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Collect to resell: a profitable strategy for green electronic product manufacturers

Wen Lin
Iowa State University

Collect to resell: a profitable strategy for green electronic product manufacturers

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Wen Lin

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Mervyn Marasinghe

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ABSTRACT

This thesis models and analyzes a closed loop supply chain for electronic products such as cell phones. In this supply chain, there exists an original equipment manufacturer (OEM) who sells new products and collects used products. Also there exists a reseller who procures the used products from the OEM. The reseller refurbishes these used products and then resells them as refurbished products. Besides reselling, the OEM is assumed to have an option to sell the used products to recyclers for recycling. In the thesis, we model this as a two-period Stackelberg game with the OEM as the leader and the reseller as the follower (OEM-Reseller model). We then compare this model with the centrally coordinated model. We solve equilibrium prices, production quantities as well as profits using backwards induction in both OEM-Reseller model and centrally coordinated model. Managerial insights are derived from numerical analysis. We show that under some conditions, OEM's profit from reselling refurbished products exceeds the loss of profit due to reduced new products sales from the competition of refurbished products. That is to say, selling the collected used products to the reseller can be a profitable strategy for the OEM.

CHAPTER 1. INTRODUCTION

By the end of 2006, there are over 233 million cell phones in use in the United States. Assuming the average life time of 18 months the annual discard amount is estimated to be 150 million (INFORM Inc 2008). These end-of-life cell phones contain hazardous materials such as lead and nickel. If buried in the landfill or burnt in the incinerator, they impose great threat to public safety (Bhuie et al. 2004).Some states such as Maine and California have issued environmental laws regarding electronic waste collection and treatment (State of Maine 2007, State of California 2003, 2004) to promote collection of end-of-life electronic products and to ensure they are treated properly after collection. As part of the corporate responsibility (Motorola 2008a), some manufacturers have launched free take back programs to collect used products over the country.

What happens to the used cell phones after they are collected from the customers? According to Motorola, these phones are sorted first. Those that are old, damaged or have little value are sent for recycling. Phones that have value will be refurbished (Motorola 2005). In the recycling process, the used phones are weighed and shredded. The useful materials, such as copper, and precious metals (Gold, Silver) are extracted, refined and eventually sold at the material market. As for the refurbishing process, there are two levels of refurbishing. If the used phones are of high quality and pass call tests, they go through cosmetic repairs and are then repackaged and resold. If they fail the call test, the reseller dissemble them and repair them as long as it is economic before reselling them as “refurbished” phones (Neira et al. 2006).

This thesis models the post-collection treatment in the following way. We assume the presence of an OEM and a reseller. The OEM is defined as a company that produces and sells its own product under its brand name, such as Motorola, LG, and Nokia (State of California SB20, Neira et al. 2004). A reseller is a company that refurbishes and resells used products. An example of the reseller is Recellular Inc.. Particularly, the OEM collects used phones from the market and sells them to the reseller. The reseller is able to refurbish all units purchased from the OEM. The collected phones that are not bought by the reseller will be sent for recycling.

Although many cell phone OEMs claim that they are environment-friendly and provide free take-back services, their post-collection treatment methods vary. For example, Samsung and Nokia recycle the collected cell phones only. Motorola not only recycles but also engages in the reselling business by working with Recellular (Neira et al. 2006).

The reason why cell phone OEMs are hesitant about engaging in reselling business might be their fear of loss of new product sales from the competition of refurbished products (Ferguson and Toktay 2006). Recycling on the other hand, seems to be a safer option for these OEMs because the used products are dissembled and scraped, posing no threat to the new product sales. For example Lexmark, an American printer manufacturer, collects empty printer cartridges by providing a discount on a new cartridge. Instead of remanufacturing them, Lexmark recycles them (Ferguson and Toktay 2006). There appears to be limited decision frameworks to advise OEMs about the economic viability of the post-collection treatment options (Guide and Wassenhove 2001).

We analyze OEM's preference for reselling and recycling quantitatively in this thesis. We hope to understand what would be the optimal post-collection treatment method for the

OEM under different conditions from the market. Specifically, we seek to answer the following questions:

1. Under what condition would the OEM allow reselling refurbished products?

2. How much collected used products would the reseller procure from the OEM? At what price would the OEM charge the reseller for each collected used product?

3. What would the equilibrium market price for new and refurbished products be?

The results of this thesis can be used as decision support model for green electronic product OEMs who wish to comply with electronic waste treatment policies where improper disposal is banned. We find that under some conditions, OEM's profit from reselling refurbished products exceeds the loss of profit due to reduced new products sales from the competition of refurbished products. If the customers value more for refurbished products, the OEM can benefit from charging higher wholesale price for each collected used product and both OEM and reseller would have a higher profit. That is to say, if the market configuration falls under these conditions, selling the collected used products to the reseller is more profitable for the OEM than selling them to recyclers.

The rest of this thesis is organized as follows. In Chapter 2 we show how our model is derived and different from previous literature. Chapter 3 presents the assumptions, notations and derivation of the OEM-Reseller model (OR model). Conditions under which the OEM will allow the reselling of used products are calculated. We solve the equilibrium production quantities and prices under each possible condition. After this we prove the existence and uniqueness of the Stackelberg game in the second period. Then by numerical analysis we discuss some managerial insights. In Chapter 4 we analyze the centrally-coordinated model (CC model), where a central planner sells both new and refurbished products. The OR model is then compared against CC model using numerical examples. Chapter 5 concludes our findings in Chapter 3 and 4, and proposes future research ideas, such as extending the planning horizon to infinite periods, making collection rate as a decision variable, and analyzing government's environmental policies' impact on the supply chain.

CHAPTER 2. LITERATURE REVIEW

Product reuse after its end-of-life has received much research attention in the recent years. There is extensive literature regarding the relationship among manufacturer, retailer, collector, and remanufacturer, analyzing how it influences the retail price, collection rate and channel profit (Majumder and Goenevelt 2001, Savaskan et al 2004, Fergusan and Toktay 2006).

Many papers assume that customers cannot distinguish a new product from a remanufactured product. Typical such products are ink-cartridges and single-use cameras (Savaskan 2004). Majumder and Groenvelt (2001) proposed a model where, although the customers cannot tell if a product is new or remanufactured by the OEM, they value products sold by the OEM more than those sold by the local remanufacturer. Ferguson and Toktay (2006) introduced the price-demand function where the customers value new products more than remanufactured products. This thesis assumes the customers can distinguish a new phone and a refurbished one by having lower willingness-to-pay for refurbished products.

To capture the essence of a finite lifetime, many authors used two-period model (Majumder and Groenevelt 2001, Ferguson and Toktay 2000, Webster and Mitra 2007). For products with relatively mature technology such as single use cameras, some literature models them by assuming a steady production and return flow of products, hence using a steady state model (Savaskan et al. 2004). This thesis uses two-period model to capture the fast obsolescence of electronic products.

Many agents, such as manufacturer, retailer, collector, and remanufacturer are involved in product recovery decisions. Majumder and Groenevelt (2001) looked at the competition between the OEM and a local remanufacturer under an exogenously imposed returned product allocation mechanism. Savaskan et al. (2004), on the other hand, looked at the vertical integration of the recovery supply chain: how would the prices and profits of each player change if different collection strategy is implemented. Bhattacharya et al. (2006) analyzed how decentralized recovery decision making affects the optimal order quantities a retailer orders from the manufacturer. Our model is similar to the Remanufacturer-Separate (REMS) configuration in this paper, where manufacturer and retailer make decisions together, and remanufacture competes with them.

In many literatures, the OEM competes with local remanufacturers (Majumder and Groenevelt 2001). Ferguson and Toktay (2006) showed that the OEM suffers detrimental loss in profit at the presence of an entering remanufacturer. They then proposed and compared two preemptive options for the OEM: 1. OEM start remanufacturering; 2. OEM collect to deter local remanufacturer's acquisition of used products. In this thesis we proposed a new option: the OEM collects used products, and then sells them to the reseller. The OEM benefits from the resale of refurbished products by charging a high wholesale price for the reseller's procurement. We found that this profit may exceed the loss of new product sales under some conditions, rendering reselling as a profitable option for the OEM.

In all the above mentioned literature, Game theory is extensively used. Most of the previous literature that uses Cournot model, where OEM and remanufacturer make decisions based on the expectation of their opponent's best responses simultaneously. This thesis, however, uses a Stackelberg game to analyze the relationship between the cell phone OEM and the reseller. The OEM, having absolute control over the collection of used product, acts as the Stackelberg leader and maximizes profit by observing the reseller's best response function. The OEM first decides on the production quantity and unit wholesale price expecting the best response from the reseller. The reseller then makes its move after observing the OEM's decisions to maximize its profit. This game is subject to non-negativity constraints of both OEM's and reseller's production quantities, and of the inequality constraint that the OEM cannot sell to the reseller more than what has been collected.

To treat electronic wastes safely and economically, many states have passed laws to prevent products containing toxic material going into landfill or being incinerated. California passed Senate Bill 20 (SB20) and Senate Bill 50 (SB50), and requires manufacturers and retailers must provide free product take-back program to customers. Although no federal law has been passed, it is likely that each state would come up with its own electronic waste treatment law in the future. Our thesis provides decision support for electronic product OEMs who wish to comply with these environmental laws.

Cell phone recovery has been studied many times by researchers. Bhuie et al.(2004)'s research provides real life data of the cell phone recycling industry. Guide et al.(2005) published a case study of Recellular, the industry leader of cell phone recovery (Recellular 2008a). Neira et al. (2006) investigated the current cell phone end-of-life management practices in the United States. According to Guide (2005) reseller does not collect directly from end customers, but procures used products from many sources including airtime providers and third party collectors. The reseller then performs testing and economic repair before selling the refurbished cell phones back to the U.S. market or exporting them to developing countries (Bhuie et al 2004). This thesis assumes that the OEM is the only procurement source for the reseller, and hopes to answer how a powerful cell phone manufacturer decides whether allow reselling used phone or not.

This thesis makes contribution to the literature in following ways. First, it extends from the preemptive relationship between manufacturer and reseller, where the manufacturer tries to drive the reseller out of the market in fear of competition (Ferguson and Toktay 2006), to a model where the manufacturer sells collected products to reseller and generates revenue from refurbished phones sales. Second, unlike the common constrained Cournot model (Ferguson and Toktay 2006, Oraiopolos 2007), this thesis analyzed the Stackelberg model with non-negativity constraints and used product supply constraint of the OEM.

CHAPTER 3. OEM-RESELLER MODEL (OR MODEL)

Following the lead by Ferguson and Toktay (2006), we propose an OEM-Reseller model. In our model, instead of trying to drive the competitor out of the market by collecting the used products without remanufacturing, the OEM sells a part of, or all of them to the reseller.

This kind of relationship between two non-cooperative entities in supply chain is rare in the previous remanufacturing literature, yet it exists in reality. Webster and Mitra (2007) discuss such a configuration of supply chain as part of the individual Waste Electrical and Electronic Equipment (WEEE) take-back implementation under the European Union WEEE Directive. Many cell phone OEMs such as Motorola, are collecting used cell phones for free to meet the environmental legislatures, for example SB20 and SB50 in California. After the collection, the OEM will either resell or recycle phones based on the conditions of the market (Motorola 2005). Recellular Inc., the biggest used cell phone reseller in the world (Recellular 2008), acquires these phones from Motorola (Recellular 2007) and service providers such as Verizon (Neira 2006). For the OEM, selling to the reseller, although increases competition with its new product sales, may still be a more profitable way than recycling given the high recycling cost. The objective of our research is to investigate the following questions:

1) Under what conditions will the OEM choose to sell to the reseller?

2) How much would the reseller purchase? How much would the OEM charge the reseller? What would the market price of new and refurbished phones be?

In order to answer these questions, we set up a two period model to analyze the closed loop supply chain.

3.1 Model Assumptions and Notation

This thesis considers a two-period model with two players: the OEM and the reseller. Here OEM is a generalization of the manufacturer and its distribution network, which includes retailers and service providers. We made this generalization because these agents generally act coordinately in the selling of cell phones (Neira 2006). They achieve profit maximization with new product sales only. This is similar to the REMS model in Bhattacharya et al.(2006), where the manufacturer and retailer make decisions as a single player against a remanufacturer.

A reseller on the other hand, acquires collected products from the manufacturer, and then refurbishes and resells these used products. Karakayali et al.(2007) discussed a similar decentralized supply chain consisting of a collector and a remanufacturer. The OEM-Reseller model is illustrated in Figure 1.

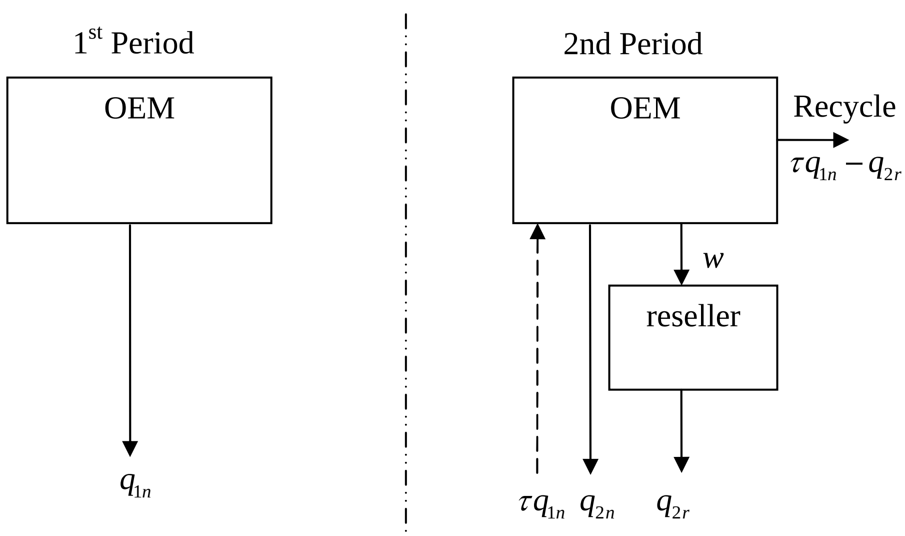


Figure 1 Illustration of the OEM-Reseller model

In the first period, the OEM sells only new products to the market as a monopolist. The OEM decides on the quantity q_{1n} to sell. The market price p_{1n} is given by the inverse demand function. There is no competition in this period and only new products are sold in the market.

At the beginning of the second period the OEM collects τq_{1n} units of used products from the customers. The OEM then sells q_{2r} of these used products to the reseller. The reseller sells them back to the market after refurbishing (Neira et al. 2006). The OEM charges the reseller a unit wholesale price w for each unit procured (Karakayali et al. 2007). In this period, the OEM continues to only sell new products to the market. The collected products that are not sold to the reseller are sold to the recyclers at the price of R per unit.

The OEM competes with the reseller in the second period for profit. The OEM, having the dominating power over the supply chain, acts as the Stackelberg leader and sets its quantity q_{2n} and unit wholesale price w first. The reseller then maximizes its profit taking the OEM's decisions q_{2n} and w, by setting the procurement and resale quantity q_{2r}. Notations used in this thesis are explained below.

q_{1n}: quantity of new products sold by the OEM in the first period, scaled to the magnitude of the normalized market size 1, $0 \le q_{1n} \le 1$ (units);

q_{2n}: quantity of new products sold by the OEM in the second period, scaled to the magnitude of the normalized market size 1, $0 \le q_{2n} \le 1$ (units);

q_{2r}: quantity of refurbished products sold by the reseller in the second period scaled to the magnitude of the normalized market size 1, $0 \le q_{2r} \le 1$ (units);

w: unit wholesale price paid by the reseller to the OEM for each used product procured, scaled to the magnitude of customer's maximum willingness-to-pay ($/unit);

p_{1n}: retail price of new products in the first period, scaled to the magnitude of customer's maximum willingness-to-pay, $0 \le p_{1n} \le 1$ ($/unit);

p_{2n}: retail price of new products in the second period, scaled to the magnitude of customer's maximum willingness-to-pay, $0 \le p_{2n} \le 1$ ($/unit);

p_{2r}: retail price of refurbished products in the second period, scaled to the magnitude of customer's maximum willingness-to-pay, $0 \le p_{2r} \le 1$ ($/unit);

C_n: marginal production cost for each new product incurred by the OEM, scaled to the magnitude of customer's maximum willingness-to-pay, $0 \le C_n \le 1$ ($/unit);

C_r: marginal processing cost for each refurbished product incurred by the reseller, scaled to the magnitude of customer's maximum willingness-to-pay, $0 \le C_r \le 1$ ($/unit);

C_c: collection cost for each used product collected by the OEM, scaled to the magnitude of customer's maximum willingness-to-pay, $0 \le C_c \le 1$ ($/unit);

R: price recyclers pay to the OEM for each used product sent for recycling, scaled to the magnitude of customer's maximum willingness-to-pay. The recyclers decides on the value of R considering the market price of recycled materials and the recycling costs, to maintain a positive profit for each recycled product. R can be both positive and negative. When R is negative the recyclers charge OEM a price for each used product sent for recycling. ($/unit);

φ: customers' scaled willingness-to-pay, $0 \le \varphi \le 1$ ($/unit);

δ : relative willingness-to pay, defined as the ratio of refurbished product value to new product value perceived by the customers, $0 < \delta < 1$;

τ : collection rate, defined as fraction of q_{1n} that was collected at the beginning of second period, $0 \le \tau \le 1$;

Π_{1O}: OEM's profit in the first period;

Π_{2O}: OEM's profit in the second period;

Π_O: OEM's total profit in two periods, $\Pi_O = \Pi_{1O} + \Pi_{2O}$;

Π_R: reseller's profit;

Π_{1C}: central planner's profit in the first period;

Π_{2C}: central planner's profit in the second period;

Π_C: central planner's profit in two periods, $\Pi_C = \Pi_{1C} + \Pi_{2C}$;

$\frac{q_{2r}}{\tau q_{1n}}$: resell rate, defined as the ratio of quantities of refurbished products sold by the reseller to the quantity of used products collected by the OEM, $0 \le \frac{q_{2r}}{\tau q_{1n}} \le 1$;

In order to model the closed loop supply chain, we made the following assumptions.

Assumption 1: The OEM-Reseller model is modeled as a two period game.

In the cell phone industry, when a newly designed product arrives in the market, the OEM enjoys the advantage of monopolist for a limited time. During this time, the customers have only two options: 1. buy a new product; 2. stay inactive. After some time, those who are first to purchase these products would seek to upgrade their gears. Many OEMs provide a discount incentive when customers return their used products at the purchase of a new one to as a promotion tool (Ray et al. 2005). From this point on, used products start to enter the market, giving those who cannot afford a new phone a cheaper option of getting a refurbished one. A two period model was able to capture the finite product life of electronics products without adding extra complexities. The first period models the monopolistic scenario, and second period models the competition between new and refurbished products in the market. Many papers on remanufacturing utilized the two-period assumption, such as Majumda and Groenevelt (2001), Debo et al. (2005), and Furguson and Toktay(2006).

Assumption 2: Useful lifespan of each product is one period. The consumers will re-enter the market after a period of usage of the products. Therefore, the market size is the same for period 1 and period 2.

This is especially true in the cell phone industry in the United States. After the service contract, typically 18 to 24 months, high end customers will seek to purchase a new phone by reentering the market (Neira et al. 2006). This assumption is appropriate if we carefully set the length of the period to be sufficient for the life span of an average cell phone. By assuming a constant market size we are able to derive inverse demand functions.

Assumption 3: Customer willingness-to-pay φ for new products is heterogeneous and uniformly distributed in the interval [0,1].

We made this assumption to capture the distinctive nature of each customer, who has a various income and purchase behavior (Ferguson and Toktay 2006, Ray et al. 2005). Although in real life the consumers' willingness to pay may be of some complicated distribution, uniform distribution captures the essence of diversity in consumers' purchasing behavior and preferences, without adding unnecessary computational complexity. In fact, this has been a classical assumption in many economic literatures.

Because of this assumption, every price and cost parameter is scaled to the magnitude of customer's maximum willingness-to-pay. For example, the marginal production cost of new product C_n should satisfy $0 < C_n < 1$ because the customers will pay at most 1 for a unit of new product, in order for the OEM to profit the cost should be smaller than the price.

Assumption 4: Each customer's willingness-to-pay for a refurbished product is a fraction δ of his/her willingness-to-pay for a new product. 0< δ <1

The customers perceive reduced quality for a refurbished product. These refurbished products are distinctively labeled as "refurbished". The relative willingness-to-pay δ can be viewed as the substitutability of refurbished products to new products. If a customer's willingness-to-pay for a new product is φ, then his/her willingness-to-pay for a refurbished product is $\delta\varphi$. That is to say, customers value refurbished products more when δ is large. When $\delta = 1$, refurbished products become perfect substitute of new products. Unlike products such as single use cameras or ink cartridges studied in many remanufacturing literatures, for instance in Savaskan et al.(2004), the customers can tell the difference between a new cell phone and a refurbished one. Due to possible concerns such as battery performance and style, customers value less for a refurbished cell phone. Under this assumption the new product is a perfect substitute to the refurbished product, while the refurbished one is an imperfect substitute to the new one (Ferguson and Toktay 2006, Debo et al. 2005).

To derive the inverse demand function we assume that each customer buys and keeps only 1 product at a time. Normalizing the market size to 1 we can derive the simple linear inverse demand function:

$$p_{1n}(q_{1n}) = 1 - q_{1n} \tag{3.1}$$

$$p_{2n}(q_{2n},q_{2r}) = 1 - q_{2n} - \delta q_{2r} \tag{3.2}$$

$$p_{2r}(q_{2n},q_{2r}) = \delta(1 - q_{2n} - q_{2r}) \tag{3.3}$$

The derivation of the inverse demand function is given in the Appendix B.

Assumption 5: OEM collects and reseller purchases used products from OEM.

OEM, together with its distribution network, such as service providers and retailers, are able to reach into the market and acquire used cell phones. Cell phone OEMs such as Motorola provides a prepaid envelope addressed to its recycling center to the customers that comes with a new phone (Motorola 2008b).OEMs also host active recycle programs such as Motorola's RaceToRecycle (Motorola 2005) to collect used phones by providing rewards or incentives. Service providers such as Verizon (2008) and retailers such as Bestbuy (2007) provide drop boxes in store and the customers can return their used cell phones at no cost. Resellers, on the other hand, are far less known to the customers compared to the OEM. According to a case study on Recellular, the majority of its procurement comes from the service providers.(Guide et al. 2005) Robotis et al.(2004) argues that the reseller can decide the procurement quantity from various sources. Therefore, we made this assumption that the OEM acts as the collector, and reseller decides how much to purchase from the OEM. Similar assumption has been made by Webster and Mitra(2007).

Assumption 6: OEM sells only new products in both periods.

Although remanufacturing is gradually recognized as a profit generating sector companies such as Caterpillar (Forbes 2005), many manufacturers, especially cell phone OEMs, are concerned with the damage from refurbished products to the new product sales. At the moment, it is not observed that a cell phone OEM directly engage in refurbishing and reselling of used products. There are many possible reasons. For example the refurbishing process is sometimes very different from the manufacturing process, resulting in a high set up cost. Cell phone OEMs need to focus on the core business that guarantees high profit to satisfy the stakeholders. The OEM wouldn't engage in product recovery activities before completely understanding its costs and benefits (Guide and Wassenhove 2001).

Assumption 7: Collected products are in good condition and have resale value. The reseller can refurbish and resell all the products procured from the OEM.

This assumption is justifiable in two folds. First the OEM as the collector can control the quality of collected used products by choosing the collection channel and return incentives. Second, the reseller maintains a quality standard for the procurement from OEM. According to Neira et al. (2006), the procurements from the service providers are usually in good condition. Motorola provides trade-in programs to acquire used phones of good condition with a higher incentive captured in the collection cost in our model (Motorola 2008c). The reseller ensures the procurement from the collectors is of high resale value (Guide and Wassenhove 2001).

Assumption 8: Only the OEM knows the amount of used phones collected at the beginning of the second period. All other variables and parameters are known to both players.

Because of the complexity and randomness of customer's returning behavior it is hard for the reseller to observe the collected quantity by the OEM. Even the OEM cannot predict precisely how much will be collected. We therefore set the collection rate τ as a given parameter instead of as a decision variable. From this assumption the OEM faces the used product supply constraint that it cannot sell to the reseller more than the amount of collected used products. The OEM sets the unit wholesale price w to be high enough to satisfy this inequality constraint. Other variables, such as production quantities in each period are known by both players. OEM knows how much the reseller will sell because reseller procures from the OEM. The reseller on the other hand, can observe OEM's production quantities because the OEM is usually a public company that publishes annual reports. Complete information is assumed in many previous literatures (Ferguson and Toktay 2006).

3.2 Model Derivation

OEM maximizes the total profits in both periods assuming the equilibrium in second period will come to pass. We analyze the model by using backwards induction, starting from the second period.

3.2.1 Second Period

In the second period, this is a Stackelberg game between the OEM and the reseller. In a general two-player Stackelberg game, player 1 (Stackelberg leader) chooses his/her strategy x_1 first. Then player 2 (Stackelberg follower) observes the leader's choice x_1 and chooses his/her strategy x_2. To determine the Stackelberg equilibrium (x_1^*, x_2^*), we first determine player 2's strategy $x_2^*(x_1)$ that maximizes his/her profit for any given x_1 by player 1. Here $x_2^*(x_1)$ is called the best response function. We then determine the player 1's strategy x_1^* that maximizes his profits $\pi(x_1, x_2^*(x_1))$. This method is called backwards induction (Cachon and Netessine 2004).

Specifically, in our model the OEM as the leader first decides on q_{2n} and w. The reseller, as the follower, takes q_{2n} and w as given and decides procurement quantity q_{2r} to maximize reseller's profit.

$$\underset{q_{2r}}{\text{Max}} \quad \Pi_{2R} = (p_{2r}(q_{2n}, q_{2r}) - C_r - w)q_{2r}$$
$$= (\delta(1 - q_{2n} - q_{2r}) - C_r - w)q_{2r} \tag{3.4}$$

$$\text{s.t. } q_{2r} \geq 0 \tag{3.5}$$

The OEM, taking the best response from the reseller $q_{2r}^{*}(q_{2n}, w \mid q_{1n})$, maximizes its profit by setting the production quantity q_{2n} and the unit wholesale price w.

$$\underset{q_{2n},w}{\text{Max}} \quad \Pi_{2O} = (p_{2n}(q_{2n}, q_{2r}) - C_n)q_{2n} - C_c\tau q_{1n} + wq_{2r}^{*} + R(\tau q_{1n} - q_{2r}^{*})$$
$$= (1 - q_{2n} - \delta q_{2r} - C_n)q_{2n} - C_c\tau q_{1n} + wq_{2r}^{*} + R(\tau q_{1n} - q_{2r}^{*}) \tag{3.6}$$

$$\text{s.t. } q_{2r}^{*} \leq \tau q_{1n}, \; q_{2n} \geq 0 \tag{3.7}$$

Here the OEM achieves revenue of $p_{2n}(q_{2n}, q_{2r})$ for each new product sold in the second period. The OEM also invests $C_c\tau q_{1n}$ in collecting used products. The OEM then sells these collected products, receiving wq_{2r}^{*} from the reseller and $R(\tau q_{1n} - q_{2r}^{*})$ from the recyclers. Notice the inequality constraint $q_{2r} \leq \tau q_{1n}$ in (3.7). This is due to the used product supply constraint of the OEM: OEM cannot sell to the reseller more than what has been collected. The OEM sets its price w so that the demand from reseller q_{2r}^{*} does not exceed the amount OEM collected at the beginning of the second period.

Reseller's Move. Let's first derive reseller's best response function. The Lagrangean is

$$L_R = [\delta(1 - q_{2n} - q_{2r}) - C_r - w]q_{2r} + \mu q_{2r} \tag{3.8}$$

First Order Necessary Condition (FONC):

$$\frac{\partial L_R}{\partial q_{2r}} = -C_r - w + \delta - \delta q_{2n} - 2\delta q_{2r} + \mu = 0$$
$$\mu q_{2r} = 0 \qquad (3.9)$$
$$\mu \geq 0$$

The Second Order Sufficient Condition (SOSC):

$$\frac{\partial^2 \Pi_{2R}}{\partial q_{2r}^2} = -2\delta \qquad (3.10)$$

is satisfied because $0 < \delta < 1$, therefore $-2\delta < 0$, indicating the reseller's profit is strictly concave in q_{2r}.

Based on whether the constraint (3.5) is inactive or active, there are two possible cases.

Case 1: q_{2r} >0

Solving FONC(3.9) we have μ =0, and $q_{2r} = -\frac{C_r + w - \delta + \delta q_{2n}}{2\delta}$.

Condition: $C_r + w - \delta + \delta q_{2n} < 0$

Case 2: q_{2r} =0

Solving FONC(3.9) we have q_{2r} =0, $\mu = C_r + w - \delta(1 - q_{2n}) \geq 0$.

Condition: $C_r + w - \delta + \delta q_{2n} \geq 0$

So the reseller's best response function can be written as:

$$q_{2r}^*(q_{2n}, w \mid q_{1n}) = -\frac{C_r + w - \delta + \delta q_{2n}}{2\delta} \qquad (3.11)$$

if $C_r + w - \delta + \delta q_{2n} < 0$ (3.12)

or $q_{2r}^*(q_{2n}, w \mid q_{1n}) = 0$ (3.13)

$$\text{if } C_r + w - \delta + \delta q_{2n} \geq 0 \tag{3.14}$$

OEM's Move. We now proceed to solve for the optimal q_{2n}^* and w^* that maximize OEM's profit in the second period. The Langrangean of OEM is:

$$L_O = (1 - q_{2n} - \delta q_{2r} - C_n)q_{2n} - C_c \tau q_{1n} + w q_{2r}^* + R(\tau q_{1n} - q_{2r}^*) + \lambda(\tau q_{1n} - q_{2r}^*) + \rho q_{2n} \tag{3.15}$$

FONC:

$$\begin{aligned} &\frac{\partial \Pi_{2O}}{\partial q_{2n}} = 0, \frac{\partial \Pi_{2O}}{\partial w} = 0 \\ &\lambda \geq 0, \rho \geq 0 \\ &\rho q_{2n} = 0, \lambda(\tau q_{1n} - q_{2r}^*) = 0 \end{aligned} \tag{3.16}$$

There are 6 possible cases based on the combination of the three inequality constraints (3.5) and (3.7). We now solve for the solution under each case. Notice that $q_{1n} > 0$ in order for our discussion to be meaningful. This is because if $q_{1n} = 0$ then the model becomes such that OEM does not produce in the first period, and therefore no used phones are collected in the second period. This is not optimal because the OEM can make more profit by selling new products in both periods.

Case1: $q_{2r} < \tau q_{1n}, q_{2n} > 0, q_{2r} > 0$

Case2: $q_{2r} < \tau q_{1n}, q_{2n} = 0, q_{2r} > 0$

Case3: $q_{2r} = \tau q_{1n}, q_{2n} = 0, q_{2r} > 0$

Case4: $q_{2r} = \tau q_{1n}, q_{2n} > 0, q_{2r} > 0$

Case5: $q_{2r} < \tau q_{1n}, q_{2n} > 0, q_{2r} = 0$

Case6: $q_{2r} < \tau q_{1n}, q_{2n} = 0, q_{2r} = 0$

When $C_r + w - \delta + \delta q_{2n} < 0$, $q_{2r}^{*}(q_{2n}, w \mid q_{1n}) = -\dfrac{C_r + w - \delta + \delta q_{2n}}{2\delta}$, (3.15) becomes

$$L_O = \frac{1}{2}({q_{2n}}^{2}(-2+\delta) + q_{2n}(2 - 2C_n + C_r + R - \delta + \lambda + 2\rho) + \frac{(C_r + w - \delta)(R - w + \lambda) + 2\tau q_{1n}\delta(R + \lambda - C_c)}{\delta}) \tag{3.17}$$

SOSC:

$$H = \begin{bmatrix} \dfrac{\partial L_O^2}{\partial^2 q_{2n}} & \dfrac{\partial L_O^2}{\partial\, q_{2n} \partial\, w} \\ \dfrac{\partial L_O^2}{\partial\, w \partial\, q_{2n}} & \dfrac{\partial L_O^2}{\partial^2 w} \end{bmatrix} = \begin{bmatrix} -2+\delta & 0 \\ 0 & -\dfrac{1}{\delta} \end{bmatrix} \tag{3.18}$$

Since $0 < \delta < 1$, $-2+\delta$ <0, $-\dfrac{1}{\delta}$ <0, and $\left|H\right| = \dfrac{2-\delta}{\delta} > 0$, the Hessian is negative-definitive. Hence Π_{2O} is strictly concave in q_{2n} and w when $C_r + w - \delta + \delta q_{2n} < 0$.Case1 to Case 4 satisfy this condition (3.12).

Case1: $q_{2r} < \tau q_{1n}, q_{2n} > 0, q_{2r} > 0$

From FONC (16) we have $\lambda = 0, \rho = 0$

$$w^{*} = \frac{1}{2}(\delta + R - C_r) \tag{3.19}$$

$$q_{2n}^{*} = \frac{2 - 2C_n + C_r + R - \delta}{4 - 2\delta} \tag{3.20}$$

Substitute (3.19) and (3.20) into (3.11) we have

$$q_{2r}^{*} = \frac{\delta C_n - R - C_r}{4\delta - 2\delta^2} \tag{3.21}$$

Conditions:

from (3.12), $\frac{C_r + R - \delta C_n}{2-\delta} < 0$,

$$\text{so } R < \delta C_n - C_r \tag{3.22}$$

$$\text{from } q_{2r} < \tau q_{1n},\ \tau q_{1n} > \frac{\delta C_n - R - C_r}{4\delta - 2\delta^2} \tag{3.23}$$

$$\text{from } q_{2n} > 0,\ R > 2C_n + \delta - 2 - C_r \tag{3.24}$$

from (3.22) $q_{2r} > 0$ is satisfied.

In conclusion the conditions for case 1 to happen is

$$\tau q_{1n} > \frac{\delta C_n - R - C_r}{4\delta - 2\delta^2} \text{ and } 2C_n + \delta - 2 - C_r < R < \delta C_n - C_r \tag{3.25}$$

Case2: $q_{2r} < \tau q_{1n}, q_{2n} = 0, q_{2r} > 0$

From FONC (3.16) We have $\lambda = 0$

$$w^* = \frac{1}{2}(\delta + R - C_r) \tag{3.26}$$

from $q_{2n} = 0$ we have $\rho = \frac{1}{2}(-2 + 2C_n - C_r - R + \delta)$

$$q_{2r}^* = \frac{\delta - R - C_r}{4\delta} \tag{3.27}$$

Conditions:

$$\text{From } q_{2r} < \tau q_{1n} \text{ and (27), } \tau q_{1n} > \frac{\delta - R - C_r}{4\delta} \tag{3.28}$$

$$\text{From } q_{2r} > 0\text{, we have } R < \delta - C_r \tag{3.29}$$

$$\text{From } \rho \geq 0,\ R \leq 2C_n + \delta - 2 - C_r \tag{3.30}$$

Condition (3.12) becomes $\frac{1}{2}(C_r + R - \delta) < 0$ is satisfied from (3.29)

Since if (3.30) is satisfied, (3.29) is met, the conditions for case 2 to happen is:

$$\tau q_{1n} > \frac{\delta - R - C_r}{4\delta} \text{ and } R \leq 2C_n + \delta - 2 - C_r \qquad (3.31)$$

Case3: $q_{2r} = \tau q_{1n}, q_{2n} = 0, q_{2r} > 0$

From FONC(3.16),

$$\rho = -1 + C_n + 2\delta\tau q_{1n} \qquad (3.32)$$

$$\lambda = -C_r - R + \delta - 4\delta\tau q_{1n} \qquad (3.33)$$

and

$$w^* = -C_r + \delta - 2\delta\tau q_{1n} \qquad (3.34)$$

$$q_{2n}^* = 0, q_{2r}^* = \tau q_{1n}$$

Conditions:

From $\rho \geq 0$ and (32), $\lambda \geq 0$ and (33) we have

$$\frac{1 - C_n}{2\delta} \leq \tau q_{1n} \leq \frac{\delta - R - C_r}{4\delta} \qquad (3.35)$$

Condition (3.12) becomes $-2\delta\tau q_{1n} < 0$ is satisfied because $q_{1n} > 0$ .In order for (3.35) to hold we must have $R \leq 2C_n + \delta - 2 - C_r$. Therefore, the conditions for case3 to exist is

$$\frac{1 - C_n}{2\delta} \leq \tau q_{1n} \leq \frac{\delta - R - C_r}{4\delta} \text{ and } R \leq 2C_n + \delta - 2 - C_r \qquad (3.36)$$

Case4: $q_{2r} = \tau q_{1n}, q_{2n} > 0, q_{2r} > 0$

From FONC (3.17) we have $\rho = 0, \lambda \geq 0$

$$\lambda = -C_r - R + \delta(C_n - 2(2 - \delta)\tau q_{1n}) \qquad (3.37)$$

$$q_{2n}^{*} = \frac{1}{2}(1 - C_n - 2\delta\tau q_{1n}) \tag{3.38}$$

$$w^{*} = \frac{1}{2}(-2C_r + \delta(1 + C_n - 2(2-\delta)\tau q_{1n})) \tag{3.39}$$

Conditions:

(3.12) is equivalent to $q_{2r} = \tau q_{1n} > 0$ is satisfied.

$$q_{2n}^{*} > 0 \text{, from (3.38) } \tau q_{1n} < \frac{1 - C_n}{2\delta} \tag{3.40}$$

$$\lambda \geq 0 \text{ and (3.37) } \tau q_{1n} \leq \frac{\delta C_n - R - C_r}{4\delta - 2\delta^2} \tag{3.41}$$

from (3.41) we have $\delta C_n - R - C_r > 0$, therefore $R < \delta C_n - C_r$

So (3.40) and (3.41) can be rewritten as

$$\tau q_{1n} < \frac{1 - C_n}{2\delta} \text{ if } R \leq 2C_n + \delta - 2 - C_r$$

$$\tau q_{1n} \leq \frac{\delta C_n - R - C_r}{4\delta - 2\delta^2} \text{ if } 2C_n + \delta - 2 - C_r < R < \delta C_n - C_r \tag{3.42}$$

Now let's consider when condition (3.14) is satisfied, then $q_{2r}^{*}(q_{2n}, w \mid q_{1n}) = 0$ .Only two cases under this category.

Case5: $q_{2r} < \tau q_{1n}, q_{2n} > 0, q_{2r} = 0$

from FONC(3.16) we know $\lambda = 0, \mu_1 = 0$

$$q_{2n}^{*} = \frac{1 - C_n}{2} \tag{3.43}$$

So $q_{2n} > 0$ is always satisfied because $0 < C_n < 1$

$q_{2r} < \tau q_{1n}$ is met too because $q_{2r} = 0$

The only constraint for this case to hold is condition (3.12):

$$w \geq \frac{1}{2}(1+C_n)\delta - C_r \quad (3.44)$$

Case6: $q_{2r} < \tau q_{1n}, q_{2n} = 0, q_{2r} = 0$

From FONC (3.16) $\lambda = 0, \rho \geq 0$

From $q_{2n} = 0$, which is equivalent to $\frac{1}{2}(1 - C_n + \rho) = 0$. This is impossible because C_n<1 and $\rho \geq 0$. Therefore the FONC is not met in this case, this means that this case will not hold.

Notice that, case 1-4 are divided by mutually exclusive boundaries consisting of values of R and τq_{1n}. Case 5, on the other hand, is not bounded by and limits on R and τq_{1n}. That is to say, given any value of R and τq_{1n}, as long as condition(14) is satisfied, the optimal solution would be that of Case5.

OEM needs to compare its maximum profit between that in Case1-4 when condition(3.12) is met, and the profit from Case5 when condition(3.12) is not met. It can be shown that when $R \leq \delta C_n - C_r$, the profit the OEM can get from setting its w to be greater than $1/2(1+C_n)\delta - C_r$, is smaller than that described in each case where condition (3.12) is met. That is to say, only when $R \geq \delta C_n - C_r$ would case 5 be optimal for OEM. To better explain the mutually exclusive boundaries we propose the following proposition.

The results can be shown in the following table.

Table 1. Solution of OR model at second period

Case	q_{2n}^*	q_{2r}^*	w^*	$\Pi_{2O}^*(q_{2n}^*, q_{2r}^* \mid q1n)$	Conditions
1	$\frac{2-2\,Cn+Cr+R-\delta}{4-2\delta}$	$\frac{Cr+R-Cn\,\delta}{-4\delta+2\delta^2}$	$\frac{1}{2}(-Cr+R+\delta)$	$-\frac{1}{4(-2+\delta)\delta}(2(-1+Cn)^2\delta+(Cr+R-\delta)(Cr+R+\delta-2\,Cn\,\delta)+4\,q1n\,(Cc-R)(-2+\delta)\delta\tau)$	$\tau q_{1n} > \frac{\delta C_n - R - C_r}{4\delta - 2\delta^2}$, $2C_n + \delta - 2 - C_r < R < \delta C_n - C_r$
2	0	$-\frac{Cr+R-\delta}{4\delta}$	$\frac{1}{2}(-Cr+R+\delta)$	$\frac{(Cr+R-\delta)^2+8\,q1n\,(-Cc+R)\,\delta\,\tau}{8\delta}$	$\tau q_{1n} > \frac{\delta - R - C_r}{4\delta}$, $R \le 2C_n + \delta - 2 - C_r$
3	0	$q1n\,\tau$	$-Cr+\delta-2\,q1n\,\delta\,\tau$	$-q1n\,\tau\,(Cc+Cr+\delta(-1+2\,q1n\,\tau))$	$\frac{1-C_n}{2\delta} \le \tau q_{1n} \le \frac{\delta - R - C_r}{4\delta}$, $R \le 2C_n + \delta - 2 - C_r$
4	$\frac{1}{2}(1-Cn-2\,q1n\,\delta\,\tau)$	$q1n\,\tau$	$\frac{1}{2}(-2\,Cr+\delta(1+Cn+2\,q1n(-2+\delta)\tau))$	$\frac{1}{4}(-1+Cn)^2-q1n\,(Cc+Cr-Cn\,\delta)\,\tau+q1n^2(-2+\delta)\,\delta\,\tau^2$	$\tau q_{1n} < \frac{1-C_n}{2\delta}$ if $R \le 2C_n + \delta - 2 - C_r$, or $\tau q_{1n} \le \frac{\delta C_n - R - C_r}{4\delta - 2\delta^2}$ if $2C_n + \delta - 2 - C_r < R < \delta C_n - C_r$
5	$\frac{1-Cn}{2}$	0	any $w \ge 1/2(1+C_n)\delta - C_r$	$\frac{1}{4}(1-2\,Cn+Cn^2-4\,Cc\,q1n\,\tau+4\,q1n\,R\,\tau)$	$R \ge \delta C_n - C_r$
6	0	0			Not Optimal

Proposition 1: There are two threshold values of R: lower threshold value $\overline{L} = 2C_n + \delta - 2 - C_r$, and higher threshold value $\overline{H} = \delta C_n - C_r$ .When $R \le \overline{L}$, the OEM might stop making new products in the second period and profit only through selling to the reseller. When $\overline{L} < R < \overline{H}$, both new and refurbished products will be in the market. When $R \ge \overline{H}$, the OEM will recycle all the collected products, thus excluding the reseller from the market. The optimal solutions in second period are shown in Table 2, 3, and 4.

Table 2. Solution when $R \le \overline{L}$ in OR model

Case	τq_{1n}	q_{2n}^*	q_{2r}^*	w^*
4	$\tau q_{1n} < \frac{1-C_n}{2\delta}$	$\frac{1}{2}(1-C_n-2q_{1n}\delta\tau)$	τq_{1n}	$\frac{1}{2}(-2C_r+\delta(1+C_n-2(2-\delta)\tau q_{1n}))$
3	$\frac{1-C_n}{2\delta} \le \tau q_{1n} \le \frac{\delta-R-C_r}{4\delta}$	0	τq_{1n}	$-C_r+\delta-2\delta\tau q_{1n}$
2	$\tau q_{1n} > \frac{\delta-R-C_r}{4\delta}$	0	$-\frac{C_r+R-\delta}{4\delta}$	$\frac{1}{2}(R+\delta-C_r)$

Table 3. Solution when $\overline{L} < R < \overline{H}$ in OR model

Case	τq_{1n}	q_{2n}^*	q_{2r}^*	w^*
4	$\tau q_{1n} \le \frac{\delta-R-C_r}{4\delta}$	$\frac{1}{2}(1-C_n-2q_{1n}\delta\tau)$	τq_{1n}	$\frac{1}{2}(-2C_r+\delta(1+C_n-2(2-\delta)\tau q_{1n}))$
1	$\tau q_{1n} > \frac{\delta-R-C_r}{4\delta}$	$\frac{2-2C_n+C_r+R-\delta}{4-2\delta}$	$\frac{\delta C_n-R-C_r}{4\delta-2\delta^2}$	$\frac{1}{2}(R+\delta-C_r)$

Table 4. Solution when $R \geq \overline{H}$ in OR model

Case	τq_{1n}	q_{2n}^*	q_{2r}^*	w^*
5	any value	$\frac{1}{2}(1-C_n)$	0	any $w \geq 1/2(1+C_n)\delta - C_r$

3.2.2 First Period

Now that we found the two plays' equilibrium in the second period, we are able to take this information to the first stage and solve for the optimal q_{1n}^*.

The OEM tries to maximize its total profit in two periods. Its optimization problem can be formulated as:

$$\underset{q_{1n}}{\text{Max}} \;\; \Pi_{1O} + \Pi_{2O}^*(q_{1n}) = (1-q_{1n}-C_n)q_{1n} + (1-q_{2n}^* - \delta q_{2r}^* - C_n)q_{2n}^* - C_c \tau q_{1n} + w q_{2r}^* + R(\tau q_{2n}^* - q_{2r}^*) \tag{3.45}$$

It is not prevalent in the cell phone industry that, the OEM will stop selling new products because of the competition from refurbished products ($q_{2n} = 0$). In order to study the competition between the OEM and the reseller, we focus on the case when both new and refurbished products are available in the market ($2C_n + \delta - 2 - C_r < R < \delta C_n - C_r$), as depicted in table 3.

It can be observed that there is a threshold value of $\overline{q_{1n}} = \frac{\delta - R - C_r}{4\tau\delta}$. When $q_{1n} \leq \overline{q_{1n}}$, the OEM will sell all the collected products to reseller; When $q_{1n} > \overline{q_{1n}}$, OEM will sell a fraction of the collected products to reseller and recycle the rest of them.

First we need to check that $\Pi^*_{2O}(q_{1n})$ is continuous at the boundary $\overline{q_{1n}}$ .In the

region $q_{1n} \le \overline{q_{1n}}$,

$$\Pi^*_{2O}(\overline{q_{1n}}) = \frac{1}{4(-2+\delta)\delta}(-C_r^2 + R^2 + 2C_nC_r\delta - 2C_c(C_r + R - C_n\delta) + \delta(-2+\delta-2C_n(-2+Cn+\delta))) \tag{3.46}$$

In the region $q_{1n} > \overline{q_{1n}}$, the limit of Π^*_{2O} at $\overline{q_{1n}}$ is

$$\Pi^*_{2O}(\overline{q_{1n}}) = \frac{1}{4(-2+\delta)\delta}(-C_r^2 + R^2 + 2C_nC_r\delta - 2C_c(C_r + R - C_n\delta) + \delta(-2+\delta-2C_n(-2+Cn+\delta))) \tag{3.47}$$

. (3.46) and (3.47) are the identical, so $\Pi^*_{2O}(q_{1n})$ is continuous at $\overline{q_{1n}}$.

When $q_{1n} \le \overline{q_{1n}}$, $\frac{\partial \Pi^*_{2O}}{\partial q_{1n}} = \tau(-C_c - C_r + \delta(C_n + 2q_{1n}(-2+\delta)\tau))$, at $\overline{q_{1n}}$,

$$\frac{\partial \Pi^*_{2O}}{\partial q_{1n}}(\overline{q_{1n}}) = \tau(-C_c - C_r + \delta(C_n + 2\overline{q_{1n}}(-2+\delta)\tau)) = \tau(R - C_c) \tag{3.48}$$

When $q_{1n} > \overline{q_{1n}}$,

$$\frac{\partial \Pi^*_{2O}}{\partial q_{1n}} = \tau(R - C_c) . \tag{3.49}$$

(3.48) is equivalent to (3.49). Therefore, not only is $\Pi^*_{2O}(q_{1n})$ continuous in q_{1n}, it is also differentiable.

Lemma 1. $\Pi^*_{2O}(q_{1n})$ *is concave in* q_{1n}.

Proof: When $q_{1n} \le \overline{q_{1n}}$, $\Pi^*_{2O}(q_{1n}) = \frac{1}{4}(1-C_n)^2 - q_{1n}(C_c + C_r - C_n\delta)\tau + q_{1n}^2(-2+\delta)\delta\tau^2$ is strictly concave in q_{1n} .when $q_{1n} > \overline{q_{1n}}$,

$$\Pi^*_{2O}(q_{1n}) = \frac{1}{4(2-\delta)\delta}(2(1-C_n)^2\delta + (C_r + R - \delta)(C_r + R + \delta - 2C_n\delta) + 4q_{1n}(C_c - R)(-2+\delta)\delta\tau)$$

is linear in q_{1n} . $\frac{\partial^2 \Pi^*_{2O}(q_{1n})}{\partial q_{1n}{}^2} \le 0$ in both regions of q_{1n}, therefore $\Pi^*_{2O}(q_{1n})$ is concave.

Lemma 2: $\Pi_{1O} + \Pi^*_{2O}$ *is strictly concave in* q_{1n}.

Proof: We have proved that Π^*_{2O} is differentiable and concave. Because

$\frac{\partial^2 \Pi_{1O}}{\partial q_{1n}^2} = -2 < 0$, $\frac{\partial^2 \Pi^*_{2O}(q_{1n})}{\partial q_{1n}{}^2} \le 0$, $\frac{\partial^2 (\Pi_{1O} + \Pi^*_{2O}(q_{1n}))}{\partial q_{1n}{}^2} < 0$. So $\Pi_{1O} + \Pi^*_{2O}$ is strictly concave.

Now let's solve the maximization problem of $\Pi_{1O} + \Pi^*_{2O}$ .The slope of $\Pi_{1O} + \Pi^*_{2O}$ at $\overline{q_{1n}}$ is:

$$1 - C_n - C_c\tau + R\tau + \frac{C_r + R - C_n\delta}{2\delta\tau - \delta^2\tau} \tag{3.50}$$

If(3.50) ≤ 0, which is equivalent to

$$R \le (C_r + \delta((-2+\delta)\tau(-1+C_c\tau) + C_n(-1+(-2+\delta)\tau))) / (-1+(-2+\delta)\delta\tau^2) \tag{3.51}$$

then the optimal $q^*_{1n} \le \overline{q_{1n}}$.

Solving $\frac{\partial\ (\Pi_{1O} + \Pi^*_{2O}(q_{1n}))}{\partial q_{1n}} = 0$ we have

$$q^*_{1n} = \frac{-1 + C_n + (C_c + C_r - C_n\delta)\tau}{-2 + 2(-2+\delta)\delta\tau^2} \tag{3.52}$$

and

$$\Pi_{1O}^{*}+\Pi_{2O}^{*}=\frac{1}{(4+8\delta\tau^{2}-4\delta^{2}\tau^{2})}(2-2C_{r}\tau+C_{c}^{2}\tau^{2}+C_{r}^{2}\tau^{2}+2\delta\tau^{2}-\delta^{2}\tau^{2}+2C_{c}\tau(-1+C_{r}\tau)$$
$$+2C_{n}^{2}(1+\delta(-1+\tau)\tau)-2C_{n}(2-\delta\tau+2\delta\tau^{2}-\delta^{2}\tau^{2}+C_{c}\tau(-1+\delta\tau)+C_{r}\tau(-1+\delta\tau)))$$

(3.53)

If $R>(C_{r}+\delta((-2+\delta)\tau(-1+C_{c}\tau)+C_{n}(-1+(-2+\delta)\tau)))/(-1+(-2+\delta)\delta\tau^{2})$

then the optimal $q_{1n}^{*}>\overline{q_{1n}}$. Similarly from $\frac{\partial\ (\Pi_{1O}+\Pi_{2O}^{*}(q_{1n}))}{\partial q_{1n}}=0$ we can find

$$q_{1n}^{*}=\frac{1}{2}(1-C_{n}+(R-C_{c})\tau) \tag{3.54}$$

$$\Pi_{1O}^{*}+\Pi_{2O}^{*}=\frac{1}{4(-2+\delta)\delta}(-C_{r}^{2}-2C_{r}R-R^{2}-4(-1+C_{n})^{2}\delta+2C_{n}C_{r}\delta+2C_{n}R\delta+$$
$$2\delta^{2}-4C_{n}\delta^{2}+C_{n}^{2}\delta^{2}+2(-1+C_{n})(C_{c}-R)(-2+\delta)\delta\tau+(C_{c}-R)^{2}(-2+\delta)\delta\tau^{2})$$

(3.55)

Proposition 2. If $R\leq(C_{r}+\delta((-2+\delta)\tau(-1+C_{c}\tau)+C_{n}(-1+(-2+\delta)\tau)))/(-1+(-2+\delta)\delta\tau^{2})$,

then $q_{1n}^{*}=\frac{-1+C_{n}+(C_{c}+C_{r}-C_{n}\delta)\tau}{-2+2(-2+\delta)\delta\tau^{2}}$. Otherwise $q_{1n}^{*}=\frac{1}{2}(1-C_{n}+(R-C_{c})\tau)$.

Detailed results are made in the form of table in Appendix A.

3.3 Existence and Uniqueness of Stackelberg Equilibrium

As demonstrated in section 3.2, the second period is a Stackelberg game between the OEM and the reseller. We have found the equilibrium under five mutually exclusive conditions in Table1. Now let's prove the existence and uniqueness of this Stackelberg equilibrium.

According to Simaan and Cruz (1973), a sufficient condition for the existence of a Stackelberg equilibrium is the continuous objective functions if the both the leader's and the follower's decision sets are compact. A sufficient condition for uniqueness of Stackelberg equilibrium is the concavity of the leader's profit function (Cachon and Netessine 2004).

3.3.1 Proof of Existence

It is easy to understand why the OEM's decisions q_{2n} and w, and the reseller's decision q_{2r} are compact (i.e. the feasible sets are closed and bounded). For example, the OEM cannot set its production quantity in second period q_{2n} to be less than zero, nor can it set to be too high because over production will decrease the retail price p_{2n} to zero, which leads to zero profit. In fact, in most literature about Stackelberg equilibrium each player's feasible action sets are assumed to be compact. We focus on the continuity of the objective functions of the two players.

First let's look at the reseller's objective function (3.4). This is a continuous function in q_{2r} in the feasible range of $q_{2r} \geq 0$. Then we can look at the OEM's objective function (3.6). There are two forms with different ranges of w, respectively.

Case 1: $C_r + w - \delta + \delta q_{2n} < 0$ (3.56)

Substitute (3.11) in (3.6) we have

$$\Pi_{2O} = \frac{1}{2}(q_{2n}{}^2(-2+\delta) + q_{2n}(2-2C_n + C_r + R - \delta) + \frac{(C_r + w - \delta)(R-w) + 2\tau q_{1n}\delta(R-C_c)}{\delta}) \quad (3.57)$$

Case 2: $C_r + w - \delta + \delta q_{2n} \geq 0$ (3.58)

Substitute (3.13) in (3.6) we have

$$\Pi_{2O} = (1 - q_{2n} - C_n)q_{2n} + (R - C_c)\tau q_{1n} \quad (3.59)$$

Condition (3.56) can be re-written as $w < \delta - \delta q_{2n} - C_r$ and (3.58) as $w \geq \delta - \delta q_{2n} - C_r$. Given and $q_{2n} \geq 0$, there is a boundary value of $\overline{w} = \delta - \delta q_{2n} - C_r$ that if $w < \overline{w}$, the OEM's objective function is (3.57), and if $w \geq \overline{w}$ the objective function is (3.59).

Substitute w with $\overline{w}$ in (3.57) we have:

$$\Pi_{2O} = (1 - q_{2n} - C_n)q_{2n} + (R - C_c)\tau q_{1n} \quad (3.60)$$

(3.60) is identical to (3.59). That is to say, the OEM's objective function Π_{2O} is continuous in w and q_{2n}. Therefore, the objective functions of the follower and of the leader have been proved to be continuous, this proved the existence of the Stackelberg Equilibrium.

3.3.2 Proof of Uniqueness

Now let us look at the uniqueness of the Stackelberg Equilibrium. In Case 1, the Hessian matrix is:

$$H = \begin{bmatrix} \frac{\partial \Pi_{2O}^2}{\partial^2 q_{2n}} & \frac{\partial \Pi_{2O}^2}{\partial\ w \partial\ q_{2n}} \\ \frac{\partial \Pi_{2O}^2}{\partial\ q_{2n} \partial\ w} & \frac{\partial \Pi_{2O}^2}{\partial^2 w} \end{bmatrix} = \begin{bmatrix} -2+\delta & 0 \\ 0 & -\frac{1}{\delta} \end{bmatrix}$$

(3.61)

Since $0 < \delta < 1$, $-2+\delta$ <0, $-\frac{1}{\delta}$ <0, and $|H| = \frac{2-\delta}{\delta} > 0$, the Hessian is negative-definitive. Hence Π_{2O} is strictly concave in q_{2n} and w in this case.

In Case 2, the Hessian matrix becomes:

$$H = \begin{bmatrix} \frac{\partial \Pi_{2O}^2}{\partial^2 q_{2n}} & \frac{\partial \Pi_{2O}^2}{\partial\ w \partial\ q_{2n}} \\ \frac{\partial \Pi_{2O}^2}{\partial\ q_{2n} \partial\ w} & \frac{\partial \Pi_{2O}^2}{\partial^2 w} \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 0 & 0 \end{bmatrix}$$

(3.62)

-2<0, $|H|$=0, the Hessian is semi-negative-definite. Therefore, Π_{2O} is concave in q_{2n} and w in this case.

In conclusion, since the leader's profit Π_{2O} is concave in both cases, the Stackelberg equilibrium is unique.

3.4 Numerical Examples and Sensitivity Analysis

We examine our results by using empirical data. We choose Motorola RAZR V3 as our research object. At MOTO Store a new V3 is sold at $170 (MOTO Store 2008). On Recellular.com a used RAZR V3 is sold at $31 (Recellular 2008b).The 6% collection rate is of the same magnitude to the published 3.32% data in 2006 (Motorola 2008d). We use $300 as the maximum willingness-to-pay by customers. Recall our assumption of customer's willingness-to-pay for new products uniformly distributed between 0 and 1. Values of prices and costs should be scaled down to the magnitude of [0, 1]. Therefore, we divide these values by the maximum willingness-to-pay of $300. In addition to that, we assume the customers' willingness-to-pay for a refurbished product is 30% of that for a new product. The values of the parameters are therefore:

C_n =$60/300=0.2, C_r =$6/300=0.02, C_c =$6/300=0.02, δ =0.3, τ :=0.06, R=$3/300=0.01

From proposition 2， substituting the above values of the parameters yields

p_{2n} =0.6*300=$180, p_{2r} =0.175*300=$52.5, p_{1n} =0.6001*300=$180.1, w=0.148*300=$44.3. This is close to real life data. We chose the price paid by the recyclers for each unit of used product R, and customer's relative willingness to pay δ for the sensitivity analysis because 1.) R directly affects OEM's preferences of resell and recycle, and 2) δ captures the competition between new products and refurbished products.

3.4.1 Variation with R

We use the same set of parameter values: C_n =$60/300=0.2, C_r =$6/300=0.02, C_c =$6/300=0.02, δ =0.3, τ =0.06 and we relax the value of R. From proposition 1 we know when R$\leq$-1.32, the OEM might stop producing new products in the second period. When -1.32<R<0.04, both new and refurbished products exist in the market. When R$\geq$0.04, OEM does not send any collected used products to the reseller, so the reseller is forced to quit the market. Let us focus on the case when both products exist in the market. We increase R from 0 to 0.04 and see how the equilibrium resell rate, $p_{1n,} p_{2n,} p_{2r,} w$ and Π_o, Π_{2r} change accordingly.

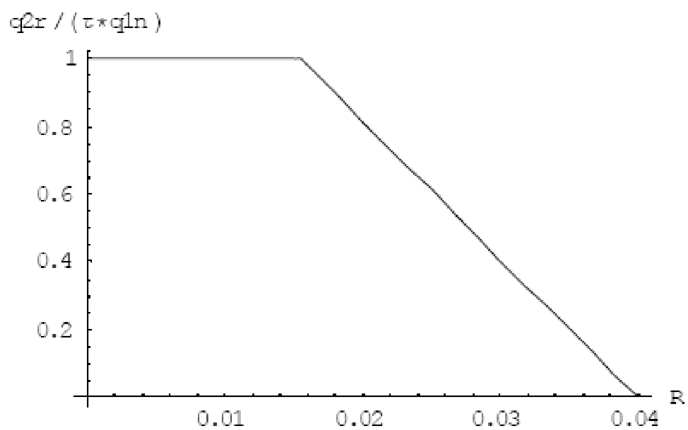


Figure 2 Resell rate with changing R

We can observe a threshold value of $\overline{R}$=0.016.When $R < \overline{R}$ the OEM sells all the collected products to the reseller, i.e. $q_{2r} = \tau q_{1n}$, no products are sent for recycling. When $R \geq \overline{R}$, the OEM sells a fraction of collected products to the reseller, and sells the rest of them to the recyclers. That is to say, when recyclers pay more for per used product, the OEM has the incentive to recycle more to earn profits from recycling the collected products. This results in a decreased resell rate when R increases. If R continues to increase to the boundary value of 0.04, the OEM will sell all the collected products to the recyclers and the resell rate becomes zero.

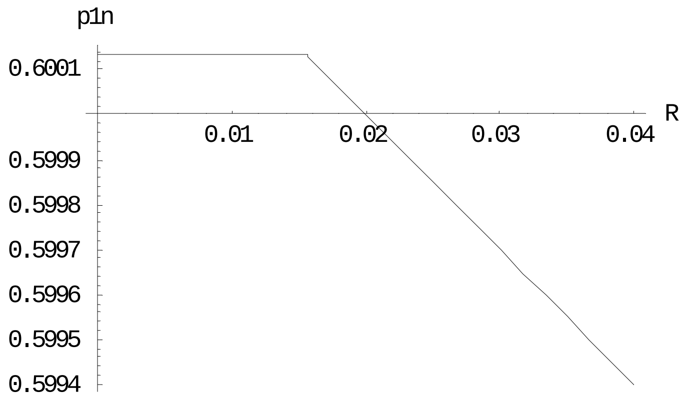


Figure 3 Price of new products in first period with changing R

The price of new products in the first period decreases linearly when R> $\overline{R}$. This is because the OEM acts strategically. As R increases, recycling becomes more attractive. When R> $\overline{R}$, the OEM starts to sell to the recyclers. Note that the OEM still needs to satisfy the reseller's demand for used products. Therefore, the OEM would increase the production quantity in the first period to make profit from recycling in addition to profits from selling to the reseller. As the production quantity increases, the price is decreased in the first period.

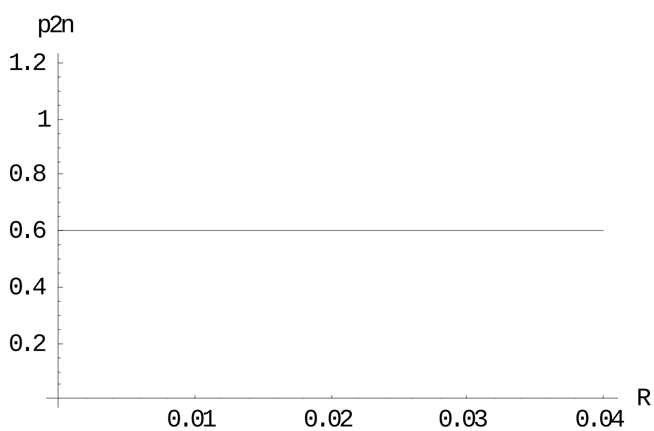


Figure 4 Price of new products in the second period with changing R

It is very interesting to see the price to new products in the second period remains constant with regard to R. When R increases, recycling becomes more attractive to reselling for the OEM. As a result, the new product sales q_{2n} increases yet refurbished product sales q_{2r} decreases. Recall the inverse demand function $p_{2n}(q_{2n}, q_{2r}) = 1 - q_{2n} - \delta q_{2r}$. Increased q_{2n} and decreased q_{2r} keep p_{2n} constant. One possible explanation is that, since OEM knows new phones produced in the second period would no longer be of salvage value when they are returned, the OEM does not have the incentive to sell more to acquire additional value after the second period. Therefore the price to new products in the second period does not change to R.

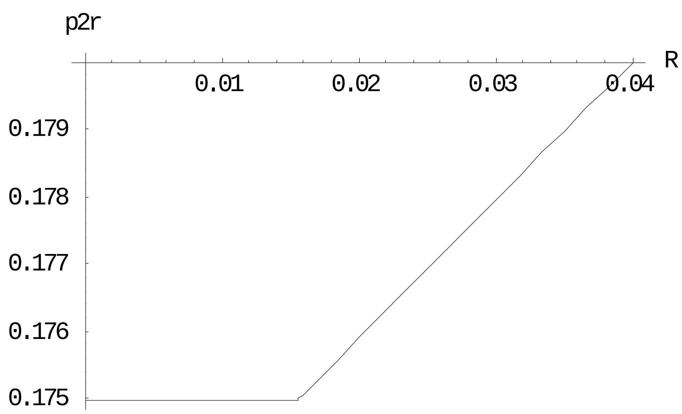


Figure 5 Price of refurbished products in second period with changing R

Compared to p_{2n}, price of refurbished product p_{2r} is directly affected by R When R increases, the OEM will want to sell less to the the reseller by charging a higher unit wholesale price. As the procurement cost of the reseller increase, the price of refurbished products increases as well.

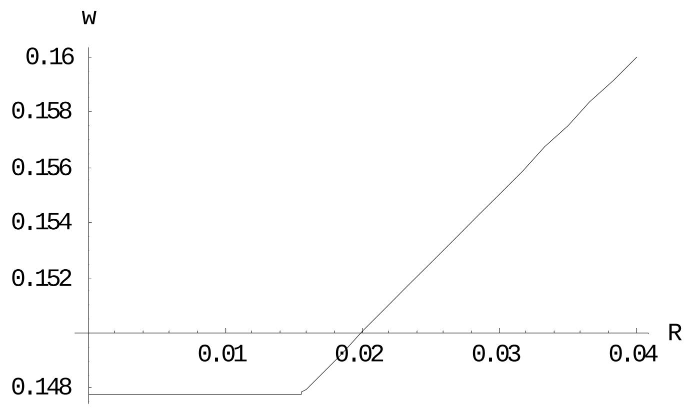


Figure 6 Unit wholesale price w with changing R

This figure illustrates that when R is large, the OEM will increase its unit wholesale price w to deter reseller's participation in the market. Accordingly, the price of refurbished products in the second period will increase as illustrated in Figure 5.

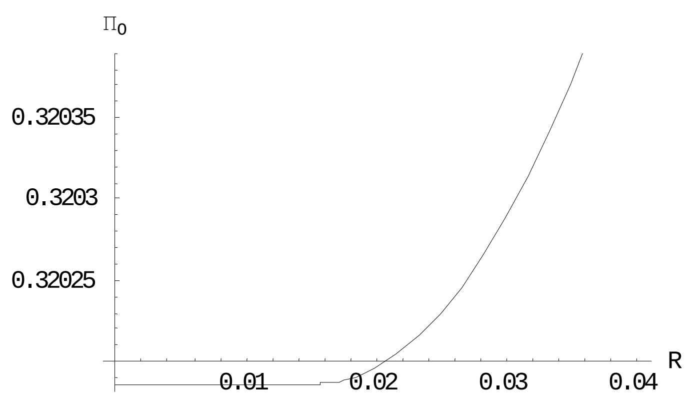


Figure 7 OEM's total profit from two periods with changing R

Now let us consider how OEM's total profit from two periods $\Pi_o = \Pi_{1o} + \Pi_{2o}$ changes when R increases. When R is small such that $R < \overline{R}$ the OEM's total profit stays constant. This is because the price the recyclers offer is not high enough to attract OEM to sell used products to. The OEM's profit from collected used product comes only from the reseller. Therefore the OEM's total profit is irrelevant to R in this case.

When $R \geq \overline{R}$, the OEM starts to profit from selling to recyclers. When R increases the OEM's total profit in two periods Π_o also increases.

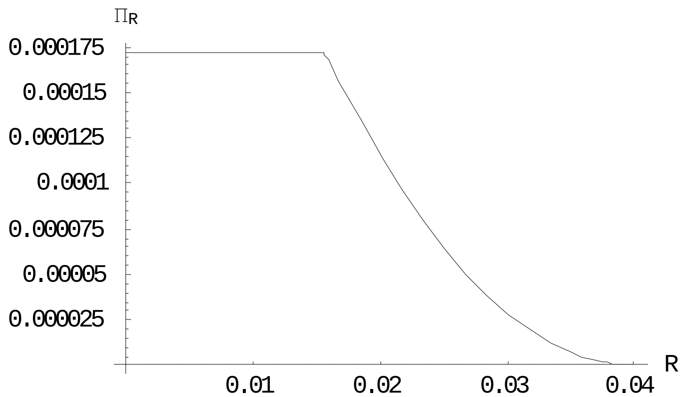


Figure 8 Reseller's profit with changing R

The reseller's profit does not change if R is smaller than the threshold value $\overline{R}$. However as R increases to greater than $\overline{R}$, the OEM will raise the unit wholesale price w to force reseller buy less so as to sell some products to the recyclers, leading to a decreased profit of the reseller. When R reaches $0.04*300=$12 the reseller will not buy anything from the OEM because w is too high, therefore forced to quit the market.

If we compare the values of the reseller's profit and the OEM's profit, we can see the reseller's profit is much less than the OEM's profit. This is because the reseller's profit is subject to 1) the return rate τ, 2) unit wholesale price w, 3) and customers' relative willingness-to-pay δ. Particularly, a large part of the reseller's marginal cost goes to the unit wholesale price w. This is in accordance with Bhuie et al. (2004), where the acquisition cost is significantly larger than the processing cost.

Because the OEM controls the used product supply, a large part of revenue from refurbished products goes to the OEM because of the high unit wholesale price. The reseller might seek other sources of used product supply to reduce the collection cost. Numerous literature deals with closed loop supply chains where both the OEM and the reseller collect used product. We do not elaborate such case in our thesis. Instead, we want to focus on the decisions of the OEM and show how reselling can be a profiting strategy for the OEM after collection.

3.4.2 Variation with δ

Now let's explore how the relative willingness-to-pay δ would affect the profits, prices and production quantity decisions of OEM and reseller. Again, we use the data set: C_n =\$60/300=0.2, C_r =\$6/300=0.02, C_c =\$6/300=0.02, τ =0.06, R=\$3/300=0.01 while changing δ .

We want to focus on the case where new and used products co-exist in the market. From the condition 2 of Proposition 1: $2C_n + \delta - 2 - C_r < R < \delta C_n - C_r$ we can see the condition for both products to exist in market is $\frac{R + C_r}{C_n} < \delta < 2 + C_r + R - 2C_n$.

With the above set of parameter values we know that δ should be from the range of (0.15, 1.63). Note that $0 < \delta < 1$, we perform the sensitivity analysis in $\delta \in (0.15,1)$.

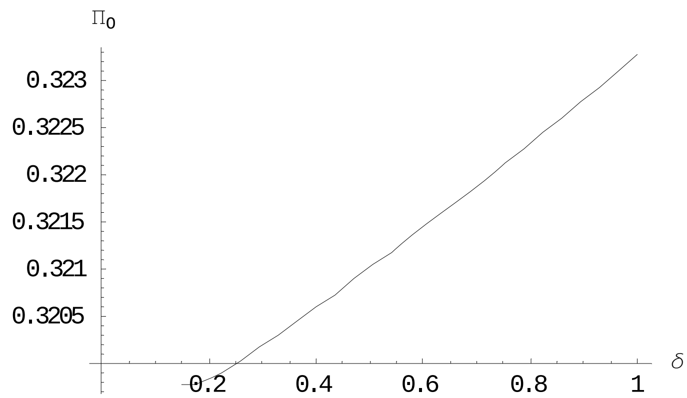


Figure 9 OEM's total profit from two periods with changing δ

Observation 1: The OEM's profit increases when the customers value more for the refurbished products.

This may seem counter intuitive to the conventional belief that when the customers value the refurbished products more, the OEM's profit would decrease. Indeed, because of the competition the OEM's new product sales are reduced. However, in our model, because the OEM sets the unit wholesale price w, the OEM can benefit from customer's higher valuation for refurbished products by charging the reseller more for each used product. The profit from selling collected products is large enough to compensate the loss of new product sales due to competition. That is to say, the OEM still manages to increase profit by allowing both new and refurbished products in the market.

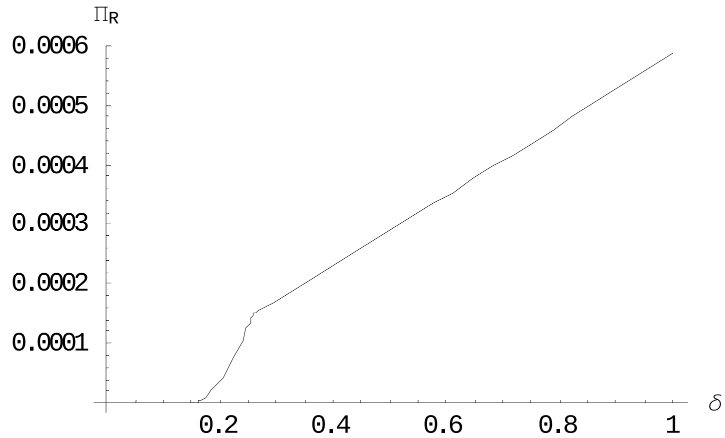


Figure 10 Reseller's profit with changing δ

The reseller's profit increases when δ increases. This is understandable because if the customers are willing to pay more for the refurbished products, the reseller would make more profits by selling at a higher price. We can observe a critical value of $\overline{\delta}$ =0.26. When $\delta < \overline{\delta}$ the reseller's profit increases faster than when $\delta \geq \overline{\delta}$. As demonstrated in Figure 12, when $\delta < \overline{\delta}$ the reseller's sales quantity q_{2r} is not constrained by number of collected used products τq_{1n} . That is to say, when $\delta < \overline{\delta}$, if δ increases, both the price and the demand of the refurbished products would increase. When $\delta \geq \overline{\delta}$, only the price of refurbished products increase, resulting in a slower growth of reseller's profit.

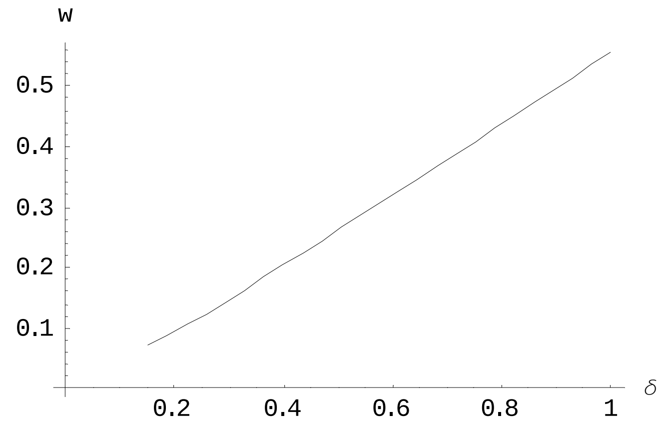


Figure 11 Unit wholesale price w with changing δ

This picture shows how the OEM would charge the reseller more for each collected product when δ increases. The increased profit from w compensates loss in OEM's decreasing new product sales in the second period.

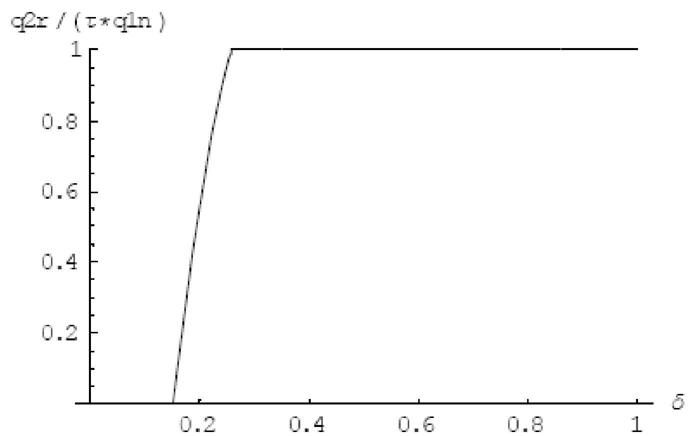


Figure 12 Resell rate with changing δ

We can observe the threshold value of $\overline{\delta}$ =0.26. when $\delta \geq \overline{\delta}$ the resell rate would become 1, meaning the OEM will sell all the collected products to the reseller. This is because the OEM is constantly comparing the margin of:

1) selling to the recyclers.

2) selling to the reseller

When $\delta \geq 0.15$, the OEM can get more margin from selling to the reseller by charging a high w. Therefore the OEM will sell all the collected products to the reseller.

The above results are based on the assumption that the OEM is the only collector of used products in the market. As δ increases, the margin from refurbished product increases. As a result, many individual collectors would try to acquire these used products directly. On seeing this, the OEM must take actions such as increasing the investment in advertising or raising the buyback price from the customers. We do not consider the entry of independent collectors to stay focused on the competition between the powerful OEM and the reseller.

CHAPTER 4. CENTRALLY COORDINATED MODEL (CC MODEL)

In the previous chapter we discussed the OEM-Reseller model where the OEM does not participate in product resale directly, but chooses to sell to the resellers. In some industries, especially copy machine and automobile industries, many companies are actively recovering and selling remanufactured products, For example at Caterpillar, more than $1billion revenue is generated from its remanufactured parts line “Reman” in 2005, and is expected to grow by 15% annually (Forbes 2005). In this chapter we use the centrally coordinated model as a benchmark to evaluate OEM’s performance in the OEM-Reseller model.

4.1 Model Configuration

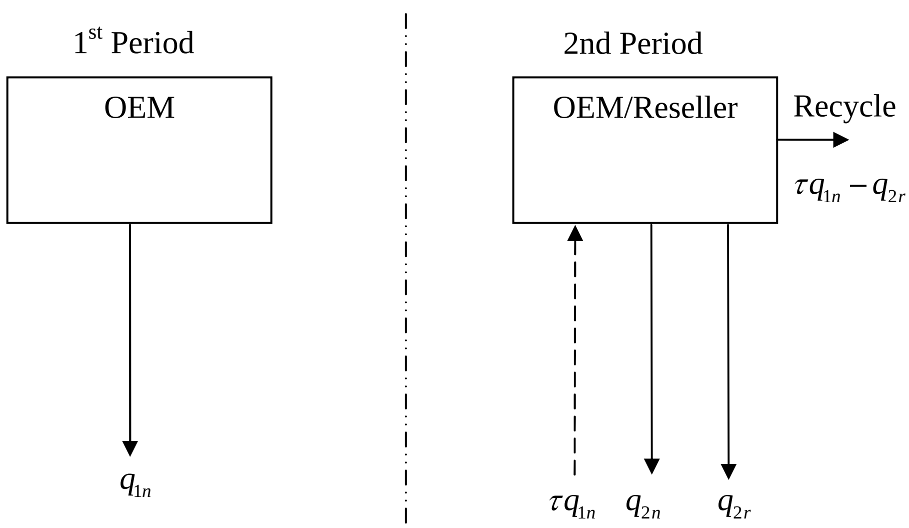


Figure 13 Illustration of the Centrally Coordinated model

The key assumptions and notations in this model are the same as those in the OEM-Reseller model in chapter 3 except OEM and reseller act coordinately in CC model. We use central planner to stand for the coordinated body of OEM and reseller in the following context. In the first period the central planner produces q_{1n} cell phones. In the second period, after the central planner collects τq_{1n} used products, the central planner refurbishes q_{2r} of them, and sells them along side with q_{2n} units of new products in second period.

To further simplify the model, let's first assume the fixed set up cost from initiating the resale program is zero for the central planner. This is because it does not affect any price or quantity decision (Majumder and Groenevelt 2001).

4.2 Model Derivation

Again since this is a sequential optimization, let's use backwards induction to find the central planner's optimal production strategies.

4.2.1 Second Period

The central planner maximizes its profit in the second period by setting the production quantities q_{2n} and q_{2r}:

$$\begin{aligned} &\underset{q_{2n},q_{2r}}{\text{Max}} \quad \Pi_{2C} = (1-q_{2n}-\delta q_{2r}-C_n)q_{2n} + (\delta(1-q_{2n}-q_{2r})-C_r)q_{2r} + R(\tau q_{1n}-q_{2r}) - C_c \tau q_{1n} \\ &\text{s.t.} \quad q_{2n} \geq 0, q_{2r} \geq 0, q_{2r} \leq \tau q_{1n} \end{aligned} \tag{4.1}$$

The Lagrangean is

$$\begin{aligned} L_{2C} &= (1-q_{2n}-\delta q_{2r}-C_n)q_{2n} + (\delta(1-q_{2n}-q_{2r})-C_r)q_{2r} + R(\tau q_{1n}-q_{2r}) - C_c \tau q_{1n} \\ &\quad + \lambda(\tau q_{1n}-q_{2r}) + \mu q_{2r} + \rho q_{2n} \end{aligned} \tag{4.2}$$

SOSC: The Hessian Matrix is

$$H = \begin{bmatrix} \frac{\partial L_{2C}^2}{\partial^2 q_{2n}} & \frac{\partial L_{2C}^2}{\partial\ q_{2n} \partial\ q_{2r}} \\ \frac{\partial L_{2C}^2}{\partial\ q_{2r} \partial\ q_{2n}} & \frac{\partial L_{2C}^2}{\partial^2 q_{2r}} \end{bmatrix} = \begin{bmatrix} -2 & -2\delta \\ -2\delta & -2\delta \end{bmatrix}$$

-2<0, -2δ <0, $|H| = 4\delta(1-\delta)$ >0, therefore Hessian Matrix is negative-definite and the central planner's profit is strictly concave in (q_{2n}, q_{2r}). As a result, the SOSC is satisfied.

FONC:

$$\begin{aligned} &\frac{\partial L_{2C}}{\partial q_{2n}} = 0, \frac{\partial L_{2C}}{\partial q_{2r}} = 0 \\ &\lambda \geq 0, \rho \geq 0, \mu \geq 0 \\ &\rho q_{2n} = 0, \mu q_{2r} = 0, \lambda(\tau q_{1n} - q_{2r}) = 0 \end{aligned} \tag{4.3}$$

There are six possible cases:

Case1: $q_{2r} < \tau q_{1n}, q_{2n} > 0, q_{2r} > 0$

Case2: $q_{2r} < \tau q_{1n}, q_{2n} = 0, q_{2r} > 0$

Case3: $q_{2r} = \tau q_{1n}, q_{2n} = 0, q_{2r} > 0$

Case4: $q_{2r} = \tau q_{1n}, q_{2n} > 0, q_{2r} > 0$

Case5: $q_{2r} < \tau q_{1n}, q_{2n} > 0, q_{2r} = 0$

Case6: $q_{2r} < \tau q_{1n}, q_{2n} = 0, q_{2r} = 0$

Now let's solve for the solution in each case that satisfies FONC respectively.

Case1: $q_{2r} < \tau q_{1n}, q_{2n} > 0, q_{2r} > 0$

From FONC (4.3) we have $\lambda = 0$, $\mu = 0$, and $\rho = 0$. Solving (4.3) yields

$$q_{2n} = \frac{1 - C_n + C_r + R - \delta}{2(1-\delta)} \tag{4.4}$$

$$q_{2r} = \frac{\delta C_n - C_r - R}{2\delta(1-\delta)} \quad (4.5)$$

Conditions:

From $q_{2r} < \tau q_{1n}$,

$$\frac{\delta C_n - C_r - R}{2\delta(1-\delta)} < \tau q_{1n} \quad (4.6)$$

From $q_{2n} > 0$,

$$R > \delta - C_r - (1 - C_n) \quad (4.7)$$

From $q_{2r} > 0$,

$$R < \delta C_n - C_r \quad (4.8)$$

Reorganize (4.6), (4.7), and (4.8) we have the conditions for Case 1 solution to hold is:

$$\frac{\delta C_n - C_r - R}{2\delta(1-\delta)} < \tau q_{1n} \text{ and } \delta - C_r - (1 - C_n) < R < \delta C_n - C_r \quad (4.9)$$

Case2: $q_{2r} < \tau q_{1n}, q_{2n} = 0, q_{2r} > 0$

From FONC (4.3) we have $\lambda = 0, \mu = 0$.

Solving (4.3) yields

$$q_{2r} = \frac{\delta - C_r - R}{2\delta} \quad (4.10)$$

$$\rho = \delta - C_r - R - (1 - C_n) \quad (4.11)$$

From $q_{2r} < \tau q_{1n}$,

$$\frac{\delta - C_r - R}{2\delta} < \tau q_{1n} \quad (4.12)$$

From $q_{2r} > 0$,

$$R < \delta - C_r \tag{4.13}$$

From $\rho \geq 0$,

$$R \leq \delta - C_r - (1 - C_n) \tag{4.14}$$

Since if (4.14) is satisfied, (4.13) is automatically satisfied, the conditions for case 2 solution to hold is:

$$\frac{\delta - C_r - R}{2\delta} < \tau q_{1n} \text{ and } R \leq \delta - C_r - (1 - C_n) \tag{4.15}$$

Case3: $q_{2r} = \tau q_{1n}, q_{2n} = 0, q_{2r} > 0$

From FONC (4.3) we have $\mu = 0$.

Solving (4.3) yields:

$$\lambda = -C_r - R + \delta - 2\delta\tau q_{1n} \tag{4.16}$$

$$\rho = -1 + C_n + 2\delta\tau q_{1n} \tag{4.17}$$

From $\lambda \geq 0$ and (4.16),

$$\frac{\delta - C_r - R}{2\delta} \geq \tau q_{1n} \tag{4.18}$$

From $\rho \geq 0$ and (4.17),

$$\tau q_{1n} \geq \frac{1 - C_n}{2\delta} \tag{4.19}$$

In order for (4.18) and (4.19) both hold, $\frac{\delta - C_r - R}{2\delta} \geq \frac{1 - C_n}{2\delta}$, equivalent to

$$R \leq \delta - C_r - (1 - C_n) \tag{4.20}$$

Therefore the conditions for case 3 solution to hold are

$$\frac{\delta - C_r - R}{2\delta} \geq \tau q_{1n} \geq \frac{1 - C_n}{2\delta} \text{ and } R \leq \delta - C_r - (1 - C_n) \qquad (4.21)$$

Case4: $q_{2r} = \tau q_{1n}, q_{2n} > 0, q_{2r} > 0$

From FONC (4.3) we have $\mu = 0, \rho = 0$

Solving (4.3) yields

$$q_{2n} = \frac{1 - C_n}{2} - \delta \tau q_{1n} \qquad (4.22)$$

$$\lambda = -C_r - R + \delta(C_n - 2\tau q_{1n}(1 - \delta)) \qquad (4.23)$$

From $\lambda \geq 0$,

$$\frac{\delta C_n - C_r - R}{2\delta(1 - \delta)} \geq \tau q_{1n} \qquad (4.24)$$

From $q_{2n} > 0$,

$$\frac{1 - C_n}{2\delta} > \tau q_{1n} \qquad (4.25)$$

For (4.24) to hold,

$$\delta C_n - C_r > R \qquad (4.26)$$

Therefore the conditions for Case 4 solution to hold are:

$$\frac{1 - C_n}{2\delta} > \tau q_{1n} \text{ if } R \leq \delta - C_r - (1 - C_n) \text{ or,} \qquad (4.27)$$

$$\frac{\delta C_n - C_r - R}{2\delta(1 - \delta)} \geq \tau q_{1n} \text{ if } \delta C_n - C_r > R > \delta - C_r - (1 - C_n) \qquad (4.28)$$

Case5: $q_{2r} < \tau q_{1n}, q_{2n} > 0, q_{2r} = 0$

From FONC (4.3) $\lambda = 0, \rho = 0$.

Solving (4.3) yields

$$q_{2n} = \frac{1-C_n}{2} \tag{4.29}$$

$$\mu = C_r + R - \delta C_n \tag{4.30}$$

Since $q_{2r} = 0$ and $q_{1n} > 0, q_{2r} < \tau q_{1n}$ is satisfied.

$q_{2n} > 0$ is also satisfied because $0 < C_n < 1$

From $\mu \geq 0$ and (4.30),

$$\delta C_n - C_r \leq R \tag{4.31}$$

So the constraint for Case5 solution to hold is (4.31).

Case6: $q_{2r} < \tau q_{1n}, q_{2n} = 0, q_{2r} = 0$

From FONC (4.3), $\lambda = 0$

Solving (4.3) yields

$$\rho = -1 + C_n \tag{4.32}$$

$$\mu = C_r + R - \delta \tag{4.33}$$

$\rho \geq 0$ is violated because $C_n < 1$.

That is to say, Case6 will never hold.

The results can be shown in the following table.

Table 5. Solution of CC model at second period

Case	q_{2n}^*	q_{2r}^*	$\Pi_{2C}^* \mid q_{1n}$	Conditions
1	$\frac{1-C_n+C_r+R-\delta}{2(1-\delta)}$	$\frac{\delta C_n-C_r-R}{2\delta(1-\delta)}$	$-\frac{1}{4(-1+\delta)\delta}((-1+Cn)^2\delta+(Cr+R-\delta)(Cr+R+\delta-2Cn\delta)+4q1n(Cc-R)(-1+\delta)\delta\tau)$	$\frac{\delta C_n-C_r-R}{2\delta(1-\delta)}<\tau q_{1n}$, $\delta-C_r-(1-C_n)<R<\delta C_n-C_r$
2	0	$\frac{\delta-C_r-R}{2\delta}$	$\frac{(Cr+R-\delta)^2+4q1n(-Cc+R)\delta\tau}{4\delta}$	$\frac{\delta-C_r-R}{2\delta}<\tau q_{1n}$, $R\le\delta-C_r-(1-C_n)$
3	0	τq_{1n}	$-q1n\,\tau(Cc+Cr+\delta(-1+q1n\,\tau))$	$\frac{\delta-C_r-R}{2\delta}\ge\tau q_{1n}\ge\frac{1-C_n}{2\delta}$, $R\le\delta-C_r-(1-C_n)$
4	$\frac{1-C_n}{2}-\delta\tau q_{1n}$	τq_{1n}	$\frac{1}{4}(-1+Cn)^2-q1n(Cc+Cr-Cn\delta)\tau+q1n^2(-1+\delta)\delta\tau^2$	$\frac{1-C_n}{2\delta}>\tau q_{1n}$ if $R\le\delta-C_r-(1-C_n)$, or $\tau q_{1n}\le\frac{\delta C_n-C_r-R}{2\delta(1-\delta)}$ if $\delta-C_r-(1-C_n)<R<\delta C_n-C_r$
5	$\frac{1-Cn}{2}$	0	$\frac{1}{4}(-1+Cn)^2-q1n(Cc-R)\tau$	$\delta C_n-C_r\le R$
6	0	0		No optimal solution

Proposition 3: There are two threshold values of R: lower threshold value $\overline{L} = \delta - C_r - (1 - C_n)$, and higher threshold value $\overline{H} = \delta C_n - C_r$. When $R \le \overline{L}$, the central planner might stop making new products in the second period and profit only through reselling. When $\overline{L} < R < \overline{H}$, both new and refurbished products will be in the market. When $R \ge \overline{H}$, the central planner will recycle all the collected products. The optimal solutions in second period are shown in Table 6, 7, and 8.

Table 6. Solution when $R \le \overline{L}$ in CC model

τq_{1n}	q_{2n}^*	q_{2r}^*
$\tau q_{1n} < \frac{1 - C_n}{2\delta}$	$\frac{1 - C_n}{2} - \delta \tau q_{1n}$	τq_{1n}
$\frac{1 - C_n}{2\delta} \le \tau q_{1n} \le \frac{\delta - C_r - R}{2\delta}$	0	τq_{1n}
$\frac{\delta - C_r - R}{2\delta} < \tau q_{1n}$	0	$\frac{\delta - C_r - R}{2\delta}$

Table 7. Solution when $\overline{L} < R < \overline{H}$ in CC model

τq_{1n}	q_{2n}^*	q_{2r}^*
$\tau q_{1n} \le \frac{\delta C_n - C_r - R}{2\delta(1 - \delta)}$	$\frac{1 - C_n}{2} - \delta \tau q_{1n}$	τq_{1n}
$\frac{\delta C_n - C_r - R}{2\delta(1 - \delta)} < \tau q_{1n}$	$\frac{1 - C_n + C_r + R - \delta}{2(1 - \delta)}$	$\frac{\delta C_n - C_r - R}{2\delta(1 - \delta)}$

Table 8. Solution when $R \geq \overline{H}$ in CC model

τq_{1n}	q_{2n}^*	q_{2r}^*
any given value	$\frac{1 - Cn}{2}$	0

In practice, it is common for the company to produce both new and remanufactured products, as in the case with Caterpillar. Henceforth we focus on the case when $L < R < H$ in the first period derivation to solve for the optimal production quantity q_{1n} .

4.2.2 First Period

The central planner tries to maximize its total profit in two periods. Its optimization problem can be formulated as:

$$\begin{aligned}\underset{q_{1n}}{\text{Max}}\ \ \Pi_{1C} + \Pi_{2C}^*(q_{1n}) &= (1 - q_{1n} - C_n)q_{1n} + (1 - q_{2n}^* - \delta q_{2r}^* - C_n)q_{2n}^* \\ &+ (\delta(1 - q_{2n}^* - q_{2r}^*) - C_r)q_{2r}^* - C_c\tau q_{1n} + R(\tau q_{2n}^* - q_{2r}^*)\end{aligned} \tag{4.34}$$

Recall $\delta - C_r - (1 - C_n) < R < \delta C_n - C_r$. There are two regions of τq_{1n} :

Region 1: $\tau q_{1n} \leq \dfrac{\delta C_n - C_r - R}{2\delta(1-\delta)}$

$$\Pi_{2C}^*(q_{1n}) = \frac{(1 - C_n)^2}{4} - \tau q_{1n}(C_c + C_r - \delta C_n) - \delta(1-\delta)(\tau q_{1n})^2 \tag{4.35}$$

Region 2: $\dfrac{\delta C_n - C_r - R}{2\delta(1-\delta)} < \tau q_{1n}$

$$\begin{aligned}\Pi_{2C}^*(q_{1n}) = \frac{1}{4\delta(1-\delta)}&((1 - C_n)^2\delta + (C_r + R - \delta)(C_r + R + \delta - 2\delta C_n) \\ &+ 4\delta\tau q_{1n}(1-\delta)(R - C_c)\end{aligned} \tag{4.36}$$

First we need to check that $\Pi^*_{2C}(q_{1n})$ is continuous at the boundary

$\overline{q_{1n}} = \dfrac{\delta C_n - C_r - R}{2\tau\delta(1-\delta)}$.

In region 1 when $q_{1n} = \overline{q_{1n}}$,

$$\begin{aligned}\Pi^*_{2C}(\overline{q_{1n}}) = {} & (-R^2 + (1-C_n)^2\delta + (C_r - \delta)(C_r + \delta - 2\delta C_n) \\ & +2\mathrm{C}_c(C_r + R - \delta C_n)) / (4(1-\delta)\delta)\end{aligned} \tag{4.37}$$

In region 2 when q_{1n} approximates $\overline{q_{1n}}$,

$$\begin{aligned}\Pi^*_{2C}(\overline{q_{1n}}) = {} & (-R^2 + (1-C_n)^2\delta + (C_r - \delta)(C_r + \delta - 2\delta C_n) \\ & +2\mathrm{C}_c(C_r + R - \delta C_n)) / (4(1-\delta)\delta)\end{aligned} \tag{4.38}$$

(4.37) is equivalent to (4.38), therefore $\Pi^*_{2C}(q_{1n})$ is continuous at $\overline{q_{1n}}$.

We then need to check if differentiable at $\overline{q_{1n}}$:

In region 1 $\dfrac{\partial \Pi^*_{2C}}{\partial q_{1n}} = \tau(-C_c - C_r + \delta(C_n - 2(1-\delta)\tau q_{1n}))$

$$\text{when } q_{1n} = \overline{q_{1n}},\ \frac{\partial \Pi^*_{2C}}{\partial q_{1n}} = (R - C_c)\tau \tag{4.39}$$

In region 2,

$$\frac{\partial \Pi^*_{2C}}{\partial q_{1n}} = (R - C_c)\tau \tag{4.40}$$

(4.39) is equal to (4.40). So $\Pi^*_{2C}(q_{1n})$ is continuous and differentiable in both regions.

Lemma 3. $\Pi^*_{2C}(q_{1n})$ *is concave in* q_{1n}.

Proof: When $q_{1n} \le \overline{q_{1n}}$, (4.35) is strictly concave in q_{1n} .when $q_{1n} > \overline{q_{1n}}$, (4.36) is linear in q_{1n} . $\frac{\partial^2 \Pi^*_{2C}(q_{1n})}{\partial q_{1n}{}^2} \le 0$ in both regions of q_{1n} , therefore $\Pi^*_{2C}(q_{1n})$ is concave.

Lemma 4: $\Pi_{1C} + \Pi^*_{2C}$ *is strictly concave in* q_{1n} .

Proof: We have proved that Π^*_{2C} is differentiable and concave.

$\frac{\partial^2 \Pi_{1C}}{\partial q_{1n}^2} = -2 < 0$, $\frac{\partial^2 \Pi^*_{2C}(q_{1n})}{\partial q_{1n}{}^2} \le 0$, $\frac{\partial^2 (\Pi_{1C} + \Pi^*_{2C}(q_{1n}))}{\partial q_{1n}{}^2} < 0$. So $\Pi_{1C} + \Pi^*_{2C}$ is strictly concave.

Now let's solve the maximization problem of $\Pi_{1C} + \Pi^*_{2C}$ .The slope of $\Pi_{1C} + \Pi^*_{2C}$ at $\overline{q_{1n}}$ is:

$$\frac{\partial\ \Pi_{1C} + \Pi_{2C}}{\partial q_{1n}} = 1 - C_n - C_c \tau + R\tau + \frac{C_r + R - \delta C_n}{\delta\tau(1-\delta)} \tag{4.41}$$

if (4.41) $\le$ 0, i.e.

$$R \le \frac{C_r + \delta((-1+\delta)\tau(-1+C_c\tau) + C_n(-1+(-1+\delta)\tau))}{-1+(-1+\delta)\delta\tau^2} \tag{4.42}$$

Optimal production quantity q^*_{1n} falls in region 1.

Solving FONC: $\frac{\partial\ \Pi_{1C} + \Pi_{2C}}{\partial q_{1n}} = 0$ yields

$$q^*_{1n} = \frac{-1 + C_n + (C_c + C_r - C_n\delta)\tau}{-2 + 2(-1+\delta)\delta\tau^2} \tag{4.43}$$

The corresponding total profit of the central planner in two periods $\Pi_C = \Pi_{1C} + \Pi_{2C} =$

$$\frac{1}{(4+4(1-\delta)\delta\tau^2)}(2(-1+C_n)^2 + 2(-1+C_n)(C_c + C_r - C_n\delta)\tau + ((C_c + C_r)^2 + (1 + C_n^2 - 2C_n(1 + C_c + C_r))\delta + (-1 + 2C_n)\delta^2)\tau^2) \tag{4.44}$$

if (4.41)> 0, i.e.

$$R > \frac{C_r + \delta((-1+\delta)\tau(-1+C_c\tau) + C_n(-1+(-1+\delta)\tau))}{-1+(-1+\delta)\delta\tau^2} \tag{4.45}$$

Optimal production quantity q_{1n}^* falls in region 2.

Solving FONC: $\frac{\partial\ \Pi_{1C} + \Pi_{2C}}{\partial q_{1n}} = 0$ yields

$$q_{1n}^* = \frac{1}{2}(1 - C_n - C_c\tau + R\tau) \tag{4.46}$$

The corresponding total profit of the central planner in two periods $\Pi_C = \Pi_{1C} + \Pi_{2C} =$

$$\frac{1}{(4(-1+\delta)\delta)}(-C_r^2 - 2C_rR - R^2 - 2(-1+C_n)^2\delta + 2C_nC_r\delta + 2C_nR\delta + 2\delta^2 - 4C_n\delta^2 + C_n^2\delta^2 + 2(-1+C_n)(C_c - R)(-1+\delta)\delta\tau + (C_c - R)^2(-1+\delta)\delta\tau^2) \tag{4.47}$$

Proposition 4. If $R \le \frac{C_r + \delta((-1+\delta)\tau(-1+C_c\tau) + C_n(-1+(-1+\delta)\tau))}{-1+(-1+\delta)\delta\tau^2}$, then

$q_{1n}^* = \frac{-1 + C_n + (C_c + C_r - C_n\delta)\tau}{-2 + 2(-1+\delta)\delta\tau^2}$. Otherwise $q_{1n}^* = \frac{1}{2}(1 - C_n - C_c\tau + R\tau)$

4.3 Comparison between OR Model and CC Model

Using CC model as a benchmark we would be able to analyze the performance of the OR model. Again we want to focus on the condition where both new products and refurbished product exist. Therefore, we use the data set C_n =\$60/300=0.2, C_r =\$6/300=0.02, C_c =\$6/300=0.02, δ =0.3, τ =0.06, and we increase R from 0 to 0.04.

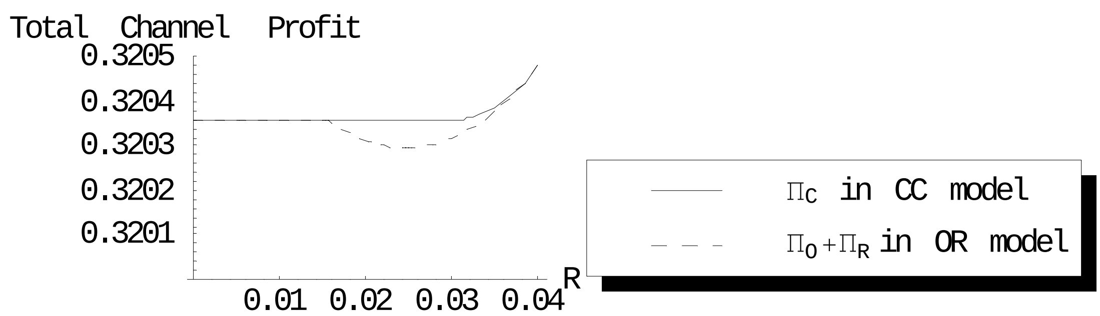


Figure 14 Comparison of total channel profit

Figure 14 shows the total channel profit's change with increasing R. The dashed line represents the sum of OEM's total profit in two periods Π_O and reseller's profit Π_R in OR model. The solid line represents the central planner's total profit in both periods in CC model. We can observe two threshold values of R: $\overline{R_{OR}}$ =0.016 and $\overline{R_{CC}}$ =0.030. When $R \leq \overline{R_{OR}}$, all the collected used products are sent for reselling in both CC model and OR model. Because none of the collected used products are sent for recycling, the profit of both players is irrelevant with R in this region. When R is between $\overline{R_{OR}}$ and $\overline{R_{CC}}$, the OEM sends some of the collected used products for recycling in OR model, yet still send all of them for reselling in CC model. If R is large so that $\overline{R_{CC}} < R \leq 0.04$, the OEM will sent some used products for recycling in both CC model and OR model. Fewer products are sent for reselling when R increases. If R continues to increase beyond 0.04, the OEM will not send any collected used products for reselling, and recycle all of them in both OR model and CC model.

In figure 14 the total channel profit in OR model appears to be the same as OEM's profit in CC model if $R \le \overline{R_{OR}}$. However, it can be calculated that

$$\Pi_C - \Pi_O - \Pi_R = \frac{\delta^2 \tau^4 [1 - C_n + (\delta C_n - C_c - C_r)\tau]^2}{4[1 + (2-\delta)\delta\tau^2]^2 [1 + (1-\delta)\delta\tau^2]} > 0$$

So when $R \le \overline{R_{OR}}$, the total channel profit in CC model is higher than that in OR model. The difference is very small given the current set of parameter values we use (to the order of 10^{-7}), therefore from the figure the two curves for total channel profit seems to be converging.

When $\overline{R_{OR}} < R \le 0.04$, the total profit of OEM and Reseller in OR model is less than OEM's profit in CC model. When R reaches 0.04 OR model converge with CC model because reseller would be excluded from the market, leaving only OEM selling new products.

Since the CC model usually yields the highest possible channel profit, we may use it as a benchmark to judge the efficiency of the OR model. The closer the total channel profit in OR model is to the CC model, the higher efficiency it is. From Figure 14 we can see that the difference between OR model and CC model is very small when the OEM sells all the products to the resellers in both models ($R < \overline{R_{OR}}$). As the value of R increases, difference between the two models first increases, the decreases. When R reaches 0.04, the OEM sells all the used products to the recyclers. Because the reseller is excluded from the market, OR model and CC model is the same when $R \ge 0.04$.

Note the value of the total channel difference is small. This is because the OEM is so powerful that it can control the decision of the reseller. Most of the channel profit goes to the OEM in the OR model. Therefore the efficiency of the OR model appears to be very high.

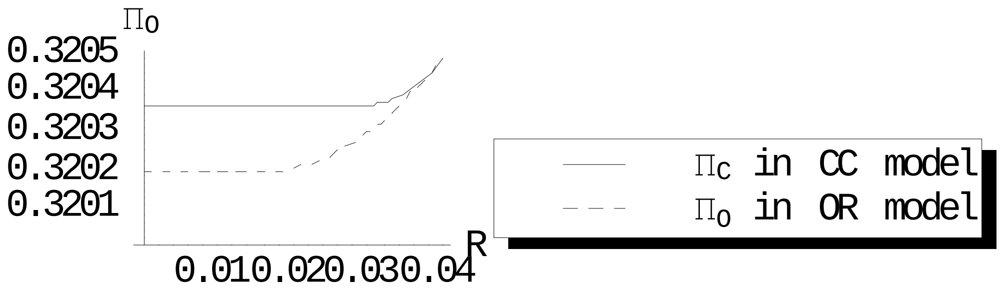


Figure 15 Comparison of OEM's total profits in two periods

The central planner in CC model makes more profit than the OEM in OR model. This is reasonable because by assuming zero set-up cost, the supply chain is always most efficient in the centrally coordinated model. However, we can see that as R increase, the difference between CC model and OR model decreases. This means when R increases, the OEM would be able to collect more profit from the channel.

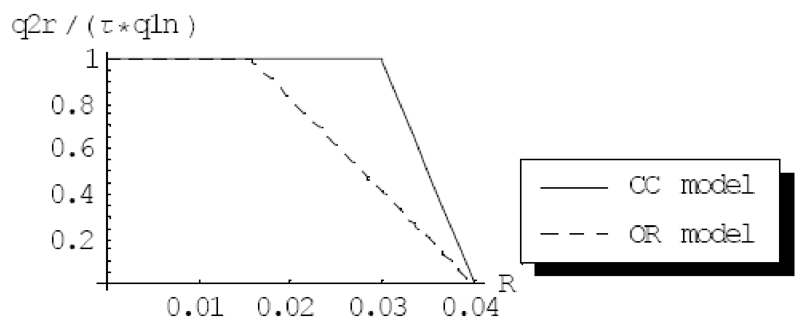


Figure 16 Comparison of resell rate

As discussed in Chapter 3, when $R < \overline{R_{OR}}$, OEM will sell all the collected used products to the reseller in OR model. Similarly in CC model the OEM will resell all the collected used products from period 1 if $R < \overline{R_{CC}}$. When R increases, the OEM starts to sell some of the collected used products for reselling. From this figure we can see that given a value of R, the resell rate in CC model is higher or equal to that in OR model. That is to say, the OEM finds reselling more profitable than recycling for a larger range in CC model given the current set of parameter values. It can be calculated analytically that,

$$\overline{R_{CC}} - \overline{R_{OR}} = \frac{\delta\tau[1 - C_n + (\delta Cn - Cc - Cr)\tau]}{1 + \delta\tau^2[3 - 2\delta + (2-\delta)(1-\delta)\delta\tau^2]}, \tag{4.48}$$

because $1+\delta\tau^2[3-2\delta+(2-\delta)(1-\delta)\delta\tau^2]>0$, if $\delta\tau[1-C_n+(\delta C_n - C_c - C_r)\tau]>0$ $\overline{R_{CC}} > \overline{R_{OR}}$, and our observation that given R, the resell rate in CC model is at least as large as that in OR model holds. Numerically, within reasonable range of parameter values (4.48)>0. The difference between $\overline{R_{CC}}$ and $\overline{R_{OR}}$ decreases when collection cost Cc or refurbishing cost Cr increases, implying the OEM (or central planner) will favor less for reselling.

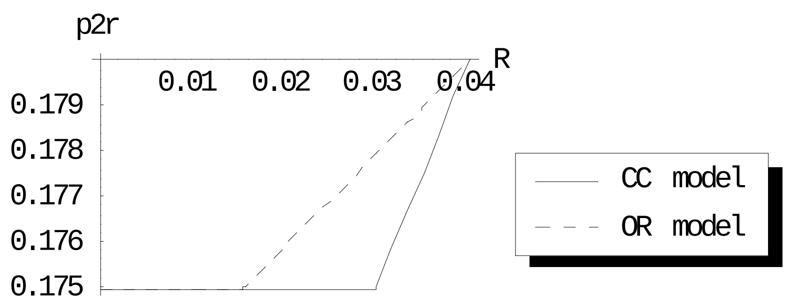


Figure 17 Comparison of refurbished product price in second period

When the R is small, OEM sells all the collected products to the reseller, the retail price is almost the same in both CC model and OR model. When R is large, the price of the refurbished products increases in both CC model and OR model, yet it is lower in OR model.

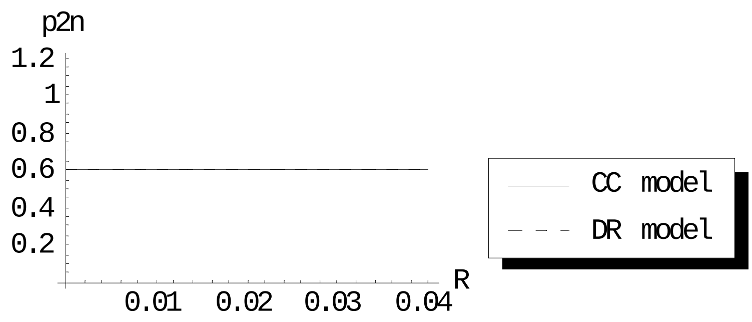


Figure 18 Comparison of new product price in second period

The retail price of new products in the second period is the same in both CC model and OR model.

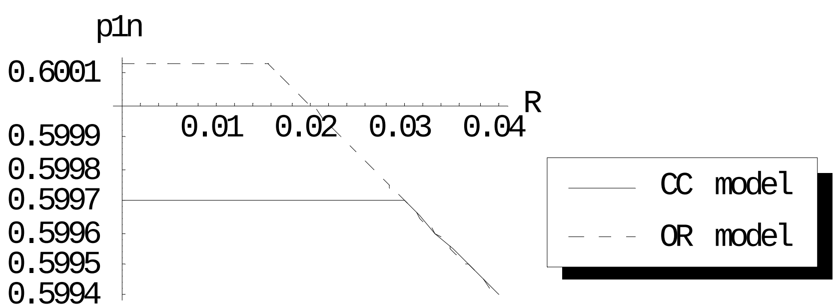


Figure 19 Comparison of new product price in first period

The new product price in CC model would be less than that in OR model in the first period. This is because the central planner would be able to sell more in this period in CC model compared to OR model. Even though the price is smaller, the total profit is still higher because of the larger sales volume in CC model.

In conclusion, in the CC model the OEM enjoys higher profit than in OR model. The customers would also benefit from CC model because of the cheaper price of products in each period.

CHAPTER 5. CONCLUSIONS AND FUTURE RESEARCH

5.1 Conclusions

This thesis extends Ferguson and Toktay's model by proposing a OEM-Reseller model. In this model, the OEM collects used products and sells them to either the reseller or recyclers. This thesis shows under what conditions would the OEM decides

1) Whether to allow refurbished products in the market or not.

2) Whether to sell all the collected products to the reseller or not.

In each condition the equilibrium retail prices and production quantities decisions are calculated using the two-period Stackelberg game with non-negativity and inequality constraints.

This thesis analyzed how the equilibrium decisions of OEM and the reseller change with different values of price the recyclers pay for each used product sent for recycling R and customers' relative willingness-to-pay δ . Some interesting managerial insights are explored in the numerical examples.

The value of R directly affects OEM's preference for resell or recycling. When R increases the OEM tend to be more inclined to sending the collected products for recycling. When R exceeds a threshold value the OEM will recycle all the collected products.

If the OEM is able to control collection of used products, under some conditions it is more profitable for the OEM to sell these collected products to the reseller for reselling compared to recycling. This is quite contrary to the OEM's traditional fear of loss in new product sales from the competition with refurbished products. The mathematical formulations of these conditions are given in proposition 1 and 3.

This thesis shows that if customers value refurbished products more, the OEM can profit from charging the reseller a higher wholesale price w. The increased profit from selling to the reseller exceeds the loss of profit of reduced new product sales, resulting in a increased profit of the OEM.

We then compare the OEM-Reseller model with the centrally coordinated model by using numerical analysis. Focusing on the case where the OEM allows both new and refurbished products in the market we found that centrally coordinated model yields higher profit for the OEM and lower prices for the consumers than OEM-Reseller model.

5.2 Discussion and Future Research

The OEM-Reseller model can be extended in many ways.

5.2.1 Infinite Period Game

In this thesis we address the cell phone recovery problem by analyzing and comparing the profit from reselling and recycling. We focused on a specific model of cell phone which will no longer be of resale value after the second period. Our model can be extended in such a way that, if there exist stable collection flow and demand flow for refurbished cell phones, an infinite period game would be able to capture the cell phone recovery process of general cell phone models. To be specific, the refurbished phones can be returned and refurbished for a second time; The new products can be collected after the second period. This kind of infinite period game can be used in products that can be remanufactured many times, such as single-use-cameras.

5.2.2 Collection Rate τ as a Decision Variable

The OEM can increase the collection rate by investing more in collection, such as advertising or providing higher incentives for each returned cell phone. If we can model the collection cost as a function of collection rate, we would be able to find the optimal collection rate for the OEM under which the OEM achieves maximized profit. This would give us a more comprehensive understanding of OEM's recovery strategies.

5.2.3 Environmental Subsidies from the Government

We have demonstrated in the previous numerical examples that, if the price recyclers pay to the OEM for each unit of collected product sent for recycling R increases, the resell rate would decrease. It is interesting to analyze how the government's subsidy for recycling, reselling, collection would affect the OEM's optimal recovery strategies. The government can be added as a player who sets up environmental policies to achieve maximum social welfare of the supply chain. Using the extended OEM-Reseller model we would be able to find the optimal subsidy the government should provide.

5.2.4 Stochastic Model

We analyze the closed loop supply chain using deterministic models. For example, we derive the simple inverse demand function by assuming customers' willingness-to-pay is uniformly distributed. In this way we are able to analyze the relationship between the OEM and the reseller without unnecessary computational complexities.

However, few things in real life can be captured by simple deterministic models. To make our findings more accurate we may use stochastic models for the analysis. For example, Ferguson and Koeningsberg (2007) uses a random variable u to capture the uncertainty in customers’ willingness-to-pay. The OEM-Reseller model in this thesis can be extended in such a fashion.

APPENDIX A. COMPARISON TABLE OF OR MODEL AND CC MODEL

	OR model($R \le \overline{R}$)	OR model($R > \overline{R}$)
$\overline{R}$	$\frac{C_r + \delta((-2+\delta)\tau(-1+C_c\tau)+C_n(-1+(-2+\delta)\tau))}{-1+(-2+\delta)\delta\tau^2}$	
Π_O	$\frac{1}{(4+8\delta\tau^2-4\delta^2\tau^2)}(2-2C_r\tau+C_c^2\tau^2+C_r^2\tau^2+2\delta\tau^2-\delta^2\tau^2$ $+2C_c\tau(-1+C_r\tau)+2C_n^2(1+\delta(-1+\tau)\tau)$ $-2C_n(2-\delta\tau+2\delta\tau^2-\delta^2\tau^2+C_c\tau(-1+\delta\tau)+C_r\tau(-1+\delta\tau)))$	$\frac{1}{4(-2+\delta)\delta}(-C_r^2-2C_rR-R^2-4(-1+C_n)^2\delta$ $+2C_nC_r\delta+2C_nR\delta+2\delta^2-4C_n\delta^2+C_n^2\delta^2$ $+2(-1+C_n)(C_c-R)(-2+\delta)\delta\tau+(C_c-R)^2(-2+\delta)\delta\tau^2)$
Π_R	$\frac{\delta\tau^2(-1+C_n+(C_c+C_r-C_n\delta)\tau)^2}{4(-1+(-2+\delta)\delta\tau^2)^2}$	$\frac{(C_r+R-C_n\delta)^2}{4(-2+\delta)^2\delta}$
w	$\frac{1}{2}(-2C_r+\delta+\delta(C_n+\frac{2(-2+\delta)\tau(-1+C_n+(C_c+C_r-C_n\delta)\tau}{-2+2(-2+\delta)\delta\tau^2}))$	$\frac{1}{2}(-C_r+R+\delta)$
p_{1n}	$1-\frac{-1+C_n+(C_c+C_r-C_n\delta)\tau}{-2+2(-2+\delta)\delta\tau^2}$	$\frac{1}{2}(1+C_n+C_c\tau-R\tau)$
p_{2n}	$\frac{1+C_n}{2}$	$\frac{1+C_n}{2}$
p_{2r}	$\frac{1}{(-2+2(-2+\delta)\delta\tau^2)}(\delta(-1-C_n(1+(1+\delta(-1+\tau))\tau)$ $+\tau(1-(C_c+C_r)\tau+\delta(-1+(-2+C_c+C_r+\delta)\tau))))$	$\frac{C_r(-1+\delta)+R(-1+\delta)+\delta(-2-C_n+\delta)}{2(-2+\delta)}$
q_{2n}	$\frac{1}{-2+2(-2+\delta)\delta\tau^2}(-1+\delta\tau(1-$ $(2+C_c+C_r-\delta)\tau)+C_n(1+\delta\tau(-1+2\tau)))$	$\frac{2-2C_n+C_r+R-\delta}{4-2\delta}$
q_{2r}	$\frac{\tau(-1+C_n+(C_c+C_r-C_n\delta)\tau)}{-2+2(-2+\delta)\delta\tau^2}$	$\frac{C_r+R-C_n\delta}{-4\delta+2\delta^2}$
q_{1n}	$\frac{-1+C_n+(C_c+C_r-C_n\delta)\tau}{-2+2(-2+\delta)\delta\tau^2}$	$\frac{1}{2}(1-C_n-C_c\tau+R\tau)$

Table 9. Solution for OEM-Reseller model

	CC model($R \le \overline{R}$)	CC model($R > \overline{R}$)
$\overline{R}$	$\frac{C_r+\delta((-1+\delta)\tau(-1+C_c\tau)+C_n(-1+(-1+\delta)\tau))}{-1+(-1+\delta)\delta\tau^2}$	
Π_O	$\frac{1}{(4+4(1-\delta)\delta\tau^2)}(2(-1+C_n)^2+$ $2(-1+C_n)(C_c+C_r-C_n\delta)\tau+((C_c+C_r)^2+(1+C_n^2$ $-2C_n(1+C_c+C_r))\delta+(-1+2C_n)\delta^2)\tau^2)$	$\frac{1}{(4(-1+\delta)\delta)}(-C_r^2-2C_rR-R^2-2(-1+C_n)^2\delta$ $+2C_nC_r\delta+2C_nR\delta+2\delta^2-4C_n\delta^2+C_n^2\delta^2$ $+2(-1+C_n)(C_c-R)(-1+\delta)\delta\tau+(C_c-R)^2(-1+\delta)\delta\tau^2)$
p_{1n}	$\frac{-1+C_n+(C_c+C_r-C_n\delta)\tau}{-2+2(-1+\delta)\delta\tau^2}$	$\frac{1}{2}(1-C_n-C_c\tau+R\tau)$
p_{2n}	$\frac{1+C_n}{2}$	$\frac{1+C_n}{2}$
p_{2r}	$\frac{1}{-2+2(-1+\delta)\delta\tau^2}(\delta(-1+C_n(-1+(-1+\delta)\tau)+$ $(-1+\delta)\tau(-1+(C_c+C_r+\delta)\tau)))$	$\frac{C_r+R+\delta}{2}$
q_{2n}	$\frac{1}{2}(1-C_n-\frac{2\delta\tau(-1+C_n+(C_c+C_r-C_n\delta)\tau)}{-2+2(-1+\delta)\delta\tau^2})$	$\frac{1-C_n+C_r+R-\delta}{2-2\delta}$
q_{2r}	$\frac{\tau(-1+C_n+(C_c+C_r-C_n\delta)\tau)}{-2+2(-1+\delta)\delta\tau^2}$	$\frac{C_r+R-C_n\delta}{-2\delta+2\delta^2}$
q_{1n}	$\frac{-1+C_n+(C_c+C_r-C_n\delta)\tau}{-2+2(-1+\delta)\delta\tau^2}$	$\frac{1-C_n-C_c\tau+R\tau}{2}$

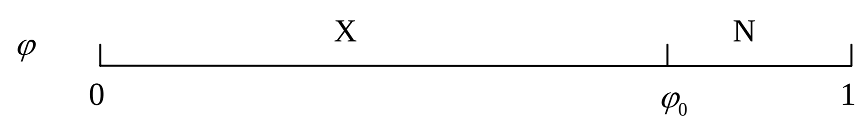
Table 10. Solution for Centrally Coordinated Model

APPENDIX B. DERIVATION OF THE INVERSE DEMAND FUNCTION

In assumption 4 we assumed the customers' willingness-to-pay φ is uniformly distributed between the interval [0,1]. We also normalized the market size to 1. Now we will be able to derive each customer's net utility (NU) following Desai and Purohit(1998):

$$NU = \begin{cases} \varphi - p_{1n} \text{ or } \varphi - p_{2n}, & \text{if bought a new product in period 1 or 2 respectively,} \\ \delta\varphi - p_{2r}, & \text{if bought a resold product in period 2,} \\ 0, & \text{if didn't buy anything.} \end{cases}$$

First let's first derive the inverse demand function in the first period. In this period, a customer has only two choices: Get a new product(N) or get nothing(X). The quantity of customers is the market size, in this case 1, times the probability of a customer getting a positive net utility. The distribution of customers can be illustrated as follows:



Considering the the customer who's indifferent between N and X, at φ_0 he/she gets the same net utility from choosing N and X:

$$\varphi_0 - p_{1n} = 0 \tag{B.1}$$

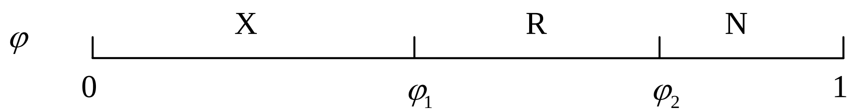
$$\text{yields} \quad \varphi_0 = p_{1n} \tag{B.2}$$

$$\begin{aligned} q_{1n} &= 1 * (1 - \varphi_0) \\ &= 1 - p_{1n} \end{aligned} \tag{B.3}$$

Inverse demand function is:

$$p_{1n}(q_{1n}) = 1 - q_{1n} \tag{B.4}$$

Now let's look at period 2. A customer faces three options: buy new product(N), buy refurbished product(R), buy nothing(X). Similarly we can illustrate the situation below:



If all three strategies are observed in equilibrium, then consumers who follow a N strategy value the product more than consumers who follow a R strategy, who value it more than consumers who follow an X strategy. (Ferguson and Toktay 2006)

Now consider the customer who is indifferent between X and R at point φ_1. He/she gets the same net utility from choosing X and R:

$$\delta\varphi_1 - p_{2r} = 0 \tag{B.5}$$

$$\text{yields } \varphi_1 = \frac{p_{2r}}{\delta} \tag{B.6}$$

Then we consider the customer who is indifferent between N and R, at point φ_2. He/she gets the same net utility from choosing N and R:

$$\varphi_2 - p_{2n} = \delta\varphi_2 - p_{2r} \tag{B.7}$$

Solving (B.7) we have

$$\varphi_2 = \frac{p_{2n} - p_{2r}}{1-\delta} \tag{B.8}$$

Therefore the number of customers are:

$$q_{2n} = 1*(1-\varphi_2)$$
$$q_{2r} = 1*(\varphi_2 - \varphi_1) \quad \text{(B.9)}$$

Solving (B.6) ,(B.8) and (B.9) we have the inverse demand functions:

$$p_{2n}(q_{2n}, q_{2r}) = 1 - q_{2n} - \delta q_{2r}$$
$$p_{2r}(q_{2n}, q_{2r}) = \delta(1 - q_{2n} - q_{2r}) \quad \text{(B.10)}$$

Notice the different effects of the relative willingness-to-pay δ on the prices. Given the production quantities, when δ increases, the price of refurbished products increases to take advantage of the increased willingness-to-pay. However, the price of new products decreases as the two products become closer substitutes and there is more competition, i.e. the cannibalization effect. (Ferguson and Toktay 2006)

proof: If all three strategies are observed in equilibrium, then consumers who follow a N strategy value the product more than consumers who follow a R strategy, who value it more than consumers who follow an X strategy.

This is equivalent to proving the net utility of customers with willingness-to-pay $\varphi > \varphi_2$ is larger if he/she chooses N than choosing R, and net utility of customers with willingness-to-pay $\varphi > \varphi_1$ is larger if he/she chooses R than being inactive X.

For customers with $\varphi > \varphi_2$

NU from N is $\varphi - p_{2n}$ (B.11)

NU from R is $\delta\varphi - p_{2r}$ (B.12)

(B.11)-(B.12) yields

$$(1-\delta)\varphi-(p_{2n}-p_{2r}) \quad \text{(B.12)}$$

When $\varphi > \varphi_2$, substitute(B.8) into this inequality (B.12) becomes

$$(1-\delta)\varphi-(p_{2n}-p_{2r}) > (1-\delta)\varphi_2-(p_{2n}-p_{2r}) = 0 \quad \text{(B.13)}$$

Hence the net utility of customers with willingness-to-pay $\varphi > \varphi_2$ is larger if he/she chooses N than choosing R.

For customers with $\varphi > \varphi_1$

NU from R is $\delta\varphi - p_{2r}$ (B.14)

NU from X is 0 (B.15)

(B.14)-(B.15) yields

$$\delta\varphi - p_{2r} \quad \text{(B.16)}$$

When $\varphi > \varphi_1$, from (B.6)

$$\delta\varphi - p_{2r} > \delta\varphi_1 - p_{2r} = 0 \quad \text{(B.17)}$$

That is to say, net utility of customers with willingness-to-pay $\varphi > \varphi_1$ is larger if he/she chooses R than being inactive X.

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