EDDY CURRENT PROBE IMPEDANCES DUE TO
INTERACTION WITH ADVANCED COMPOSITES

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INTRODUCTION

Electromagnetic fields in uniaxial conductors have been studied in order to examine the induced current distributions excited by external alternating current sources. The problem is motivated by the need to evaluate eddy current testing techniques for the inspection of composites such as graphite-epoxy or boron-epoxy. In particular it is important to understand how eddy currents behave in anisotropic materials, especially uniaxial conductors in which the conductivity in the axial direction may be several hundred times greater than the transverse conductivity[1].

In eddy current testing the principle problem is to infer information about the material under test from probe responses such as the impedance changes $\Delta Z$, due to the interaction with induced currents. We have calculated probe interaction impedances for cylindrical air-cored coils, assuming the coil axes are normal to the surface of a homogeneous uniaxial conductor and with the axes assumed to be tangential to the surface. These impedances are found from closed form expressions for both the normal coils and for the tangent coils. Before giving the impedance equations, we shall first briefly examine how the electromagnetic field in the presence of a uniaxial conductor may be calculated.

There are a number of possible approaches that may be used to determine the electromagnetic field. For example, in the formalism used by Roberts and Sabbagh[2], the problem is solved for a uniaxial slab using a four-vector whose components are the components of the electric and magnetic fields tangential to the interfaces. Although only two of the four components are independent, the interface conditions are simply satisfied by requiring that the four-vector is continuous. One can reduce the number of unknown functions in each layer from four to two by analyzing the problem in terms of transverse electric (TE) and transverse magnetic (TM) scalar potentials. However, the scalar decomposition is obviously simpler when there is only
one preferred direction, as in the case where the material axis is normal to the interfaces[3].

Rather than calculating the field in a multilayered structure, we consider a uniaxial conducting half-space and determine the TE and TM potentials for arbitrary current source using Green's functions. The potentials are defined with respect to the interface normal in air and the direction of the material axis in the conductor. Because we are interested in low frequency applications where the wavelength of radiation is large compared with all other finite parameters of the problem, displacement current in air will be neglected.

SCALAR POTENTIALS

Assuming a time dependence $\exp(-i\omega t)$, the electric field in air may be expressed using a TE potential $W'$ and a TM potential $W''$ as[4]

$$E_1(r) = i \omega \mu_0 [\nabla \times \hat{z} W'(r) - \nabla \times \nabla \times \hat{z} W''(r)],$$

(1)

where $\hat{z}$ is a unit vector normal to the plane of the interface ($z = 0$). For the uniaxial region ($z < 0$) the TE potential $\psi'$ and the TM potential $\psi''$ may be defined with respect to the direction of the material axis, the $x$-direction say. The electric field may then be expressed as[5]

$$E_2(r) = i \omega \mu_0 [\nabla \times \hat{z} \psi'(r) - (\nabla \times \nabla \times \hat{z} + \hat{p} \hat{z} \nabla^2) \psi''(r)],$$

(2)

where $\nabla_{zx}$ is the gradient transverse to the $x$-direction and hence $\nabla^2_{zx} = \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$. $\hat{p}$ is a measure of the anisotropy given by

$$p = 1 - \frac{\epsilon}{\epsilon_{xx}},$$

(3)

where $\epsilon$ and $\epsilon_{xx}$ are the components of the complex permittivity tensor having the form

$$\bar{\epsilon} = \bar{I}\epsilon + (\epsilon_{xx} - \epsilon)\hat{z}\hat{z}$$

(4)

for a uniaxial medium, where $\bar{I}$ is a unit tensor. The definitions of potential implied by (2) differ from those commonly used in time-harmonic radiation theory[5] by constant factors since we wish to match the solution in the conductor to a quasi-static solution in air.

Adopting the quasi-static approximation in air does not mean that displacement current is necessarily totally neglected since it may be taken into account in the conductor through the real part of the permittivity tensor. In materials such as graphite-epoxy, the real part of the permittivity is, at least in part, a manifestation of capacitive coupling between fibers. However, because cross-fiber conduction tends to dominate, the effects of the displacement currents within the material are difficult to measure. Although our analysis considers $\bar{\epsilon}$ as complex, it is taken to be purely imaginary in our numerical calculations because the real part is usually unknown in many cases of practical interest and because it is probably small at typical inspection frequencies ($<10MHz$).

A notable consequence of the quasi-static limit is that the normal current (displacement plus charge current) and hence $\hat{z}E_2$ is zero at the interface. This means that the TE and TM potentials of equation (2) are constrained by the relationship
at $z = 0$, and are, therefore, coupled at the surface. Although the two modes combine to give a zero normal electric field at the surface, the TM mode attenuates more rapidly with depth than the TE mode since the latter depends only on the transverse conductivity. Thus below the surface, the combined electric field has a net $z$-component. In contrast the electric field in an isotropic half-space conductor is wholly transverse to the interface for all forms of excitation in air generating a quasi-static field above the conductor. The presence of $z$-directed eddy currents in anisotropic materials has an important practical significance in that they may provide a useful mechanism for the detection of delaminations.

**COIL IMPEDANCE**

The coil impedance $\Delta Z$, due to the presence of the material, is related to the electric field reflected at the surface of the conductor, $E_1 - E_0$, by

$$I^2 \Delta Z = -\int_{\Omega_0} J(r) [E_1(r) - E_0(r)] dr$$

(6)

where $E_0$ is the free-space field due to the current source $J$, $I$ is the coil current and the integral is over the source region $\Omega_0$. Because the TM part of the reflected field is conservative in the quasi-static limit and $J$ has zero divergence, it turns out that the TM term makes no contribution to $\Delta Z$. From the TE part of the field we get

$$I^2 \Delta Z = -i\omega \mu_0 \int_{\Omega_0} j'(r) [W'(r) - W'_0(r)] dr$$

(7)

where $W'_0$ is the free-space TE potential due to $J$, and following an integration by parts, we have defined a scalar source $j'$ given by

$$j'(r) = \hat{z} \nabla \times J(r).$$

(8)

A more convenient expression for $\Delta Z$ is found using an integral form for $W'$ found by solving the field equations using half-space Green's functions. There is not space to develop the full solution here but we shall state the main results needed for efficiently computing impedances. This result will be expressed in terms of the two dimensional Fourier transform, denoted by a tilde and defined by

$$\tilde{W}(u, v) = \iint_{-\infty}^{\infty} W(r)e^{-i(uz + vz)} du dv,$$

(9)

$u$ and $v$ being Fourier space coordinates. The two-dimensional Fourier transform of the TE potential in air may be written in terms of the scalar source defined in (8) as

$$\tilde{W}'(z) = \tilde{W}_0(z) + \int_{0}^{\infty} \tilde{U}(r)(z, z')j'(z') dz'$$

(10)

where

$$\tilde{U}(r)(z, z') = \frac{1}{2\pi^3} \Gamma e^{-s(z + z')}$$

(11)

with $s = (u^2 + v^2)^{1/2}$ and the reflection coefficient, $\Gamma$, given by
\[ \Gamma(u, v) = \frac{\nu^2 k_+^2 - u^2 \gamma'' + \beta \gamma' (\gamma'^2 - \nu^2)}{\nu^2 k_+^2 - u^2 \gamma'' - \beta \gamma' (\gamma'^2 - \nu^2)} \]  

(12)

where \( k_+^2 = \omega^2 \varepsilon_0 \mu_0 \), \( k_-^2 = \omega^2 \varepsilon_{xx} \mu_0 \), \( \gamma' = (u^2 + v^2 - k_+^2)^{1/2} \), \( \gamma'' = (\frac{5\pi}{\varepsilon} u^2 + v^2 - k_-^2)^{1/2} \) and roots with positive real parts are taken.

Transforming (7) using Parseval's theorem and combining the result with (10) gives

\[ I^2 \Delta Z = -i \omega \mu_0 \int_{-\infty}^{\infty} \tilde{j}_e(-) \tilde{\sigma}(h, h) \tilde{j}_e^* \, du \, dv \]

(13)

\( \tilde{j}_e \) is defined as an equivalent surface source at some suitable characteristic height \( h \), above the interface. Thus

\[ \tilde{j}_e^* e^{-sh} = \int_0^\infty e^{-sz} \tilde{j}^*(z) \, dz \]

(14)

with a similar expression used to define \( \tilde{j}_n(-) \) except that the two-dimensional Fourier-transform of \( j'(x, -y, z) \) replaces \( \tilde{j}(z) \) in the \( z \)-integral. In some cases, such as the air-cored normal coil or a cylindrical tangent coil discussed in the next section, one can express \( j'(z) \) in functional form and carry out the \( z \)-integration of (14) analytically[6]. Then the integration with respect to the Fourier-space variables, equation (11), may be further transformed or done numerically as necessary.

RESULTS AND DISCUSSION

\( \tilde{j}_e \) has been evaluated for air-cored normal and tangent coils enabling us to compute the electromagnetic field in the material and the interaction impedances, \( \Delta Z \). A very useful property of the transverse potentials is that field line diagrams can be drawn by plotting contour maps of these functions using data computed in the plane normal to the preferred direction. This means that we can display the TE field lines or the TM field lines in an arbitrary transverse plane. Figure 1 shows field line diagrams in a plane parallel to the \( yz \)-plane intersecting the \( x \)-axis at \( x = 10 \) mm., due to a normal air-cored pancake coil (\( O/D \) 20 mm., \( I/D \) 6.67 mm., axial length 2 mm. and lift-off 0.1 mm.) of the type analysed by Dodd and Deeds for isotropic conductors. The excitation frequency is assumed to be 1 MHz and the conductivities, 100 mhos/m and 20,000 mhos/m, correspond to skin depths of 50 mm. and 3.5 mm. respectively, the smaller value being the transverse conductivity.

The TE mode in the material is characterised by the transverse conductivity and consequently has a much greater penetration depth than the TM mode (Figure 1). In materials such as graphite-epoxy the TE mode is associated with cross-fiber conduction but the induced current of the TM mode meets less resistance since it flows and returns mainly via the fibers, although some cross-fiber conduction is also necessary. The intensity of the TM field is greatest in the plane which is transverse to the material axis and passes through the coil axis; in our coordinate system that is the \( yz \)-plane. The TE mode in this plane on the other hand, is found to have zero intensity for normal coil excitation.

In a uniaxial conductor the \( z \)-directed electric field is zero at the surface (\( z = 0 \)) but the combined effect of the TE and TM fields produces a non-zero \( z \)-component of \( E_2 \) below the surface as shown in Figure 2. This effect is evidently more extensive in the fiber direction for the case of our unidirectional material model.
FLUX LINE DIAGRAMS

Field in a uniaxial conductor due to a normal pancake coil.
(plane of the field shown, x=10mm.)

(a) Transverse Magnetic Field

(b) Transverse Electric Field

Figure 1. (a) TM and (b) TE fields in a uniaxial conductor where the conductivity ratio, $\sigma_{xx}/\sigma$, is 200. The field is excited by a normal pancake coil, 0/D 20mm.

Figure 2. Electric field component $E_z$ in a plane parallel to and 5mm below the surface of a uniaxial conductor. The field is excited by an air-cored pancake coil.
Equation (13) has been used, assuming a homogeneous source region, to compare the normalised impedance--plane diagrams of a square section coil (O/D 60mm., I/D 40mm., axial length 10mm.) with the pancake coil described above. In each case the coil interacts with a uniaxial half-space and the lift-off is taken to be zero. The results, Figure 3, show that one can obtain something approaching ideal coupling using the air-cored pancake coil.

Figure 4 illustrates the variation of normalised impedance with tangent coil orientation and also shows the limited coupling of air-cored tangent-coils.

Figure 3. Computed normalize impedance variation with frequency of (a) a square-section normal coil (b) a pancake coil above a uniaxial conductor. Points correspond to a successive doubling of the frequency up to 1.024 GHz.
Figure 4. Tangent coil impedance variation with frequency. (a) square section coil, O/D 60mm., I/D 40mm., axial length 10mm. $\alpha$ (angle between the coil axis and the material axis) = 0 deg, (b) square section coil, $\alpha = 90$ deg, (c) solenoidal coil, O/D 42mm., I/D 38mm., axial length 20mm., $\alpha = 0$ deg, (d) solenoidal coil, $\alpha = 90$ deg.

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REFERENCES


