THE INFLUENCE OF INTERFACIAL CONDITIONS ON THE ULTRASONIC WAVE INTERACTION WITH MULTILAYERED MEDIA

Adnan H. Nayfeh and Timothy W. Taylor
Department of Aerospace Engineering and Engineering Mechanics
University of Cincinnati
Cincinnati, OH 45221

INTRODUCTION

Interfaces, both external and internal, constitute an integral part of the structure of materials, and so they enter directly into the structure/property/performance linkage which forms the central theme of materials science and engineering. These "boundary aspects" of bulk materials, because of their relatively discontinuous character, are not only difficult to study and elucidate, but they can also exert a major influence on the behavior of materials and, hence dominate their utility in many applications.

Since multilayered structural components (composites) are subjected to harsh environments which can lead to degradation in load carrying capability, an understanding of the role interfaces play on degradation processes (mostly in the forms of fiber-matrix and inter-ply interfacial delaminations) is very essential in determining the survivability of the structure. Since modern composites are man-made structures, interfaces are naturally created as a consequence of the manufacturing processes. These include, besides the adhesive bonding, diffusion bonding, open or closed extended cracks, smooth interfaces and interfacial roughness. Regardless of the bonding process the composite's interface is complicated.

It has recently been recognized that ultrasonic wave propagation might offer an alternative technique which is capable of assessing the influence of a wide variety of interfacial conditions [3,4]. In this paper we develop analytical solutions for the interaction of ultrasonic waves with multilayered material plates immersed in liquid. A variety of interfacial continuity or discontinuity conditions will be examined. Such interfacial conditions which include rigid, smooth, adhesive and diffusion bondings will be identified below in the Theoretical section.

In order to solve the problem we shall generalize our single layer's result to the multilayered case by using the matrix transfer technique introduced originally by Thomson [3] and somewhat later on by Haskell [4] for applications in geophysics and acoustics. According to this technique we construct the propagation matrix for a stack of arbitrary number of layers by extending the solution from one layer to the next while satisfying the appropriate interfacial continuity conditions.
THEORETICAL DEVELOPMENTS

Consider a laminated plate consisting of an arbitrary number, \( n \), of elastic isotropic layers which could be either rigidly or smoothly bonded at their interfaces. This plate is assumed to be totally immersed in liquid. The problem then is to study the reflected beam from the fluid-plate interface for an incident wave originating in the fluid at an arbitrary angle from the normal to the interface.

Guided by our single layer plate analysis of [2], in order to facilitate the present analysis, we shall use two sets of two-dimensional coordinate systems \((x,z)\), as illustrated in Figure 1. One system is global which has its origin at the bottom of the plate such that \( x \) denotes the propagation direction and \( z \) is normal to the interfaces. Here the layered plate will then occupy the space \( 0 \leq z \leq d \) where \( d \) denotes the total thickness of the plate. The second system is local for each sublayer of the plate. Since the plate is made of \( n \) layers, the \( k \)th layer will then have its local coordinates \( x \) and \( z^{(k)} \) with local origin at the interface between layers \( k-1 \) and \( k \). Hence layer \( k \) occupies the space \( 0 \leq z^{(k)} \leq d^{(k)} \), where \( d^{(k)} \) is its thickness.

With this choice of coordinate systems all motions will be independent of the \( y \)-direction and the relevant elastodynamic equations for each layer can be written in the form of a coupled system of equations for the displacements \( u \) and \( w \)

![Layered media model](image.png)

Fig. 1. Layered media model.
Supplemented with the constitutive relations

\[ \sigma_x = (\lambda + 2\mu) \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}, \quad \sigma_z = (\lambda + 2\mu) \frac{\partial w}{\partial z} + \frac{\partial u}{\partial x} \]  

(3, 4)

\[ \sigma_{xz} = \mu \left( \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} \right) \]  

(5)

Due to the absence of viscosity in the fluid (water) its relevant field equations take the form of the corresponding equations (1-5) once \( \mu \) and \( \sigma_z \) are set equal to zero.

Equations (1-5) and those corresponding to the fluid must be supplemented with the appropriate interfacial continuity conditions. For rigid bonding between the individual layers of the plate these are

\[ \phi^{(k)} = \phi^{(k+1)} \quad u^{(k)} = u^{(k+1)}, \quad w^{(k)} = w^{(k+1)} \]  

(6)

at \( z^{(k)} = d^{(k)} \) (or \( z^{(k+1)} = 0 \)), \( k = 1, 2, \ldots, n-1 \).

Similarly, at the lower and upper fluid-plate interfaces, respectively, we have

\[ \phi^{(1)} = 0, \quad \phi^{(1)} = \phi^{(1)} \quad u^{(1)} = u^{(1)}, \quad w^{(1)} = w^{(1)} \]  

at \( z^{(1)} = 0 \) (or \( z = 0 \)).  

(7)

\[ \phi^{(n)} = 0, \quad \phi^{(n)} = \phi^{(n)} \quad u^{(n)} = u^{(n)}, \quad w^{(n)} = w^{(n)} \]  

at \( z^{(n)} = d^{(n)} \) (or \( z = d \)).  

(8)

where superscripts \( u \) and \( 1 \) designate upper and lower fluids, respectively.

In what follows we shall describe the propagation process in the plate by solving the field equations in each of its layers and the fluids and satisfying the interfacial continuity conditions. For waves whose projected wave vector is along the x-axis, equations (1) and (2) admit the formal solutions

\[ (u, w)_k = (U, W)_k e^{iq(x-ct+az)} \]  

(9)

where \( U \) and \( W \) are constant amplitudes, \( q \) is the wave number, \( c \) is the phase velocity and \( \alpha \) is the ratio of the \( z \) and \( x \)-directions wave numbers. By satisfying equations (1) and (2), followed by solving for the four roots of \( \alpha \), using superposition and substituting in the constitutive relations (3)-(5) we obtain an equation relating the displacements and stresses in each layer, say \( k \), to its formal wave amplitudes \( (U_1, U_2, U_3, U_4)_k \). If this resulting equation is specialized to the \( k^{th} \) layer boundaries, \( z^{(k)} = 0 \) and \( z^{(k)} = d^{(k)} \), followed by satisfying the interfacial conditions, we obtain a differential equation in the wave number \( q \) with the formal wave amplitudes as its coefficients.
conditions (6) from one layer to its neighbors we finally relate the field variables at the top of the total plate to its bottom face variables

\[
\begin{bmatrix}
    (n) u \\
    w(n) \\
    - (n) \sigma_z \\
    - (n) \sigma_{xz}
\end{bmatrix} =
\begin{bmatrix}
    A_{11} & A_{12} & A_{13} & A_{14} \\
    A_{21} & A_{22} & A_{23} & A_{24} \\
    A_{31} & A_{32} & A_{33} & A_{34} \\
    A_{41} & A_{42} & A_{43} & A_{44}
\end{bmatrix}
\begin{bmatrix}
    u^{(1)} \\
    w^{(1)} \\
    - (1) \sigma_z \\
    - (1) \sigma_{xz}
\end{bmatrix}
\]

where

\[
[A_{ij}] = [a_{ij}] [a_{ij}]_{n-1} \cdots [a_{ij}]_1
\]

\[
[a_{ij}]_k =
\begin{bmatrix}
    B_1 & B_2 & B_3 & B_4 \\
    \alpha_1 B_1 & - \alpha_1 B_2 & \frac{1}{\alpha_2} B_3 & \frac{1}{\alpha_2} B_4 \\
    D_1 B_1 & D_1 B_2 & D_2 B_3 & D_2 B_4 \\
    D_3 B_1 & - D_3 B_2 & D_4 B_3 & - D_4 B_4
\end{bmatrix}
\begin{bmatrix}
    1 & 1 & 1 & 1 \\
    1 & - \alpha_1 & \frac{1}{\alpha_2} & \frac{1}{\alpha_2} \\
    1 & D_1 & D_2 & D_2 \\
    D_3 & - D_3 & D_4 & - D_4
\end{bmatrix}
\]

\[
B_1 = e^{i \alpha_1 d(k)}, \quad B_2 = \tilde{B}_1, \quad B_3 = e^{i \alpha_2 d(k)}, \quad B_4 = \tilde{B}_3
\]

\[
\alpha_1^2 = \frac{c^2}{c_L^2} - 1, \quad \alpha_2^2 = \frac{c^2}{c_T^2} - 1
\]

\[
D_1 = \mu \left( \frac{c_L^2}{c_T^2} - 2 \right), \quad D_2 = -2 \mu, \quad D_3 = 2 \mu \alpha_1, \quad D_4 = \frac{\mu}{\alpha_2} \left( \frac{c_L^2}{c_T^2} - 2 \right)
\]

\[
\sigma_z = \sigma_z / lq \quad \text{and} \quad \sigma_{xz} = \sigma_{xz} / lq
\]

Invoking the continuity conditions (7) and (8) we finally get the reflection and transmission coefficients

\[
R = \frac{R_{21} - Q R_{11}}{R_{21} + Q R_{11}} \quad ; \quad T = \frac{2 \alpha_f^2 Q}{R_{21} + Q R_{11}}
\]

where

\[
Q = \frac{\rho_f c^2}{\alpha_f}, \quad \alpha_f^2 = \left( \frac{c^2}{c_f^2} \right) - 1
\]

922
SMOOTH INTERFACE CONDITIONS

Now, if a smooth contact interface is introduced within the plate at an arbitrary location, say the interface between layers \( m \) and \( m+1 \) then the previous analysis must be modified. In this sense, we may now consider the plate to be composed of two subplates, the top subplate with \( n-m \) layers and the bottom one with \( m \) layers, where \( 1 < m < n \). The appropriate interfacial conditions for the smooth contact surface are

\[
\begin{align*}
\omega^{(m+1)} = \omega^{(m)}, & \\
\sigma_{z}^{(m+1)} = \sigma_{z}^{(m)}, & \\
\sigma_{xz}^{(m+1)} = \sigma_{xz}^{(m)} = 0, & \text{at } z^{(m)} = d^{(m)}
\end{align*}
\]

(12)

First, we construct the top subplate's characteristic matrix by truncating \( A_{ij} \) starting from the top as

\[
[A_{ij}]_{T} = [a_{ij}]_n [a_{ij}]_{n-1} \ldots [a_{ij}]_{m+1}.
\]

(13)

Next, we construct the bottom subplate's characteristic matrix from the remaining part of \( A_{ij} \) as

\[
[A_{ij}]_{B} = [a_{ij}]_m [a_{ij}]_{m-1} \ldots [a_{ij}]_1.
\]

(14)

Hence, it is clear that \( [A_{ij}] = [A_{ij}]_{T} [A_{ij}]_{B} \). Notice that when the smooth contact surface is at the plate-substrate interface \( [A_{ij}]_{T} \) in (13) becomes \( [A_{ij}] \) and \( [A_{ij}]_{B} \) in (14) becomes \( I_{4 \times 4} \).

By invoking the interface condition (12) on the subplates followed by satisfying the plate-fluid continuity conditions we construct expressions for the reflection and transmission coefficients similar to those of rigid conditions (11a)-(11e) with the new parameter definitions

\[
\begin{align*}
\begin{vmatrix}
M_{11} & M_{12} \\
M_{21} & M_{22}
\end{vmatrix} &= \begin{vmatrix}
T_{11} & T_{12} \\
T_{21} & T_{22}
\end{vmatrix} \begin{vmatrix}
B_{11} & B_{12} \\
B_{21} & B_{22}
\end{vmatrix}
\end{align*}
\]

(15)

where the \( [T_{ij}] \) and \( [B_{ij}] \) matrices are defined as in (11d) and (11e) except that the \( [A_{ij}] \) in (11) are replaced by \( [A_{ij}]_{T} \) and \( [A_{ij}]_{B} \), respectively.

Insight on the problem of nonspecular reflection of finite acoustic beams from fluid-solid interfaces can be gained from an examination of the
reflection coefficient $R$ as a function of angle of incidence and frequency.

NUMERICAL RESULTS

For our material menu we choose steel, copper and epoxy; properties of which are collected below as:

<table>
<thead>
<tr>
<th>Material</th>
<th>$C_L$ (km/sec)</th>
<th>$C_T$ (km/sec)</th>
<th>$\rho$ (g/cm³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel</td>
<td>5.69</td>
<td>3.13</td>
<td>7.9</td>
</tr>
<tr>
<td>Copper</td>
<td>4.76</td>
<td>2.32</td>
<td>8.9</td>
</tr>
<tr>
<td>Epoxy</td>
<td>3.45</td>
<td>1.28</td>
<td>1.25</td>
</tr>
</tbody>
</table>

Without any loss of generality the thickness $d$ of the plate will be kept constant, and the plate's constituents (layers) will be assigned volume fractions adding to unity. Numerical results are presented below in the form of variations of the absolute value of the reflection coefficient with $Fd$ for a given phase velocity (or through Snell's law of angle of incidence) where $F$ is the frequency and $d$ is the thickness measured in MHz and mm, respectively. In all subsequent numerical illustration we choose the phase velocity to be 3 km/sec. (angle of incidence 30°).

In order to assess the influence of smooth interfaces on the behavior of otherwise homogeneous plates we consider the situation where one has two subplates of identical material whose thicknesses sum to $d$. In Figure 2 we compare results for two steel subplates in smooth contact whose volume fractions are given respectively by (.1,.9), (.3,.7), (.4,.6), (.5,.5), and (0.,1.). This is done in order to simulate the influence of smooth interface locations on the propagation characteristics. In Figure 3 results are presented in the form of comparisons for a 1% epoxy layer bonding two steel subplates whose volume fractions are given by (.095,.01,.895); (.495,.01,.495) and (0.,0.,1.). This figure clearly demonstrates not only the dramatic influence of the thin epoxy bond but also the importance of its location within the material. In Figure 4 comparisons are presented for rigid, smooth, and 1% epoxy bonding at the interface of two equal volume fraction copper (top) and steel subplates.

Finally, in Figure 5 we compare rigid bonding to that of diffusion bonding of two equal thickness copper and steel plates. For diffusion bonding we approximate the bonding region by four intermediate layers reflecting linear variation of the individual properties from that of copper to that of the steel. In order to show any appreciative influence results are depicted for a bonding region occupying 60% of the total thickness. By examining Figure 4 we see that diffusion influences the behavior at relatively high frequency ranges where microstructure is expected to play important roles. Diffusion bonding regions of less than 60% obviously lead to results which are closer to those of rigid bonding (i.e. the solid line).
Fig. 2. Comparison of results obtained for a smooth interface located at .1d, .3d, .4d, or .5d from the top of a steel plate with those of the single steel plate.

Fig. 3. Comparison of results for 1% epoxy located at .1d or at .5d from the top of a steel plate with those of the single steel plate.
Fig. 4. Equal thickness copper and steel plate with, ----- rigid bonding; --- smooth bonding; ---- 1% epoxy.

Fig. 5. Equal thickness copper and steel plates (50% each) with, ----- rigid bonding (0% diffusion region) and ---- 60% diffusion region.

REFERENCES