase in interest costs of 10 basis points (one-tenth of a percentage point) on $13 billion amounts to $13 million additional costs a year.

The large volume and increased cost of debt have been accompanied by fluctuations in interest rates and funding needs. During 1978, the interest rate on FICB consolidated 9-month bonds, a primary source of funds, fluctuated between 7.15 and 10.00 percent. During the same year, the total consolidated bonds outstanding for the Omaha FICB (one of 12 district banks) ranged from $1,193 million to $1,299 million, a difference of $106 million.1 Because of the fluctuation in interest rates and funding needs, and the large volume of debt outstanding, a procedure that can determine optimal funding strategies in an uncertain environment and reduce the average cost of funds would be a useful management aid.

**Debt Management Decisions**

To provide adequate credit at a reasonable cost, a FICB must make two principal debt management decisions. They are:

1. The amount of debt that it should participate in at a specific time to meet the anticipated needs of the PCAs and to refinance maturing debt instruments before another opportunity to issue debt arises.

2. The term structure of the debt participation. The decision to participate in any given debt issue will be influenced by past debt issuance and the possible participation in future debt issues.

The first decision is difficult because of the uncertainty in the needs of the PCAs in future periods between debt issuances. Debt participation in any amount below evolving actual needs of the PCAs would require short-term borrowing, often at a cost above System-wide debt cost. In contrast, debt in an amount greater than actual needs requires excess funds to be invested, normally at a rate below the cost of the funds.

The second decision is difficult because of the uncertainty in future interest rates. The debt term structure selected depends not only upon the present known yield curve, but also upon yield curves that may develop in the future and affect future financing and refinancing decisions.

The objective of this study is to develop a liability management model that can aid in the debt management decisions of a FICB. The analysis will be structured to determine the optimal borrowing activities that would minimize the expected cost of credit at various levels of cost variance. More specifically, given the expected value of cost and variance-covariance of cost of various debt instruments that can be issued, and the stochastic demand for funds by the PCAs in the future, the optimal (in terms of minimum cost at various levels of cost-risk) maturity distribution and time issuance of debt will be determined for a multiperiod planning horizon.

**Federal Intermediate Credit Banks**

The FICBs are part of the cooperative Farm Credit System, which consists of 37 banks and numerous local associations organized to facilitate the movement of funds from the national money market to farmers, ranchers, commercial fishermen, and their cooperatives. The System consists of 12 district Federal Land Banks and their local Associations, 12 district Federal Intermediate Credit Banks and the local Production Credit Associations, 12 district Banks for Cooperatives, and a Central Bank for Cooperatives.

Each district FICB is organized as an incorporated cooperative. The voting stock is held by the PCAs in the district. The PCAs in turn are owned by user-members. The FICB advances loan funds to PCAs and other qualified financing institutions (OFIs) as required. Loans

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1 These figures were obtained from the Federal Intermediate Credit Bank of Omaha, "Consolidated Bonds and Farm Credit Investment Bonds Outstanding," monthly issues for 1978, unpublished.

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made to the PCAs are similar to an open-end revolving line of credit. The terms provide for payment on demand, and interest accrues on the daily outstanding balances. Loans are collateralized by assets of the PCAs. Credit arrangements with OFIs are similar, but loans are normally discounted and held as security. Interest charges to PCAs and OFIs are variable and are based upon the average cost of bonds and notes outstanding to the FICB. These rates are exempt from any state's usury laws.

Loanable funds are obtained by the FICB from a number of sources. Most funds are obtained by participating in Farm Credit System debt instruments issued in the national money market. These securities include Consolidated System-wide Bonds and Consolidated System-wide Discount Notes. The System-wide Bonds are joint issues of all 37 banks and include monthly issued 6-month and 9-month bonds and quarterly issued long-term bonds of various terms to maturity. The bonds are issued in book entry form, and bond certificates are not issued. The short-term bonds are typically priced and sold at the end of each month and delivered at the first of the following month. The long-term bonds are typically priced and sold during the middle of the month and delivered about the twentieth of the month. Interest payments on the long-term bonds are normally semiannual; interest payments on the 6-month and 9-month bonds are made at maturity.

The System-wide Notes are joint issues for short-term financing needs and are issued in maturities of 5 to 270 days. The notes are similar to commercial paper offerings of other businesses. The decision to participate in a note issue is announced on Friday for notes to be issued during the next week. Since most needs of the FICB are for the seasonal crop production cycle, the majority of funds historically have been obtained from 9-month FICB bonds. All the securities are issued through the Fiscal Agency of the Farm Credit Banks in New York City.

Other sources of funds include lines of credit established with commercial banks, federal funds, investment bonds purchased by members of PCAs, and borrowings from other banks of the System through the Central Bank for Cooperatives. These sources are used sparingly and primarily for short-term financing needs between bond issues.

Because loan demands are stochastic, liquidity is necessary and provided by the aforementioned short-term debt instruments and by holding excess funds from debt issues in a liquid form until they are needed. Some of these funds are held in cash, but a substantial portion is held in U.S. Treasury and U.S. government agencies' securities. Excess funds will also be offered to the other 36 banks in note form.

**Previous Studies**

A number of studies have analyzed the debt selection activities and policies of the Farm Credit System. Hollenhorst (8) evaluated the Federal Land Banks' debt management policies for the period 1947 to 1961. He examined the conventional rules-of-thumb and practices used in debt management and found that they were relatively ineffective in deriving low-cost debt structures for the FLBs.

Brake (4) in a 1968 study for the Farm Credit Administration projected loan demands of the Farm Credit System to 1980. Boger (3) completed a similar type of aggregate projection for the Central Bank for Cooperatives. Each bank in the System, however, makes its own financing decisions within System policy guidelines, so loan projections are necessary for each district bank. Swortzel and Jensen (14) constructed a short-term econometric forecasting model to forecast new money requests 2 months in advance for the Baltimore FICB. The estimated equations of the model explained most of the variation in new money requests during the sample period and were fairly accurate in identifying future turning points.

Bildersee (1, 2), Percival (11), Morris (10), and Smith (13) of the Wharton School of Finance analyzed the financing needs of the Farm Credit System and made recommendations on financing techniques in 1973. They concluded that, because there are temporary differences in interest rates for various maturity segments of the market, fixed participation in selected maturities may not minimize the average cost of debt. To exploit these market imperfections, the System must maintain access to all segments of the market and vary financing to any advantageous market. These decisions require judgements concerning the long-run trend in interest rates and funding requirements. The evidence presented by the Wharton School researchers argues against formalized financing plans and for the value of market advice and research and flexibility in the timing and placement of issues. The model developed in this study can aid in making optimal decisions as to the timing and maturity of funding activities.

**The Conceptual Framework**

The analytical procedure used in this study is a mathematical optimization model (quadratic programming) similar to the asset portfolio procedure developed by Markowitz (9) and used in formulating stock and other investment portfolios. In this study, the procedure is applied to the liability rather than the asset side of the balance sheet. The model is multiperiod. Stochastic interest rates are incorporated into the model by estimating both expected cost coefficients and a variance-covariance cost matrix for the various periods' debt activities. Probabilistic need for funds (demand) for the various periods are converted to optimal deterministic values by an inventory-type model and used as the right-hand side of the programming model.

**Expected-cost—variance-of-cost model**

The mathematical form of the expected-cost—variance-of-cost model (EC-VC) is identical with the expected-return—variance-of-return model (E-V) presented by Markowitz except that now the objective is to minimize the variance of cost subject to an expected level of debt cost. Rather than having a given level of funds to invest, it is now necessary to generate a speci-
fied level of funds to meet loan demands. Mathematically, the model can be stated as:

Minimize: \[ Z = X'QX \] (1)

subject to: \[ AX = B \] (2)
\[ CX = k \] (3)
\[ X \geq 0 \] (4)

where
\[ C(i,m) = \text{the expected discounted cost vector for the planning horizon,} \]
\[ Q(mm) = \text{the discounted variance-covariance matrix of } C, \]
\[ A(n,m) = \text{the technical matrix,} \]
\[ B(n,1) = \text{the funding requirements and debt policy constraints,} \]
\[ k = \text{the cost constant, which is varied parametrically,} \]
\[ X(m,1) = \text{the debt activity levels found by solution after each change in } k. \]

Because the procedure is now applied to the liability rather than the asset side of the balance sheet, at any level of expected cost one unique liability structure is determined that minimizes the variance of cost. Aversion to risk, which produces a direct relationship between return and variance on an E-V frontier, produces an indirect relationship between cost and variance on an EC-VC frontier (Figure 1). The shape of the frontier results from applying quadratic programming to the feasible space of debt portfolios. The feasible space is a convex set in the expected-cost and variance-of-cost space. Because the quadratic program minimizes variance at a fixed level of expected cost (varied parametrically), the solution set or efficient frontier traced out lies along the lower left border of the feasible space.

The right-hand side (the B vector) of the EC-VC model contains the deterministic estimates of the financing requirements of the FICB. But since the financing needs are stochastic, it is necessary to convert these stochastic variables into deterministic values. One possibility is to use the expected values of these needs as the values for the right-hand side. In some instances, however, it may be optimal to plan for bond and note debt outstanding to be an amount greater or less than expected debt needs. This would depend upon the cost of short-term debt and short-term investment return.

**Inventory model**

The function of a FICB is to market money. It buys on a volume basis in the national money market and repackages the funds into smaller loans, which are sold to PCAs and OFIs. As a marketing firm, the FICB faces many of the decisions faced by other marketing firms. One of these decisions is to determine the optimal quantity of funds to purchase per bond or note issue to provide for the stochastic demand for its product until another bond or note issue occurs. Because it is necessary between bond and note issues to maintain some level of inventory, the FICB faces an inventory quantity decision.

With the use of an inventory model, it is possible to estimate the optimal bond and note sales for a time period, given an estimate of probable demand, expected cost of bonds and notes, cost of inventory (funds) deficits, and return from excess inventory balances. After the optimal bond and note quantities are determined for each time period, these values can be inserted as the right-hand side of the mean-variance model, and the optimal term structure of bond and note debt can be determined.

Debt demand for each period is defined as the amount of debt funds necessary to service the loans outstanding for that period. This definition of demand involves a stock rather than a flow concept. Debt outstanding during any period depends not only upon new loans granted during the period, but also upon loans made in previous periods that have a maturity of more than one period. Because demand is defined as a stock, the vast majority of demand occurs instantaneously at the start of the period as outstanding loans are carried into the new period. The model assumes for simplicity that all the stochastic debt demand for each period occurs immediately after a bond or note is issued at the beginning of the period. Thus, after demand occurs there will be either an excess or a shortage of funds for the remainder of the period. Excess funds are invested in short-term investments; deficits are covered by short-term borrowing. The objective is to minimize the expected cost of funds for the period. The control variable is the quantity of bonds and notes to be outstanding for the period. More explicitly, we want to minimize:

\[ \text{minimize: } Z = X'QX \]
\[ E[c(y)] = c \cdot y + p \int_{y}^{\infty} (v-y) f(v) \, dv - h \int_{-\infty}^{y} (y-v) f(v) \, dv \quad (5) \]

where
- \( v \) = amount of debt demanded for a given time period,
- \( f(v) \) = probability density function for the possible values of \( v \),
- \( c \) = average bond and note cost,
- \( p \) = short-term debt cost,
- \( h \) = short-term excess fund return,
- \( y \) = amount of bonds and notes outstanding.

The first derivative of equation (5) with respect to \( y \) set equal to zero is:

\[ \frac{dE[c(y)]}{dy} = c - p \int_{y}^{\infty} f(v) \, dv - h \int_{-\infty}^{y} f(v) \, dv = 0. \quad (6) \]

By definition,

\[ \int_{y}^{\infty} f(v) \, dv = 1 - \int_{-\infty}^{y} f(v) \, dv. \]

Inserting this identity into equation (6) and solving for the minimum cost yields:

\[ \int_{-\infty}^{\infty} f(v) \, dv = \frac{p-c}{p-h}. \quad (7) \]

If \( f(v) \) is estimated, then \( y^* \) can be determined as the optimal quantity of bonds and notes outstanding for the period. But \( y^* \) is only defined if

\[ 0 \leq \frac{p-c}{p-h} \leq 1. \]

This can occur only under either of the two conditions:

(a) \( p > c > h \),
(b) \( p < c < h \).

For a minimum cost, the second derivative of equation (6) valued at \( y^* \) must be greater than zero, or

\[ \frac{d^2E[c(y)]}{dy^2} = (p-h) f(y^*) > 0. \quad (8) \]

Since \( f(y^*) > 0 \), for the second-order condition of (8) to be fulfilled, \( p > h \), so condition (a) must hold and \( p < c < h \).

The expected inventory (funds) shortage for a given period will be

\[ E(s) = \int_{y}^{\infty} (v-y^*) f(v) \, dv. \quad (9) \]

This shortage can be multiplied by \( p \) for the expected funds shortage cost. The expected cost of not financing all debt by the lower bond and note cost is \((p-c) E(s)\).

Similarly, the expected excess funds for a given period will be

\[ E(x) = \int_{-\infty}^{y^*} (y^*-v) f(v) \, dv. \quad (10) \]

This excess can be multiplied by \( h \) for the expected return on excess funds. The expected cost of overfinancing with bonds and notes will be \((c-h) E(x)\).

The FICB is assumed to meet all the financing needs of the PCAs so that none of the debt-financing activity levels will affect the probability distribution of debt demand in any successive periods. Thus, there will be no correlation between \( y^* \)'s.

### The Empirical Model

An empirical model structured to be solved as a quadratic programming problem was specified to obtain numerical information concerning the debt management problem of a FICB. Data for model construction were obtained from the Omaha FICB. The following discussion presents the basic structure of the empirical model and the coefficient estimation procedures. Further details are available in Tauer (15).

#### Planning horizon

The planning horizon of the model is 3 years. Three years enables analysis of the impact of sequential funding with discount notes, 6-month and 9-month bonds. The sequential impact of the longer-term bonds with various terms to maturity (2-year to 12-year bonds have been used) would have required a substantially longer planning horizon.

The model is multiperiod; the first 18 periods are monthly periods, the last six periods are quarterly periods. Monthly periods were selected because the 6-month and 9-month bonds are issued at the beginning of each month. The last half of the planning horizon was separated into quarters to reduce the number of activities in the model and still provide adequate detail. Transition to quarters required aggregating the monthly funding activities into quarters during the last half of the 3-year planning horizon. The model terminates at the end of the 3-year horizon. Termination activities were not included.\(^4\)

#### Activities

Eighteen 9-month bond activities were defined for the 18 monthly periods, and six 9-month bond activities were defined for the last six quarters. Six-month bond activities were defined in a similar manner. Long-term bond activities were defined as bonds issued on a...
The theory states that the current long-term spot rate is documented in the interest rate literature (Sharpe, 12) and that minimum size.

### Constraints

The model contains 24 rows, which incorporate the funding needs (loan demand) of the FICB. The first 18 rows correspond to the first 18 monthly periods; the last six rows correspond to the six quarterly periods of the last half of the planning horizon. Transfer rows and columns were used to bring into solution the debt structure outstanding at the beginning of the planning horizon because initial outstanding debt obligations will provide for some of the funding needs of the bank.

Additional constraints were placed on selected activities in some of the solutions to ascertain the effects of various debt management policies. One policy restriction is that no more than 10 percent of debt outstanding can be acquired by a single bond issue. Also, no more than 10 percent of the debt can be held as discount notes. A third restriction is that at least 30 percent of debt must be held in long-term bonds.

### Coefficient estimation procedures

It was assumed that the debt cost probability density function is multivariate normal; thus, estimation of expected values of debt costs and the variance-covariance of these costs will completely define the probability density function. Although it is not necessary, inasmuch as any functional form can be used, the probability density function for future FICB debt needs was also assumed to be normally distributed.

**The Variance-Covariance Matrix:** The expectations theory of the term structure of interest rates was used to derive the variance-covariance matrix. Monthly observations of secondary market yields on all federal government agency securities from 1965 to 1977 were used in this derivation. Secondary market yields were used rather than initial placement rates because initial rates were not available for all currently used securities. A 3-year monthly moving observation from the 13 years of data was used to generate 120 3-year observations for actual rates. The expectations theory was then used to obtain expected values for interest rates in each of the 120 observation periods.

The expectations theory and its variations are well documented in the interest rate literature (Sharpe, 12). The theory states that the current long-term spot rate is the geometric mean of the current short-term spot rate and future short-term rates expected to occur during the duration of the long-term security. The rate thus derived is the expected rate determined by the market as market participants price various terms to maturity.

To compute the expected interest rate values for the 9-month bonds into the 3-year future for one of the 120 observations, the following formula was used:

\[
E(r_{o,k,k+9}) = \left( \frac{1 + r_{o.o,k}^{(k/12)} + 9/12}{1 + r_{o.o,k}^{(3/12)}} \right)^{(12/9)}
\]

where

- \( r_{t,nt,tn} \) = an interest rate,
- \( t_x \) = the date on which the rate is computed,
- \( t_2 \) = the date on which the bond is issued,
- \( t_3 \) = the date on which the bond matures,
- \( k \) = the month into the future that the bond will be issued (varies from 1 to 35).

Similar formulas were used to compute the expected rates for the 1-month, 6-month, and 3-year securities. The actual interest rates that occurred and the estimated expected rates were converted into actual and expected costs per $1,000 of debt. The deviations of the actual costs from the expected costs were then squared and divided by 120 to obtain the coefficients of the variance-covariance matrix.

The variance-covariance matrix thus calculated is 84 by 84, corresponding to the 84 activities of the model. It is assumed that the variance-covariance matrix is the same regardless of the expected values of bond and note costs. Interest rates for the first month are considered known with certainty, so any variance or covariance term involving any of the first months' debt activities is zero. As debt activities occur further into the 3-year future, the variance of those activities becomes larger. This increase occurs because there is more uncertainty concerning the expected interest rate that will occur. Covariances of activities in different periods approach zero as the periods become more separated by time. This occurs because any economic condition that affects interest rates in one period will have a small impact on interest rates in a different time period. The impact becomes smaller as the time between periods becomes greater.

**Expected Costs:** Two projections of interest rates for the analysis period of January 1979 to December 1981 were used. Both forecasts were obtained from the same

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5 The security yield data were obtained from the Fiscal Agency of the Farm Credit System.

6 Calculated forward rates were used as expected rates. A liquidity premium was not included in the calculation because its existence, value, and stability are debatable. In addition, the forward rates calculated did not have a positive (or negative) bias when compared with the actual rates. A positive bias should occur with a liquidity premium.

7 The projections were provided by the FICB of Omaha.
national econometric model. The "most probable" interest rate forecast calls for interest rates to decrease during 1979, increase during 1980, and then fall again during 1981 (Figure 2, Panel A). The "recession" forecast simulated interest rates to increase the first two quarters of 1979, to fall drastically during the third and fourth quarters of 1979 as the recession develops, and then decrease moderately during 1980 and 1981 (Figure 2, Panel B). To derive expected debt costs, the cost of debt issuance in basis points was added to the interest rates. These adjusted rates were multiplied by $1,000, and then by the term to maturity of the debt activity, or the time left until the end of the planning horizon if that time was less than the term of the debt activity.

**Discount Coefficients:** The purpose of discounting is to convert money flows during various periods into a constant dollar value for one period, typically the first period. The conversion adjusts for the time value of money, inflation, and risk. In the EC-VC model, the risk of unexpected inflation and the unexpected change in default risk are reflected in the variance-covariance measure of cost. This is because the expectations theory of interest rates was used to compute the variance-covariance measures, and the time preference of money, expected inflation, and the expected component of the default risk premium would be inherent in the forward interest rates. Deviations (variance-covariance) from the forward rates would include unexpected changes in debt cost due to unexpected inflation and the unexpected variation of the default risk premium. The risk that money flows (costs) will not materialize because of changes in loan demand is incorporated in the inventory component of the model procedure. The only elements of the discount rate that are not incorporated in the coefficients are the pure time preference of money and expected inflation. Hence, the discount rates should reflect those two components.

The interest rate that most closely approximates only the pure time preference of money and expected inflation would be the rate on short-term U.S. Treasury Bills. In this study, projected 3-month bill rates were used. A separate bill-rate projection was used for each projection of expected interest rates.

The activities' expected costs and the variance-covariance matrix were discounted by the appropriate rate depending upon when interest payments are made on the various debt securities. Although the fixed costs of debt issuance occur earlier than interest payments, fixed costs are almost a negligible percentage of total debt cost; thus, for simplicity, they were discounted by using the same procedures as for the interest costs. Debt repayment flows were not discounted because the net flow would be the increase or decrease in total debt outstanding from period to period regardless of the selection of debt activities.

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8The expected component of the default risk premium is a cost item for the debt-issuing agency, but it does not affect temporal comparisons because it is not a risk incurred by the agency, but instead by the investor. It is therefore not included in the discounting procedure.

**Debt Requirements:** It is necessary to estimate probability density functions of FICB debt for each month for 18 months into the future, and then for each quarter for an additional six quarters into the future. To formulate normal probability density function estimates, the two parameters of the distribution, the mean and the standard deviation, were obtained by a linear regression of FICB debt on selected regressors. The forecasted values from the estimated regression equations were used as the means for the future periods. The variances of the error of forecast were used as the measure of variances for the distributions.

Two separate linear regressions were estimated to obtain two different forecasts of FICB debt. The first equation fitted was an ordinary least squares time-series equation. FICB bond debt from the first quarter of 1970 to the third quarter of 1978 (35 observations) was used to fit the equation. Bond debt rather than total liabilities was used because it was easily obtainable, and over the observation period, it typically represented more than 99 percent of total liabilities. Time was used as a regressor as well as three dummy variables for the first three quarters of each year. Since a constant term was estimated, the effect of the fourth quarter is implicit in the equation. The estimated equation was:
The model does not forecast FICB debt outstanding. However, since PCAs acquire nearly all of their funds for loans from the district FICB (the remainder is acquired from their capital accounts) and the FICB, in turn, acquires most of its funds by debt participation, it was possible to use the forecasted values of PCA loans outstanding to derive forecasted levels of FICB debt outstanding.

Actual PCA loans outstanding were regressed linearly on actual FICB bond debt outstanding for the end of each quarter from the second quarter of 1970 to the second quarter of 1978, which is the same data period used by Farmbank Research to fit their equation. The estimated equation is:

\[ Y = 37.38 + 0.85 \times PL \]

where

\[ Y = \text{FICB debt in millions of dollars}, \]
\[ PL = \text{PCA loans outstanding in millions of dollars.} \]

The t-statistics are listed in parentheses below the coefficient values. Both are significant at the 0.025 level. The F-value also is significant at that level. The standard error of the regression is $16,866 million. Because the explanatory variable, PCA loans, is not known with certainty during the forecast period, conditional forecasts are made. The values of this forecast also are listed in Table 1.

Each equation generated a slightly different estimate and allowed testing the sensitivity of the model to various debt projections. The time-series equation provided a projection of debt need that increased every month, with the greatest increase occurring the first quarter of each year as farmers prepared for the crop season. The second equation projects debt to increase generally over the 3-year horizon, with larger increases again occurring during the first quarter and decreases occurring during the fourth quarter of each year. The decrease results as farmers sell part of their crops to reduce their debts at the end of the crop season.

The probability density functions of FICB debt along with the average cost of bond debt, short-term debt cost, and excess funds return were used to derive the optimal level of bond and note debt for each period with application of the inventory model. With this model, the optimal FICB bond and note debt for each period was determined exogenously from the quadratic program, and the values were inserted into the program. The optimal bond and note debt for each period was first solved in terms of standard deviations to the right or left of the means of a standard normal curve. Then the number of standard deviations was multiplied by the value of the standard deviation for the specific period and added to or subtracted from the mean for that specific period to obtain the optimal value for bond debt outstanding.

As a proxy for short-term investment return, the forecasted yield was used on 90-day Treasury Bills for the various periods and forecasts. Although higher rates may be available via other investment avenues, Treasury Bills would always be available for any foreseeable amount of excess funds of the bank. As a proxy

\[
\begin{align*}
Y &= -1610.10 - 68.065 Q_{1} - 40.910 Q_{2} \\
(-25.39) & \quad (-3.706) \quad (-2.207) \\
-19.218 Q_{3} + 101.269 TM \\
(-1.046) & \quad (40.01) \\
R^{2} &= 0.98
\end{align*}
\]

\[
\begin{align*}
Y &= 37.38 + 0.85 PL \\
(3.75) & \quad (86.68) \\
R^{2} &= 0.99,
\end{align*}
\]

where

\[
\begin{align*}
Y &= \text{FICB debt in millions of dollars}, \\
PL &= \text{PCA loans outstanding in millions of dollars.}
\end{align*}
\]

The t-statistics are listed below the coefficient values. Both are significant at the 0.025 level. The F-value also is significant at that level. The standard error of the regression is $16,866 million. Because the explanatory variable, PCA loans, is not known with certainty during the forecast period, conditional forecasts are made. The values of this forecast also are listed in Table 1.

Table 1. FICB debt forecasts, beginning of the month (in millions of dollars).

<table>
<thead>
<tr>
<th>Quarter</th>
<th>First</th>
<th>Second</th>
<th>Third</th>
<th>Fourth</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>1326.701</td>
<td>41.490</td>
<td>1288.486</td>
<td>17.801</td>
</tr>
<tr>
<td>February</td>
<td>1367.769</td>
<td>41.608</td>
<td>1294.432</td>
<td>17.820</td>
</tr>
<tr>
<td>March</td>
<td>1468.837</td>
<td>41.726</td>
<td>1300.379</td>
<td>17.839</td>
</tr>
<tr>
<td>April</td>
<td>1533.905</td>
<td>41.940</td>
<td>1331.373</td>
<td>17.947</td>
</tr>
<tr>
<td>May</td>
<td>1468.837</td>
<td>41.726</td>
<td>1300.379</td>
<td>17.839</td>
</tr>
<tr>
<td>June</td>
<td>1429.321</td>
<td>42.442</td>
<td>1361.557</td>
<td>17.853</td>
</tr>
<tr>
<td>July</td>
<td>1408.752</td>
<td>42.198</td>
<td>1331.732</td>
<td>17.839</td>
</tr>
<tr>
<td>August</td>
<td>1415.148</td>
<td>42.731</td>
<td>1382.831</td>
<td>17.966</td>
</tr>
<tr>
<td>September</td>
<td>1421.564</td>
<td>42.865</td>
<td>1383.970</td>
<td>18.062</td>
</tr>
<tr>
<td>October</td>
<td>1421.564</td>
<td>42.865</td>
<td>1383.970</td>
<td>18.062</td>
</tr>
<tr>
<td>November</td>
<td>1421.564</td>
<td>42.865</td>
<td>1383.970</td>
<td>18.062</td>
</tr>
<tr>
<td>December</td>
<td>1421.564</td>
<td>42.865</td>
<td>1383.970</td>
<td>18.062</td>
</tr>
</tbody>
</table>

The second estimate of debt needs was obtained by regressing FICB debt on PCA loans outstanding. Farmbank Research, a research agency of the Farm Credit System, has developed an econometric model to forecast PCA loans outstanding for the Omaha district (Farmbank Research and Information Service, 6). The forecasts are generated for the end (beginning) of the quarter. The model does not forecast FICB debt outstanding. However, since PCAs acquire nearly all of their funds for loans from the district FICB (the remainder is acquired from their capital accounts) and the FICB, in turn, acquires most of its funds by debt participation, it

The probability density functions of FICB debt along with the average cost of bond debt, short-term debt cost, and excess funds return were used to derive the optimal level of bond and note debt for each period with application of the inventory model. With this model, the optimal FICB bond and note debt for each period was determined exogenously from the quadratic program, and the values were inserted into the program. The optimal bond and note debt for each period was first solved in terms of standard deviations to the right or left of the means of a standard normal curve. Then the number of standard deviations was multiplied by the value of the standard deviation for the specific period and added to or subtracted from the mean for that specific period to obtain the optimal value for bond debt outstanding.

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\[
\begin{align*}
Y &= \frac{-1610.10 - 68.065 Q_{1} - 40.910 Q_{2}}{(-25.39) \quad (-3.706) \quad (-2.207)} \\
&\quad -19.218 Q_{3} + 101.269 TM \\
&\quad (-1.046) \quad (40.01)
\end{align*}
\]

\[
R^{2} = 0.98
\]
for the short-term borrowing rate, the forecasted federal funds rate was used. To obtain projected average debt cost, a weighted average was used of forecasted 9-month and 3-year bond rates. The weighting was 0.7 for the 9-month bonds and 0.3 for the 3-year bonds. These weights reflect the approximate historical debt distribution of the FICB of Omaha.

The FICB was restricted to a limit of planned excess funds or deficit funds of $25 million from the expected value for each period. The $25 million is the line of credit that the FICB has established with commercial banks. Although there is no formal limitation to the investment portfolio of the bank, the same $25 million restriction was applied to excess funds.

### Empirical Results

A number of applications of the model were performed with different projections of expected debt cost and optimal debt requirements. Two applications used historical debt costs and debt requirements for the periods 1975-1977 and 1976-1978 to validate the model. Two applications to the future period 1979-1981, one with debt policy constraints and the other without debt policy constraints, used the most probable forecast of interest rates and the debt forecast from PCA loan projections to obtain expected debt cost and optimal bond and note debt requirements. Another two applications to the 1979-1981 period, one with and the other without the debt policy constraints, used the recession forecast of interest rates and the time-series forecast of debt. The same variance-covariance matrix was used for all applications. The expected cost and variance-covariance coefficients were discounted by using U.S. Treasury Bill rates corresponding to the appropriate forecast of expected debt cost.

#### Model validation

To verify the realism of the model, it was applied to the two historical periods of 1975-1977 and 1976-1978, the two most recent periods for which information on actual bond issuances was available.9 For the first test period of 1975-1977, seven unique portfolios were generated. They ranged from a high expected cost of $190,263 million (standard deviation of $26,323 million) to a low expected cost of $167,532 million (standard deviation of $35,313 million).10 The actual cost that the bank incurred during this time to meet new financing needs was $176,032 million. Because the standard deviation of the lowest expected-cost solution is $35,313 million, if normality is assumed, there is a 60-percent probability that the low expected-cost portfolio's actual cost would have been lower than the $176,032 million cost actually incurred by the bank. Similarly, there is a 40-percent probability that the high expected-cost portfolio's actual cost would have been higher than the bank's actual cost.

Because actual historical debt amounts and costs were used in the validation tests, the portfolios correspond closely to the actual debt issued by the bank. One major difference between the portfolios generated and the actual debt issuance of the bank is that the bank typically would participate in the term bond and the 9-month bond when both were offered. In the portfolios generated by the model, typically only one of the two bond types was selected during the months that both were available. The term bond was selected for the portfolios at the high standard deviation levels, and the 9-month bond was selected at the low standard deviation levels. At first glance, it would seem that the model is not diversifying debt as well as the bank actually did, but diversification can be accomplished over time with a similar or different bond type as well as by the use of different bond types at a point in time. Since interest rates are more variable over time than between securities at a point in time, it would be natural for the diversification of debt to occur over time more than by maturity at issuance. This will become more obvious when the additional types of debt maturities are added to the model.

#### Most probable expected debt cost

The following two efficient frontiers were produced by using the most probable forecast of interest rates to derive expected debt costs combined with debt forecasts from PCA loan projections to obtain optimal bond and note debt requirements. The first frontier includes no debt policy constraints, and the second frontier includes debt policy constraints.

**No Debt Policy Constraints**: The model excluding the debt policy constraints generated 25 individual portfolios on the frontier, ranging from a low expected discounted cost of $226,602 million (high discounted standard deviation of $28.195 million) to a high expected discounted cost of $237,035 million (low discounted standard deviation of $18.805 million). The efficient frontier is plotted as curve A in Figure 3. This frontier illustrates the tradeoff between expected cost and standard deviation—as a movement up the frontier to a lower expected-cost portfolio occurs, a higher level of standard deviation of cost must be assumed.

The portfolios with the highest expected cost, an intermediate expected cost, and the lowest expected cost are shown in Table 2. The highest-cost portfolio

---

9Because only 9-month and term bonds were available to the FICB during these two periods, only these two bond maturities were included in the historical tests.

10To facilitate presentation of results on the empirical work, the standard deviation rather than variance is used to measure risk.
Figure 3. Expected cost and standard deviation efficient frontiers (in millions of dollars).

A = Most probable forecast of expected debt cost and debt requirements from PCA loan projections, no debt policy constraints
B = Most probable forecast of expected debt cost and debt requirements from PCA loan projections, with debt policy constraints
C = Recession forecast of expected debt cost and debt requirements from time series forecast, no debt policy constraints
D = Recession forecast of expected debt cost and debt requirements from time series forecast, with debt policy constraints

(lowest standard deviation) entails extensive use of term bonds the first year. For the months in the first year that term bonds are not available, discount notes are used to provide funds until another term bond can be used. The second year of this portfolio involves the use of some 9-month bonds as well as term bonds and discount notes. The term bonds and some of the 9-month bonds are carried into the last quarter of the second year, but no new debt is issued that quarter. Indeed, during the fourth quarter of the second year, discount notes and some 9-month bonds will mature and not be refinanced. This occurs because the optimal debt needs of the bank decrease from the third to the fourth quarter. During the third year of the planning

Table 2. Debt issuance for the planning horizon: most probable expected debt cost and debt requirements from PCA loan projections, no debt policy constraints (in millions of dollars).

<table>
<thead>
<tr>
<th>Year</th>
<th>Term Bonds</th>
<th>Nine-month Bonds</th>
<th>One-month Discount Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1979</td>
<td>126.073</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>305.441</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>358.906</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>169.762</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>107.472</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>1980</td>
<td>107.472</td>
<td>38.163</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>25.797</td>
<td>12.366</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>1981</td>
<td>0.0</td>
<td>0.0</td>
<td>433.578</td>
</tr>
<tr>
<td></td>
<td>0.0</td>
<td>0.0</td>
<td>756.909</td>
</tr>
<tr>
<td></td>
<td>0.0</td>
<td>0.0</td>
<td>965.069</td>
</tr>
<tr>
<td></td>
<td>0.0</td>
<td>0.0</td>
<td>520.291</td>
</tr>
<tr>
<td></td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Expected Discounted Cost = 237.062
Standard Deviation = 18.803

Expected Discounted Cost = 234.684
Standard Deviation = 19.924

Expected Discounted Cost = 226.602
Standard Deviation = 28.195
horizon, only discount notes are used. The dominance of discount notes during the last year, and especially in the last two quarters of the planning horizon, is evidenced here and in portfolios presented later. This phenomenon may be myopic; tests indicate, however, that the myopic terminal-year condition does not seem to be transmitted into the first and second years.

In the intermediate cost and standard deviation portfolio, as in the highest-cost (lowest standard deviation) portfolio, only term bonds and discount notes are used the first year; indeed, there are no differences in the debt portfolios for the first 6 months of the first year between these two portfolios. However, in July of the first year, discount notes are substituted for term bonds in the intermediate-cost portfolio. Then in August, a greater use of discount notes occurs. Discount notes issued in September mature at the end of that month, and a large volume of term bonds is used to refinance the debt in October. Although various factors influence the choice of activities, it seems that inasmuch as interest rates are projected to reach their lowest levels during October of 1979, the discount notes are issued the months immediately preceding October so that a large amount of debt can be refinanced with the term bond at the lowest interest rate of the planning horizon. This phenomenon did not occur in the earlier low-variance portfolio because the discount notes have high variances. During the second year, fewer 9-month bonds and more discount notes and term bonds are used than in the lowest-variance portfolio. For the first quarter of the third year, 9-month bonds and 6-month bonds replace the discount notes of the low-variance portfolio. This occurs because projected interest rates rise slightly during the early part of 1981, and the 6-month and 9-month issues lock in a low debt cost before rates begin to rise.

For the lowest-cost portfolio of the frontier, activities are selected on the basis of their expected cost without regard to variance. Therefore this portfolio is identical to one that would result from linear programming. For the first year, only discount notes are used during the first 9-months, and then, in October, $960.182 million of term bonds are used to refinance all new debt accumulated since the beginning of the planning horizon. Discount notes again are used during November and December until another term bond can be issued in January of the second year. In February of the second year, a 9-month bond is issued before interest rates begin to increase. As new debt needs increase early in the second year, a term bond and a small volume of discount notes are first used; then discount notes are issued during the third quarter because debt needs fall at the end of the year, and the discount notes will mature at that time. For the third year, discount notes are again used extensively along with the 9-month bond.

A summary of activities used the first year for the 25 individual portfolios on the efficient frontier is shown in Table 3. The table indicates the monthly average percentage of bond and note debt acquired the first year from using the four types of debt securities. Every fifth portfolio is summarized, with the portfolios listed in order of descending expected costs and ascending standard deviation. A movement from higher to lower expected cost portfolios results in a shift from term bonds to discount notes augmented with 6-month bonds and then to discount notes. Nine-month bonds are never included in the portfolios during the first year on this efficient frontier.

If the model is used on an operational basis, a bank would be especially interested in the activities for the first period because a decision to participate in the debt issues of that period would be imminent. Many of the portfolios on the efficient frontier have the same first-period debt activities (Table 4), and the range of expected cost and standard deviation is quite large before there is a change in the first-period's activities. For example, expected cost varies from the highest cost of $237.062 million to $231.446 million before a change occurs in the activities for the first period; this change is from $126.073 million in term bonds to $96.665 million in term bonds and $29.408 million in 6-month bonds. The next change, at an expected cost of $228.733 million, is to $126.073 million in 6-month bonds. The final change at the lowest expected-cost solution is to $126.073 million in discount notes.

For a bank to select a debt portfolio from the set of 25, it must assess its preference for risk and cost combinations. If the bank desires a low expected-cost liability structure, it must be willing to assume the risk that cost may be more variable (larger standard deviation) compared with a higher expected-cost debt structure. All other things equal, a bank that chooses a portfolio on the lower right section of an EC-VC efficiency frontier can be referred to as more risk averse than a bank that selects a portfolio on the upper section of the frontier.

If the probability density function of cost is multivariate normal, then some probability ranges of cost can be calculated to guide a bank in the selection of a

<table>
<thead>
<tr>
<th>Portfolio number</th>
<th>Term bonds</th>
<th>Nine-month bonds</th>
<th>Six-month bonds</th>
<th>Discount notes</th>
<th>Expected cost</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 to 20</td>
<td>237.062 to 231.446</td>
<td>22.906 to 29.408</td>
<td>96.665 to 126.073</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>21 to 25</td>
<td>228.733 to 228.400</td>
<td>25.733 to 27.910</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>26 to 25</td>
<td>228.400 to 28.195</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>126.073</td>
<td></td>
</tr>
</tbody>
</table>

Table 4. Bond participations the first period: most probable expected cost and debt requirements from PCA loan projections, no debt policy constraints.
With Debt Policy Constraints: The model incorporating the debt policy constraints generated 116 individual portfolios on the efficient frontier. The portfolios ranged from a low expected discounted cost of $234,310 million (high standard deviation of $285,787 million) to a high expected discounted cost of $237,184 million (low standard deviation of $22,021 million). This efficient frontier is plotted as curve B in Figure 3. As illustrated, the addition of debt policy constraints shifts the efficient frontier to the right; at any level of standard deviation, the portfolio with policy constraints has a higher expected cost than the portfolio without policy constraints. At the low standard deviation of $22,021 million, the increase in expected cost is approximately $5 million. At the high standard deviation of $285,787 million, the increase in expected cost is approximately $7.7 million. With the projections of expected costs and debt requirements used in this application, it seems that the addition of policy constraints imposes a greater penalty cost at the greater standard deviation levels.

One purpose of debt policy constraints is to reduce the volatility of the cost of debt. Application of the policy constraints, however, does not necessarily accomplish this objective. The greatest standard deviation for the portfolios generated with the constraints is $28,578 million, which is 1 percent larger than the greatest standard deviation ($28,195 million) without the debt policy constraints. Unfortunately, in addition to failing to limit potential high cost-volatility, the constraints raise the minimum standard deviation attainable from $18,803 million without the debt policy constraints to $22,021 million with the constraints, a 17-percent increase.

Portfolios for the highest expected cost, an intermediate expected cost, and the lowest expected cost on the efficient frontier generated with constraints are shown in Table 5. The low standard-deviation portfolio with the constraints, like the analogous portfolio without the constraints, includes a large amount of term bonds the first year of the planning horizon. The volume, however, now is reduced because no more than 10 percent of the total debt can be held in any specific bond issue. As a substitute for the term bonds, the next lowest-variance bonds are selected, which in this instance are the 9-month bonds. Some discount notes also are included in the portfolio, mostly entering the month before a term bond is issued. The debt acquired by the discount notes is again refinanced with term bonds when possible. In the second year of the constrained low standard-deviation portfolio, more debt issuance activities are included than in the analogous nonconstrained portfolio. This occurs because the 9-month bonds issued during the first year must be refinanced. When these bonds mature, they are refinanced with additional 9-month bonds and discount notes. For the last year of the portfolio, because discount notes are constrained, some 6-month bonds and 9-month bonds are included.

A move from the high expected-cost to the intermediate expected-cost portfolio results in a shift from 9-month bonds to 6-month bonds and discount notes for the first year (Table 5). Except for the first
month, the amount of term bonds to be issued the first year in this portfolio is not altered from the high-cost (low standard deviation) portfolio. In the second year, a move to the intermediate from the high expected-cost portfolio entails very little change in term bond usage but includes more 9-month bonds, fewer discount notes, and no 6-month bonds. The debt issuance of the third year also is restructured with the inclusion of 9-month and term bonds because more maturing debt is refinanced and the use of the discount notes is constrained.

The first year of the low expected-cost portfolio on this frontier differs substantially from the unconstrained low expected-cost portfolio (Table 5). Now, not only are discount notes included in the first year, but because of the 10-percent constraint limiting the use of any specific issue, 6-month bonds are included as well inasmuch as they are the next lowest-cost debt activity. Term bonds also are included in the portfolio during the first year because of the requirement to maintain 30 percent of the debt in term issues. Unlike the unconstrained portfolio where $960.182 million of term bonds were issued in October because of that issue's low cost, now only $129.673 million can be issued because of the 10-percent constraint. Because of the limits on the issuance of term bonds, beginning with November of the first year and continuing into the second year, 9-month bonds are issued, subject to the 10-percent constraint. Interest rates are projected to increase, and low debt costs are being locked in with the long-term securities—term and 9-month bonds. During the third year, low-cost notes, term bonds, and 9-month bonds are used subject to the 10-percent-limit constraint.

Similar to the first period's activities for the unconstrained model, a movement to lower costs entails a shift from term bonds to 6-month bonds to discount notes for the first period. Now, however, at least $93,479 million of term bonds always is issued the first period to comply with the 30-percent minimum debt to be held in term bonds. Although there are 116 different portfolios for the entire planning horizon, there are a significantly reduced number of first-period options, in this case, five. Three of the five first-period options are indicated in Table 5 as the amount of debt issued for January 1979. The other two first-period options involve issuing $118.135 million of term bonds and $7.938 million of 6-month bonds (with an expected cost of $23.084); or $93.479 million of term bonds, $30.458 million of 6-month bonds, and $2.136 million of discount notes (with expected costs ranging from $26.581 to $26.700 million).

The high expected-cost portfolio has a 90-percent probability that the actual discounted cost will fall between $208.997 million and $265.371 million; for the low-cost portfolio, the 90-percent confidence interval is $197.730 million to $270.890 million. Again, there is considerable overlap in these confidence intervals.

Recession forecast of expected debt cost

Two additional efficient frontiers were produced by using the recession forecast of interest rates to derive expected debt cost combined with forecasts from the time-series regression to derive optimal bond and note debt requirements. These coefficients were first used in the model with no debt policy constraints and then with debt policy constraints. No Debt Policy Constraints: The model with no debt policy constraints generated 74 unique portfolios on the efficient frontier. The portfolios ranged from a low expected discounted cost of $236.870 million (high discounted standard deviation of $44.934 million) to a high expected discounted cost of $275.332 million (low discounted standard deviation of $17.775 million). The efficient frontier is plotted as curve C in Figure 3. The cost difference between the low and high expected-cost portfolios is $38.462 million. With the efficient frontier produced by using the most probable debt cost forecast, the difference between the low and high expected-cost portfolios was only $10.433 million. The range in standard deviation for the efficient frontier under the recession forecast also has a larger spread, $27.159 million, relative to the spread of $9.39 million for the most probable forecast. The larger range and higher levels of expected cost and standard deviation can be attributed both to the greater variability in interest rates of the recession forecast and the greater debt needs of the time series forecast.

The highest expected-cost, an intermediate expected-cost, and the lowest expected-cost portfolios are shown in Table 6. The low standard deviation portfolio is similar in debt issuance to the low standard deviation portfolio under the most probable debt cost forecast (Table 2). Term bonds are used primarily the first year, with discount notes meeting the new debt needs that occur between term bond issues. Because the total debt requirements of the bank remain relatively constant for the first part of the second year, no additional debt is issued during that time except for a small amount of discount notes. During the last half of the second year and into the third year, discount notes are issued to meet slightly increasing debt needs. The discount notes have the lowest interest rates, and although their variance-covariance coefficients are larger than for the other securities in the third year, the relative differences are not as great the third year as during the first and second years.

A movement to the intermediate-cost portfolio results in a drastic switch from term bonds to 9-month bonds early the first year, and to discount notes for the last half of the first year. Nevertheless, the debt needs of the first month still are completely fulfilled by term bonds. Because interest rates rise sharply in the early months of the first year, 9-month bonds are issued before the period when interest rates peak and then are refinanced at lower rates when interest rates decrease. Although this strategy is not the lowest-cost strategy, it does have a medium standard deviation value. During the last half of the first year, discount notes are used to take advantage of the continuous decrease in interest rates. Interest rates slowly decrease the second year, and there is a tendency for short-term securities to be used to take advantage of this decrease. Nevertheless, because the variance is at an intermediate level, there are still some term bonds included in the portfolio.

The low-cost portfolio in Table 6 includes 9-month
bonds for January and February of the first year, 6-month bonds for March and April, and then discount notes for the rest of the planning horizon. This debt issuance pattern is a direct response to the movement of interest rates during the planning horizon. At the beginning of the planning horizon, interest rates are increasing, so the 9-month bonds are used to lock in an attractive rate for the duration of the higher-interest-rate period. Later, when higher interest rates will last only 4 to 5 months, 6-month bonds are used. Two months before interest rates peak for the planning horizon, the use of discount notes begins and continues for the rest of the planning horizon. At this time, because interest rates will soon decline, it would be more costly to use one of the other three securities, which all have longer terms than the discount note.

A summary of funding activities used the first year is shown in Table 7. Every twelfth portfolio is summarized. The low standard deviation portfolios involve extensive use of term bonds. A movement to higher standard deviation portfolios induces a shift from term bonds to 9-month bonds and then to discount notes. In the highest standard deviation portfolios, 6-month bonds are also used. With the most probable forecast of interest rates discussed earlier, the movement from low to high standard deviation portfolios involved a shift from term bonds to 6-month bonds to discount notes.

Table 6. Debt issuance of the planning horizon: recession forecast of expected debt cost and time series debt forecast, no debt policy constraints (in millions of dollars).

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>178.867</td>
<td>0.0</td>
<td>0.0</td>
<td>381.868</td>
<td>381.868</td>
<td>381.868</td>
</tr>
<tr>
<td>February</td>
<td>0.0</td>
<td>1.706</td>
<td>0.0</td>
<td>98.879</td>
<td>98.879</td>
<td>98.879</td>
</tr>
<tr>
<td>March</td>
<td>0.0</td>
<td>1.540</td>
<td>0.0</td>
<td>114.883</td>
<td>114.883</td>
<td>114.883</td>
</tr>
<tr>
<td>April</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>93.926</td>
<td>93.926</td>
<td>93.926</td>
</tr>
<tr>
<td>May</td>
<td>0.0</td>
<td>164.275</td>
<td>0.0</td>
<td>100.062</td>
<td>100.062</td>
<td>100.062</td>
</tr>
<tr>
<td>June</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>105.741</td>
<td>105.741</td>
<td>105.741</td>
</tr>
<tr>
<td>July</td>
<td>284.152</td>
<td>112.877</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>August</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>137.227</td>
<td>137.227</td>
<td>137.227</td>
</tr>
<tr>
<td>September</td>
<td>0.0</td>
<td>253.623</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>October</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>173.753</td>
<td>173.753</td>
<td>173.753</td>
</tr>
<tr>
<td>November</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>December</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>10.374</td>
<td>10.374</td>
<td>10.374</td>
</tr>
</tbody>
</table>

Table 7. Average monthly new debt outstanding for the first year: recession forecast of expected debt cost and time series debt forecast, no debt policy constraints.

<table>
<thead>
<tr>
<th>Portfolio number</th>
<th>Term bonds</th>
<th>Nine-month bonds</th>
<th>Six-month bonds</th>
<th>Discount notes</th>
<th>Expected Discounted Cost</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>85.3</td>
<td>3.7</td>
<td>0</td>
<td>11.9</td>
<td>275.332</td>
<td>17.775</td>
</tr>
<tr>
<td>12</td>
<td>75.6</td>
<td>9.0</td>
<td>5.8</td>
<td>9.7</td>
<td>271.102</td>
<td>18.027</td>
</tr>
<tr>
<td>24</td>
<td>65.4</td>
<td>17.2</td>
<td>8.1</td>
<td>9.3</td>
<td>266.533</td>
<td>18.929</td>
</tr>
<tr>
<td>36</td>
<td>33.3</td>
<td>34.6</td>
<td>0.0</td>
<td>11.9</td>
<td>256.383</td>
<td>23.139</td>
</tr>
<tr>
<td>48</td>
<td>24.1</td>
<td>58.8</td>
<td>0.0</td>
<td>17.1</td>
<td>250.492</td>
<td>26.689</td>
</tr>
<tr>
<td>60</td>
<td>24.1</td>
<td>42.4</td>
<td>7.1</td>
<td>26.4</td>
<td>244.666</td>
<td>31.593</td>
</tr>
<tr>
<td>72</td>
<td>0.0</td>
<td>42.0</td>
<td>13.1</td>
<td>44.9</td>
<td>236.986</td>
<td>44.334</td>
</tr>
<tr>
<td>74</td>
<td>0.0</td>
<td>29.6</td>
<td>14.6</td>
<td>55.8</td>
<td>236.870</td>
<td>44.934</td>
</tr>
</tbody>
</table>

Although the model generates 74 individual portfolios, the first period decisions involve only two possibilities, term bonds in the amount of $178,867 million at the low standard deviation levels, or 9-month bonds in the same amount at the high standard deviation levels. The high expected-cost portfolio has a 90-percent probability that the actual discounted cost will fall between $252,580 million and $298,084 million; for the low expected-cost portfolio, the range is $179,354 million to $294,386 million.

With Debt Policy Constraints: The model with debt constraints generated 141 unique portfolios on the efficient frontier. The portfolios ranged from a low expected discounted cost of $247,961 million (high discounted standard deviation of $31,738 million) to a high expected discounted cost of $264,536 million (low discounted standard deviation of $21,590 million). The 141 portfolios on the frontier are plotted as curve D in Figure 3.
frontier with constraints is smaller compared with the frontier without the constraints. The largest standard deviation for a portfolio on the frontier derived with the debt constraints is $31,738 million, compared with $44,934 million without the debt constraints. Thus, it seems that, with the recession forecast of debt cost, the debt constraints are effective in limiting the risk exposure of the bank. However, that accomplishment is achieved with an increase in the lowest expected cost and standard deviation obtainable. The lowest-cost portfolio on the frontier generated by the model with debt constraints has a cost $11.091 million higher than the lowest-cost portfolio on the frontier derived by the model without the debt constraints. Also, the lowest standard deviation on the frontier derived with the debt policy constraints is $21,590 million, which is $3,815 million greater than the lowest standard deviation on the frontier obtained without the debt policy constraints.

The highest expected-cost, an intermediate expected-cost, and the lowest expected-cost portfolios on the efficient frontier are shown in Table 8. The difference between these portfolios and the analogous portfolios obtained without the debt policy constraints (Table 6) can be attributed to the constraints.

In the low standard deviation portfolio with the debt constraints, term-bond volume is restricted, so a large amount of 9-month bonds also are included in the portfolio in the first year of the planning horizon. This adjustment, which occurs with the addition of the debt policy constraints, is completely analogous to the changes that occurred using the most probable rate forecast when debt policy constraints were added. As with the most probable forecast, the second year of the low standard deviation portfolio involves the continued use of term bonds and some 9-month bonds to refinance maturing 9-month and 6-month bonds issued in the first year.

The debt-issuance pattern for the first two years of the intermediate-cost portfolio with the debt policy constraints is very similar to the portfolio without the constraints, with a few exceptions. Because of the policy limiting the use of any debt activity to 10 percent of total debt, the dollar volume of some of the activities is reduced. This occurs with the discount notes at the end of the first year, so other bonds, mostly 9-month bonds are substituted for notes. The same policy limits the use of term bonds in January of both the first and second years. In the first year, 9-month bonds are substituted for term bonds; in the second year, the substitute is discount notes. In the third year, 6-month and 9-month bonds are substituted for term bonds. In the third year, 6-month and 9-month bonds are used because the discount notes are limited by the 10-percent constraint.

In the low-cost portfolio with debt constraints, 9-month bonds again are issued the first few months of the planning horizon to lock in a low debt cost before interest rates increase. As interest rates decrease beginning the middle of the first year, discount notes are used so that debt can be refinanced each month at a lower cost. Because the 10-percent debt constraint limits the exclusive use of discount notes, 6-month bonds also are used as interest rates fall. Because the 6-month bond's maturity is six times that of the 1-month discount note, debt cannot be refinanced as rapidly when interest rates decrease. The result is that the cost of the lowest-cost portfolio with the debt constraints is $247,961 million compared with $236,870 million without debt constraints, an increase of 5 percent. Another difference between the portfolios is that term bonds now are used because of the constraint to keep 30 percent of the debt in term bonds. The largest amount of term bonds permissible, subject to the 10-percent constraint, is issued the first month. This term debt could have been spread over the planning horizon within the limitations of the debt constraints, but be-

Table 8. Debt issuance for the planning horizon: recession forecast of expected debt cost and time series forecast, debt policy constraints (in millions of dollars).

<table>
<thead>
<tr>
<th></th>
<th>Lowest Standard Deviation Portfolio</th>
<th>Intermediate Standard Deviation Portfolio</th>
<th>Highest Standard Deviation Portfolio</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Term bonds</td>
<td>6-month bonds</td>
<td>One-month discount notes</td>
</tr>
<tr>
<td>1979</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>January</td>
<td>134.288</td>
<td>44.579</td>
<td>0.0</td>
</tr>
<tr>
<td>February</td>
<td>114.883</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>March</td>
<td>78.564</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>April</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>May</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>June</td>
<td>83.912</td>
<td>0.0</td>
<td>21.329</td>
</tr>
<tr>
<td>July</td>
<td>141.206</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>August</td>
<td>109.269</td>
<td>27.958</td>
<td>0.0</td>
</tr>
<tr>
<td>September</td>
<td>0.0</td>
<td>20.952</td>
<td>93.474</td>
</tr>
<tr>
<td>October</td>
<td>143.005</td>
<td>1.386</td>
<td>0.0</td>
</tr>
<tr>
<td>November</td>
<td>108.463</td>
<td>0.0</td>
<td>11.611</td>
</tr>
<tr>
<td>December</td>
<td>0.0</td>
<td>0.0</td>
<td>94.886</td>
</tr>
<tr>
<td>1980</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>January</td>
<td>144.603</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>February</td>
<td>86.887</td>
<td>0.0</td>
<td>39.750</td>
</tr>
<tr>
<td>March</td>
<td>0.0</td>
<td>0.0</td>
<td>144.468</td>
</tr>
<tr>
<td>April</td>
<td>144.294</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>May</td>
<td>81.662</td>
<td>0.0</td>
<td>29.893</td>
</tr>
<tr>
<td>June</td>
<td>0.0</td>
<td>0.0</td>
<td>38.964</td>
</tr>
<tr>
<td>Third Quarter</td>
<td>144.333</td>
<td>0.0</td>
<td>65.354</td>
</tr>
<tr>
<td>Fourth Quarter</td>
<td>93.988</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>1981</td>
<td></td>
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<td></td>
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<tr>
<td>First Quarter</td>
<td>0.0</td>
<td>11.288</td>
<td>451.276</td>
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<tr>
<td>Second Quarter</td>
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<td>81.541</td>
<td>387.911</td>
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<td>Third Quarter</td>
<td>0.0</td>
<td>469.384</td>
<td>0.0</td>
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<tr>
<td>Fourth Quarter</td>
<td>0.0</td>
<td>112.535</td>
<td>475.892</td>
</tr>
</tbody>
</table>

Expected Discounted Cost = 250.678
Standard Deviation = 27.808

Expected Discounted Cost = 247,961
Standard Deviation = 31.738

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cause interest rates were rising, the lowest-cost option is to issue the maximum amount of term bonds the first month before interest rates increase. Later the first year and in the second and third years, additional term bonds are issued to maintain the 30-percent minimum constraint.

Although there are 141 individual portfolios for the 3-year planning horizon, there is only one debt issuance option for the first period—$44,579 million of 9-month bonds and $134,288 million of term bonds. The term bonds are at the maximum 10-percent limit. They are at that constrained level in the low standard deviation portfolio because the standard deviation of that debt activity is zero. In the low-cost portfolio for which standard deviation levels are higher, the term bonds remain at the same value because of the 30-percent requirement constraint.

The highest expected-cost portfolio has a 90-percent probability cost range of $236,901 million to $292,171 million. The range for the lowest expected-cost portfolio is $207,336 million to $288,586 million.

Comparison of the efficient frontiers

As noted, changes in the coefficients of the model and the addition of debt policy constraints shift the efficient frontier. The addition of debt policy constraints shifts the frontier to the right such that, at any level of standard deviation, a greater expected cost is incurred. This is reflected by a shift from frontier A to frontier B (Figure 3), the frontiers obtained by using the most probable debt forecast without and with the debt policy constraints, respectively, and from frontier C to D, the frontiers obtained by using the recession debt forecast without and with the debt policy constraints, respectively.

The shift from frontier A to frontier B is not a parallel shift; the increase in expected cost is greater at high standard deviation levels. This occurs because the term bond in October of the first year is used at a volume as large as $960.182 million in the unconstrained frontier, but is restricted to a maximum of $129.673 million in the constrained portfolio. Thus, the increase in expected cost is greater at the higher standard deviation levels when the use of that bond in the unconstrained portfolio is extensive because of its low expected cost. The shift from frontier C to D is more nearly parallel because no prevalent bond is used on the unconstrained frontier.

The shift from C to D also entails a reduction in the highest-cost risk exposure (standard deviation) of the bank. This did not occur with the shift from A to B. The reduction occurs because a large volume of discount notes, which have a high standard deviation, was used in the low-cost portfolio in frontier C but was limited in use on frontier D.

The shift from A to C, which results from a change in expected debt cost coefficients and debt requirements, is much more drastic than the shift due to the debt constraints. This implies that the debt policy constraints have a relatively small impact on the portfolios on the efficient frontiers compared with the effects of a change in the projected expected debt cost coefficients and debt needs.

References


