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Michelle Cirillo
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On becoming a geometry teacher:
A longitudinal case study of one teacher learning to teach proof

by

Michelle Cirillo

A dissertation submitted to the graduate faculty
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

Major: Education

Program of Study Committee:
Beth Herbel-Eisenmann, Co-major Professor
Corey Drake, Co-major Professor
Leslie Bloom
Katherine Richardson Bruna
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Iowa State University
Ames, Iowa

2008

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ABSTRACT

My experiences as a teacher and as a researcher led me to pursue the topic of this dissertation – how a new teacher learns to teach proof to high school geometry students. Through my work on a research project, I became interested in this 10th grade teacher after watching him teach proof for the first time. I anticipated that, not unlike my own experience as a teacher of geometry, this teacher’s proof-related teaching and discourse practices would look different in subsequent years. This classroom provided a context for me to study how a beginning teacher, using a conventional textbook, develops his practice related to teaching proof.

In order to pursue this dissertation topic, I designed a longitudinal case study in which I collected data across three years, including classroom observation data, interviews, reflective writings on the daily lessons, and instructional materials that were used in the classroom (e.g., the textbook). In addition, interviews conducted for a larger research project provided background information that was relevant to this study.

Drawing from the literatures on new teacher induction, curriculum enactment, proof in school mathematics, and classroom discourse, I discuss how and why this beginning teacher’s practice changed across time. More specifically, in this dissertation, I describe how and why the teacher: supplemented the written curriculum with additional proofs and tasks; explicitly increased his focus on the process of proving; and created more space for student participation. I argue that the teacher purposefully made these changes in order to enact a practice that was more like what he called “real math” than like school mathematics.
Three significant findings of this study are related to issues of curriculum use, authentic mathematical practices, and professional development in the area of classroom discourse. For example, the teacher explored the use various proof forms, and his participation in a professional development experience motivated him to experiment with various discourse moves. This study has implications for teacher educators, curriculum developers, mathematics supervisors, and researchers interested in understanding issues related to the teaching and learning of proof in school mathematics.
CHAPTER 1: INTRODUCTION

As I sat in a high school geometry class and observed a novice teacher, Matt, teaching proof for the first time in September of 2005, I was reminded of my own experiences of teaching formal proof to secondary students. I noticed that Matt seemed to struggle with some of the same things that I struggled with when I started teaching proof. For example, Matt seemed to struggle with knowing which definitions and theorems should be emphasized based on which ones would appear consistently throughout the course. Such things are difficult to determine until “curricular knowledge” (Shulman, 1986) is acquired either through the experience of using particular curriculum materials over time or through other capacity-building experiences.

During the first year of this study, I remember thinking that Matt’s practice would likely look quite different after he taught the course a few times. I thought that Matt would have a much better vision of where he was headed with proof in his second year than he did the first time he worked through the course. Because I found teaching formal proof in geometry to be one of the greatest challenges that I encountered during my eight years as a secondary mathematics teacher, I decided that studying Matt’s teaching of proof in geometry during his induction years would make an interesting and worthwhile research project. From a research perspective, following Matt’s evolution from a beginning to a more experienced teacher of proof provided a context to better understand how one becomes a geometry teacher.

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1 This teacher’s name and all students’ names are pseudonyms.
By drawing on literature related to learning to teach, Fuller and Brown (1975) explored the experience of “Becoming a Teacher.” In their chapter, these authors discussed the notion that those who become teachers pass through several stages of concerns as they learn to teach. Since this early book chapter was published, the idea of “becoming” a teacher has been the topic and title of other books and articles on the subject of beginning teachers (see, e.g., Grossman, 1990; Huberman, Grounauer, & Marti, 1993; Kremer-Hayon & Ben-Peretz, 1986; Paoni, 1995). This literature sometimes addresses the process of becoming a certified teacher, and in other cases, addresses how one becomes an established teacher.

Some researchers suggest that the process occurs on a continuum over time. For example:

Research on teachers’ careers suggest that the first three to five years of teaching are a period of time in which an individual moves from “novice” to “established” teacher status. In a sense, the new teacher continues to become a teacher [italics added] throughout that period, acquiring contextually useful knowledge, skills, and values; and refining, detailing, and deepening the image of self-as-teacher. (Mager, 1992, p. 5)

Conducting research in Matt’s geometry classroom over the course of three years, therefore, provided a context to help me understand some of the challenges of becoming a geometry teacher.

Acknowledging that geometry and proof are not synonymous, I write about teaching proof in the context of the geometry course because in schools in the United States, it is in the geometry course that proof is typically addressed as a stand-alone topic. In fact, the paucity of proof throughout the school mathematics curriculum contributes to the difficulty of teaching it in geometry (Clements, 2003). This lack of substantive attention to proof is also problematic because, in many advanced university mathematics courses, mathematical proof plays an important role, reflective of the discipline of mathematics (Conner, 2007). For
the purpose of this study, I use the definition of proof that was proposed by Movshovitz-Hadar in the *Encyclopedia of Mathematics Education* (2001):

> ...an ideal mathematical proof displays in a systematic way a finite sequential set of statements that leads from definitions, axioms (i.e., statements the truth of which is unquestioned in a given theory) and theorems (i.e., statements the truth of which has already been proved) to a conclusion, in such a way that as long as the axioms are accepted and the definitions are agreed upon, the conclusion is inevitable and its validity must be recognized. (p. 585)

I chose to define proof in this way because this definition was very similar to the one offered by the teacher in this study.

Researchers have pointed out that not only do students find the learning of proof to be challenging (Farrell, 1987; Usiskin, 1980), but teachers also find the teaching of proof to be a difficult endeavor (Knuth, 2002c; Wu, 1996a). In 1971, for example, Stone pointed out that there was a “general agreement that in all of school mathematics there is no subject more difficult to learn or to teach than axiomatic geometry” (p. 91). More recently, Wu (1996a) claimed that Euclidean geometry is perhaps the most vilified portion of school mathematics. Finally, Knuth (2002b) concluded that teachers’ conceptions of proof are somewhat limited, and that their conceptions of proof need to be enhanced if they are to be successful in enhancing the role of proof in secondary school mathematics. These researchers point to a need to better prepare and support teachers to teach proof.

Currently, preservice teacher preparation often does not do an adequate job of equipping new teachers to handle the demands of daily classroom life (Liston, Whitcomb, & Borko, 2006) let alone prepare them to handle particular curricular issues, such as the teaching of proof. For example, teachers have the responsibility of making daily decisions about what and how to teach, but they are frequently given little guidance to make such
decisions. One novice teacher described the new teacher experience as feeling “lost at sea without any map or anything…to figure out where you were going” (Kauffman, Johnson, Kardos, Liu, & Peske, 2002, p. 281). This “lost” feeling sometimes extends to new teachers’ content understandings. More specifically, even after earning an undergraduate degree in mathematics, there is no guarantee that beginning teachers have conceptual understandings of the topics taught in the secondary mathematics curriculum (Grossman & Stodolsky, 1994).

A lack of conceptual understanding is especially problematic in an era of standards-based instruction. In recent reforms of school mathematics (National Council of Teachers of Mathematics [NCTM], 1989, 1991, 2000), attention to both pedagogy and content is evident in the new standards for teaching mathematics (Grossman & Stodolsky, 1994). These Standards have reframed the issue of mathematical understanding around constructivist ideas and have served as the impetus for the creation of reformed, understanding-based curricula (Bohl, 2000). One of the first dilemmas that new teachers face is deciding what to teach and how to structure their curriculum (Grossman & Thompson, 2004). The role of curriculum materials has been viewed as increasingly important over the last two decades (Remillard, 2005). This is evidenced both by the development of Standards-based curriculum materials as well as the abundance of studies on teachers’ use of these materials. In the case of geometry proof, the curriculum has been noted as a primary cause of students’ poor performance, both in what topics are addressed and how they are communicated (Jaime, Chapa, & Gutierrez, 1992). The NCTM recommends that teachers at every level “help

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2 Throughout this study, when I refer to “the Standards” without citing a particular document, I do so to reference the collective group of documents (1989, 1991, 2000). Since the 2000 document “builds on and consolidates the messages from the previous documents” (NCTM, 2000, p. 6), it is not always necessary to distinguish between the three. When providing a direct quotation, however, I specifically cite the particular document by referencing the date.
students make, refine, and explore conjectures on the basis of evidence and use a variety of reasoning and proof techniques to confirm or disprove those conjectures” (NCTM, 2000, p. 3).

As pointed out by Stylianides (2007), a major reason that proof and proving have been given increased attention in recent years is because proof and proving are fundamental to doing and knowing mathematics and communicating mathematical knowledge. When describing proof as the “guts of mathematics,” Wu (1996b, p. 222) argued that anyone who wanted to know what mathematics was about needed to learn how to write, or at least understand, a proof. These comments complement the call to bring students’ experiences in school mathematics closer to the practices of mathematicians (Ball, 1993; Ball, Lubienski, & Mewborn, 2001; Lampert, 1992; NCTM, 2000). This idea is not new, as a number of curriculum theorists from Dewey (1902) to Schwab (1978) have argued that the disciplines should play a critical role in the elementary and secondary curriculum. Therefore, by engaging students in ‘authentic mathematics,’ where they are given opportunities to refute and prove conjectures (Lakatos, 1976; NCTM, 2000; Lampert, 1992) teachers can create genuine small mathematical societies in the classrooms (Brousseau, 1997).

Despite the call to elevate the importance of proof with the goal of making students’ experiences in school mathematics more authentic, teachers are often ill-prepared to carry out the vision of the Standards (Smith III, 1996). Ideally, by the time students leave high school, their “explanations should become more mathematically rigorous and students should increasingly state in their supporting arguments the mathematical properties they used” (NCTM, 2000, p. 62). In this vein, students should be given opportunities to become critical
thinkers of mathematics by carefully listening to, and thinking about the claims made by others (NCTM, 2000).

Over the last few decades, increased attention has been given to discourse processes such as careful listening and thinking about others’ claims. Purposefully shaping mathematical discourse in the classroom is a significant aspect of a teacher’s work (Franke, Kazemi, & Battey, 2007) because, without explicit attention to patterns of discourse, long-established discourse norms of mathematics classrooms (e.g., triadic dialogue or IRE) are likely to prevail (Ball, 1991). Studies that explicitly focus on classroom discourse can contribute to the field as they allow researchers and teacher education communities to engage with these studies, unpack them, and learn from them (Franke et al., 2007). This dissertation contributes to this body of work because the data were collected during a period of time when the participating teacher was purposefully attending to his discourse practices through participation in a larger study.

Because of the high status that proof has in mathematics, it is important that we gain a better understanding of the challenges related to teaching it. Through this research project, I sought to understand the process of developing the practice of teaching formal proof in high school geometry. If we wish to better prepare teachers to address the goals of the Standards, and therefore change the nature of school mathematics, we must first address the factors that shape it (Grossman & Stodolsky, 1994). Factors that shape the nature of school mathematics that are addressed in this study include the teacher’s use of curriculum materials and his facilitation of classroom discourse. In addition, this study aims to provide information about

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3 These concepts will be discussed in greater detail in the next chapter.
some of the issues and challenges that geometry teachers face as they attempt to incorporate Standards–based reform recommendations into their instruction.

This study is unique on several levels. Few, if any, studies investigate a geometry teacher teaching proof for the first time. Although there are many studies of teachers using reform-oriented curriculum materials, there is a paucity of work that explicitly considers how a teacher who was attempting to enact a Standards-based practice of teaching proof attempted to do so using a conventional textbook. Finally, beginning secondary teachers are notably absent in much of the literature in mathematics education. The research reported here is significant because it: (a) helps us understand some of the ways that a conventional mathematics textbook constrained a beginning teacher’s attempts to engage his students with authentic mathematics, (b) suggests that attention to one’s discourse practices can be useful both in engaging students with proof and in helping teachers attend more carefully to their students during the induction phase of teaching, and (c) provides insights into the types of support both beginning and experienced teachers may need whenever they are enacting new curriculum materials for the first time. These findings have implications for mathematics education researchers, geometry teachers, mathematics supervisors, teacher educators, professional developers, and curriculum developers.

Research Questions

Acknowledging that there is much to learn related both to how mathematics teachers “become” teachers of proof and to the particular issues that arise as one teaches proof for the first time, this case study addresses one small part of these larger issues by answering the following research questions:
- How did Matt’s teaching of geometry proof change across three years?
  
  - How did Matt develop pedagogically?
  
  - How did his teaching change as he acquired curricular knowledge?

- To what did Matt attribute these changes?

  - In what ways did Matt’s participation in a larger Discourse Study influence the ways in which he taught proof?

To answer these questions, I conducted a qualitative study of one novice teacher teaching proof across three years.

**Overview of the Study**

I designed this study as a longitudinal case study in which the changing practice of one secondary teacher is reported. Because the change process is slow, it is important that researchers study teachers over time – certainly for more than one year, and preferably for several years after their preservice teacher preparation (Borko & Putnam, 1996). It is also important for researchers to:

…capture the complexity of changes in teachers’ knowledge, beliefs, and practices across the domains of the professional knowledge base. Doing so entails the use of multiple data sources such as observations of teachers in workshop and classroom settings; interviews about knowledge, beliefs, and practices; and teachers’ reflective writings. This triangulation to support claims with data from a number of sources is important because of the fairly high levels of interpretation and interference involved in understanding changes in the mental lives of teachers – what they know and believe and how they think about what they do. (Borko & Putnam, 1996, p. 703)

As recommended above, in this dissertation, the evolution of Matt’s practice is described through the analysis of various data sources including interviews, classroom observations, and artifacts such as curriculum materials and written reflections. An analysis of Matt’s use of curriculum materials, his more explicit emphasis on proving theorems, and his classroom
discourse provide evidence that Matt’s practice changed across time in ways that more closely aligned his professed beliefs with his practice. In addition, I argue that Matt intentionally made these changes to his practice in order to work toward providing a more authentic mathematical experience for his students.

The next chapter expands on the ideas introduced in this chapter by providing a theoretical and empirical grounding for this study. I review literature on teacher induction, curriculum in school mathematics, proof, and classroom discourse to provide a lens to consider the findings of this study. In Chapter 3, I discuss the theoretical perspectives, the context of the study and my reflections on the fieldwork, and the research methods. Chapter 4 helps set up the findings by using interview data to explore Matt’s beliefs about mathematics and proof, as well as discuss his philosophy of teaching. I then present the findings in the three chapters that follow. These three chapters are related to the particular changes that were observed and factors that motivated Matt to make the changes described. Finally, concluding comments and implications of the study are discussed in Chapter 8.
CHAPTER 2 : LITERATURE REVIEW

In this chapter, I situate the study by presenting the empirical, theoretical, historical, and curricular issues that have stimulated and influenced my thinking about this research project. The chapter consists of four main sections that correspond with the primary domains of this inquiry: the induction phase of teaching, issues related to curriculum in school mathematics, mathematical proof, and classroom discourse. In the first section, I discuss literature on new teacher induction and the kind of research that is needed to understand experiences of beginning mathematics teachers. Second, I discuss teachers’ use of curriculum materials and the historical place of proof in the school mathematics curriculum. I argue that studies examining the geometry curriculum are warranted. In the third section of the chapter, I summarize the vast body of literature on proof, giving special attention to qualitative studies of proof practices in secondary mathematics classrooms. Through a review of this literature, I contend that more studies are needed to understand the process of teaching proof in secondary classrooms in the context of the classroom. Fourth, I discuss various features of classroom discourse that have been identified in the literature. More specifically, I review how researchers have conceptualized the following discourse features: revoicing, teachers’ use of pronouns, teacher questions, and pace and wait time. I argue that these particular features of classroom discourse can close down or open up space for student participation. Finally, I revisit the research questions introduced in Chapter 1.
New Teacher Induction

Learning to teach involves the ongoing attainment of professional expertise over a long period of time (Morine-Dershimer, 1992). When novice teachers accept a teaching position, however, they are given the same responsibilities as veteran teachers. More specifically, teachers face two major tasks when they begin teaching: to do the job they have been hired to do and to learn how to do the job (Wildman, Niles, Magliaro, & McLaughlin, 1989). Despite this enormous challenge, schools virtually ignore the latter task. Perhaps this is because learning how to teach has no official status in the current U.S. education system (Wildman et al., 1989). That is, district support through professional development, formal induction programs, or mentoring programs is not mandated. As a result, few structures are put in place to assist with this learning or even to ensure that it is carried out (Wildman et al., 1989). The task of learning how to do the job can be overwhelming as new teachers are expected to learn, for example, the curriculum, the social and political contexts of the school and the community, the procedures for reporting attendance and grades, how to use their textbook and other materials provided for instruction, including technology, and, most importantly, the learning styles, needs, and abilities of their students. This time of transition from student of teaching to teacher of students is often referred to as the induction phase (Feiman-Nemser, 2001).

The Induction Phase

The shift from student to teacher is the most dramatic transition during the induction phase (Morine-Dershimer, 1992). In light of the fact that beginning teachers have much to learn about teaching and little knowledge related to their new role, they must learn quickly if
they are going to survive (Wildman et al., 1989). Whether novice teachers’ early experiences are easy or painful, they typically operate in survival mode during the initial months of teaching (Liston et al., 2006). The first few months of the induction phase are often called the survival stage because the new teacher is “fighting for his or her professional life, and often for a sense of worth and identity as well” (Ryan, 1986, p. 13). Depending on how well the teacher manages this stage, students may also suffer, and the survival stage can affect the ways in which the teacher views teaching in the future (Ryan, 1986).

Some sources of struggle faced by teachers during the induction phase include: a deficient theoretical grounding learned in teacher preparation that does not adequately equip teachers for the demands of daily classroom life, the emotional intensity of teaching, and workplaces that do not sufficiently support teacher learning (Liston et al., 2006). These first years of teaching are intense periods of experimentation and modification, as teachers are constantly encountering new experiences, which, in turn, impact the formation of their philosophies, knowledge bases, dispositions, and abilities (Luft, 2007). Teachers need to know about many things, including pedagogy, curriculum, and their subject matter (Feiman-Nemser, 2001). As they attempt to figure out these larger issues, they must tend to the day to day tasks of teaching such as: marking papers, assigning grades, attending meetings, responding to parents and guardians, completing required paperwork, and fulfilling other “duties,” which make for long days, evenings, and weekends (Liston et al., 2006). While it is the case that all teachers need to tend to these day to day tasks, accomplishing all of them is more intense for beginning teachers.

In addition to learning the procedures and practices of a new school, novice teachers are also confronted with the process of tapping into and acquiring various kinds of
knowledge. Frequently they discover that whole areas of knowledge and skill were left out of their teacher preparation. A challenge for all teachers is integrating the many different kinds of knowledge in the context of particular situations (Ball et al., 2001). In the sections that follow, I discuss some of the different kinds of knowledge that teachers need to integrate. I then discuss the need for more studies on the induction phase in the field of mathematics education.

Knowledge for Teaching

Shulman (1986) described three types of knowledge that are necessary for effective teaching: subject matter content knowledge, pedagogical content knowledge, and curricular knowledge. According to Shulman (1986, 1987), to present specific content to particular students, teachers need a special blend of content and pedagogy that he referred to as “pedagogical content knowledge.” Pedagogical content knowledge (PCK) goes beyond knowledge of subject matter to the dimension of subject matter knowledge for teaching (Shulman, 1986). This includes the ways of representing and reformulating the subject that make it comprehensible to students (Shulman, 1986). Sparked, in part, by Shulman’s conceptualization of pedagogical content knowledge, researchers in the 1980s and 1990s sought to identify what teachers know (or should know) to teach mathematics (Hill, Sleep, Lewis, & Ball, 2007).

Mathematical Knowledge for Teaching

Although many researchers, teachers, policymakers, and teacher educators have acknowledged the importance of teachers’ content knowledge, there remains an unsolved problem for the improvement of mathematics teaching and learning. That is, what is the
nature and role of “mathematical knowledge for teaching” (MKT) and how can it be
developed effectively (Ball & Bass, 2003; Ball et al., 2001, p. 440; Hill et al., 2007)? The
construct of MKT is multifaceted:

[It includes] not only teachers’ ability [sic] to solve the problems that their students
are expected to solve, but also to understand the content in the particular ways needed
for teaching it, to understand what students are likely to make of the content, and to
craft instruction that takes into account both students and the mathematics. (Hill et al.,
2007, p. 125)

The description provided here explicitly points to a need for teachers to consider their
students as they plan for and craft instruction.

It is unclear, however, how much MKT beginning teachers have and whether or not
MKT can be employed by novice teachers. Some researchers claim that novice teachers tend
not to consider students or student learning very extensively, very carefully, or in very
sophisticated ways (see, e.g., E. A. Davis, Petish, & Smithey, 2006; Kagan, 1992; Veenman,
1984). There is a widespread belief that until beginning teachers learn to manage students,
they cannot concentrate on teaching them (Feiman-Nemser & Parker, 1990). For example,
Veenman (1984) documented the preoccupation that new teachers had with problems of
classroom management and student motivation. Other studies, however, provide evidence
that beginning teachers do pay attention to students as learners. For example, both Brown
(1993) and Wildman et al. (1989) found that the novice teachers in their studies focused
strongly on students as they thought about curriculum planning. More research is needed on
how beginning teachers develop in these areas.

**Needed Research on Beginning Mathematics Teachers**

Although there is widespread agreement that novice teachers lack a number of
important skills, only a few researchers have sought to understand how beginning teachers
develop their knowledge of and for teaching (D. S. Brown, 1993; Clandinin & Connelly, 1986). Several of the studies conducted on novice secondary teacher planning have been done in the content area of language arts (D. S. Brown, 1993). More recently, researchers in the area of science education are beginning to explore the challenges that new science teachers face as they begin their teaching careers.

In a review of the literature, Davis et al. (2006) found that there were few studies on how new science teachers use curriculum materials or how they understand scientific inquiry. The authors concluded that providing support focused on “real” challenges is critical to retaining highly qualified new science teachers. They also called for more research across the other content areas, including mathematics, because although many challenges of new teachers will be similar across subject areas, some will not. As Luft (2007) claimed, these types of studies allow those who work directly with beginning teachers to understand the experiences of these new teachers in their particular contexts.

Studies on mathematics teachers during the induction phase are important because they provide unique insights into topics that are pressing to new mathematics teachers. This is critical since research shows that teachers of mathematics and science are more likely to leave their teaching jobs than other teachers (Ingersoll, 2000). Poor administrative support and poor salaries are the top reasons for mathematics teachers’ dissatisfaction with their jobs (Ingersoll, 2000). Perhaps if we found better ways to support teachers during the induction phase, they would be less likely to leave the profession. Thus, similar to Luft’s (2007) work with new science teachers, studies such as this one are important because they reveal the complexity of being a first-year mathematics teacher in the context and setting in which the new teacher works. Because teacher educators play a critical role in helping prospective
teachers construct more realistic images of their lives as beginning teachers (Friedrichsen, Chval, & Teuscher, 2007), the descriptions provided in this dissertation can be helpful to teacher educators as well. Studies on beginning mathematics teachers can also tap into the cognitive side of teaching which can include the thinking, learning, and decision-making processes of new mathematics teachers. This type of research can not only benefit teacher educators preparing secondary teachers, but also those working with elementary teachers who teach multiple subjects (E. A. Davis et al., 2006).

In mathematics education, it is easy to find studies that involve, as participants, either preservice (e.g., Cooney et al., 1998; Lloyd, 2005; Mingus & Grassl, 1999; Vacc & Bright, 1999) or experienced teachers⁴ (e.g., Clarke, 1997; Herbel-Eisenmann, Lubienski, & Id-Deen, 2006; Lloyd, 1999; Nathan & Knuth, 2003; Remillard, 2000; M. S. Smith, 2000). Only a few studies can be found in the mathematics education literature that include novice teachers as participants (e.g., Drake, 2002; Middleton, 1999). In seeking literature for this review, I could not locate studies that solely focused on novice secondary mathematics teachers. Perhaps this is because, as pointed out by Luft (2007), these beginning teachers comprise a population that is difficult to study. Even the most confident first-year teacher might find the added expectation of participating in a research study to be daunting and overwhelming. As a result of hearing stories about the enormous task of learning to do the job of teaching and the accompanying exhaustion that goes with it, new teachers begin their first year worried about survival. In addition, well-meaning, experienced teachers and

⁴ By “experienced,” I mean at least three years of experience, however, most of the teachers in these studies were classified as veteran teachers and had more than ten years of experience.
administrators advise newcomers to steer clear of such participation (Luft, 2007). This results in researchers’ limited understanding of newly qualified teachers’ experiences.

Studies that address the issues and challenges of learning to teach mathematics should seek to address this gap in the mathematics education literature. Understanding the process of learning to teach requires that researchers pay attention to the preservice, induction, and inservice years (Roehrig & Luft, 2006). In the absence of knowledge and recommendations for mathematics teachers in the induction phase, mathematics teachers (and their students) receive a compromised educational experience. This study, therefore, can provide important information for professional developers who work with beginning teachers.

My work builds on the studies conducted in science education, but also contributes to the mathematics education literature through its longitudinal nature. Not only will issues related to the ways in which the novice teacher in this study was able to increase his focus on students across time be explored, but also the particular challenges and issues that this new teacher faced when teaching mathematical proof for the first time. Some challenges of working with mathematics curriculum materials will be explored as well. This is the first study of its kind in several ways. First, it is the first longitudinal study of a beginning secondary mathematics teacher across three years. It is the first study of a beginning teacher learning to teach proof for the first time. This is also the first study of a mathematics teacher attempting to enact a Standards-based pedagogy using a conventional geometry textbook.

These findings can inform teacher educators, those who facilitate induction or mentoring programs, researchers who wish to understand the challenges of teaching proof in school mathematics, as well as curriculum developers. In the next section, I discuss issues
related to the school mathematics curriculum and the historical place of geometry proof in school mathematics.

**Curriculum in School Mathematics**

Research has indicated that most teachers rely heavily on commercial curriculum materials such as textbooks and teachers’ guides (Goodlad, 1984; Kauffman et al., 2002; Woodward & Elliott, 1990). Even novice teachers who began student teaching with negative opinions about commercial textbooks turned to those materials in the absence of other support (Ball & Feiman-Nemser, 1988). In mathematics, in particular, it has been said that teachers rely heavily on textbooks (Gehrke, Knapp, & Sirotnik, 1992; Jackson, 1992; Nicol & Crespo, 2006; Remillard, 2005). Throughout this study, I use the terms, “curriculum,” “textbooks” and “curriculum materials.” Before proceeding, I offer definitions for these and other important terms related to curriculum.

The term “curriculum” has multiple meanings (Jackson, 1992; Posner, 2004; Remillard, 2005). Broadly, the term curriculum refers to “the substance or content of teaching and learning – the ‘what’ of teaching and learning (as distinguished from the ‘how’ of teaching” (Stein, Remillard, & Smith, 2007, p. 321). Textbooks and other curriculum materials are said to influence the ‘what’ of teaching and learning.

Although the terms “curriculum materials” and “textbook” are used somewhat interchangeably to refer to the printed, published materials designed for use by teachers, many teachers and mathematics educators distinguish between the two (Stein et al., 2007). During the latter part of the 20th century, mathematics “textbooks” used in U.S. classrooms were viewed as collections of explanations and exercises for the students to complete. Unlike
the term textbook, the term “curriculum materials” has been used recently to refer to something that was “anti-textbook” because these resources offered programs that rejected the notion that learning mathematics involved decontextualized exercises from a book (Stein et al., 2007). For most Standards-based curriculum materials,\(^5\) textbooks were replaced by thin, student workbooks that were designed to support students’ investigative work (Stein et al., 2007).

A major goal of Standards-based curriculum developers was to create materials that engaged students in ways that traditional programs had not. Battista (1999) contended that in order to remain competitive, all major textbook companies would produce textbooks that claimed to fall in line with the call for change. He warned, however, that unlike the NSF-funded, Standards-based curriculum programs, most of the commercially available textbooks that claimed to be consistent with the NCTM Standards were actually “traditional curricula with enough superficial changes tacked on so that the publishing companies can market them as ‘new’ and consistent with reform” (Battista, 1999, p. 433). For the most part, many of these textbooks are “mere caricatures of genuine reform curricula” (Battista, 1999, p. 433), and are more like textbooks that precede the Standards than not.

Throughout this study, I use the terms “textbook” and “curriculum materials.” In this dissertation, the term “curriculum materials” can be taken to include any commercially developed materials that are used in classrooms. In this sense, I see textbooks as a subset or an element of the curriculum materials that are given to the teacher. I also use the term “written curriculum” to refer only to the printed materials that are given to the students (e.g.,

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\(^5\) The term Standards-based curriculum materials refers to curriculum materials that were developed during the 1990s with the NCTM Standards in mind. Development of the early Standards-based materials was funded by the National Science Foundation (NSF).
the textbook). Finally, the “enacted curriculum,” sometimes referred to as the “experienced” or “lived” curriculum, is used to refer to what actually takes place in the classroom (Gehrke et al., 1992).

In the sections that follow, I first discuss research related to the interactions between teachers’ beliefs, curriculum, and practice to help the reader understand why the teacher in this study may have changed his practices over time. I then discuss curriculum adaptation, followed by the ideas of “curriculum vision” and “curriculum trust” (Drake & Sherin, in press) in order to provide a framework for the analysis and findings related to the teacher’s evolving relationship with his textbook. Finally, I address the historical place of geometry proof in the U.S. curriculum.

**Interactions between Beliefs, Curriculum, and Practice**

Every mathematics class has at least two curricula: the intended and the enacted curriculum (Philipp, 2007). An important aspect of understanding a teacher’s practice involves attending to how the curriculum materials are enacted by a teacher, with students, in a particular context (Ball & Cohen, 1996). This teacher-curriculum relationship is influenced by teachers’ beliefs (Philipp, 2007). The work presented in this study is grounded in research on the interaction between teachers’ practices and beliefs about mathematics, teaching, and learning (e.g., Philipp, 2007; Remillard & Bryans, 2004).

Cooney (2001) contended that it is important to think of beliefs as clusters of “dispositions to act, which include both utterances and actions” (p. 21). The beliefs that teachers draw on (considering the range of beliefs one holds) depend on what is happening at that point in time and with the particular set of students with whom they are working.
Researchers have found that close analyses of teachers’ beliefs help explain how teachers structure their lessons (A. G. Thompson, 1984) and receive and interpret curriculum (Putnam, Heaton, Prawat, & Remillard, 1992).

Many researchers have also pointed out that teachers’ knowledge, beliefs, and practices can be influenced by their interactions with curriculum materials (Collopy, 2003; Lloyd, 1999, 2002, 2008; McLaughlin, 1976). This relationship between teacher and text is reciprocal:

Just as the knowledge and beliefs teachers hold about mathematics, teaching, and learning affect the ways in which they enact curricula, so too should we expect that teachers might learn from the curricula they use. That is, teachers not only adapt and change curricula, but also are changed by the curricula they use. (Philipp, 2007, p. 287)

An example of how curriculum use can influence teachers is provided in Lloyd’s study of Anne, a novice teacher. Lloyd (2008) found that Anne’s enactment of curriculum materials during her student teaching influenced the ways that she adapted a different set of materials during her first year of teaching. Some of the changes that Anne made to the lessons provided her students with increased opportunities to engage with mathematical ideas, particularly through activities. Lloyd’s study suggests that interactions with previous curriculum materials can influence the ways that even new teachers use subsequent curriculum materials. This finding differed from what one might expect in the case of a beginning teacher (Lloyd, 2008). For example, Remillard and Bryans (2005) found that less experienced teachers tended to be thorough pilots of their curriculum materials. This notion of how closely teachers use and adapt their curriculum materials is explored in the next section.
Curriculum Adaptation

Research on teachers’ use of curriculum materials is often concerned with the ways in which teachers *adapt* their curriculum materials (Sherin & Drake, in press). Recently, some researchers have begun to develop frameworks to describe teachers’ general tendencies and patterns of curriculum activity. Both Remillard and Bryans (2004) and Brown and Edelson (2003) have characterized teachers’ degrees of *appropriation* of curriculum materials. As illustrated by Figure 2-1, both sets of researchers suggested that these characterizations lie along a continuum (Lloyd, 2008). Most relevant to this study is Brown and Edelson’s (2003) description of “adaptation”:

Curricular adaptations are instances where teachers adopt certain elements of the curriculum design, but also contribute their own design elements to the implementation. Most instances of curriculum use involve some sort of adaptation, be it deliberate or unintentional. Adaptations are characterized by a ‘shared’ responsibility for curriculum design, distributed between the teachers and the materials. (p. 5)

Some reasons that teachers have given for adapting their materials are: to respond to perceived needs of students, to address some of their own interests, and to “cover” material

![Figure 2-1. Categories of Teachers' Curriculum Use from Lloyd (2008)](image-url)
to prepare students for upcoming exams (Ben-Peretz, 1990).

Findings from various studies suggest that beginning teachers’ curriculum use may be distinctly different from that of more experienced teachers (M. W. Brown & Edelson, 2003; Remillard & Bryans, 2004; Sherin & Drake, in press). That is, with the exception of Lloyd’s study, beginning teachers tend to thoroughly pilot rather than adapt curriculum materials when they first begin to use them. In time, teachers’ decisions about adaptation are likely influenced by their philosophical beliefs about mathematics and learning. Other related influences include teachers’ curricular knowledge, “curriculum vision” and whether or not they develop “curriculum trust” (Drake & Sherin, in press).

**Curricular Knowledge, Curriculum Vision, and Curriculum Trust**

The absence of curricular knowledge likely results in a lack of curricular coherence (Schmidt, Wang, & McKnight, 2005). For example, in one study of novice elementary teachers, Westerman (1991) concluded that the novice teachers in the study did not have enough knowledge about the overall curriculum nor sufficient awareness of student characteristics to allow them to “perform an adequate cognitive analysis of the lessons they were planning” (p. 296). The novices rarely discussed integrating the students’ prior knowledge with the present lesson, but, rather, they focused narrowly on the curriculum objectives for their grade. Novice teachers also planned to teach sub-skills without focusing on how the sub-skills fit into the bigger picture (Westerman, 1991). Other research (see, e.g., Drake, 2002) suggests that early career teachers rely heavily on external sources of information (i.e., curriculum materials) to determine what mathematics they should teach,
and to a lesser degree, how they should teach it. This finding points to the level of depth with which beginning teachers “read” their curriculum materials.

Ricoeur (1974) distinguished between teachers’ “reading of” and “philosophical interpretation” of the curricular text. Unlike philosophical interpretations, “reading” is the activity of attempting to understand what is in the text rather than imposing one’s convictions to take a position toward the work (Ricoeur, 1974, p. 160). Teachers’ beliefs about mathematics and how it is learned influence the ways that teachers use and interpret curriculum materials (Stein et al., 2007). When conflicts exist between teachers’ beliefs and the ideas embraced by the curriculum designers, this influence is particularly evident (Stein et al., 2007).

It has been noted that to be successful, teachers need to develop a “curriculum vision” (Darling-Hammond et al., 2005; Zumwalt, 1989). Darling-Hammond et al. (2005) claimed that “well-prepared teachers have developed a sense of ‘where they are going’ and how they and their students are going to get there” (p. 177). Building on this work, Drake and Sherin (in press) argued that a key phenomenon of interest to teachers is how they use curriculum materials, and, subsequently, the development of curriculum vision is an important aspect of teacher expertise. Connected to Ricoeur’s (1974) conceptualization of “reading” and “philosophically interpreting” their texts, the development of a teacher’s curriculum vision is reflected in the ways teachers read and evaluate their curriculum materials (Drake & Sherin, in press).

Over time, teachers may or may not also develop “curriculum trust.” By curriculum trust, Drake and Sherin (in press) referred to “a set of teacher beliefs and practices that reflect an understanding that the curriculum materials, as written, provide a developmental
mathematics trajectory that will support students in achieving the mathematical goals defined by the curriculum vision” (p. 7). In most cases, a curriculum vision must be in place for development of curriculum trust to occur. Curriculum trust can be developed over time, even as teachers adapt their curriculum materials to meet the needs of their particular students and contexts (Drake & Sherin, in press). Brown and Edelson (2003) described such adaptations as “productive changes” to curriculum materials (p. 9). One such productive change could be the supplementation of curriculum materials with additional activities or tasks designed to engage students with mathematics. More specifically, teachers can create supportive learning environments by “knowing the students, the situation, and the content, and then making decisions that support interaction that productively engages students in moving their ideas forward” (Franke et al., 2007, p. 228).

The Role of Tasks in Mathematics Classrooms

Researchers have recently taken a closer look at mathematical tasks because tasks are central not only to students’ learning, but also to their view of mathematics (National Research Council, 1989). Thus, teachers, their students, and the tasks interact in dynamic ways that shape students’ learning (National Research Council, 1989). I discuss this here because, in this dissertation, across time, the teacher supplemented the curriculum materials with additional tasks that were not included in the textbook. In this study, a mathematical task is defined as “a classroom activity, the purpose of which is to focus students’ attention on a particular mathematical idea” (Stein, Grover, & Henningsen, 1996, p. 460).

Traditionally, the kinds of tasks that students are asked to engage with in school mathematics tend to be more procedural than conceptual. As teachers begin to think about
engaging their students with mathematics, they examine the tasks that they are using and often discover that these tasks lend themselves to using a single procedure (Franke et al., 2007). As a result, they often end up supplementing their curriculum materials with additional tasks that they either gather from other materials or they create on their own (NCTM, 1991). Selection of tasks is important because Silver and his colleagues, in their focus on mathematical tasks in the QUASAR (Quantitative Understanding: Amplifying Student Achievement and Reasoning) project, found that worthwhile tasks can engage students in meaningful ways (Silver & Smith, 1996; Silver & Stein, 1996; Stein et al., 1996; Stein & Smith, 1998). Although selecting and setting up a high-level task does not automatically yield high-level engagement, starting with a good task is necessary for providing opportunities to engage students in high-level thinking (Franke et al., 2007). Cognitively demanding tasks allow teachers to engage students in sharing their thinking, comparing different approaches, making conjectures, and generalizing (Silver & Smith, 1996).

So far, in this section, I summarized some of the literature on teachers’ use of curriculum materials. This literature is important because “what [teachers] do with curriculum resources matters” (Lloyd, Remillard, & Herbel-Eisenmann, in press). In order to understand a teacher’s practice, it is important to study how the teacher enacts the curriculum in their classroom. Teachers’ knowledge and beliefs influence the relationship that they develop with the curriculum materials. As teachers work with these materials, they may or may not develop curriculum trust, depending on whether or not their curriculum vision is aligned with their interpretation of the curriculum developers’ vision. In the absence of curriculum trust, teachers are likely to adapt their curriculum materials to better align with
their own vision or to meet the needs of their students. One way that teachers may choose to adapt their curriculum materials is through the addition of activities or tasks not found in their materials. In the section that follows, I shift to a more explicit discussion of the geometry curriculum in the United States. This discussion takes into account the historical place that geometry proof has had in school mathematics and the written curriculum currently available to teachers.

**Proof in the Geometry Curriculum**

The study of geometry proof in school mathematics was one of the most controversial issues debated by educators and mathematicians during the 20th century (Fehr, 1973). Despite many changes in the school curriculum over time, however, the place of Euclidean geometry in school mathematics in the United States has remained relatively constant (Sinclair, 2008). Historically, the 10th grade course has been dedicated to a systematization and extension of geometric knowledge with primary emphasis on the concept of formal deductive proof based on an axiomatic structure (Brumfiel, 1973).

Textbooks were instrumental in making explicit a process for doing proofs (Herbst, 2002b). Textbook authors, Schultze and Sevenoak (1913) were the first to write proofs in two columns of *statements* and *reasons* divided by a vertical line (Shibli, 1932).\(^6\) This form emphasized the necessity of providing a reason for each statement and assisted the teacher in inspecting and correcting student work (Shibli, 1932). The two-column form remained an important scaffold in assisting students with proving as is evident through its existence in

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\(^6\) See Appendix A for an example of a two-column proof.
textbooks across time.\textsuperscript{7} Herbst (2002b) argued that “the two-column proof format brought stability to the geometry curriculum by providing a way to meld the proofs given by the text and the proofs asked from students” (p. 304).

Despite the calls to de-emphasize the two-column form (NCTM, 1989, 2000; Usiskin, 1980), this form continues to thrive in school mathematics (Weiss & Herbst, in review). Even though some of the teachers in Weiss and Herbst’s study reported that their textbooks provided examples of the paragraph proof and flow proof, and the teachers claimed to model these other forms, the two-column form has remained the default mode of proving. A primary criticism of this form is that it portrays an “inauthentic” view of mathematics (Weiss & Herbst, in review). In addition, many have lamented the scarcity of proof outside of the geometry class (see, e.g., Schoenfeld, 1994; Wu, 1996b). Schoenfeld (1994) suggested that “proof is not a thing separable from mathematics as it appears to be in our curricula; it is an essential component of doing, communicating, and recording mathematics” (p. 76). This suggests a lack of connectivity and cohesion within the curriculum.

Recent reform efforts have called for substantial changes in the nature and role of proof in school mathematics. According to Knuth (2002a), the position of proof was elevated significantly by the most recent Standards document. Although documents such as Principles and Standards for School Mathematics ([PSSM] NCTM, 2000) provide a comprehensive overview of the goals and methods agreed upon by many different stakeholders in mathematics education, they do not necessarily provide detail about the typical geometry curricula most students in the United States encounter (Sinclair, 2008). However, Clements

\textsuperscript{7} An example of a two-column proof is provided in Appendix A. Further discussion of this and other proof forms will be found in subsequent chapters.
(2003) claimed that “traditional approaches dominate today’s geometry instruction, and they are ineffective” (p. 169). Traditional approaches, such as the use of the two-column form, are likely perpetuated through the curriculum resources made available to teachers.

In a review that addressed how “reform-oriented” secondary mathematics curriculum materials “stacked up” against the Standards, the authors found that there was a limited treatment of formal proof in all five of the programs that were reviewed (Martin et al., 2001). This finding is important because it indicates that if a goal of the 10th grade course is to teach deductive proof, these “reform-oriented” materials may not very useful in accomplishing this goal.

In the first chapter of Teachers’ Use of Curriculum Materials (Remillard, Herbel-Eisenmann, & Lloyd, in press), the editors explained that the high prevalence of chapter proposals that investigated experienced teachers using Standards-based curriculum materials in elementary and middle school settings were reflective of much of the research currently available in the field. As a study of a novice secondary teacher’s enactment of a non-Standards-based geometry textbook, this dissertation addresses several gaps in the literature on teachers’ use of curriculum materials. Similar to Remillard et al.’s edited book, this study is important because “understanding what teachers [at every level] do with [various kinds] of curriculum materials and why, as well as how their choices influence classroom activity is critical for informing ongoing work surrounding the development of new programs and their adoption in the world of practice” (p. 1). Additionally, the longitudinal nature of this study provides a glimpse into the changes across time in the enacted curriculum as one teacher
teaches proof using a conventional textbook. In the next section, I review literature on the teaching and learning of proof in order to situate this study with the work done by other researchers. First, I provide an overview of the major categories of study, including what we currently know about the teaching and learning of proof. Then, I give particular attention to studies that provide information about what “doing” proofs looks like in school mathematics.

**Mathematical Proof**

A review of the literature on the teaching and learning of proof illustrates that there are a variety of ways that proof has been studied and theorized. People working in the fields of mathematics, mathematics education, and cognitive science have taken an interest in mathematical proof, particularly in the context of geometry instruction (Herbst & Brach, 2006). In the next section, I provide an overview of the categories of study and briefly discuss what is currently known about the teaching and learning of proof. I then afford greater detail to the final category of study because it is most closely related to this dissertation work.

**Overview of the Categories of Study**

One large category of research is about the history of proof, the role of proof, and what counts as a proof (see, e.g., Hanna, 1995; Herbst, 2002b; Hersh, 1993; Stylianides, 2007; Wu, 1996b). Although mathematics educators cannot come to a consensus on a specific definition of proof, many agree that the two primary roles of proof are to convince and to explain. Another broad category of research is about students' and teachers’ beliefs or

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8 I explain my use of the term “conventional” in the next chapter when I give more detail about this particular textbook.
conceptions about proof (see, e.g., Chazan, 1993; Knuth, 2002c; Schoenfeld, 1988). This research shows that both students and teachers have a wide variety of ideas about what a proof is and what counts as a proof. This is not particularly surprising since there does not seem to be a consensus on these things in the field. A third area of research is about proof schemes or categories of justification used by students (Balacheff, 1988; Bell, 1976; Marrades & Gutierrez, 2000; Sowder & Harel, 1998). For example, Sowder and Harel (1998) determined that students’ proof schemes could be organized into three broad categories: externally based, empirical, and analytic. A fourth area of research addressed the cognitive levels of development that were categorized by Dutch educators, Pierre van Hiele and Dina van Hiele-Geldof and the difficulties that students have with proof (van Hiele-Geldof, 1957, 1984/1957; van Hiele & van Hiele-Geldof, 1958). Several researchers have used the van Hiele model as a framework to study classrooms in the United States (see, e.g., Burger & Shaughnessy, 1986; Fuys, Geddes, & Tischler, 1988; Senk, 1985, 1989).

Much of the previous research on proof related to school mathematics points to one unfortunate finding: The teaching of proof in the United States is not done in a meaningful and productive way. More specifically, several findings that provide contextual background for this study are:

- Exposure to proof at the K-12 level is mainly limited to a brief topic taught in the 10th grade (Fuys et al., 1988; Mingus & Grassl, 1999; Newton, in press)

- Teachers’ conceptions of proofs are limited, both at the elementary and the secondary level (Knuth, 2002c; Mingus & Grassl, 1999).

- Students have difficulty with proof, and even after the 10th grade course, most students are not capable of completing an appropriate proof (McCrone & Martin, 2004; Schoenfeld, 1985; Senk, 1985; Williams, 1980).
• Students’ beliefs about proof are often unproductive and unsupportive of a positive attitude toward writing proofs (Chazan, 1993; Schoenfeld, 1988; Solomon, 2006).

• Teachers are not necessarily prepared to teach proof in a manner that is advocated by the Standards (Herbst, 2002a; Knuth, 2002c).

More recently, researchers have begun to examine classroom practices. In these studies, the authors tend to view teaching and learning as a social experience. As pointed out by Herbst and Brach (2006), the work of documenting the role that proof plays in school mathematics points to the importance of studying proof and proving in context. In other words, if we are to better understand the issues and challenges of teaching proof, and subsequently make improvements, we need to study the teaching of proof in the classroom. In the section that follows, I discuss and afford greater detail to this final category of studies since it is most closely related to the study reported here.

“Doing Proof” in School Mathematics

This group of studies is unlike other studies in that rather than theorizing about proof or assessing students’ and teachers’ knowledge and beliefs about proof, the studies outlined here are qualitative studies that provide us with information about what “doing proofs” is like in school mathematics (Herbst & Brach, 2006; Weiss & Herbst, in review). A goal of these studies was to determine what actually happens when teachers teach proof in secondary mathematics. Here I summarize some of these studies because they are especially relevant to this dissertation study. I then discuss how the dissertation study is related to and builds on the work described below.

Schoenfeld (1988) described a year-long case study of the teaching and learning that took place in a 10th grade geometry class. He argued that although the class was “well
taught” (the curriculum was followed, the class was well managed, and students scored well on standardized tests), the students in the class learned some inappropriate and counterproductive ideas about the nature of mathematics as a direct result of their mathematics instruction. Schoenfeld elaborated on four practices that he found to be problematic. For example, students viewed themselves as passive consumers of other’s mathematics, and students perceived that the form of a mathematical proof was what counted most. More specifically, Schoenfeld conjectured that “in the vast majority of high school geometry classrooms across the country,” a proof that was written in any form other than the two-column form would not be accepted as adequate or appropriate (p. 157). In other words, even a paragraph proof, the form that most mathematicians would consider fully adequate, would be deemed inappropriate in school mathematics classrooms.

In her study on teachers’ thinking about students’ thinking in geometry, Lampert (1993) outlined what doing a proof in high school geometry typically entails. According to Lampert, students are first asked to memorize definitions and learn the labeling conventions before they can progress to the reasoning process. They are also taught how to generate a geometrical argument in the two-column form where the theorem to be proved is written as an if-then statement. After students write down the “givens” and determine what it is that they are to prove, they write the lists of statements and reasons to make up the body of the proof. In this context, there is never any doubt that what needs to be proved can be proved, and because teachers rarely ask students to write a proof on a test that they have not seen before, students are not expected to do much in the way of reasoning.

Martin et al. (2005) examined the interplay of a second year teacher and his students in the teaching and learning of geometric proof. Martin and colleagues connected with
Sfard’s (2000) notion that proof is a form of discursive activity and that secondary school students’ classroom conversations are typically a cross breeding between everyday discourse and modern mathematical discourse. This work allowed these researchers to “identify patterns of teacher and student actions that expose some potential connections between teachers’ practice and students' developing understanding of proof” (p. 121). They found that the teacher’s choice to pose open-ended tasks, engage students in dialogue that places responsibility for reasoning on the students, and coach students as they reason, created an environment in which students who actively participate in the classroom discourse are able to engage in proof development activities. In other words, the teacher and the activities of the classroom are critical components to students’ understanding of proof. Martin and her colleague concluded that more research is needed to establish a more robust connection between the pedagogical choices of teachers and individual understandings of proof.

Much of the recent work of Herbst and colleagues is also focused on classroom interactions and proving in geometry at the secondary level (Herbst, 2002a, 2006; Herbst & Brach, 2006; Herbst et al., in press; Weiss & Herbst, in review). In the paragraphs that follow, I review some of these studies as they are theoretically relevant to this dissertation research and have greatly influenced my thinking and the process I used in the analysis of data.

Herbst (2002a) drew from the work of Brousseau (1997), who proposed the idea of the didactical contract. According to Brousseau, a relationship of reciprocal obligation exists between teachers and their students. Each partner in the relationship has the responsibility for managing the acquisition of knowledge and is responsible to the other person to fulfill their own obligations. Herbst used this idea to study the ways in which students and their teachers
negotiate the didactical contract in the acquisition of geometry knowledge that contributes to writing a proof. His analysis suggested that the traditional custom of engaging students in doing formal two-column proofs places contradictory demands on the teacher. Herbst concluded that if the process of proving is to play a more instrumental role in school mathematics (as it does in the discipline of mathematics), then alternative ways of engaging students in this process must be found.

In another study, Herbst and Brach (2006) analyzed transcripts from 29 interviews in which 16 students commented on various aspects of proving in their high school geometry classes. These researchers highlighted some of the ways that “doing proofs” is similar to and different from situations in which students do regular exercises or solve problems. For example, when students provide details and justify their steps in a proof, they exhibit logical thinking and communication skills that are dissimilar from other occasions in which they are required to “show their work” (Herbst & Brach, 2006, p. 106). In addition, the interview data explored in Herbst & Brach’s study helped establish a model of the division of labor between students and their teacher in the situation of doing proofs. Some examples of students’ dispositions about the division of labor in the classroom were: (a) It is customary that the “given” and “prove” will be provided for students; (b) Proof problems usually include a labeled diagram given at the outset of the problem; and (c) Students would only be asked to prove statements that are true. Herbst and Brach argue that in the activity of proving what makes sense for students to do with particular mathematical tasks and what they think they are supposed to do in instructional situations are sometimes incongruent.

Herbst et al. (in press) built on this work by describing instances of student engagement with proof in various geometry courses in a high school. Through this work they
unearthed a system of norms that appear to regulate the activity of “doing proofs” in geometry class. The authors contend that a collection of actions related to filling in the two-column form are regulated by norms that express how labor is divided between teacher and students and how time is organized as far as sequence and duration of events. They argued that despite the superficially different episodes in which doing proofs were observed, there were deep similarities among those events. As an example, I list the first 6 of 25 norms reported by Herbst et al. (in press):

The work to be done, producing a proof, consists of (1) writing a sequence of steps (each of which consists of a “statement” and “reason”), where (2) the first statement is the assertion of one or more “given” properties of a geometric figure, (3) each other statement asserts a fact about a specific figure using a diagrammatic register and (4) the last step is the assertion of a property identified earlier as the “prove”; during which (5) each of those asserted statements are tracked on a diagram by way of standard marks, (6) the reasons listed for each of those statements are previously studied definitions, theorems, or postulates, as well as the “given”…(pp. 14-15)

This model of the instructional situation of doing proofs in terms of a system of norms is helpful to those who wish to investigate what it might mean to create a different place for proof in geometry classrooms (Herbst et al., in press).

Finally, Weiss and Herbst (in review) explored geometry teachers’ perspectives on how the two-column form can be a resource or a constraint in engaging students in proving. Through the analysis of teachers’ responses to a videotaped classroom episode in a secondary geometry classroom, Weiss and Herbst found that the teachers, as a group, held two contradictory dispositions regarding the value of the two-column proof form. First, the two-column form prohibits students from making assertions without immediately providing a justification for it. On the other hand, the form simultaneously acts as a resource enabling students to make claims that are, for the moment, unjustified, for the purpose of seeing
whether the claims may prove to be valuable in the final product. Weiss and Herbst concluded that the two-column form can play a dual role. Although in many cases teachers use the two-column form in a way that limits students’ ability to think flexibly when formulating an argument, this form can also be used in a way that enables greater flexibility in reasoning and proof (Weiss & Herbst, in review).

The documentation of classroom norms in the process of doing proofs is relevant to this dissertation study because the norms provide a frame for examining the practices of the teacher in this study. The fact that the participating teacher in this study was teaching proof for the first time is important because I am able to use the work of Herbst and colleagues to determine whether or not the beginning teacher in this study was carrying out norms that were similar to the ones reported from classrooms of established teachers. This study also builds on the work of Herbst and colleagues as well as some of the other studies by examining possibilities outside of the use of the two-column form. More specifically, as the data will show, some of the norms described in the previous paragraphs were breached by the participating teacher as he attempted to provide students with a more authentic mathematical experience.

The data for Schoenfeld’s (1988) and Lampert’s (1993) studies were collected prior to the publication of the Standards. This is relevant because one would hope that the teaching practices in a pre-Standards classroom would look different from the practices in a post-Standards classroom. These studies allow me to consider the impact of the Standards and, in particular, the call to deemphasize the two-column form, on a contemporary classroom.

Much of the work done by Lampert, Herbst and colleagues, and Martin et al. can be categorized as studies in geometry classrooms that use a discourse lens. With the exception
of Martin et al.’s (2005) study, particular features of discourse that were used to engage students were not described. This study builds on and extends the work of Martin et al. by analyzing discourse moves that are different from some of the ones addressed in their study.

Next, I describe the discourse in traditional mathematics classrooms. I also describe four “discourse moves” that can be used by teachers to create an open space for student participation in the lesson.

**Classroom Discourse**

*‘Who needs the most practice talking in school? Who gets the most?’*

*Exactly. The children need it, the teacher gets it.*


As was mentioned previously, the manner in which mathematics teachers are now expected to teach is likely different from what they experienced as students. Teachers interested in providing opportunities for students to think must tend to the types of tasks they assign, the materials and equipment they use, their grouping practices, the nature of classroom discourse, and their assessment practices (C. L. Thompson & Zeuli, 1999). Because “telling” was the dominant mode of teaching that most teachers experienced in school, in our families, in sports, and in other settings, changing these habits creates additional challenges for today’s teachers. Also, since reform-oriented pedagogies require that teachers adapt a different role in mathematical activity than the one they observed as students, attempting to enact the Standards can pose additional challenges (Clarke, 1997; Smith III, 1996). One challenge for teachers interested in enacting a Standards–based
pedagogy is paying close attention to the ways that one uses language to communicate mathematics.

Language can be viewed as “a highly specific and dynamic tool in human interaction” (van der Aalsvoort & Harinck, 2000, p. 16). It is not just vocabulary and grammar but, rather, a system of resources for making meanings. Mathematics teachers use mathematical language in an attempt to make sense of each topic in a particular way. Students, however, use their own language to construct a view of mathematics that can be very different from the one being communicated by the teacher (Lemke, 1990). This disjuncture points to one reason that communicating mathematics can be so difficult. As Lemke (1990) pointed out for science teaching, we must learn to view mathematics teaching as a social process and find ways to bring students into the community of people who talk mathematics. Despite the fact that learning in school is done primarily through language, however, the language of school tasks is seldom explicitly discussed or taught in classrooms (Schleppegrell, 2004) or in teacher preparation programs. Drawing on research that has described linguistic features of classroom discourse, I argue that teachers’ language use can close down or open up space for student participation in lessons.

By discourse, I refer not only to the full range of language use that can be employed in a classroom (Lee, 2006), but also to the ideas and ways of thinking about language (D. Barnes, 1990; Edwards & Mercer, 1987). I also include gestures, pictures, and visual images – anything deliberately or artificially employed as a sign (Dewey, 1933; O'Halloran, 2000; Rasmussen, Stephan, & Allen, 2004). However, I focus primarily on the oral language in Chapter 7. I begin by describing the discourse pattern that is typically found in mathematics classrooms. Then, I describe four particular features of classroom discourse. I argue that if
teachers pay attention to the forms of these four features of classroom discourse, they can
open up space for students in the classroom. I begin by describing the discourse pattern that
is found in typical mathematics classrooms.

**Classroom Talk in Mathematics Classrooms**

Classroom behavior is guided by rules or norms that are implicitly taught, tacitly
agreed upon, and cooperatively maintained (Cazden & Mehan, 1989). A number of
researchers have summarized some of the implicit rules of classroom discourse (D. Barnes,
1976, 1990; Cazden, 2001; Dewey, 1933; Edwards & Mercer, 1987; Mehan, 1979). For
example, in their succinct summary of implicit rules, Edwards and Mercer (1987) noted: (a)
It is the teacher who asks the questions; (b) The teacher knows the answers; (c) Repeated
questions imply wrong answers. (p. 45). The most dominant classroom discourse pattern has
been described as an unfolding series of *initiation-reply-evaluation* (IRE) *sequences*
(Cazden, 2001; Edwards & Mercer, 1987; Mehan, 1979). This pattern places the teacher
firmly in control of the talk by dictating the question, selecting the interlocutor, and voicing

As Dewey (1933) pointed out, teachers have a habit of monopolizing continued
discourse, and many, if not most, teachers would be surprised if informed of the amount of
time that they have talked as compared with their students. Student talk in classrooms is often
confined to answering through brief phrases or in single disconnected sentences. Exposition
and explanation are often reserved for the teacher who frequently hints at the answer and
then “amplifies what he supposed the child must have meant” (Dewey, 1933, p. 245). This
75-year old description of classroom discourse provided by Dewey remains an appropriate
descriptor of the kind of classroom talk that occurs in many contemporary classrooms. More recently, however, teachers and researchers have begun to explore and research discourse forms that differ from the IRE sequence. In the next section, I describe and compare the ways that teachers can open up or limit discourse during whole-class talk.

**Open versus Closed Discourse**

Many studies tend to refer to any kind of teacher-student talk as *discussion* (Dillon, 1984). Several researchers, however, have distinguished between two different teacher-student talk forms: recitation and discussion (see, e.g., Dillon, 1984; Gall & Gall, 1976; Nystrand, 1997; Stodolsky, Ferguson, & Wimpelberg, 1981). Dillon (1984) described *recitation* as follows:

> *Recitation* describes the familiar form, which is characterized by (among other aspects) recurring sequences of teacher question plus student answer, where students “recite” what they already know or are coming to know through the questioning. (pp. 50-51, italics in original)

In contrast, *discussion* describes group interactions that are *not* of this character, where students and the teacher “discuss” what they do not know (Dillon, 1984). For Stodolsky et al. (1981), discussion involves longer exchanges and questions soliciting students’ opinions and thoughts rather than just correct answers. These exchanges occur both among students as well as between teacher and students. In a review of research on classroom discussions Dillon (1984) concluded that true discussions are difficult to conduct and rarely found in classrooms. Similarly, Grant and McGraw (2006) found that fostering student-to-student discussions was challenging.

A related conceptualization of talk forms is the distinction between *univocal* and *dialogic* discourse. As with recitation sequences, discourse characterized as “univocal”
(Lotman, 1988) emphasizes “the transmission of information from a sender to a receiver” (Wood, 1998, p. 168). In contrast, like discussion, text that acts as a generator of new meanings has a “dialogic” function (Wertsch, 1991). It serves the purpose of “enabling students to generate new meanings for themselves” (Wood, 1998, p. 168). A dialogic view of communication is one that is advocated by the Standards.

For the purpose of this research, “discussion” will be considered a form of teacher-student talk that opens up space for student participation, and a “recitation sequence” will be viewed as a talk form that is more closed. Bridges (1979) described various ways that true discussions must be open. For example, the discussion must be open to all arguments, the discussion is open to any person, and the discussion is open-ended, not required to come to one single conclusion (Bridges, 1979). Acknowledging that talk formats operate on a continuum, many researchers have pointed out that most classrooms operate somewhere between recitation and discussion (Herbel-Eisenmann, 2001) with the discourse being at times univocal and dialogic (Knuth & Peressini, 2001). Cazden (1988) contended that within a matter of moments, a lesson can move from recitation to discussion, and the activity that students are engaged in can determine the form of the lesson. There are various ways that teachers can facilitate open discourse.

**Discourse Moves**

Although the traditions of school mathematics already position students and teachers in certain ways relative to each other (e.g., teachers are viewed by their students as the authority of the mathematics), discourse moves made within particular instances of the discourse can substantiate and have the potential to alter these structures (Wagner & Herbel-
Eisenmann, 2008). Krussel, Springer, and Edwards (2004) defined a *discourse move* as “a *deliberate* action taken by a teacher to participate in or influence the discourse in the mathematics classroom” (p. 309, emphasis added). Despite the fact that the move may have been made deliberately, many of the consequences of a teacher’s discourse moves may be unintended and even potentially “at direct odds with the teacher’s purpose” (Krussel et al., 2004, p. 307) or beliefs. Chapin, O’Connor, and Anderson (2003) similarly write about “talk moves” that support mathematical thinking and productive mathematical talk. And although Chapin et al. state that each move can serve various purposes, I would argue that not all of these purposes support mathematical thinking, nor do they necessarily support productive mathematical talk. In the same vein, I would argue that not all of these moves are necessarily “deliberate” as claimed by Krussel et al. (2004). Therefore, I use the term “discourse moves,” but I define these as: actions taken by the teacher that influence the discourse in the classroom. These actions may or may not be deliberate, and I consider the absence of talk (i.e., silence or wait time) to be a discourse move. In the sections that follow, four discourse moves are discussed. I argue that these four moves can have the function of opening up or closing down dialogue. Here, I discuss revoicing, pronoun use, pace and wait time, and questions. I begin by defining and discussing revoicing.

**Revoicing**

Simply stated, *revoicing* (a term coined by O’Connor & Michaels (1993)) is the act of reuttering someone else’s talk (Franke et al., 2007). Both Cazden (2001) and Lemke (1990) have noted that the final move in an IRE sequence provides the teacher with an opportunity for “recontextualization” or “retroactive contextualizing,” respectively. This discourse move,
now called “revoicing,” has been defined as “the reuttering of another person’s speech through repetition, expansion, rephrasing, and reporting” (Forman, Larreamendy-Joerns, Stein, & Brown, 1998a, p. 531; O'Connor & Michaels, 1993). Revoicing serves many goals, including: to create alignments and oppositions in an argument; to clarify or amplify content; to explain reasoning further; to introduce particular ideas; or to redirect discussions (Forman et al., 1998a; O'Connor & Michaels, 1993). These functions noted in the literature could potentially open up space for student participation. For instance, consider the following example of revoicing:

Teacher: Can somebody explain how they solved the problem? Jenny?
Jenny: I divided the two numbers and then I added 13.
Teacher: Jenny said that she divided and then added 13. Did anybody else solve this problem differently or do what Jenny did?

In this example, rather than the teacher offering an immediate evaluation, she revoiced Jenny’s answer in order to amplify it and then solicited other strategies. As other students offer their solutions, alignments or oppositions with Jenny’s solution strategy can be created.

Studies of teachers in mathematics classrooms have shown that revoicing can often play a supportive role both in terms of mathematical support for ideas and the development of students’ identities around mathematics (Franke et al., 2007). In this sense, revoicing can open up space for student participation in the discourse by placing students on more equal footing with their teacher by “allowing them the opportunity to evaluate other interpretations or extensions of their utterances” (R. A. J. Brown & Renshaw, 2000, p. 65). Similarly, revoicing can make students’ ideas available to others, give students time to hear ideas again, and position a student’s claim with respect to another student’s claim in order to create the
basis for an ongoing discussion (Chapin et al., 2003). Each of these consequences can give students more student agency than they would have in more teacher-centered talk formats.

We can never know for sure, however, whether or not students are interpreting the act of revoicing as a discourse move that is creating space for student participation. Other consequences of revoicing could serve the unintended purpose of closing down the discourse. For example, the classroom teachers in Herbel-Eisenmann, Drake, and Cirillo’s (in review) study realized that revoicing could have negative functions, such as: controlling students, putting the teacher (rather than the student) at the center of an idea, or allowing students not to listen to each other. Thus, revoicing can accomplish many goals, and can either support or limit productive discourse (Franke et al., 2007). Although we can never really know for sure how students are interpreting the revoicing moves of their teachers, I argue that revoicing is a more open form of discourse than IRE because it is usually less evaluative, and, therefore, may encourage more student participation.

**Pronoun Use**

Various researchers have studied teachers’ use of first and second person pronouns in classrooms (Fortanet, 2003; Pimm, 1984, 1987; Rounds, 1987a, 1987b; Rowland, 1999, 2000). In this review, I specifically focus on the use of the pronoun “we” because it is most relevant to this study. Pimm (1984, 1987) reviewed various uses of “we” both in and out of school. He pointed out that “we” does not always mean “a plural group which includes the speaker” as recognized by linguists (1984, p. 39). For example, a teacher could tell her students, “This is how we solve this problem.” In fact, the teacher, herself, may not have solved the problem in the same way that she proposed to her students. The use of we rather
than I could also be intended as a clue to generality (Pimm, 1987). In this case, the teacher could be referring to the we who solve mathematics problems or we mathematicians. Similarly, textbook authors frequently use we in statements such as “we have just found that…” (Pimm, 1987).

Some researchers have distinguished between the “inclusive-we” (I + you) and the “exclusive-we” (I + they) (Haas, 1969; Pimm, 1987; Rounds, 1987b; Spiegelberg, 1973). Both of these uses include the speaker (I). Teachers can use the inclusive-we to identify themselves as members of the group (e.g., What should we do next?). On the other hand, teachers can use the exclusive-we and align themselves with mathematicians (e.g., We call this a prime number). Through her work Rounds (1987a) concluded:

Thus, we is an egalitarian pronominal choice: Teachers may include themselves in the group consisting of mathematicians or in the group consisting of students without change of referential sign. By using we in this fashion, the teacher can avoid constantly reminding students of their relative differential status. (p. 649)

That is, through the use of we, teachers can signal solidarity with their students and covertly maintain a certain semblance of power without changing pronoun use. Therefore, some research (Bailey, 1984; Rounds, 1987b) suggests that by developing an atmosphere of communality through strategic use of pronouns, teachers can encourage their students to work side by side with them on mathematical problems.

As was previously mentioned, one use of we rather than I could be a clue to generality (Pimm, 1987). On the other hand, we can also be employed as a means of spreading responsibility within members of the group (Pimm, 1987). For example, teachers’ use of we could be viewed as an invitation that opens up the discourse to the students in the classroom (e.g., “What should we do next?”). Successful classroom discourse can, therefore,
be characterized by what emerges from the teachers’ ability to “develop an atmosphere of cooperative interaction and consensus – a sense of working together to achieve a common goal” (Rounds, 1987a, p. 666).

Questions and Interaction Patterns

The primary function of a discussion is to construct group knowledge (Bridges, 1987), and questions are the key to fruitful discussions. A single, well-formulated question can be sufficient for an hour’s discussion (Dillon, 1983). However, many studies have shown that while teachers ask a lot of questions, these questions frequently call for specific factual answers, resulting in lower cognitive thought (Gall, 1984; Perrot, 2002). The hundreds of questions asked by the typical teacher on a typical day reflect the popularity of the recitation method (Gall, 1984). The types of questions asked are central to the type of learning that takes place in the classroom (Manouchehri & Lapp, 2003). More specifically, some interaction patterns promote deeper mathematical thinking than others (Herbel-Eisenmann & Breyfogle, 2005; Martens, 1999), and skillful questioning of student thinking can provide the teacher with valuable knowledge about students’ developing mathematical ideas (Martino & Maher, 1999).

Many researchers have offered different ways of categorizing and describing questioning patterns (Ainley, 1987, 1988; Bellack, Kliebard, Hyman, & Smith, 1966; Herbel-Eisenmann & Breyfogle, 2005; Manouchehri & Lapp, 2003; J. Smith, 1986; Wood, 1998). Some question-types open up discussion, while others are more “closed” (Ainley, 1987). For example, one type of question takes the form of part-sentences “left hovering in mid-air for the student to supply the missing word or phrase” (Ainley, 1987, p. 24). An example of this
type ‘fill-in-the-blank’ type of question is: “This polygon has three sides so we call it a …”

This kind of question is closed, both because it relates to matters of established fact and because the teacher has one “right” answer in mind. On the other hand, it creates the illusion of participation and cooperative activity. It is also one way to hold students’ attention (Ainley, 1987). This kind of questioning is a linguistic activity that is almost entirely restricted to classrooms or teaching situations (Ainley, 1987; Mehan, 1979). While both “open” and “closed” styles of questioning play important roles, Smith (1986) advocated for more open questions:

Both open and closed questions have a part to play, but the open question with its multiplicity of possible correct responses tends to be under-used. This is probably because it is more difficult to manage in the classroom context. Yet the benefits of opening up discussion and maximising the chance of individuals being able to contribute are often greater with a more open question. (p. 47)

In the sections that follow, I briefly describe various types of interaction patterns found in the literature that are relevant to this study. I begin by discussing focusing and funneling.

**Focusing and funneling.** A common form of interaction that limits students’ responses is called *funneling* (Bauersfeld, 1980; Wood, 1998). In the funnel pattern, an incorrect answer is the starting point from which the teacher leads a student through a series of explicit questions until the student provides the correct answer (Wood, 1998). Wood (1998) provided this example of funneling which followed after a student (Jim) was asked to give the answer to 9 + 7.

Jim: 14.
Teacher: OK. 7 plus 7 equals 14. 8 plus 7 is just adding one more to 14, which makes ___? (voice slightly rising).
Jim: 15.
Teacher: And 9 is one more than 8. So 15 plus one more is ____?
Jim: 16. (pp. 170-171)
In this line of questioning, the teacher asked Jim closed questions that were similar to the fill-in-the-blank type questions described by Ainley (1987). This recitation pattern is univocal in the sense that “the teacher is engaged in cognitive activity and the student is merely answering the questions to arrive at an answer, often without seeing the connection among the questions” (Herbel-Eisenmann & Breyfogle, 2005, p. 485).

An alternative to the funnel pattern is the focusing pattern (Wood, 1998). This pattern was described as follows:

A focusing-interaction pattern requires the teacher to listen to students’ responses and guide them based on what the students are thinking rather than how the teacher would solve the problem. This pattern of interaction serves many purposes, such as allowing the teacher to see more clearly what the students were thinking or requiring the students to make their thinking clear and articulate so that others can understand what they are saying. (Herbel-Eisenmann & Breyfogle, 2005, p. 486)

Because this type of interaction values student thinking and encourages students to participate in the discussion (Herbel-Eisenmann & Breyfogle, 2005), the focusing pattern is more dialogic, and, therefore, a more open interaction pattern than funneling.

**Tag questions.** A tag is a question that is added to a declarative statement to either indicate uncertainty or to request confirmation (Dennis, Sugar, & Whitaker, 1982). An example of a tag question that requests confirmation is: “So the first thing that we have to do is draw a diagram, right?” When the source of a tag question is low in credibility, then tag questions tend to decrease persuasion (Blankenship & Craig, 2007). However, when the source is credible (e.g., the teacher in the classroom) and the tag is paired with a strong argument, then the listener is persuaded more so than when a tag is not used (Blankenship & Craig, 2007). Because the credible person (teacher) presumably knows what s/he is talking about, the message recipients (students) are reminded of the source’s credibility and perceive
the tag question as anticipating an affirmative response (Blankenship & Craig, 2007). In other words, used by credible sources, tag questions are meant to persuade and cajole agreement from the listeners. The use of tag questions, thus, serves a univocal rather than dialogic function. This type of questioning closes off communication between student and teacher and should not be viewed as a true question in the sense that the teacher is not seeking information from the students.

If a teacher’s goal is to open up discussion by providing an atmosphere where students have time to organize thoughtful responses, then it is imperative that an appropriate questioning method be utilized. The art of questioning, however, may take years to develop as it requires an in-depth knowledge of both mathematics and students’ learning of mathematics (Maher & Martino, 1996). Once it is enacted, the teacher has a powerful tool to support students as they work to build mathematical ideas (Maher & Martino, 1996). One skill that teachers can use to encourage longer and more thoughtful responses is the use of wait time.

**Pace and Wait Time**

An important variation in classroom discourse is the temporal relationship between a teacher’s question, the student’s response, and the teacher’s subsequent feedback (Cazden, 2001). Rowe (1974) identified two important points in the IRE sequence. *Wait Time I* was identified as the period that followed the teacher’s question. *Wait Time II* is the period that followed the student’s answer before the teacher began speaking again. In her review of the literature, Rowe (1974) concluded that the quality of discourse can be improved markedly by increasing Wait Time I and II. She found that by increasing wait time, students were more
likely to react to each other and the length of students’ responses increased. In addition, extending wait time was shown by Tobin (1986) to improve the cognitive level of questions asked by teachers in mathematics classrooms. From this seemingly small change in pace, many important social and cognitive outcomes have been noted (Cazden, 2001; Rowe, 1986).

If teachers are interested in having their students respond to questions with more than simple answers, they must give their students time to think (Lee, 2006). Students need at least three to five seconds of wait time in whole-class questioning, and at least 30 seconds if they are supposed to speak to a partner (Lee, 2006). Although the research on the value of wait time is clear, it can be difficult to use this discourse move consistently (Cazden, 2001; Chapin et al., 2003). Similar to wait time, “deliberate silence” is one of seven possibilities offered by Dillon (1981) as alternatives to questioning. The goal of this discourse move is to remain silent after a student response to encourage either the student to continue speaking or another student to enter the conversation. In either case, the discourse would then be more like discussion (an “open” form of discourse) than recitation.

In this section, I argued that the discourse moves described here can have the effect of either opening up or closing down discourse. When the teacher closes the discourse through lack of wait time or tag questions, for example, then the talk will be more univocal and the format will be more like recitation. When the discourse is more open or dialogic, then whole-class talk format can move away from recitation toward discussion. Although both of these talk formats can be seen as appropriate depending on the instructional goals and the context of the classroom activity at a specific point in time, the latter goal is closer to the heart of Standards-based instruction. This dissertation study contributes to this line of research because the data suggest that across time, Matt’s deliberate attention to his discourse helped
him create space for increased student involvement through the particular discourse moves described here.

**Summary**

In this chapter, I reviewed literature from four domains of inquiry: the induction phase of teaching, issues related to curriculum in school mathematics, mathematical proof, and classroom discourse. Although I reviewed this literature as separate categories, these four domains of inquiry are very much related. First, it has been said that mathematical proof is a form of discourse (see, e.g., Wheeler, 1990). Through the literature, I argued that teachers can use particular discourse moves to open up space to involve students in the discourse. Additionally, literature was reviewed that highlighted the role that mathematical tasks and curriculum materials play in the quality of the discourse. Research that addressed the ways in which teachers’ beliefs and curriculum visions can impact the enactment of these materials was also discussed. Last, the literature reviewed on beginning teachers pointed to a need to better support teachers as they are learning to teach. More specifically, there is a need to learn more about teachers in specific content areas during the induction phase.

At the same time there has been a call to bring the curriculum of school mathematics closer to the discipline. Among other things, this requires adjustments to the ways in which teachers facilitate the construction of knowledge. One such recommendation involves an emphasis on conjecturing and proving through productive discourse. Although many of the proof-related studies reviewed here were related to discourse, some of the studies (i.e., Lampert, 1993; Schoenfeld, 1988) were conducted before the publication of the *Standards*, and other studies (e.g., Herbst & Brach, 2006; Weiss & Herbst, in review) were based on
what teachers and students reported was happening in their geometry classrooms. Martin et al. (2005) studied interactions between a teacher and his students by analyzing the discourse over the course of a few months. There is a need, however, for more studies that look into geometry classrooms and examine the work of teachers in practice (Herbst, 2006; Knuth, 2002c). Additionally, “research is needed to understand the conditions in which teachers work and how those conditions impact the mathematical work that teachers can sustain” (Herbst, 2006, p. 314). One such condition involves the curriculum materials that are made available to them.

This study, therefore, addresses several gaps in the literature presented here. Through the design of a longitudinal case study, I sought to understand the issues and challenges that a beginning teacher encounters as he learns to teach proof. By studying his discourse practices and his use of curriculum materials over time, I sought answers to the following questions: How did this teacher’s practice change over time, and why, from his perspective, did he make these changes? In the next chapter, I describe the method and procedures used to carry out this study.
CHAPTER 3: METHOD AND PROCEDURES

In this chapter, I describe my methodology by first discussing the theoretical perspectives that inform this work. I then describe the context of the study, some reflections on the fieldwork, the collection of data, and the analytic procedures. The methodology described here allowed me to investigate one teacher’s development as he learned to teach proof. More specifically, I was able to attend to the ways in which Matt’s teaching practice changed across time and investigate why, from his perspective, these changes were made. My interest in this study developed through my own experiences as a student of mathematics and as a secondary teacher who encountered dilemmas and challenges when teaching proof. I begin by describing the theoretical perspective that informed this study.

Theoretical Perspectives

Due to the longitudinal nature of this study, I found it appropriate to employ ethnographic techniques, as well as discourse analysis, to study Matt’s induction into the teaching of geometry proof. Here, I briefly discuss ethnography, discourse analysis, and why pairing these two methodologies was appropriate for this study. In subsequent sections, I provide detail about how the methodologies informed the process of data collection and analysis.

Ethnography

Ethnography has become a popular methodology in qualitative research inquiry. Rather than trying to understand “strange” and “exotic” cultures, ethnographers outside of anthropology traditionally are interested in familiar cultures and people in their everyday
work, community, and institutional settings (Prasad, 2005). Schools and classrooms are such everyday settings:

From an ethnographic perspective, a classroom culture is always constructed in our classrooms whether we realize it or not. Students and teachers come together…every weekday for nine months and create patterned and shared ways of interacting, understanding, and believing (Bloome, 1985). An ethnographic perspective provides a lens to understand these particular patterns of classroom life which often become invisible because they become so regular, patterned, and ordinary. (Frank, 1999, pp. 2-3)

Ethnographic research has proved to be popular and successful in developing an understanding of the social and cultural processes in educational settings (Jeffrey & Troman, 2004).

The term *ethnography* is sometimes used in a superficial manner to describe any type of lengthy period of field observation (Prasad, 2005). While an appropriate amount of time in the field is necessary to reveal both the depth and complexity of relations and social structures (Jeffrey & Troman, 2004), not every study involving lengthy periods of field observation should be considered an ethnography. Claiming that methods alone are not enough, Wolcott (2002) urged researchers to distinguish between “borrowing ethnographic techniques in their data gathering and doing ethnography” (p. 41, italics in original). Because I did not seek to understand the relationship between culture and the behavior of a group, and because I took more of an observational than participant role, I save the label of *ethnography* for appropriate studies. Rather, I claim that I borrowed and adapted ethnographic fieldwork techniques (e.g., observation and interviewing) to conduct this research. In addition to using the tools of ethnography, I also made use of analytic methods from discourse analysis.
**Discourse Analysis**

A major focus of this study is analyzing the teacher’s pedagogical development across time. It has been noted that discourse analysis can be used to study growth in teachers’ knowledge. Although I am not studying teachers’ knowledge, per se, this quotation points to the value of discourse analysis for research in mathematics education:

[Discourse analysis] is a promising approach because teachers’ use of mathematical language is both an indicator and target of growth in knowledge and skills; teachers need, in classrooms, to be able to use mathematical terms accurately and precisely…. [however], such measurements do not scale easily, and they require finely calibrated frameworks for interpreting and analyzing discourse patterns… That said, this remains an intriguing possibility, one that should be investigated formally. (Hill et al., 2007, p. 137)

Despite the recent emphasis on discourse and communication (both in the literature and in the *Standards*), communication about mathematical ideas may not always be recognized as a central part of mathematics education due to the fact that mathematics is so often conveyed in symbols, oral, and written communication (NCTM, 2000). In this sense, mathematical language can be considered to be multisemiotic because the linguistic, visual and symbolic semiotic systems differentially contribute to the meaning of the text (O’Halloran, 2000). The functions of these three semiotic resources in mathematics can be summarized as follows:

- The mathematical symbolism contains a complete description of the pattern of the relationship between entities.
- The visual display connects our physiological perceptions to this reality.
- The linguistic discourse functions to provide contextual information for the situation described symbolically and visually. (O’Halloran, 2000, p. 363)
In this study, although the emphasis is on the linguistic discourse, I attend to all three of these semiotic resources by not only analyzing the spoken words, but also the board work created by the teacher which frequently made use of mathematical symbolism.

**Pairing Ethnography with Discourse Analysis**

One area of connection between ethnography and discourse analysis is their shared emphasis on culture. If school ethnographers are interested in the familiar culture of the institutional school setting, and discourse analysts view cultures, social groups, and institutions as ones that are produced, reproduced, and transformed through human activities (e.g., language), then it seems appropriate to make use of both of these methodologies. The tools of ethnography can help researchers gain a “thick description” (Geertz, 1973) of a particular mathematics classroom culture, and discourse analysis allows researchers to look at the language patterns carried out in this culture. Together these techniques can be valuable in shedding light on ways that we can understand the teaching of proof.

This study was designed as a case study in the sense that I was interested in “gain[ing] an in-depth understanding of the situation and the meaning for those involved” (Merriam, 1998, p. 19). In this case, by adapting ethnographic field techniques and analyzing the classroom discourse, I sought to understand how one particular teacher “became” a geometry teacher. Lemke (1998) pointed out that when rich contextual information can be factored into the analysis of text, discourse analysis produces its greatest insights:

Longitudinal designs or case studies are well suited for discourse analysis methods because we can learn a great deal about a particular class, seeing repeated patterns within the data and a variety of strategies which create variations on those patterns. (Lemke, 1988, p. 1184)
This statement provides further support for my claim that a discourse lens is appropriate for this work since it is both longitudinal and a case study.

**Guiding Questions**

The questions that guided this study include:

- How did Matt’s teaching of geometry proof change across three years?
- To what did Matt attribute these changes?

Acknowledging that these questions are broad, I discuss here some of the things that I attended to over the course of the study. As stated earlier, I was interested in how Matt developed pedagogically. I was also interested in understanding how his teaching would change as he acquired curricular knowledge after teaching the course for the first time. Additionally, I was aware that some of the changes that I noticed were likely due to Matt’s participation in a larger discourse study. I was, therefore, also interested in understanding if and how this participation impacted Matt’s practice. Next, I describe the context of this study, attending to the case study teacher, the research site, and the mathematical context.

**Context of the Study**

**Gaining Access to the Site**

Through my work as a research assistant, I met the case study teacher, Matt, in September, 2004. Matt volunteered to participate in a 5-year study (Herbel-Eisenmann, PI) which was funded through an NSF grant (#0347906). This study was designed to learn how doing action research on one’s classroom discourse might impact a teacher’s beliefs and practices over time. Eight teachers from seven different schools in a Midwestern state were
selected to vary gender, context of teaching situation, certification level, and years of teaching experience. Other papers that provide additional detail on this study include: Cirillo & Herbel-Eisenmann (in review); Herbel-Eisenmann et al. (2008); Herbel-Eisenmann et al. (in review); Herbel-Eisenmann & Schleppegrell (2008); Wagner & Herbel-Eisenmann (2008).

Baseline data related to each participant’s practice were collected during the 2005-2006 academic year. Participating teachers were then involved in study groups where they read and discussed discourse-related book chapters and articles from research and practitioner journals. As a participant in this study, Matt spent approximately 54 hours in the study/discussion group. It was through my work as a research assistant on this project that I gained access to the research site. Some of the data from this larger discourse study were used for this dissertation study. I refer to these two studies as the “Discourse Study” and the “Proof Study,” respectively.

The Research Site

The high school where Matt taught geometry was located in a city (approximately 50,000 residents) in a metropolitan area in a Midwestern state. There was a 15 to 1 student/teacher ratio in the high school, and the ethnic makeup of the school was as follows: 85.6% White, 4% African American, 4.7% Asian, and 5.5% Hispanic. Approximately 1900 students attended this school which housed students in grades 10-12. This school was chosen by the U.S. Department of Education as a “model American school.” This is somewhat
ironic, given that the school population is not reflective of the U.S. population. Facilities in this district include an Olympic-size indoor swimming pool, a weight room, an aerobics room, and a new, state-of-the-art 8000-seat football stadium. I mention this only to point out that the district had resources that could be considered above average for most schools.

The Case Study Teacher

During the first year of the Discourse Study (2004-2005), Matt taught seventh and eighth grade mathematics in a middle school in the aforementioned Midwestern state. The 2004-2005 school year was Matt’s first year as an in-service teacher. In the second year of the Discourse Study (Year 1 of the Proof Study [Y1])

I refer to the three years of the Proof Study as Y1, Y2, and Y3 throughout the rest of this dissertation.

Matt changed schools and began teaching geometry and precalculus at a high school. At this stage in his teaching, I considered Matt to be a novice teacher because even though Y1 of the Proof Study was actually Matt’s second year as an in-service teacher, he was in his first year at a high school. In this sense, almost everything was new to Matt, who was still in the induction phase: The high school was located in a different town, in a different school district, with different students, and, for Matt, the curriculum was also new. It was during this school year as I collected baseline data in Matt’s geometry classroom for the Discourse Study that I decided to continue observing him as he learned to teach proof. There are several reasons why Matt was an appropriate choice for this study. After providing more background information, I discuss the choice of Matt as the teacher for this case study.

Before Matt graduated from high school, he completed Calculus III at a university. The fact that Matt was two years ahead of the ‘typical’ student in school mathematics

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9 I address the issue of why the participant was chosen despite these atypical school demographics in the next section.
10 I refer to the three years of the Proof Study as Y1, Y2, and Y3 throughout the rest of this dissertation.
provided evidence of his propensity for mathematics. Prior to his first year of teaching, Matt earned a Bachelors degree in mathematics and a Masters degree in Mathematics Education. The preparation path that Matt took is said to be the best-case-scenario for beginning teachers. More specifically, it has been noted that students who complete a Bachelors degree in their content area followed by a fifth year for certification tend to bring to the profession a solid background in content, preparation in pedagogy, and various field experiences (Gay, 1994).

Despite the fact that Matt was a beginning teacher, he was highly respected for his mathematical content knowledge by the other teachers in the Discourse Study (see Cirillo & Herbel-Eisenmann (in review) for more information). Matt, himself, indicated that he was confident in his knowledge of mathematics. For example, in an interview, Matt said, “The mathematical knowledge, I have the sense that I know a lot more mathematics than most other people who teach high school math” (Interview D-3 11, 4/14/06). Matt’s strong learning disposition inspired him to read texts on the history of mathematics and pure mathematics, as well as mathematics education literature in his spare time. His participation in the 5-year Discourse Study at the beginning of his career not only provided further evidence of his learning disposition and his interest in improving his practice, but also his confidence in being able to manage his time at the beginning of his teaching career.

Matt was an appropriate participant for this study for several reasons. As noted in the literature review, many beginning teachers struggle with their content knowledge as well as issues of classroom management. Content knowledge did not seem to be an issue for Matt,

\[11\] All interviews are labeled using the letters “P” and “D” to differentiate between interviews that were conducted for this Proof Study and the larger Discourse Study, respectively.
and he took a laid-back approach to classroom management. During the three years that I was in his classroom, I never observed Matt become flustered by student behavior, nor did I observe students being outright disrespectful or uncooperative. This is not too surprising because it has been noted that student disruption of teaching tends to occur less frequently in academically successful schools\textsuperscript{12} than in others (Rosenholtz, 1989).

Taken together, Matt’s laid-back approach to classroom management, coupled with his strong mathematical content knowledge, made him an atypical beginning teacher. The choice of Matt as a participant, therefore, provided me with a teacher who could be viewed as a competent novice working in a situation where he was not bogged down by discipline issues. Having to pay such little attention to discipline issues is atypical of the new teacher experience (e.g., see Hogan, Rabinowitz, & Craven III, 2003; Rosenholtz, 1989; Wildman et al., 1989). In addition, there is evidence to suggest that having a strong content background enables teachers to deal flexibly with the mathematical demands of a new curriculum (Lloyd, in press; Lloyd & Wilson, 1998).\textsuperscript{13} With these things in place, Matt was able to focus primarily on enacting the curriculum and teaching his students. Stripping away some of the typical concerns of new teachers, therefore, allowed me to study Matt’s pedagogical development in a way that is not typically possible.

Additionally, in a review of the literature on teaching and learning proof, Harel and Sowder (2007) concluded: “Many teachers are unlikely to teach proof well since their own grasp of proof is limited. It is important to determine better the extent to which teachers are

\textsuperscript{12} Evidence that this school was academically successful includes: (a) approximately 88% of Advanced Placement (AP) students receive better than a three on AP Exams, (b) an ACT Mean Score: Composite 24.3, and (c) approximately 87% of students continue their education after graduation.

\textsuperscript{13} The teacher in Lloyd’s studies was an experienced teacher implementing Standards-based curriculum materials. This was not the case for Matt, but, for him, the curriculum was “new” in the sense that he had not had any prior experience with it.
equipped to deliver a curriculum in which proof is central” (pp. 836-837). In the case of Matt, because his grasp of mathematics (and therefore proof) did not seem to be limited, I was able to tease out some of the curricular issues and pedagogical dilemmas that other teachers might face as they learn to teach geometry proof in an era of standards-based reform. The case of Matt, therefore, can result in data from which generalizations to theory are possible (Eisenhart, 1999). In the next section, I describe the organization of the school and the students in Matt’s focus classes. I then discuss the mathematical context of this study.

**School Context and Students**

During all three years of this study, Matt taught geometry to students in the middle level of three tracks. The students in these classes were on grade level, meaning that they were not accelerated or behind grade level. During each year of the study, Matt selected a focus class for observation that he believed to be fairly typical of geometry classes in that particular school. The scheduling of classes was set up by semesters. Since the observations for this study were conducted during the first semester only, the student population remained fairly stable across the semester. The classes ran on a modified block schedule. Students met for 44 minutes three days a week and 79 minutes once a week. For the purpose of this study, all of the lessons that I selected for analysis took place during 44 minute periods. The number of students in each class across the three years are given in Table 3-1.

<table>
<thead>
<tr>
<th>School Year</th>
<th>Female Students</th>
<th>Male Students</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y1 2005-2006</td>
<td>13</td>
<td>14</td>
<td>27</td>
</tr>
<tr>
<td>Y2 2006-2007</td>
<td>12</td>
<td>14</td>
<td>26</td>
</tr>
<tr>
<td>Y3 2007-2008</td>
<td>9</td>
<td>10</td>
<td>19</td>
</tr>
</tbody>
</table>
Mathematical Context

In this section, I describe two sets of curriculum materials. First, I describe the *Mathematics in Context* ([MiC]; National Center for Research in Mathematical Sciences Education & Freudenthal Institute (Eds.), 1997-1998) curriculum program that Matt used during his first year of teaching when he taught in the middle school. I do so because throughout the study, Matt frequently referenced MiC. This description will assist the reader in understanding these references in the interview data. I then describe the geometry textbook that Matt used throughout this study. This will help the reader understand the differences between NSF-funded materials (e.g., MiC) and more traditional mathematics programs, which have been characterized as “drill and kill” (Senk & Thompson, 2003, p. 16) or “parrot math” (O’Brien, 1999). A description of Matt’s textbook, *Geometry* (Larson, Boswell, & Stiff, 2001), will also be useful because learning more about the written curriculum helps the reader understand how the different features of the textbook “may act as constraints and affordances in different situations and for different teachers” (Lloyd, 2008, p. 66).

The Written Curriculum

*Mathematics in Context (MiC).* The MiC curriculum is 1 of 15 different *Standards*-based mathematics curriculum programs developed during the 1990s through funding from the National Science Foundation (Senk & Thompson, 2003). The goal of the NSF-supported program was to create comprehensive instructional materials that were consistent with the calls for change in the *Standards* and other major policy reports (Senk & Thompson, 2003). An international advisory committee of mathematics educators, mathematicians, scientists,
curriculum supervisors, principals, and teachers was formed to ensure that MiC was aligned with the goals and philosophy of the *Standards*.

MiC, a middle school curriculum, is organized into units that:

[include] tasks and questions designed to engage students in mathematical thinking and discourse….Students are expected to explore mathematical relationships, develop their own strategies for solving problems, use appropriate problem-solving tools, work together cooperatively, and value each other’s strategies. They are encouraged to explain their thinking as well as their solutions. (Romberg, 2001, p. 2)

This description highlights the ideas of the process standards, namely, that students should be engaged in collaborative problem solving and communication of mathematical ideas.

Through an analysis of the curriculum, one can also see how the content standards are also present throughout the materials as middle school students are expected to explore and connect the mathematical strands (Romberg & Shafer, 2003).

Enactment of curriculum materials such as MiC demands that the teacher assumes a role that differs from the traditional role of the teacher. Citing several studies of MiC curriculum enactment, Romberg (1997) discussed the teacher’s role as shifting from a “review, presentation, study/assistance period” (Weller, 1991, p. 128) and feeling the need to “cover” certain materials, to a new role where the teacher and the text are no longer the authority of the mathematics. Rather, the students, using MiC, read, discuss, and make sense of tasks in their student booklets under the guidance of the teacher with the help of their peers (Romberg, 2001). The creation of such a learning environment, however, requires considerable knowledge (of mathematics content, pedagogy, and student learning) and places great demands on a teacher in terms of energy levels and time (Clarke, 1997). In the next section, the textbook that Matt used in his geometry class is described. This geometry textbook is much more conventional than the MiC materials.
**Geometry.** During all three years of this study, Matt used the textbook, *Geometry* (Larson et al., 2001). The only curricular objective communicated to Matt from his department was to “cover” Chapters 1-6 during the first semester. Matt said: “that's the only directive I'm given, pretty much” (Interview D-2, 4/13/06). The tendency for school districts to regulate mathematics teaching practices by mandating the use of a single textbook or set of curriculum materials is a trend in mathematics education (Remillard, 2005). As Matt said, however, within that context, he was given “a lot of freedom” (Interview D-2, 4/13/06). The way that Matt took advantage of this “freedom” will be explored in-depth in Chapter 5.

Chapters 1-6 of *Geometry* was the official curriculum for the first semester. In addition to the textbook serving as the official curriculum, the content of the common exams that were used by every geometry teacher in the department also influenced Matt’s planning of lessons. This assertion will also be discussed in more detail in Chapter 5.

The six chapters that were to be “covered” during the first semester are listed in Table 3-2. These chapters include: an introduction to deductive reasoning and logic; definitions and postulates; and theorems about perpendicular and parallel lines, triangles, and quadrilaterals.

<table>
<thead>
<tr>
<th>Chapter 1</th>
<th>Basics of Geometry</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chapter 2</td>
<td>Reasoning and Proof</td>
</tr>
<tr>
<td>Chapter 3</td>
<td>Perpendicular and Parallel Lines</td>
</tr>
<tr>
<td>Chapter 4</td>
<td>Congruent Triangles</td>
</tr>
<tr>
<td>Chapter 5</td>
<td>Properties of Triangles</td>
</tr>
<tr>
<td>Chapter 6</td>
<td>Quadrilaterals</td>
</tr>
</tbody>
</table>

15 Even though the classroom data only come from Chapters 2, 4, and 5, I mention all of the chapters covered in the first semester here because the first six chapters were described by Matt as being about proof, and he mentioned all of these chapters in the interviews.
I refer to Geometry as “conventional” because as with “traditional textbooks,” this book is organized in such a way that it presents the content without much guidance as to what is important to emphasize or how to teach it (Posner, 2004). Geometry developed Euclidean geometry as an axiomatic system. More specifically, students were provided with undefined terms, definitions, and postulates. After a brief treatment of deductive reasoning and an introduction to proof, theorems were usually presented in the if-then format. The authors either led the reader through the proofs of these theorems or left the proofs as exercises.

A typical chapter in Geometry was parsed into sections, and each of these sections had section goals. For example, in the section called “Deductive Reasoning,” Goal 1 was “Using Symbolic Notation,” and Goal 2 was “Using the Laws of Logic.” For students, there were boxes containing information labeled: “What you should learn,” and “Why you should learn it.” The answers to the “Why you should learn it” questions usually involved some “real world” connection.

Ideas that were deemed to be important by the authors (e.g., theorems, corollaries) were highlighted in green boxes. Matt frequently referred to these “green boxes” in his discussions of the textbook. Examples were provided for the student, and the following groups of exercises appeared at the end of each section: Guided Practice, Practice and

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16 Because the authors of Matt’s textbook do include real world applications and occasional activities, I would not characterize this book as completely traditional. However, on a continuum of traditional and reformed, I would place it closer to the former than the latter. Therefore, I use the word conventional rather than traditional, because, as pointed out by Herbel-Eisenmann et al. (2006), “traditional” and “reform-oriented” practices need not be dichotomous. And while Herbel-Eisenmann et al. were talking about enacted curriculum but this is also true of written curriculum.

17 Using the typology of Krulik & Rudnick (1980), it is unlikely that all students would experience what was presented on these pages as “exercises.” Rather, for some students, these “exercises” would be “questions” or “problems.”
Applications, and Mixed Review. Also, in the student edition, Chapter Summaries, Chapter Reviews, Chapter Tests, Chapter Standardized Tests, Algebra Reviews, and Quizzes appeared throughout the textbook. Each chapter contained two or three brief “concept” or “technology” activities.

In Chapters 2-6 of Geometry, the “Practice and Applications” that appeared at the end of each section typically consisted of a combination of proofs and applications of theorems. When students were asked to write a proof, the setup for the proof typically included the diagram, the “givens,” and the “prove.” Many of the proofs provided a “Plan for Proof” or were fill-in proofs. Fill-in proofs are two-column proofs that provide the basic structure for the proof, but students are expected to fill-in-the-blank for some of the statements and reasons. With a few exceptions, students were usually asked to “Write a two-column proof.” Occasionally, but rarely, students were asked to write a flow proof or a paragraph proof. Descriptions and examples of these proof forms appear in Appendix B.

The descriptions of MiC and Geometry were provided here to help the reader see the ways in which Standards-based curriculum materials differ from conventional textbooks. Unlike reform-oriented curricula whose goals are to involve students in problem solving, sense making, and complex reasoning, conventional textbooks position the teacher and the textbook as sources of intellectual authority and knowledge, and the students as receptacles to be filled (Remillard, 1991). In addition, the teacher’s role in enacting these different materials would likely require different skills and different curriculum visions. The ways that Matt used Geometry as a tool to enact the curriculum will be explored in depth in Chapter 5.

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In a “Plan for Proof” the authors outlined a “sketch” of a basic path to follow in order to prove a proposition.
Before proceeding with descriptions of data collection and analysis, it is important to acknowledge that when conducting qualitative research, “pure objectivity is impossible” (Esterberg, 2002, p. 11). Next, I reflect on the fieldwork because as the “research instrument” (Esterberg, 2002, p. 61) in this study, my personal experiences, values, and beliefs influenced the study and the relationship that I formed with the participant (Toma, 2000). This reflexivity is important because in qualitative research, the researcher should disclose some of the experiences and attitudes that were brought to the data collection and analysis process.

Reflections on Fieldwork

“Objectivity” is perhaps best seen as a label to hide problems in the social sciences.


One challenge to doing school ethnography is the familiarity of the setting. As Frank (1999) noted, “The practices of everyday life in classrooms become so routine that they become implicit” (p. 3). However, a certain amount of bias lends focus to our work and helps us get it done (Wolcott, 1995). My experiences as a classroom teacher both assisted me in having an insider’s perspective and challenged me to look beyond classroom routines that seemed familiar. For example, the content of the mathematics was very familiar to me. In that sense, during Y1, I was probably more familiar with Matt’s geometry curriculum than he was. This insider’s perspective helped me select the topic of study and then focus my study on the changes that I anticipated. Due to my own experiences as a geometry teacher, I expected that Matt’s teaching would change across time. In particular, I anticipated that Matt would become more skillful at making certain aspects of proving more explicit to his
students. On the other hand, I may have missed some of the things that changed – things that I was not looking for.

Because school ethnographers usually share many elements of the school culture with the “natives” under observation, ethnographers conducting research in schools may find it difficult to notice the more ordinary aspects of the culture itself (Frank, 1999). For example, one of the things that I did not notice until I was writing this chapter was the mismatch of the demographics of the school with the demographics of Matt’s classroom. More specifically, even though the school demographic was about 86% White, from my memory, I believe that most of the students in the classes that I observed were White. This now makes me wonder why the approximately 14% minority students were not represented in these classes which were at the middle level of three tracks. I am sorry to say that it was likely due to my “white privilege” (McIntosh, 1988) that I did not notice this during observations or analysis. In addition, the school was not very demographically different from one of the high schools that I worked at as an in-service teacher. In terms of class, my presence in the building, as an educator whose socio-economic status was lower than most of the students in the school, was not unfamiliar to me. Because I did not attend a school such as this one, however, I sometimes wondered if the students at the school appreciated the many resources that were at their disposal.

One of the things that I noticed as I analyzed the data was that I seemed to pay more attention to the mathematics than to other social things in the classroom. For example, in a conversation with Dr. Herbel-Eisenmann about how we were collecting field notes for the Discourse Study, I realized that she was attending more to the discourse, and I was attending more to the mathematics. It seemed that I was more interested in working through some of
the problems and thinking about the mathematics rather than the ways in which the mathematics was being communicated to students. As Dr. Herbel-Eisenmann pointed out to me, the mathematics could be documented in other ways (e.g., by capturing the board work with the video camera or photocopying textbook pages). Through this conversation, I gained an appreciation of the importance of attending to the teachers’ interactions with the students to better understand how knowledge was being socially constructed in this particular classroom. Using the video to analyze the mathematics at a later time, however, is not without its limitations. For example, where the video camera is pointed or what the researcher chooses to focus on in the transcription or the analysis can impact the final product (Lemke, 1998). Certainly, this kind of bias influenced the outcome of this study. For example, my initial focus on the mathematics (rather than other things such as the discourse) influenced the kinds of questions that I asked in the interviews. When researchers accept that involvement and bias are inevitable, they can find meaning by working to build close relationships with their informants. In the next section, I discuss how I negotiated a research relationship with Matt.

**Negotiating the Research Relationship**

In my experience as a teacher, the only time that anybody ever visited my classroom was for the purpose of evaluating me. For this reason, I felt a little bit uncomfortable when I first started observing classrooms. In particular, I was concerned that Matt would feel that I was evaluating him. The fact that I had already taught geometry both concerned me (because I was worried that Matt might think that I was judging him), but also helped me build a rapport with Matt. Perhaps because his father was a university professor and he grew up in a
university town, Matt did not seem to be uncomfortable with the process at all. In fact, he made jokes that indicated that he understood the process quite well. For example, as I was leaving his classroom after conducting an interview, Matt once said, “Have fun transcribing!”

In order not to interfere with the outcome of the study, I made a conscious effort not to offer suggestions for improvement or to comment on the quality of the pedagogy. In some ways, I was surprised that as a novice teacher, Matt did not ask me, an experienced teacher, for any sort of feedback or guidance as he was teaching proof for the first time. Every now and then I wondered if this had something to do with gender. On the other hand, Matt’s familiarity with the research process likely influenced how he negotiated a relationship with me. Casual conversations that occurred before and after the lessons seemed to help with rapport and the development of a mutual respect for each other. Even though he did not ask me for feedback or advice, I always felt that he spoke to me in ways that acknowledged his perception of my understanding of the mathematics. Due to the development of this comfortable relationship and my interest in the topic that I was studying, I very much enjoyed my visits to Matt’s classroom - with one exception. The only thing that I did not enjoy was the guilt that I experienced about taking up Matt’s time.

As a former classroom teacher, I knew how busy Matt was, especially as a beginning teacher. I felt a little guilty every time I asked him for an interview. He gave of his time generously, however, and he seemed to enjoy taking part in my research. Sometimes I would leave the room feeling badly that an interview took longer than I had anticipated, only to realize that it took as long as it did because Matt had a lot to say. Still, I experienced dilemmas related to taking up his time. One such dilemma was related to reciprocity. More specifically, in my first research class, it was said that researchers should not give any kinds
of rewards to study participants until the study was over, if at all. The idea being that remuneration could influence the outcome of the study because the participant would want to please or get something from the researcher. However, I did not feel comfortable taking something for nothing. Esterberg (2002) advocated for doing small favors such as buying meals. That is exactly what I did. When I came to visit for classroom observations, I frequently brought cookies or brownies, which Matt seemed to appreciate. Other times I bought him lunch when he gave up his time during a meal. When Matt’s daughter was born, I bought a gift for the baby. Because Matt was the expert on his own experiences, I was interested in learning from him, building rapport, and expressing appreciation for his time. I tried to show an interest (which was easy to do) and remain open to what Matt had to say.

Because the creation of meaningful results involves validity (i.e., whether or not research findings seem accurate or reasonable to the research participants; LeCompte, 2000), I asked Matt to read the findings chapters of this dissertation. For the most part, he seemed to agree that what was written was an accurate representation of what occurred in his classroom and what he said in the interviews. There were only one or two minor disagreements with my interpretations, but these were adjusted for this final product. With these things disclosed, I now describe the process of data collection.

**Data Sources and Collection**

The primary data sources for this study were classroom observations and interviews with Matt. In this section, I discuss the collection of these data across the three years of the study. I organized this section by data type. For example, I begin by describing how classroom observation data were collected. I then explain more specifically what data were
collected for each of the three years of this study. I then describe the collection of interview data, followed by the collection of other sources. In addition to the descriptions provided here, these data sources are outlined in Tables 3-3 and 3-4.

**Table 3-3: Data Sources from 2004-2005 and Y1 2004-2005**

<table>
<thead>
<tr>
<th>Source</th>
<th>Timing</th>
<th>Address</th>
<th>Data format</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nature of Mathematics Survey (Adamson et al., 2002)</td>
<td>May, 2005</td>
<td>Beliefs and philosophy of teaching and mathematics</td>
<td>Likert scale survey</td>
</tr>
<tr>
<td>Interview for larger Discourse Study (1)</td>
<td>March, 2005</td>
<td>Belief and practices, specifically</td>
<td>Audio recording</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Description of a typical lesson</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Teaching philosophy</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Use of textbook, technology</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Learning activities</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Role of problem solving versus basic skills</td>
<td></td>
</tr>
</tbody>
</table>

**2005-2006 (Y1 of Proof Study)**

<table>
<thead>
<tr>
<th>Source</th>
<th>Timing</th>
<th>Address</th>
<th>Data format</th>
</tr>
</thead>
<tbody>
<tr>
<td>Classroom Observations (8)</td>
<td>4 consecutive days in September, 4 consecutive days November, 2005</td>
<td>Geometry lessons, specifically</td>
<td>Audio recordings</td>
</tr>
<tr>
<td>Daily reflections (8)</td>
<td>Daily, after each lesson</td>
<td>• Deductive Reasoning and Writing a Proof</td>
<td>Video recordings</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Triangle congruence proofs and bisectors</td>
<td>Field notes</td>
</tr>
<tr>
<td>Interviews for larger Discourse Study (3)</td>
<td>After all observations were completed April, 2006 May, 2006</td>
<td>General questions about his teaching, specifically</td>
<td>Audio recordings</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Context of teaching situation</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Role of the teacher</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Central mathematical ideas</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Use of curriculum materials</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Beliefs about mathematics</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Philosophy of teaching</td>
<td></td>
</tr>
<tr>
<td>Interviews for dissertation – Proof Study (1)</td>
<td>June, 2006</td>
<td>Questions about proof and teaching proof, specifically</td>
<td>Audio recording</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Definition of proof</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Role of proof</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Goals for students in geometry</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Goals for teaching proof next year</td>
<td></td>
</tr>
</tbody>
</table>
### Table 3-4: Data Sources from Y2 and Y3

<table>
<thead>
<tr>
<th>Source</th>
<th>Timing</th>
<th>Address</th>
<th>Data format</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>2006-2007 (Y2 of Proof Study)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Classroom Observations (13)</td>
<td>7 consecutive* days in September, 6 consecutive days Oct/Nov, 2006</td>
<td>Geometry lessons, specifically</td>
<td>Audio recordings Video recordings Field notes</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Deductive Reasoning and Writing a Proof</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Triangle congruence proofs</td>
<td></td>
</tr>
<tr>
<td>Daily reflections (14)</td>
<td>Daily, after each lesson</td>
<td>Daily reflection on the lesson, specifically</td>
<td>Electronic via email</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Objective for the day</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Mathematical goals</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Description of the lesson</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Classroom discourse</td>
<td></td>
</tr>
<tr>
<td>Interviews for larger Discourse Study (1)</td>
<td>September, 2006</td>
<td>Reflection on watching a video of his teaching from the previous year, specifically,</td>
<td>Audio recording</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Things noticed about his classroom discourse</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Other things noticed</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Reaction to quantitative data</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Thoughts about potential action research project</td>
<td></td>
</tr>
<tr>
<td>Interviews for dissertation – Proof Study (2)</td>
<td>September, 2006 June, 2007</td>
<td>Questions about changes noticed from the previous year, specifically</td>
<td>Audio recordings</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Things deliberately done different from the previous year</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Use of various proof formats</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Questions about proof and teaching proof, specifically</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Definition and role of proof</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Goals for students in geometry</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Goals for teaching proof next year</td>
<td></td>
</tr>
<tr>
<td><strong>2007-2008 (Y3 of Proof Study)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Classroom Observations (9)</td>
<td>6 consecutive* days in September, 3 days in November, 2007</td>
<td>Geometry lessons, specifically</td>
<td>Audio recordings Video recordings Field notes</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Deductive Reasoning and Writing a Proof</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Triangle congruence proofs</td>
<td></td>
</tr>
<tr>
<td>Interviews for dissertation – Proof Study (2)</td>
<td>September, 2007 November, 2007</td>
<td>Questions about changes noticed from the previous year, specifically</td>
<td>Audio recordings</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Things deliberately done different from the previous year</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Use of various proof formats</td>
<td></td>
</tr>
<tr>
<td>Final Interview for Proof Study (1)</td>
<td>December, 2007</td>
<td>Questions about changes noticed across time, specifically</td>
<td>Audio recording</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Reactions to video clips from Y1 and Y3</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Changes to classroom discourse</td>
<td></td>
</tr>
</tbody>
</table>

*I did not observe on days when new material was not being presented because students were taking a quiz. This occurred once in this sequence of observations.
Classroom Observations

According to the ethnographic tradition, time is the critical attribute of fieldwork (Wolcott, 1995). For this study, I made use of a selective intermittent time mode: one where the length of time spent at the research site is longer, but with the frequency of visits dependent on the particular focus that the researcher develops (Jeffrey & Troman, 2004). In this case, I was interested in visiting the classroom for three years during lessons when the teacher introduced proof and taught triangle congruence proofs. During each classroom visit, I took field notes on my laptop from the back of the classroom. The field notes included descriptions of the mathematics and, increasingly, the classroom talk. Usually within 24 hours, I wrote a summary of the lesson. I also noted questions for future interviews.

Each lesson was audio and video recorded. A camera was set up in the back of the classroom. Matt wore a wireless microphone, and an additional microphone was placed in the room to capture student-talk. After it was determined that it was difficult to hear the students' voices on the video-recordings, a more sensitive microphone was used during Y2 and Y3. Because there was only one camera, I primarily attempted to follow the actions of the teacher. At times, however, I focused the camera on students as well. I was also careful to zoom in to capture the work that was written on the white board. I did this to assist me in the analysis of the lessons. The audio-recordings were used as a back-up of the video data and were sometimes useful in capturing additional sound that was not audible on the video. I rarely interacted with the students during the lessons but occasionally had brief conversations with Matt while students were engaged in seatwork or group work. Rather than being a participant observer, my presence in the classroom was more of an “observer as participant” (Glesne, 1999, p. 44). That is, I primarily observed but had some interactions
with Matt and his students. In the sections that follow, I provide more detail about the particular observation data that were collected across the three years of the study.

**Observation Data from Y1**

The data collected during Y1 of this study were collected for the larger Discourse Study. Matt selected a week in September and a week in November that worked with his schedule. That is, for the larger study, we asked the teachers to select a week when we could come in and observe teaching rather than students taking tests or attending assemblies. I was the member of the research team who collected the data for Matt’s lessons in the first semester of the Discourse Study. During the observations in September, Matt taught lessons on deductive reasoning and simple mathematical proofs. For the November observations, he taught lessons on triangle congruence proofs and bisectors. Because Matt taught on a modified block schedule, I observed four lessons (rather than five) during each of these two weeks, for a total of eight lessons. More detail about some of these lessons will be provided later in the chapter.

As Biklen and Casella (2007) pointed out, no field project is ever perfect. Researchers often do not make perfect decisions, and sometimes things go wrong. In the case of this study, I erred in my video collection by taping over one of the lessons. Fortunately, through the audio recording and my field notes, I was able to analyze this lesson anyway. In addition, since the last five minutes of the video were not erased, I had a record of most of the board work from that day. Together, these three things helped me reconstruct the lesson with only a few issues, which are noted in this paper when appropriate.
**Observation Data from Y2**

After deciding to return to Matt’s classroom the following year for this dissertation study, I decided to attend two full units of lessons, rather than two weeks of lessons. At that point, the focus of my study was not entirely clear so I wanted to spend more time in Matt’s classroom. Because I anticipated that the focus would be related to change across time, I scheduled these visits so that I could observe some of the same lessons that I observed the previous year. I observed seven lessons in September and six in November.

**Observation Data from Y3**

By Y3 of the study, I had decided that I would compare particular lessons across time. (I explain how I came to this decision in the analysis section.) Therefore, I returned in September of Y3 to observe the same four lessons that I observed in Y1. I also observed three of the four Y1 November lessons, skipping one of the days when Matt was giving a quiz. In the next section, I describe the interview data that were used for this study.

**Interview Data**

In addition to the data that I collected for this dissertation study, interview data from the larger Discourse Study were also a valuable source of information. All interviews were semi-structured and audio recorded. Here I describe the interview data that were collected for both of these studies because they were all used in the analysis.

**Interview Data from the Discourse Study**

All of the interviews from the Discourse Study were used in this study because they provided valuable contextual information, as well as information about Matt’s beliefs and
teaching philosophy. In addition to the data that were collected during this dissertation study, one interview that was conducted prior to Y1 was also used. This interview was primarily about Matt’s beliefs and practices. More detail on all of the interviews described here is outlined in Tables 3-3 and 3-4.

Interview Data from Y1. During Y1 of the study, three interviews were conducted for the larger study. These interviews addressed general questions about Matt’s teaching, including his beliefs about mathematics and the role of the teacher. Matt also answered questions about his particular context and his use of curriculum materials.

Interview Data from Y2. During Y2 of the larger study, one interview was conducted. Prior to this interview, Matt watched a video of his teaching from the previous year. In the interview, Matt described some of the things that he noticed in the video. He also reacted to quantitative information that was provided to him about his teaching. More specifically, Matt was provided with analysis about the sequencing of his lessons and the amount of student and teacher talk that occurred during these lessons. No additional interview data for the Discourse Study were collected during Y3. Next, I describe the interview data that were collected for the Proof Study.

Interview Data from the Proof Study

Interview Data from Y1. By the end of the 2005-2006 school year, I decided to study Matt’s practice. Therefore in June, 2005, I conducted the first interview for the Proof Study. I asked Matt about his beliefs regarding the nature and role of proof. Ideas from Knuth’s (2002c) study on teachers’ conceptions of proof helped me formulate questions for
this part of the interview. I also asked Matt about his goals for teaching proof the following year.

**Interview Data from Y2.** During Y2, I interviewed Matt three times. The first two interviews were conducted after the observations in September and November. I asked questions about the changes that I observed. At the end of Y2, I, again, asked Matt questions about the nature and role of proof, as well as his goals for the following year. I repeated many of the same questions because I was looking for change from Y1.

**Interview Data from Y3.** During Y3, I conducted three interviews. The first two interviews followed the observations in September and November. I asked questions about the changes that I observed, including the use of various proof forms. I also conducted one final interview. Due to the length and importance of this interview, I explain it separately.

**The Final Interview.** The final interview took place over four hours. In this interview, Matt watched video clips of the lessons that I selected for analysis (as described in the analysis section). Matt reacted to these video clips, describing the changes that he made across time and offering reasons for making these changes. This interview was audio and video recorded because I was interested in capturing Matt’s reactions as he watched the video clips.

**Other Data Sources**

Other data sources, including a survey, Daily Reflections, the geometry textbook, and additional classroom handouts were collected for this study.
**Nature of Mathematics Survey for Teachers**

The *Nature of Mathematics Survey for Teachers* (Adamson *et al.*, 2002) was administered in May, 2005 and again in May, 2007 for the Discourse Study. One purpose of this instrument is to determine whether the teacher’s views come more from the “absolutist” or the “fallibilist” philosophy of mathematics (Ernest, 1994). The second purpose of the instrument is to understand the teacher’s philosophy of mathematics education. That is, how is it that they teach mathematics? What is the best way for students to learn mathematics?

**Daily Reflections**

Daily Reflections were collected during Y1 and Y2 of this study. After each lesson, Matt was asked to describe the lesson and reflect on how it went. He also wrote about the mathematical goals for the lesson and the classroom discourse. Matt answered the predetermined questions on a word processor and sent his responses via email. In this sense, these reflections could be considered structured interviews. I did not ask Matt to write Daily Reflections during Y3 for two reasons. First, these reflections were time consuming for Matt. Second, even though these reflections were valuable, I did not think that the information gathered during Y3 from this data source would be helpful enough to impose on Matt’s time.

**Textbook and Other Handouts**

Matt’s textbook turned out to be an important actor in both his planning of and enactment of the lessons. For this reason, I obtained a copy of this textbook so that I could analyze it alongside the other data. In addition, I collected all handouts, quizzes, and planning sheets that Matt handed out to his students. The planning sheets were intended to help Matt
and his students keep track of the lessons and the daily homework. These proved useful to me as well. In the next section, I describe how I analyzed the data.

**Data Analysis Procedures**

**Analysis of Classroom Observations**

The first thing that I did was catalogue the observation data so that I could determine which lessons appeared consistently across the three years (see, e.g., the September data table in Appendix C). Only four lessons appeared across all three years. This was due to the planning changes that Matt made from year to year and the fact that he took a planned family leave during the November lessons in Y2. This leave impacted the schedule because I did not collect data while the substitute was teaching Matt’s classes.

With the vast amount of data that I had collected, it was useful for me to focus the study through “data reduction” (Esterberg, 2002, p. 166). Therefore, I decided to compare the lessons that appeared consistently in Y1 and Y3. I did so because there were five lessons that appeared consistently across Y1 and Y3 and comparing across Y1 and Y3 (rather than Y1 and Y3) would allow me to see the greatest amount of change across time. This decision reduced the observation data to a total of 10 lessons. All of the selected data were then imported into Transana (Fassnacht & Woods, 2005), a computer software program that allows researchers to transcribe and analyze video and audio data. Next, I watched the videos and made timelines of the lessons. Methods outlined by Lemke (1990, 1998) were useful in this analysis.

Lemke (1990, 1998), a science educator, contended that in classrooms, the largest unit of analysis for a spoken discourse text is a socially recognized *activity type* such as the
classroom lesson. A smaller *episode*, or sub-unit of that activity such as Group Work or Seatwork can form a more immediate context. From a discourse perspective, the behavior of a teacher and his students during these activity types is quite different. For example, when students engage in Seatwork, they typically work independently and do not talk to each other or the teacher. On the other hand, when students work in groups, they are expected to interact with one another. Lemke’s descriptions of these different activity types in sciences classrooms assisted the research team for the Discourse Study in constructing adaptive descriptions of activity types for mathematics classrooms. This typology assisted me in making the timelines for the lesson. An example of a timeline from one of the Y3 lessons is provided in Figure 3-1.

<table>
<thead>
<tr>
<th>1:38</th>
<th>3:15</th>
<th>23:50 …</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>BID TO START</strong></td>
<td><strong>GOING OVER HOMEWORK</strong></td>
<td><strong>WHOLE-CLASS WORK</strong></td>
</tr>
<tr>
<td>Matt: Okay, so we have a lot of stuff to do today…</td>
<td>Matt: So if you can get out your homework assignment…</td>
<td>12:51 The key on this proof is to look at which triangles…</td>
</tr>
<tr>
<td>2:18 Today I want to do two main things…</td>
<td>5:32 Okay, questions, did we reach a consensus on…</td>
<td>18:30 So let’s give those numbers so we don’t have to say these and those…</td>
</tr>
<tr>
<td>24:25 …for instance, how can we prove the Base Angles Theorem?</td>
<td>32:10 The key question is, how do we draw the line at B?...</td>
<td></td>
</tr>
</tbody>
</table>

**Figure 3-1: Example of Lesson Timeline (Y3 Special Triangles, 11/8/07)**

I further reduced the data by focusing the study on the Whole-class work activity. Using the typology developed for the Discourse Study, Whole-class work was defined as follows: “This activity is mainly teacher-led and involves the entire class rather than smaller groupings of students. It mainly takes place in teacher exposition or triadic dialogue (I-R-E),
but can also take place in other formats” (Herbel-Eisenmann, Cirillo, & Skowrons, unpublished). I selected this activity type for the analysis because this is the place where the teaching of new content primarily took place. I believed that analyzing this activity type would be the best way for me to observe changes in Matt’s pedagogy. Next, I watched the Whole-class work portion of the five lessons that appeared in Y1 and Y3 (10 total). After taking detailed notes on these lessons, I then decided to analyze four of the five lessons from Y1 and Y3 because these particular lessons allowed me to tease out some of the differences that I noticed across time. Throughout this dissertation, I refer to these four focus lessons by the following names: Deductive Reasoning; First Proofs; Special Triangles; and Bisectors. Tables that provide additional information about the textbook’s stated goals in these lessons appear in Appendix D.

Because the Y1 lessons were transcribed for the larger study, I already had transcripts for some of the lessons. I went through these transcripts, however, and fine-tuned them for the analysis. More specifically, because these lessons were transcribed by outside transcribers, some of the mathematical language was inaccurate, and some of the student turns were not accounted for. I then transcribed the Y3 lessons in Transana. Transcription, a preliminary kind of data analysis, is critically important in discourse analysis (Lemke, 1990). There is a tendency in transcription to ‘clean up’ a text. When transcripts are cleaned up in this fashion, some of the features dismissed as irrelevant (e.g., tag questions such as, “okay?”) very often turn out not to be irrelevant at all, namely because meaning is only occasionally constructed through single words (Lemke, 1998). In order to maintain fidelity to the discourse, I attempted to produce exact transcripts for the analysis of the classroom observations. This was necessary because looking at the language choices made by the
teacher (e.g., tag questions and pronouns) was important to understanding when and how the discourse was closed down or opened up.

The analysis for this study was conducted on three levels. The first level was the more macro level, where I examined Geometry and the ways that Matt said he worked with this textbook. The second level was an analysis of the geometry talk in the enactment of the lessons. I was then able to compare the geometry talk in the enacted lessons with the written curriculum. The third level was a micro analysis of the discourse. I conducted this more fine-grain analysis of the discourse by identifying particular features of the discourse (e.g., revoicing and use of pronouns) and looking at how those features changed across time.

A major strength of the approach described by Lemke (1990) in Talking Science is that it allowed me to pay attention to the geometry in the lesson rather than some of the other social aspects of the discourse. Acknowledging that a lesson is a social activity (Lemke, 1990) where knowledge is created within human interaction (Cobb, Yackel, & Wood, 1993; Esterberg, 2002), it is important that I am clear on this point. When I say social aspects of the discourse, here, I refer only to non-mathematical talk about things such as acquisition of hall passes or taking attendance. Other examples include, but are not limited to: management of student behavior, directions to take out or put away supplies, and transitional periods that occur between activity shifts. Before continuing with a description of my analysis, I explain what I meant when I wrote that these methods “allowed me to pay attention to the geometry in the lesson.”

In order to be successful in mathematics and learn how to “talk geometry,” students need to be able to find the mathematics in the dialogue. Here, I cite Lemke’s explanation (changing science to geometry):
The [geometry] in the dialogue is not just a matter of vocabulary. Classroom language is not just a list of technical terms, or even just a recital of definitions. It is the use of those terms in relation to one another. Students have to learn how to combine the meanings of different terms according to the accepted ways of talking [geometry]. They have to talk and write and reason in phrases, clauses, sentences, and paragraphs of [geometric] language. (Lemke, 1990, p. 12, emphasis added)

As an example, to be successful in writing a proof, it is not enough for students to know only the definition of perpendicular. It is also useful to know how to read, write, and interpret the symbolic representation that denotes the perpendicular relationship between two lines (i.e., $AB \perp CD$). Lemke referred to the patterns of connections among the meaning of the words in a particular field of [mathematics] as “thematic patterns” (Lemke, 1990, p. 12). All thematic patterns are small pieces of the enacted curriculum (Lemke, 1990). Lemke’s methods of discourse analysis make rich descriptions of the enacted curriculum possible (Lemke, 1998).

Using ideas from Lemke’s methods, I made thematic mappings of the lesson content for the focus lessons. This allowed me to compare the geometry talk in the enacted lessons from Y1 to Y3. Doing so helped highlight some of the changes that I noticed across the lessons. Examples and further descriptions of these mappings are provided in Chapter 6. While this analysis was useful, it only partially helped me document the breadth of change that I observed.

I then watched the episodes again, setting them alongside the transcripts. Writing “analytic memos” helped me to think about the categories and themes that I was developing (Esterberg, 2002, p. 165). A useful piece of advice that I received was to look not only for particular changes, but also for things that went away in Y3 (David Pimm, personal

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19 I adapted Lemke’s (1990) construct of “thematic diagrams” to construct simpler “thematic mappings” because proper construction of thematic diagrams is extremely time consuming and requires specialized training. As Lemke (2000) explained, each thematic formation is like a concept web but are about language rather than concepts.
communication, November 16, 2007). This piece of advice enriched my analysis greatly. I then looked more closely at the ways that particular features of the discourse changed across time. More specifically, I examined the ways that Matt’s use of pronouns, revoicing, questioning, and wait time changed across time because I noticed that by changing them, Matt seemed to have opened up more space for student participation.

Once I did the preliminary analysis that helped me determine the three levels of change that I would focus on, I conducted the final interview. To do this, I selected video clips from the four focus lessons. Conducting this interview helped with triangulation. I asked Matt to watch, for example, a clip from the Y1 First Proofs lesson and then to read his Daily Reflection from that lesson. After we discussed his reactions to these two data sources, I then showed him a clip from the Y3 First Proofs lesson. I did so to provide Matt with an opportunity to notice changes in his practice. After this final interview was finished, I then turned to the collection of interviews to help me further understand why Matt made the changes that I identified in the classroom observations.

**Analysis of Interview Data**

As was the case with the observation data from the Discourse Study, all of the interviews from the larger study were already transcribed by outside transcribers. With my themes in mind, I made timelines of the five interviews from the larger study and the six interviews from the Proof Study. I transcribed only the bits from the Proof Study interviews that would be useful in helping me understand why Matt made the changes that I observed. Again, I transcribed these interviews as faithfully as possible. However, because I was not using discourse analysis to analyze the interview data, when I report the interview data in
subsequent chapters, I sometimes present a ‘cleaned up’ version. I do so because, as Glesne (1999) noted, authenticity can be overdone, and the reader only needs to suffer so many “you know’s” and “umm’s” (p. 171). Because Matt frequently said “you know,” spoke in fragments, and had many false starts during the interviews, I conservatively cleaned up some of these transcripts. An example of an original and a reported excerpt is provided in Appendix E to illustrate the kinds of changes that were typical.

Even though I did not analyze the Y2 classroom data, some of the interview data from Y2 was useful in helping me document Matt’s reasons for making the noted changes. With my themes in mind, I went through each interview transcript, line by line, and did a focused coding of the data. This helped me to explain, in Matt’s words, why he made the changes observed in his teaching. In addition, interview data were used to provide contextual information for the study and to describe Matt’s beliefs and teaching philosophy to set up the findings.

Analysis of Other Data Sources

The Nature of Mathematics Survey for Teachers was used to provide contextual information about Matt and to look for change across time. The textbook and other handouts were used on an as-needed basis. I frequently referenced Geometry to compare the written and enacted curriculum. I used these materials for triangulation since Matt frequently referenced the textbook and the handouts in class and during interviews. This analysis helped me to better understand how Matt planned the lessons and developed a curriculum vision (Chapter 5). I also analyzed Geometry to provide the descriptions of the written curriculum that appeared in an earlier section of this chapter.
Finally, Daily Reflections were used for various purposes. First, these reflections provided Matt’s perspective on how the enacted lessons went. Second, comparing the Y1 and Y2 reflections helped document change. Third, as was previously mentioned, Matt reread these reflections and reacted to them in the final interview. These reflections were useful in helping Matt discuss and reflect on his practice.

**Summary**

My experiences as student and a teacher of mathematics have led me to pursue the topic of this dissertation – how does a teacher learn to teach geometry proof? The teacher who was the informant for the case study was a beginning teacher, teaching proof for the first time. I followed his progression across three years as he taught proof using a conventional textbook. This teacher’s classroom offered me a context to study the issues and challenges that arise when a competent teacher is teaching proof in the Standards-based era using a conventional textbook. This work can help the field gain insight into the following question: What happens when a beginning teacher attempts to enact a Standards-based pedagogy while using conventional materials?

The changes that I noticed across time were difficult for me to articulate at first, but I had a strong feeling that there were differences in the Y1 and Y3 classrooms. The ethnographic tools used to collect the data along with the tools of discourse analysis assisted me in articulating some of the differences that I noticed.

Most of the Y1 data for this study came from the larger Discourse Study. Additionally, interviews conducted for the Discourse Study were also used to provide contextual information and to help me understand how Matt worked with his curriculum
materials. Additional data were collected for the Proof study, including two additional years of classroom observations and interviews. Most of these interviews were conducted to understand why Matt was making the changes that were noticed across time.

The ideas that I investigate in this dissertation include Matt’s development of a curriculum vision (Chapter 5), his more explicit emphasis on proof across time (Chapter 6), and the changing patterns in Matt’s discourse (Chapter 7). Through a discussion of Matt’s teaching philosophy and beliefs, the next chapter sets up the findings that are discussed in Chapters 5-7. These three findings chapters explore the changes that Matt made across time and explain why, from his perspective, these changes were made.
Prior to the upcoming description and analysis of Matt’s teaching, it will be helpful to understand more about Matt by sharing details about his experience with and his beliefs about proof. Understanding Matt’s experience with proof is important because existing research shows that “the feelings teachers experienced as learners carry forward into their adult lives, and these feelings are important factors in the ways teachers interpret their mathematical worlds” (Philipp, 2007, p. 258). Although in this dissertation, I focus more on Matt’s beliefs, past experiences, and philosophy (not feelings), Philipp’s point that teachers’ experiences can carry forward into their adult lives (and subsequently, their classrooms) is relevant to this study. In this chapter, I draw on interview and classroom data to offer some detail about Matt’s philosophy of teaching, in general, and about teaching proof specifically.

In order to understand how Matt’s prior experiences with proof influenced the ways in which he interpreted his mathematical world, I first discuss his early experiences as a student learning proof and then describe his beliefs about “real math.” Next, I discuss the way in which Matt spoke about the “magic” of doing proof in order to help the reader understand how Matt viewed the process of coming to know mathematics. Finally, I describe the goals that Matt had for his students and what he believed that he could and could not do to help students learn “real math.” By introducing the findings in this manner, I hope to impart the beliefs and philosophies that motivated Matt to make the changes described in the subsequent chapters. I argue that the changes found in his teaching of proof across time were motivated by Matt’s desire to create experiences for his students that were more closely
related to “real math” than to the “school math” experience provided through lecturing from his textbook. I begin by describing the challenges that Matt faced when he was first asked to write a proof.

**Early Experiences with Proof**

As described in Chapter 3, Matt did not follow a traditional path through mathematics in high school. He completed geometry in the 8th grade as an independent study which was two years earlier than most students in the United States. Matt cannot specifically recall the content of this course, but he recalled being taught by a graduate student at the local university. Matt said that he was never asked to do a proof during his school mathematics experience, and he did not recall even being shown a proof in high school. As a mathematics major in college, however, Matt said:

**Example 4-1**

I was immediately asked to do all sorts of proofs, which, you know, now, looking back at it, I can see as not being so bad, but at the time I’m like, this is a joke. I’m like, this is impossible. You know, you can’t do this…How can people prove things about eigenspace values and all this other kind of stuff?…It was very much like what you thought was real is now no longer true.

(Interview P-1, 6/21/06)

The difficult transition that Matt experienced from school to undergraduate mathematics is not uncommon. The paucity of proof in school mathematics coupled with the fact that even in the lower-level university courses (e.g., calculus and linear algebra), few, if any, proofs are required of students (Moore, 1994) helps us understand why Matt felt that doing proofs was “impossible.” During the first year of this study, Matt compared the challenge of doing his first proof (as a student) to walking through a wall. This, he said, caused him to rethink his

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20 I write about doing proofs as opposed to writing proofs, because, as will be explained later, to Matt, these were two different activities.
major in mathematics. When I asked Matt about this experience, he said, “Yeah, actually I really questioned if I was going to keep going to college or not, but yeah…because I was like, this is imposs-, I mean, this is a joke” (Interview P-1, 6/21/06). These comments may seem surprising given that Matt was clearly above-average in school mathematics, evidenced by (among other things) his being two years ahead in his studies prior to graduating from high school. As Moore (1994) explained, however, “This abrupt transition to proof is a source of difficulty for many students, even for those who have done superior work with ease in their lower-level mathematics courses” (p. 249). Matt said that even though he did not take any sort of an introductory proof course, eventually he was “able to do it” and “able to understand or believe that this was something that [he] could do” (Interview P-1, 6/21/06).

This idea of believing that proofs could be done will become important later in this chapter.

According to Matt, his experience as a student influenced his beliefs about what it meant to know and do mathematics. After learning how to do proofs, he found himself dissatisfied with classes that were more procedural. Once he started doing proofs, Matt found classes like Ordinary Differential Equations “really boring” because there were no proofs. He said, “It was all procedural knowledge now. You know? And I’m like, this is stupid…this is dumb. Why am I doing these procedures without being told…how to prove them” (Interview P-1, 6/21/06). The experiences described here caused Matt to begin to think about mathematics in new ways. In the next section, I first present some data from a classroom episode where Matt spoke to his students about “real math” and then use interview data to better articulate what Matt meant when he talked about doing “real math.”
“Real Math” Must Involve Proof

“Real Math” versus School Math

The classroom episode described in this section took place in Y3 when students were introduced to proof. After the completion of the first proof of the school year, a brief conversation occurred about the class working on proofs over the course of the next few months. After this conversation, students could be heard saying, “I’m going to get sick of this” and “this is going to get very tedious very quickly.” In the transcript that follows, Matt responded to his students’ comments by referring to proof as being “real math” several times.

Example 4-2
Matt: It’s not tedious. It’s exciting….It’s fun…
   MS: I like numbers. Numbers are fun. This is harder.
   MS: This takes too long.
Matt: No. No. This is math. Everything that you were taught before with numbers that you thought was math, that’s not really math. Okay. That’s arithmetic. Right? This [gestures toward the proof on the board] is math.
   MS: It’s geometry. It’s not math. Math’s a bigger category. It has all of those in it.
Matt: Okay. This is more like real math.
   MS: How is it more like real math?
   MS: Define real math.
Matt: This [gestures, again, to the proof on the board] is like math.
   MS: Well how?
   MS: Why is this like math?
Matt: Because, because we’re proving, but there’s no numbers.
(Y3 First Proofs Transcript, 9/21/07)

As can be seen in this transcript, Matt explained that he saw “this,” meaning proof, as “real math.” When a student challenged him, saying, “It’s geometry…math’s a bigger category. It has all of those in it,” Matt conceded on this point slightly when he said, “This is more like

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In all transcripts, MS is male student, FS is female student, S is a student whose gender was not determined, Ss is a collective group of students, and I is interviewer.
real math.” So for Matt, it seemed that “real math” had to involve some sort of proof. In addition, he told his students that all of the mathematics that they learned before the geometry course was arithmetic. One might question this claim since these 10th grade students would have taken algebra the previous year. Algebra, however, is sometimes described as an extension and a generalization of arithmetic (Leitzel, 1989).

This conversation about “real math” continued with Matt explaining to students that they were working in a mathematical system that was “arbitrary.”22 He said that instead of the symbol that was used for the number seven, he “could have like r plus horseshoe equals four leaf clover” (Y3 Transcript, 9/21/07). Returning to the idea of proof being “real math,” Matt said, “It's a system. Right? It's a system. But this [gesturing toward the proof on the board]23 has to be true no matter what system you're in. That's what I'm trying to point out” (Y3 Transcript, 9/21/07).

During an interview three months later, Matt talked about supplementing the textbook with additional proofs so that his students could experience “real math.” When asked why he believed that it was important to supplement his curriculum with additional proofs, Matt said:

**Example 4-3**

‘Cause it's actual math. Like it's real. It's like real math. You know…going to college and getting a degree in mathematics, that's the biggest thing is that, you know, computationally, we just don't care….So, I mean it's real math. That's what people really did…it was cutting edge math 3000 years ago, but it was still cutting edge to a certain point, and it can be seen. It's relatively straight-forward to them….they can still draw the picture, they can still see it, and they can see how we can apply the

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22 At this point, Matt had read Hewitt’s first (1999) *Arbitrary and Necessary* article which distinguished between arbitrary and necessary aspects of the mathematics curriculum. Responding to the article, Matt said that he found the article “very interesting.” He also added: “I definitely agree with the conceptual aspect of the article, and would even go farther to include in a definition of mathematics that mathematics is the only subject which has necessary elements, all other disciplines being far more arbitrary.” (personal communication, July 7, 2006)

23 The proof that Matt was gesturing toward was the first proof that students did this year. This proof, which was a proof involving segment addition, appears in Appendix F.
structure that we've agreed upon to reach that conclusion. So in that sense, it's sort of, yeah, it's real mathematics...there's no numbers in it. They actually can write a proof. And they can write a pretty rigorous proof...from a pretty well agreed upon set of axioms and definitions. (Interview P-6, 12/15/07)

The statement “computationally, we don’t care” seemed similar to the statement that Matt made in class (Example 4-2) related to arithmetic not really counting as mathematics. In Matt’s view, since mathematicians would only be concerned with “real math” (i.e., proof), then they could not be bothered with simple computations. Matt’s observation that the students “can still draw a picture” and “can still see it” has been noted in the literature. In a historical examination of the justification question of the high school geometry course in the United States, Gonzalez and Herbst (2006) identified the intuitive argument as one of four competing arguments that emerged during the 20th century:

The core idea sustaining proponents of the intuitive argument was the principle that geometry provides lenses to understand, to experience, and to model the physical world by forging stronger connections between experiences, intuitions, skills, and geometrical notions….Unlike other branches of mathematics, geometry was said to merge empirical knowledge about physical objects and abstract ways of dealing with those objects. (p. 20)

In this same interview, Matt also explained that doing proofs was more like “real math” because proof “hasn't been curriculumified as much” (Interview P-6, 12/15/07). In this sense, Matt seemed to be claiming that compared to other things that appear in the school curriculum, proof has undergone less of a transformation from “real math” to school mathematics. He believed that the proof that we would write today to show that the sum of the interior angles of a triangle is equal to 180 degrees would be “pretty similar to whatever was written” a long time ago (Interview P-6, 12/15/07). Despite Matt’s claim that proof had not been “curriculumified” as much, he did say that he was constrained in the classroom by time. Matt explained that, historically, it took
human beings hundreds of years to come up with the first simple proofs. Therefore, it would be difficult to expect students to discover mathematics in a truly authentic manner. Matt also discussed the inauthenticity of the two-column proof form.

**“Real Proof” versus Two-Column Proof**

Matt claimed that the *form* of a two-column proof was inauthentic:

**Example 4-4**

Nobody writes a mathematical proof in two columns with a t-chart. I mean that's a joke...like I, I open up my *Introduction to Abstract Algebra I* book, there's no t-charts there. It's not, you know, here are my statements, here are my reasons. It's a paragraph explaining my proof or some kind of combination of figures and words. (Interview P-1, 6/21/06)

When he said that “nobody” writes a mathematical proof in two columns, one can assume that by “nobody,” Matt was referring to mathematicians and those who write undergraduate mathematics textbooks. In this sense, Matt was noting that the two-column form lacked what Weiss and Herbst (in review) referred to as “fidelity to mathematics as an intellectual discipline” (p. 5). In other words, because mathematicians would never write a proof in two columns, in school mathematics, even proof is not as authentic or as “real” as proof in the discipline of mathematics.

In the same interview, Matt further commented on the way in which the two-column structure pretended to be about “real proof” when actually it is not:

**Example 4-5**

I think that when people try to teach geometry proofs, they try to routinize it and to, okay, you can do this first...I mean that's the basis for the two-columnness is to try to give it a structure that you can then teach someone in a formal ‘I'm simply going to show you how to do this,’ and you can't do that with a real proof. (Interview P-1, 6/21/06)
Here, Matt essentially was saying that although doing proofs was more like “real math,” the two-column structure typically found in school mathematics inaccurately portrayed how a “real proof” was actually done.

Returning to his personal experience, Matt talked about the “huge” difference between the mathematics that he experienced in secondary school and the mathematics that he was asked to do at the university:

**Example 4-6**

It was basically like you get [to the university] and everything that you thought you knew...it's a big lie. It doesn't matter if you can, you know, being able to factor the difference of cubes doesn't really matter that much...there's a whole other skill-set of things that we're actually gonna look at and be interested in.

(Interview P-6, 12/15/07)

Matt, noting the abrupt transition to proof, referred to the disconnect between the mathematics that he was asked to do in high school and the mathematics that he was asked to do at the university as “a big lie.” Earlier, in Example 4-1, Matt said that when he was introduced to proof he felt that “what [he] thought was real [was] now no longer true” (Interview P-1, 6/21/06). Also, in Example 4-2, Matt explained to his students that what they were doing before proof was arithmetic, not “real math.” Here, however, Matt explicitly referred to his school mathematics experience as a “big lie,” saying that in higher level mathematics, there was a different skill set that “we’re actually gonna look at and be interested in.” This description of how Matt felt lied to when he was first presented with proof helps explain why he introduced proof the way that he did the first time he taught it.

**Teaching Proof in School Mathematics**

During Y1, the first time that Matt introduced proof to his students (and the very first lesson on proof that he ever taught), Matt told his students, “…now the magic begins…you
are about to enter the Matrix. Everything you’ve been told is a lie. Well that’s not exactly true” (Y1 First Proofs, 9/30/05). Matt seemed to be acknowledging that what he was about to ask students to do was going to be different from the mathematics that they had been doing since kindergarten. It seemed that, to Matt, the lie that he was implicitly told was that there was only one way to think about and do mathematics – and that one way involved arithmetic and procedures such as “being able to factor the difference of cubes” (Example 4-6), not proof. It was not until he was asked to do proofs that Matt saw a different side of mathematics.

In the paragraph above, I wrote that Matt told students, “now the magic begins,” and earlier, data was presented to show that when Matt was first asked to do a proof, he likened the experience to being asked to walk through a wall. In the next section, I discuss the “magical” quality that mathematics and doing proofs, in particular, had for Matt. This description will help the reader understand how Matt viewed the process of coming to know mathematics.

**Mathematics and Magic**

In addition to the two aforementioned examples, there were other instances where Matt talked about mathematics as “magic.” This analogy can also be found in the literature. For example, Barnes (2002) wrote about a student named Naidra who described his lack of insight on one particular day as not having “anything magical” happen. When pressed

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24 By “the Matrix,” Matt was referring to a science fiction action film from 1999 where humans learned that their perceived “reality” was actually the Matrix, “a simulated reality created by sentient machines in order to pacify and subdue the human population while their bodies' heat and electrical activity are used as an energy source” (Wikipedia, 2008).
further, Naidra said that “flashes of understanding can happen” and “lots of different things can spark that off” (M. Barnes, 2002, p. 83).

This sudden flash of understanding that Naidra described as magical is often referred to as an ‘Aha!’ or ‘Eureka!’ experience (Barnes, 2002). Mathematicians writing about the creative process have also described these kinds of moments. For example, Polya (1965) wrote about “a sudden clarification that brings light, order, connection and purpose to details which before appeared obscure, confused, scattered, and elusive” (p. 54). Similarly, Davis and Hersh (1983) said that “the flash of insight, the breakthrough, the ‘aha’ symbolizes that something has been brought forth which is genuinely new, a new understanding for the individual, a new concept placed before the larger community” (pp. 283-284). When Matt used the analogy of mathematics as magic, he sometimes seemed to use the idea of magic as something that was not really possible, but he also used it in the same way that was described above in the literature. The way that Matt spoke about mathematics as magic seemed to shift slightly across the time.

As I already mentioned, the first time that Matt taught proof, he said, “Now the magic begins.” On that same day, after the bell rang, as students left the room, Matt called out, “We’ll do more magic on Monday” (Y1 First Proofs, 9/30/05). During Y3, however, I did not hear Matt speak about proof as magic at all. After I present additional data and interpretation, I suggest a reason for why this shift may have occurred.

At the end of Y1, Matt talked about the magic of proof:

**Example 4-7**
The first time somebody will see something, and they'll rattle off six steps to the proof, right? And we'll get the whole proof done. I'll be like okay, Billy, how did you know that? And they have no idea, right? And they kind of fumble around for a little bit. And I'm like, I think here's what Billy's trying to say, and you just write 'magic
happens' on the board, right, and draw the arrow to the end. Because it is. I mean there's nothing, you can't systematize it. It's not about steps. It's about seeing the whole thing and then writing it down...and people confuse the writing it down with the proving, right? I mean you've already proved it. It's simply the recording of the proof, and that's what they see, and so they think proof has steps but really it doesn't. It's, there's a leap and then what has steps is your recording of what you've understood. But those are two separate things. (Interview P-3, 4/19/07)

Matt said several things that give us insight into how he viewed the proof process. First, it seemed that, to Matt, “seeing the whole thing” was the magical part of doing a proof. He was likely talking about “seeing” the logical chain of reasoning which some geometry textbooks (including Matt’s) refer to as the “Plan for Proof.” After the “leap” of understanding occurred, according to Matt, the next step was to record the proof. So first, one must see the proof. Then, one “simply” records the proof, and as Matt said, “those are two separate things.” He added, “It’s not about steps.” This idea that doing proofs is not typically a linear process as the deductive style of presentation implies is an important contribution made to the literature by Lakatos (1976). As Lakatos said: “Deductivist style hides the struggle, hides the adventure” (p. 142). In Matt’s view, “seeing” the plan for a proof cannot be systematized in the same way that much of arithmetic and algebra can. Therefore, there is a major shift in the expectations of and the cognitive demands placed on students when they are asked to do a proof. This explanation helps us further understand why Matt spoke of learning proof as entering “the Matrix” or “magic.”

In Matt’s view, a major part of opening one’s self up to the magic or the “leap” to understanding was having the belief that the proof could be done. Therefore, one of Matt’s goals became helping students believe that they could do a proof:

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25 In Matt’s geometry textbook, the first “Plan for Proof” was accompanied by this study tip: “When you write a complicated proof, it may help to write a plan first. The plan will also help others to understand your proof” (Larson et al., 2001, p. 137).
Example 4-8
They [have to] believe that they can…do a proof because they all walk in thinking that proofs are impossible, and if they leave believing that they can do it, that's far more important to me…I mean that just coincides with my own experience…that's the first biggest thing you have to get over is the belief that you can[not] do it. You know, and to understand that it's not magic, or that it is magic but it's not just all made up. That's the first biggest thing that they have to believe. (Interview P-3, 4/19/07)

Here, again, Matt drew on his own early experience as an undergraduate mathematics major (“that just coincides with my own experience”) when he said that it was important that he helped students understand that doing proof was possible. In this interview (which occurred toward the end of Y2) Matt said that proof was not magic in the sense that it was not “just all made up.” There seemed to have been a slight shift in Matt’s thinking at this point, indicated by the statement “that it’s not magic, or that it is magic but it’s not just all made up.” Perhaps this shift explains why I did not hear Matt talk about proof as magic in Y3.

So far, I have described the ways in which Matt’s early difficulties with proof influenced his beliefs about what it means to do proof. First, we saw that Matt felt that the mathematics he experienced in tertiary school was a “lie” because proof or “real math” was missing from this experience. Second, I explored the ways in which Matt described the process of proving as being like “magic.” Third, Matt said that students needed to “see” the proof before performing the simpler act of writing the proof. Finally, in Matt’s opinion, students needed to believe that proving could be done before they could successfully prove. Together these four big ideas help the reader understand Matt’s beliefs about what it meant to know and do “real” or more authentic mathematics. This description can also help us understand why Matt made the changes (described in Chapters 5 - 7) that were found across the three years of this study. In this last section, I use interview data to describe what Matt believed he could and could not do in order to help students “see” a proof.
“Seeing It”

In her work, Burton (1999a, 1999b) described the feelings of joy and excitement that accompany ‘Aha!’ moments. Describing this work, Barnes (2002) said:

Making a new discovery, or finding a new connection were compared with a light switching on, climbing a mountain and seeing the view from the top, finding a new path through unfamiliar terrain, or seeing how to fit pieces into a jigsaw. (p. 84)

During interviews, Matt talked about discovery in mathematics in ways that were very similar to the description above. In this section, I first discuss Matt’s first analogy which is similar to the above quotation where mathematicians’ insights were described as a “light switching on.” Next, I discuss Matt’s second analogy which is similar to “climbing a mountain.” Discussion of these analogies is provided here to help the reader understand how Matt viewed his role as the facilitator of mathematics in the classroom.

A Room in the Dark

Similar to describing moments of mathematical insight as “a light switching on,” Matt described the process of teaching proof as being like teaching a blind person:

**Example 4-9**

It's like trying to describe to a blind person how to find something in a room that they've never been in before....They're stumbling around in the dark.  
(Interview P-6, 12/15/07)

What struck me most about this description was its similarity to famous contemporary mathematician Andrew Wiles’ description of his work on *Fermat’s Last Theorem*:26

Perhaps I can best describe my experience of doing mathematics in terms of a journey through a dark unexplored mansion. You enter the first room of the mansion and it's completely dark. You stumble around bumping into the furniture, but gradually you

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26 *Fermat’s Last Theorem*, a famous problem in mathematics, was unsolved for more than 300 years until Wiles proved the theorem in 1995.
learn where each piece of furniture is. Finally, after six months or so, you find the light switch, you turn it on, and suddenly it's all illuminated. (PBS, 2008)

Matt, Wiles, and the mathematicians cited by Burton all described the process of working on a proof as an experience that was like being in the dark and in need of illumination. This was similar to the way that Matt spoke about how students would either see the path to conceptualizing a proof or not. He said that he cannot teach his students how to see. Next, I further discuss how Matt spoke about students seeing or not seeing the path to a proof which is related to the mathematicians’ descriptions (captured by Burton (1999a, 1999b)) of climbing a mountain and seeing the view from the top.

“I’m Like the Sherpa Guide”

During an interview at the end of Y1, Matt discussed how students either see or do not see how a particular proposition can be proved.

Example 4-10
To do a proof in a real mathematical way is very, it's very isolating. You can't teach somebody how to do a proof….I mean if a student's really gonna do a mathematical proof, you look at the problem and you either see how you do it or you don't. After that, the writing it down, although an important exercise in communication really is sort of pointless. I mean it's not pointless, but it's trivial. You know. If you can see how to prove something, then you can see how to explain it to somebody else and the seeing or not seeing it is nothing that I can teach you.

(Interview P-1, 6/21/06)

Matt noted that there were two phases involved in doing a proof: the “seeing it,” or thinking through a plan for the proof, and the “writing it down,” or communication of the proof in a written form. The latter, Matt claimed, was “trivial.” The idea that doing a proof and writing a proof are two different activities has been noted in the literature. Unlike Matt’s claim that writing the proof was “trivial,” Farrell (1987) portrayed both of these activities as important. The doing requires good problem solving skills, and the writing requires rigor and precision.
Farrell did say, however, that prospective teachers needed to learn that the writing takes a back seat to the generation of ideas. Clearly, Matt articulated this distinction.

After hearing Matt say that “seeing it is nothing that I can teach you,” I asked him if there was anything that he could do, as a teacher, to provide students access so that they could progress at the pace that was dictated by the demands of the school context. To this question, Matt replied:

**Example 4-11**
I mean you don't want to go so far as to say it doesn't matter what I do, but the reality is that I can't prove it for them. You know, simply showing somebody how to do a proof will help, but only up to a certain point. Only until they understand…the way in which a proof becomes a proof. (Interview P-1, 6/21/06)

Matt expressed what he saw as a limitation for him as the teacher. That is, Matt believed that merely “showing somebody a proof will help” but only until they understood the process of proving. Similarly, Matt once noted that “math is not a spectator sport” and “you're not gonna learn…by sitting there and watching me do it” (Interview D-5, 9/25/06). The phrase “math is not a spectator sport,” which can also be found in the literature (see, e.g., Pimm, 1987; Polya, 2004 (1945)), can be taken to mean that students must participate in mathematics, not simply watch others do mathematics.

After teaching proof for the second time, I, again, asked Matt about the comment (from Example 4-10): “seeing it is nothing that I can teach you.” I was curious as to whether Matt still believed that there was nothing or even very little that he could do to help students learn to prove. I was interested in his answer to this question because, at that point, I had observed Matt teach proof for the second time, and he had made changes²⁷ that I thought

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²⁷ These changes will be explored in-depth in Chapters 5 - 7.
might be designed to help his students “see it,” whereas the previous year he said that there was very little that he could do. I was trying to understand if there was a shift in his thinking.

**Example 4-12**

I: Last spring, you said, ‘You can’t routinize proof. You either see it or you don’t. Seeing it or not seeing it is not something I can teach you.’
Matt: Right.
I: Stand by that?
Matt: I agree. Yes, it cannot be systematized. There are no steps.
I: ‘Seeing it or not seeing it is not something I can teach you.’
Matt: No. I cannot teach you that. I can be there with you when you see it, and I can try to help you see that you have seen it.
I: So I, I’m not really clear on what, what do you see, why are you there?...Why are you in the room with them?
Matt: Why am I in the room with them? [laughs]
I: ‘Cause it sounds like what you, there’s nothing that you can do, and so...why not throw the [physical education] teacher in the room with them?
Matt: ‘...’cause the [physical education] teacher doesn’t believe that it’s possible, right?

During this exchange, I remember feeling frustrated with what Matt was saying, probably because I disagreed that there was nothing that he could do, but also because I was not sure if he even believed it himself. If, in fact, there was nothing that he could do, why then did he make changes in his practice that, by his own words, were motivated by his desire to help students understand proof? Glesne (1999) pointed out that just as in ordinary conversations, during interviews, respondents sometimes make contradictory statements. If the topic generating the contradictions is worthy of clarification, then the interviewer needs to probe further into the respondent’s most recent statement, “right then and there” (Glesne, 1999, p. 91). So, in a continued attempt to understand Matt’s perspective, the following exchange occurred:28

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28 Most of this exchange involves overlapping talk with both of us ‘interrupting’ each other. This more challenging interview style was atypical of the way that I tried to conduct interviews. I seemed to shift into a
Example 4-13
I: What is it that you're doing? You must, I mean otherwise, why do you get paid whatever you get paid--
   Matt: --Well I don't know.
I: --and why does it require that you have a degree in mathematics?
   Matt: Who knows?
I: There's nothing that you can do?
   Matt: [Laughs]
I: [Laughing] I challenge that idea!
   Matt: I mean you, okay. I mean but, [pause]. There really isn't though. I mean it's just that they wouldn't--
I: --So send your wife\(^\text{29}\) in to teach for, to teach geometry next year.
   Matt: Because they have to--
I: --And tell her, here's the problem that I want them to know how to do.
   Matt: They have to think that it's possible in order for the majority of them to complete it in the required time. (Interview P-3, 4/19/07)

During this exchange, I was really trying to understand and, as I said, “challenge” the idea that there was “nothing” that Matt could do to help his students learn how to prove.

Matt’s claim seemed to contradict the fact that he always had well-thought out reasons to explain the changes that I saw him making in Y2. This part of the interview continued in the same manner as I introduced the analogy of sports coaches who cannot go on the field and play the game for the athletes but do attend practice and are present on the sidelines trying to help their athletes succeed.

After discussing the analogy of teacher as coach, which did not seem to resonate with him, Matt initiated a new analogy:

Example 4-14
I'm like a Sherpa. Okay? That's the word I'm looking for. So...you know, I've been up and down the mountain 50 times. And if you didn't have me, you could make it to the top of the mountain. ‘Cause I'm not a requirement, right? But it'll probably be a lot

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\(^{29}\) Matt’s wife was also a teacher, but certified to teach elementary school.
uglier and take a lot longer. And, there's a good possibility that you would freeze to
death and never get to the top. Right? So. Yeah, I'm like the Sherpa guide who like,
you know, just walks with you up the mountain, but then at base camp I just, I go off
and meditate somewhere else and I really don't pay attention to what you're doing.
Right?....And I don't just have one person, right? I'm trying to herd like 30 people to
the top of the mountain before next Friday. (Interview P-3, 4/19/07)

When Matt said that he was “not a requirement,” he may have been referring back to the idea
that humans on their own could eventually ‘discover’ mathematical truths. Matt once said
that you have to “trust the Platonic\textsuperscript{30} world of mathematics” (Interview D-1, 3/9/05). So,
although, Matt could not climb the mountain for his students in the same way that he could
not “see it” for his students or “prove it for them,” he seemed to view his role as one of being
there and knowing (or believing) from experience that it was possible to get to the top of the
mountain. He also noted the reality of the classroom when he said that he had to herd 30
people to the top of the mountain “before next Friday.”

Summary

In this chapter, I first described Matt’s experiences with school mathematics and
undergraduate mathematics to help the reader understand his beliefs about mathematics and
proof. I also discussed how these experiences provided Matt with a new conception of
mathematics. These descriptions were provided to assist the reader in understanding how
Matt’s feelings and experiences carried forward and influenced his teaching practices. I then
presented classroom and interview data to help describe Matt’s view of “real math.” In this
discussion I described how Matt viewed doing proofs as synonymous with doing “real math.”
As will be shown in the upcoming analysis, the idea that students should experience “real

\textsuperscript{30} Platonism is the view that mathematics exists independently of human beings and is “out there somewhere”
floating around eternally (Davis \& Hersh, 1983, pp. 68-69)
“real proof.” Next, I explored the analogy of mathematics as “magic,” and that the path to a proof could either be seen or not.

Finally, I discussed the analogies of students being in a dark room in need of illumination and at the bottom of a mountain trying to get to the top. These analogies help us see how Matt viewed the process of coming to know mathematics and his role as the teacher in the classroom. Matt seemed to see his role in this process as one of “being there” with the students to point out when they have “seen” and helping them “believe” that it was possible. These ideas were presented prior to the presentation of the findings to help the reader understand Matt’s beliefs about mathematical proof and what he saw as a limitation of his role in leading students to the top of the mountain. These understandings shed light on the changes in Matt’s teaching that will be described in the upcoming chapters.

In the chapters that follow, I answer the research questions:

- How did Matt’s teaching of geometry proof change across three years?
  - How did Matt develop pedagogically?
  - How did his teaching change as he acquired curricular knowledge?

- To what did Matt attribute these changes?
  - In what ways did Matt’s participation in the larger Discourse Study influence the ways in which he taught proof?

I argue that Matt purposefully made the changes that I observed across time in order to provide students with opportunities to experience “real math.” First, in an effort to provide students with an experience that was more like “real math” than “school math,” Matt
supplemented his curriculum materials. Second, supplementing the curriculum materials was one of the many ways that Matt more explicitly focused on proof. Finally, across time, Matt focused on students in more sophisticated ways in order to help them reach the top of the mountain. I proceed next by describing how and why Matt supplemented the written curriculum.
CHAPTER 5: USING GEOMETRY TO TEACH “REAL MATH”

A teacher’s curriculum use is conceptualized as an interaction between the teacher and the curriculum materials: Teachers’ individual views and experiences impact their implementation of curricula and features of the curriculum materials influence teachers’ unique implementations. (Lloyd, 2008, p. 66)

In the previous chapter, evidence related to Matt’s beliefs about proof and his philosophy of teaching was presented. Matt emphasized that, unlike doing arithmetic or algebra, doing proofs was “real math.” After using his geometry textbook for the first time, Matt decided that he needed to supplement the textbook (Geometry) with additional proofs in order to help his students experience “real math.” Connecting to his past, Matt said that he was not exposed to proof until he was enrolled in undergraduate mathematics courses. Because most of what he learned prior to these mathematics courses involved procedures and formulas, Matt felt that his school mathematics experience was “a big lie.” So it seemed that as a result of his experience of feeling lied to and shortchanged of an exposure to “real math,” Matt found it necessary to expose his students to additional proofs beyond those that were provided in his textbook. Thus, as might be expected from the opening quotation, Matt’s individual views and experiences impacted how he enacted the curriculum materials.

In this chapter, I explore how Matt’s beliefs (or his “dispositions to act” (Cooney, 2001, p. 21)) about teaching were also influenced by his prior experience with the Mathematics in Context (MiC) curriculum materials that he used in his first year of teaching. I present data to provide evidence that suggests that Matt’s experience with MiC helped create an image of what his beliefs would look like in practice. As the data show, Matt’s experience with MiC raised his awareness of curricular possibilities that were different from
teaching with the conventional textbook that he was assigned to use when he was teaching high school geometry. This awareness, brought about through his experience with MiC, seemed to contribute to Matt’s inclination to adapt the written curriculum (Lloyd, 2008). Understanding Matt’s past experiences helps us better understand Matt’s unique enactment of the written curriculum.

Extended descriptions of MiC and Geometry (Larson et al., 2001) were provided in Chapter 3 because learning more about the written curriculum helps the reader understand how the different features of the textbook “may act as constraints and affordances in different situations and for different teachers” (Lloyd, 2008, p.66). One of the findings of this dissertation study is that Matt’s relationship with this textbook changed from Y1 to Y3. In this chapter, interview data are used to lay out what this process of change was like. I first begin by exploring the impact that teaching with the MiC materials (as a first year teacher in another school) had on the way that Matt thought about teaching mathematics. I then discuss Matt’s opinions and beliefs about his current textbook and how his “curriculum vision” (Drake & Sherin, in press) was not aligned with that of his textbook. As a result, Matt did not have “curriculum trust” (Drake & Sherin, in press). This work also helps us gain insight into the following question: What happens when a beginning teacher attempts to enact a Standards-based pedagogy while using conventional materials? In terms of the research questions, I argue that one of the things that changed across time was Matt’s relationship with Geometry. An important aspect of teaching is how teachers use and interact with their curriculum materials. The ways in which Matt used and supplemented Geometry to enact the curriculum complemented the increasing importance that Matt placed on proof (as described in Chapter 6).
In the next section, I describe the ways in which Matt’s earlier experience with the MiC materials influenced his beliefs about teaching mathematics and his planning of geometry lessons. This is important because, as Lloyd (2008) posited, teachers’ interactions with textbooks “may influence their ways of developing and enacting mathematics instruction in the future” (p. 65).

The Influence of MiC on Matt’s Practice

During his first year as a classroom teacher, Matt used the MiC materials to teach his seventh and eighth grade classes. At the end of that school year, Matt reported that he liked these materials, and that “the textbook is what we do everyday basically” (Interview D-1, 3/9/05). Matt said that he used the teacher’s guide rarely and only to help him with pacing. When asked how he selected which problems from the book he would have students work on, Matt said, “I pretty much do everything unless it’s something I don’t like,” but “I haven’t not liked most of it” (Interview D-1, 3/9/05). Since using MiC in this way was “working,” Matt said that, for the most part, he did the same thing everyday – put the students in groups and had them work on the problems from the booklets.

Even after changing jobs from a middle school to a high school position and teaching new courses, Matt continued to refer back to how much he liked the MiC materials:

Example 5-1
MiC was very interesting to use. Basically MiC is like the model for me of what any textbook series should in theory look like. So pretty much when I write something, actually for geometry I blatantly stole a ton of stuff out of the MiC book and copied it and gave it to them for when we talked about similar triangles. ‘Cause I'm like,

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31 Lloyd’s (2008) study was conducted with a novice teacher who was influenced by her experience with a textbook during student teaching. Here, I describe the way in which Matt was influenced by the MiC curriculum materials used during his first year as an inservice teacher.
they're not gonna understand this unless they have some kind of basis for it. So pretty much when I write something, that's kind of what I try to do. (Interview D-3, 4/14/06)

Here, Matt said that MiC was a model of what any textbook series should look like and that he believed that students needed to be given some kind of activity to engage with in order to understand the material. He said that whenever he “[wrote] something” for geometry, he modeled it after MiC, and he even “stole” material from MiC to use with his 10th grade geometry students.

When pressed further, Matt described what he meant when he said that MiC was the ideal “model.” One important aspect of the model, Matt explained, was that students should be engaged:

**Example 5-2**
It's all about giving them a situation that they're involved in mathematically somehow and then asking them a series of questions to either try to get them to extend what they just did or to try to get them to apply what they just did or to try to get them to synthesize. Here's all these pictures of similar figures, write me, in one sentence, what the relationship between the scale factor and the perimeter is.
(Interview D-3, 4/14/06)

Matt explained that students should be engaged with mathematics by answering questions and summarizing information. He also appreciated the connected nature of the MiC materials:

**Example 5-3**
MiC tends to….have a lot more sets of problems that all chain together. [But in geometry] what I'm taking is like an individual thing. Here's this topic, how does this topic make sense? Here's this next topic, how does this next topic make sense? And MiC is very, very well constructed if you actually look at it…the students will go from talking about one topic to talking about the next topic, and…they don't really understand that they just changed topics. So it's pretty well written in terms of the sequencing of how the questions will lead into the next questions. And even on how the units…because I taught both seventh and eighth grade, how much even between the years, how much this block of questions is an extension of this other block of questions that they did last year. (Interview D-3, 4/14/06)
Because of the connections between topics and between years, Matt found MiC to be “very well constructed.” In contrast, Matt noted that his geometry textbook presented the information as disjoint topics. As the “curriculum maker” (Clandinin & Connelly, 1986), in Matt’s view, it was up to him to take each topic and address the question, “How does this topic make sense?”

When asked how his experience with the MiC curriculum materials impacted his teaching of geometry at the new school, Matt answered, “It makes it really different. I mean there’s a different way” (Interview D-3, 4/14/06). In the sections that follow, additional data will be presented to help the reader understand how Matt, influenced by MiC, tried to teach geometry in “a different way.” Throughout the study, Matt referenced MiC and compared his geometry textbook to MiC. In the next section, additional data is presented to illustrate the comparisons that Matt made between MiC and Geometry. This helps us understand how the features of MiC influenced Matt’s enactment of the written curriculum in geometry.

**MiC versus Geometry**

Even after working in the high school for two years, Matt still mentioned how much he liked MiC and why teaching from MiC was much easier for him than using Geometry:

**Example 5-4**

So I didn’t fight [MiC] and then…you go from that to like here’s this crappy traditional book….It sucks…when you compare it, when you come straight from that, because there’s nothing that you can do. Like I was used to just coming in and being like…here’s what we’ve been talking about. Now do these…problems, walk around helping people, [and try] to have a discussion at the end. And you can’t do that with this book at all. (Interview P-3, 4/19/07)

Matt described the challenge of using a conventional textbook after having enjoyed the

*Standards*-based curriculum materials. In contrast, the literature is filled with studies that
describe the opposite situation. That is, teachers frequently struggle when shifting from conventional curriculum programs to enactment of reform-oriented materials (see, e.g., Clarke, 1997; Forman, McCormick, & Donato, 1998b; Van Zoest & Enyart, 1998). Factors such as mathematics teachers’ long history as lecturers (Zoest & Enyart, 1998), the “apprenticeship of observation” (Lortie, 1975), and teacher knowledge (Hill et al., 2007; Shulman, 1986) contribute to this struggle. In contrast, Matt said that he did not “fight” with MiC at all. He also found it much easier to use the problems in MiC as the actual lesson plan, but as he said, “you can’t do that with this [geometry] book at all.”

During Y3 (three years after Matt used MiC), Matt still referenced MiC during interviews. Here he discussed how much easier lesson planning was with MiC:

Example 5-5
Coming from MiC, all I looked at when I saw the section [was]….okay, here’s the unit, what are the four or five key [ideas]….So it was more about pacing and about how long does it take them to understand these certain things. And then selecting…the really big things that…they need to be able to know how to do and explain. And I did not fight with MiC at all. I liked the book. (Interview P-6, 12/15/07)

Except that he did not make full use of the teacher’s manual, Matt’s description of how he used the MiC curriculum materials reminds me of Brown and Edelson’s (2003) notion of “offloading.” Curricular offloading involves teachers relying significantly on the curriculum materials to support instruction (Brown & Edelson, 2003). Because the philosophy of MiC, namely, that students should be engaged with the material, was closely aligned with his own philosophy, Matt did not “fight with MiC.” In other words, Matt’s curriculum vision was aligned with the MiC curriculum program.
In contrast, Matt described his adjustment to *Geometry* as difficult. During the final interview in Y3, Matt reacted to a video clip from a lesson (filmed in Y1) that showed him at the board delivering a long monologue:

**Example 5-6**
I had already taught a year in MiC and then I still come here and stand up in the middle of the room and blah, blah, blah, blah....and I was already all about having kids think about things, and that they can do more than we think they can. Then you get here and hand me a traditional textbook and first year it's still me standing up in front of the room talking to myself. (Interview P-6, 12/15/07)

Matt explained that even though he had a great experience with MiC, and even though he “was already all about having kids think about things,” he had difficulty doing anything beyond lecturing in front of the room during his first year of teaching geometry. When asked why he thought he changed his pedagogy, Matt replied, “It’s just ingrained. Like, that’s what everybody does” (Interview P-6, 12/15/07). Matt also said that he was “in survival mode” the first year. So it seemed that because the curriculum vision of *Geometry* was not aligned with his own vision, Matt defaulted to what he experienced during his “apprenticeship of observation” (i.e., lecturing; Lortie, 1975) to enact the curriculum. This example helps us understand how (using Lloyd’s (2008) language), for Matt, MiC acted as “an affordance,” while his geometry textbook seemed to act as a “constraint” in enacting the written curriculum in the different contexts.

Further evidence related to how close Matt’s enacted practices were to the written materials was provided through his answers to one of the questions on the *Nature of Mathematics for Teachers Survey* (Adamson et al., 2002). In 2005, when Matt was using MiC, on the continuum, he placed himself closer to “I generally follow the textbook…” (see Figure 5-1). However, in 2007, when he was using *Geometry*, Matt placed himself closer to
“I generally modify the textbook approach and supplement it…” Data will be presented later to help the reader understand how Matt modified the textbook approach and supplemented it with additional activities.

<table>
<thead>
<tr>
<th>When preparing mathematics lessons, I generally follow the textbook and/or the prescribed curriculum.</th>
<th>X</th>
<th>X</th>
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<td>2005</td>
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When preparing mathematics lessons, I generally modify the textbook approach and supplement it with additional problems and/or activities.

Figure 5-1: Matt's Textbook Use across Time

In this next section, I briefly shift away from a focus on MiC to describe the ways that Matt viewed conventional textbooks in general. This will further help us see how, for Matt, the curriculum vision of conventional textbooks (such as Geometry) acted as a constraint on his teaching and was not aligned with his general philosophy that students should be engaged with mathematics and that mathematics should make sense. After exploring Matt’s description of conventional textbooks and what he believed needed to be changed, I then discuss the challenges that Matt had in using his conventional geometry textbook. These descriptions are provided to help the reader understand how Matt’s relationship with Geometry changed and to understand why he made the changes related to enacting his curriculum materials.

**Using a Conventional Textbook**

Matt said that most of the textbooks that he has used to teach high school (including Geometry) were written in a style that was “confusing” and “tried to undermine people’s understanding of it” (Interview P-3, 4/19/07). He characterized the presentation of material in high school textbooks in the following way:
Example 5-7
Here’s the formula. It comes from on high. Here are some examples, oh lowly one who can never understand where the formula came from. All you can do is try to manipulate this wonderful formula that somebody else came up with or was manna from heaven. All you are able to do is actually manipulate this formula to maybe get some answers to some piddily little problem which we do not concern ourselves with. (Interview P-3, 4/19/07).

Matt articulated his objection to the presentation of formulas in conventional textbooks. As he said, being given the formula implicitly communicated to students that they were not capable of “understand[ing] where the formula comes from.” Then students were given examples and expected to apply a procedure to “some piddily little problem.” In this sense, Matt objected to the “school math” presentation-style, which was not like “real math.” Matt believed that this style of presentation negatively impacted students’ beliefs about what they were able to do mathematically. He also said that this style sent a message to students that they were “not smart enough” to understand the concepts:

Example 5-8
And that’s the message that students get. You know? It’s impossible for me to get where this comes from because all I’m ever asked to do is use it and apply it in a particular situation. As opposed to the students actually being able to understand where it came from, not only use it [in] new situations but understand why that particular thing was constructed in the first place. (Interview P-3, 4/19/07)

In this excerpt, Matt expressed his belief that mathematics should make sense to students and that they should be able to “understand” where formulas and theorems came from. This idea of the authoritative “voice” of mathematics textbooks has been noted in the literature. For example, Herbel-Eisenmann (2007) found that even in some NSF-funded curriculum materials, the language choices supported the mystique of mathematics.
Although Matt clearly preferred the MiC materials over his geometry textbook, he did not believe that it was the *content* of high school mathematics that needed to “undergo a drastic change.” Rather, the way that the content was taught needed to change:

**Example 5-9**
A lot of people, like you read a lot of stuff about the NSF [curriculum programs], and basically everybody, from what I can tell, [said] all of this is crap. We need to throw it out and start over. And I don't really think you have to....there isn't very much content in my geometry book or in any of the books [or classes]...that I look at and think, that's worthless. There are some things, but overall it's not the content that I'm upset about. It's the way in which it's approached. The way in which it's taught. We're not gonna teach our students why formulas make sense. We're just gonna tell them the formulas and stuff like that. So if they give me a book and say do these six chapters in the first semester. As long as I don't think it's unreasonable,...I don't really care what they tell me to teach. No, I'll teach whatever topics that they want me to teach, but I'm gonna do it my way. (Interview D-3, 4/14/06)

Matt’s experience with MiC, and knowing that “there’s a different way” seemed to cause him to feel frustrated with the way that the content in his textbook was “approached” and “the way in which it’s taught.” He seemed to object to an approach that is just about “tell[ing] them the formulas” rather than teaching the students “why formulas make sense.” And, as Matt said, he was less concerned about the content than he was about the conventional approach that was typically used to teach it. As a result, Matt said that he would “do it [his] way,” indicating a misalignment of his own curriculum vision and that of *Geometry*.

The data provided above help us to further understand Matt’s philosophy of teaching and his beliefs about curriculum materials. It should not be a surprise then that this misalignment of visions created challenges for Matt when using *Geometry*. In the next section I explore some of those challenges, which will also explain why Matt looked for alternate approaches to working with his textbook.
Ideally, Matt said that he would like to “take the MiC curriculum and rewrite it for geometry.” “But,” he added, “that takes a long time, so I’m not gonna do it now” (Interview D-2, 4/13/06). The issue of time and rewriting curriculum materials has also appeared in research on experienced mathematics teachers. For example, Herbel-Eisenmann et al.’s (2006) Jackie, a teacher enacting different practices in two curricular contexts, expressed a similar sentiment regarding teaching algebra out of a conventional text:

The Standards fit the way I like to do things…[but] it is difficult for me to redesign curriculum materials that are not Standards-based. Time is a huge factor in that….If I wanted to teach Algebra as a Standards-based course, I’d have to design everything from the ground up. (pp. 332-333, emphasis in original)

As both Matt and Jackie said, time is a factor in redesigning curriculum materials. Although Matt said that he did not have time to “MiC” his curriculum materials, data will be presented in future chapters to show that Matt did try to incorporate many of the principles from the Standards into his teaching.

It was not just that Matt did not like his particular textbook, however. Matt said that he really had not seen any geometry textbooks that he thought were very good. This comment was directed more toward the presentation style than to the content. Similar to his description of what most high school textbooks are like, Matt said that 90% of the sections in Geometry were set up as “here’s your green box [of definitions, theorems, postulates, etc.] that tells you what you’re supposed to know” (Interview D-3, 4/14/06).

Unlike the way that he used the MiC materials, in geometry Matt needed to design lessons and activities to have the students engage with during class. More specifically, in terms of the operational curriculum (the content included and emphasized by the teacher in
class), teaching out of his geometry textbook placed very different demands on Matt’s time and planning focus. As a result, the way that Matt tried to approach his lessons was to figure out a way that the lesson would “make actual sense to them instead of just being some random text in the green box” (Interview P-3, 4/14/06). For example, in Chapter 7, I present data to show that in Y3, Matt decided that he needed to show a video clip to help the students understand the laws of logic in the *Deductive Reasoning* lesson. This shows that Matt placed importance on mathematical thinking and on tasks that involved the students in discovering the mathematics. In the subsections that follow, additional interview data is used to explore, in more detail, the challenges that Matt faced with using *Geometry* as a tool to teach proof.

**The Cliffs Notes: “It Doesn’t Tell a Story”**

In an interview during Y3, Matt compared *Geometry* to *Cliffs Notes.* He talked about the difference between the written curriculum (i.e., his textbook) and the curriculum that he enacted in his geometry classroom:

**Example 5-10**

The written curriculum from the book has no connection between the different things that are taught, or very little. Here's this random thing. Very good. Here's this random thing. Very good. There's no, it doesn't tell a story. It's like being given the *Cliffs Notes,* like here's the highlights. You know? So that's a lot of what I try to do is you try to make it seem cohesive and actually let it tell the story and go somewhere instead of just, this highlight, this highlight, this highlight. Let's see if we can figure out how these different pieces are related to each other. (Interview P-6, 12/15/07)

Through curricular adaptation, Matt attempted to make the enacted curriculum more cohesive than the written curriculum. Rather than presenting the material as “this highlight, this highlight,” Matt wanted his students to see the different connections that exist within mathematics. Even though he said, “Let’s see if we can figure out...,” indicating that the

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32 *Cliffs Notes* are a popular series of student study guides.
collective we (I + them) needed to figure out the connections, he also said “what I try to do is you try to make it seem cohesive.” So it seemed that Matt viewed curricular cohesion as something that he needed to facilitate. This idea that mathematics curriculum should be coherent has been advocated in the Standards: When planning lessons, “teachers should strive to organize the mathematics so that the fundamental ideas form an integrated whole” (NCTM, 2000, p. 15). Ideally, Matt’s textbook would present the material in a coherent way (as he believed that MiC did), but if they did not, Matt seemed to believe that it was his job to “tell a story” that was aligned with his curriculum vision. Because Matt was not able to develop curriculum trust exclusively with his geometry textbook, it is not surprising that he looked outside of his textbook for other alternatives that were more aligned with his curriculum vision.

Looking for Alternatives

Rather than lecturing and teaching as telling because it was “ingrained” (as mentioned after Example 5-6), Matt sought out other resources to better align his practice with his beliefs and curriculum vision:

Example 5-11
I have the Key Curriculum Press book. I look through that and I steal from that occasionally. The thing that I find really interesting about that book is all of the proofs are done at the very end….So if you actually taught that book you….would spend seven-eighths of the year having them measure things. The whole time….they're making a conjecture about, the sum of the interior angles of a polygon or something. I mean they're writing formulas and stuff as they go and using those, but in terms of an actual formal proof, they didn't do any of that until the very end of the entire year which is sort of, would be interesting. (Interview P-2, 9/26/06)

The Key Curriculum Press book that Matt referred to in this passage is Discovering Geometry: An Investigative Approach (Serra, 2004). Matt explained that having students
measure, conjecture, and write formulas prior to engaging in formal proof “would be interesting.” Even though Matt said, “I steal from that book all the time,” due to his school context, Matt did not believe that he would be able to use this investigative approach because it did not address formal proof until the end of the course:

Example 5-12
I can't try that here unfortunately because our proofs in our book are in the first semester. So to give the first semester final, they have to know how to do proofs. And there are no proofs on the second semester final. But I have thought about teaching the first semester in reverse order, so that I do the proofs at the end of the first semester instead of at the beginning. (Interview D-2, 4/13/06)

Matt explained here that he did not have the freedom that he would need to use the investigative approach advocated by Discovering Geometry which he thought “would be interesting.” Due to the fact that he had to give a common semester final exam that had proofs on it, he would not be able to wait until the end of the year to introduce proofs. This is evidence of some of the ways that teachers can be constrained by their curriculum materials or their school context. Matt also spoke about alternative ways of using his own textbook, such as “teaching the first semester in reverse order.”

Even though Matt could not restructure the entire course, he did feel that he had “a lot of freedom” within the semester. Basically, he said that he had “pressure from within the department” because of the end-of-semester final exam. He needed to “get through Chapters 1 through 6 by the end of the first semester” (Interview D-3, 4/14/06). In this sense, “coverage” took priority over using an approach that Matt thought would work best for students. He seemed to be able to live with the curricular demand of teaching the first six chapters in the first semester, however, as long as he had some freedom to enact the curriculum the way that he wanted to (Interview D-3, 4/14/06). Yet, Matt also said that
“we’re not gonna fight that battle.” The battle that Matt referred to here was the battle to entirely restructure his curriculum materials and go against the status quo at his school. Instead, Matt talked about reorganizing the sequential order of topics in the textbook made available to him.

**Reorganizing Geometry**

Matt offered several reasons to explain why, as he taught the six chapters, he moved toward curricular adaptation:

**Example 5-13**
If I think they spend too little time on something or if they don't emphasize a particular concept enough…if the order in which it's presented, I don't think will make logical sense to the students…[or] if the logical sequence of these things doesn't make any sense to me, that gets changed. (Interview D-4, 5/22/06)

Matt said that he would spend more or less time on a concept, and he might switch the order around if he thought “the order in which it’s presented” might not make sense to the students or did not make sense to him. Earlier, in Example 5-3, Matt mentioned that, unlike MiC, which chained the topics together, *Geometry* seemed to present topics in isolation. Matt emphasized the “logical sequence” and stressed that in order to have geometry make sense, he sometimes needed to change things around. Again, Matt’s curriculum vision was not always aligned with the vision of his textbook.

In the beginning of this section, I wrote about how Matt said that he would like to “take the MiC curriculum and rewrite it for geometry” (Interview D-2, 4/13/06). He also said that he was not going to do that because it would take too much time. After teaching with *Geometry* for the first year, however, Matt talked about writing his own textbook:
Example 5-14
I don't know if I could push it that far this soon, but in three years, if I wanted to write my own textbook that I could show covered the exact same material as their textbook and it just did it in a totally different order or a totally different way, but if I could show that the content lined up, I don't think it would be a problem. I mean I think they would, that's the ultimate bottom line here is [students] have to know how to do these little skills because we're still skill based. (Interview D-4, 5/22/06)

Again, Matt said that he needed to make sure to “cover” the prescribed content, but that he was given the freedom to cover the topics in whatever way he wanted to. He indicated that in three years (after his probationary period ended), he might think about writing his own textbook that covered the “exact same material.” Matt’s new-teacher status seemed to influence his decision (at least for now) to enact the curriculum with some sort of fidelity. His desire to write his own materials made sense given that Matt said that he had not “really seen anything…in terms of a geometry book, that I thought was good” (Interview D-2, 4/13/06). Matt suggested that, in the future, he might move beyond textbook adaptation and eventually pursue a path of “curricular improvisation,” in which teachers pursue curriculum of their own design (Brown & Edelson, 2003). Later in this chapter, however, data will be presented that shows that Matt eventually abandoned this idea.

In the next section, I highlight one of the four focus lessons that was more closely examined for this study. I do this for two reasons. First, highlighting the curricular content of the Special Triangles section will help the reader understand in more detail some of the challenges that Matt faced when using his textbook. Second, the detail provided here will help the reader understand the enactment of this particular lesson, which will be described in the next chapter.
A Closer Look at *Special Triangles*

During the final interview of the study, after watching video clips of the *Special Triangles* lesson from Y1 and Y3, Matt reacted to the lesson goals of this particular textbook section. These lesson goals (as shown in Appendix D) were: “Using Properties of Isosceles Triangles” (including the Base Angles Theorem) and “Using Properties of Right Triangles” (Larson et al., 2001, pp. 236-238). After watching the video clips (the content of which will be explored in more detail in Chapter 6), Matt said:

**Example 5-15**

Our book is actually pretty good, but I feel like I'm being given the *Cliffs Notes* to like, like there's all this stuff, seriously, like Angle-Side-Side is not mentioned in our book anywhere. Like there is this whole back story about Angle-Side-Side working in some cases and not others, and one of those cases is if you're told that it's a right angle. Then what you end up with is Hypotenuse-Leg. And that's all left out. (Interview P-6, 12/15/07)

In this interview, Matt, again, compared his textbook to the *Cliffs Notes*, saying that there was a “whole back story” about Angle-Side-Side (or SSA) that was left out. An analysis of Matt’s textbook confirmed that he was correct in saying that there was no mention of SSA. In fact, SSA is left out of many geometry textbooks (Usiskin, Peressini, Marchisotto, & Stanley, 2003). Perhaps this is because a thorough explanation of the *ambiguous case* requires knowledge of trigonometry, which is typically taught the year after the geometry course.

Rather than avoiding the SSA situation, Matt said that the authors of *Geometry* should have included it, at least for the teacher, but really for both the teacher and the students. He said: “I mean we should have a section. How hard would it be to just make a section on angle-side-side? It would not be that hard” (Interview P-6, 12/15/07). In the absence of a discussion of SSA, Matt seemed to struggle with figuring out how to teach the Hypotenuse-Leg (HL) theorem, a special case of SSA. Classroom data that shows how the absence of the
“whole back story” of SSA played out in the enacted curriculum is included in the next chapter. In this last example, there was evidence to suggest that Matt’s opinion of his textbook may have shifted after using it for three years. He began the passage (Example 5-15) saying, “Our book is actually pretty good…” This is different from what Matt said about Geometry in Y1. In the next section, through a discussion on how Matt used the textbook to assign homework, we find that toward the end of Y1, Matt said that he “very rarely” directed students’ attention to their textbooks.

Using Geometry for Homework

Earlier, in Example 5-4, Matt described using the MiC materials with the students almost every day by having students work in groups solving the problems from their book. In contrast, Matt said that he almost never directed the students’ attention to look at Geometry:

Example 5-16
Very rarely. Other than to talk about the homework problems. Almost never. Occasionally there's a picture. Every once in a while they do have in their lame attempt to try to be like the Standards. Every once in awhile they will have an activity for a particular section. Sometimes that activity isn't so bad and there have been times when I'm just like, it's 9:30 at night and I don't want to write anything. It's like oh crap, I have nothing for them to do tomorrow. Hey look, the book does this. These questions aren't horribly written and it might make sense. And they could maybe follow it and do this and get what I want out of it. So I'll say go to this page and do this activity or whatever. That's happened maybe four or five times this year.

(Interview D-3, 4/14/06)

Matt’s language choices in this interview indicate that he reluctantly directed students’ attention to their textbook if he had nothing else planned for students to do. Aside from assigning homework problems from the textbook, Matt said that he would only “rarely” have the students do an activity from the textbook or look at one of the pictures. It is also interesting to note that Matt referred to Geometry’s “lame attempt to try to be like the
Standards,” which indicates that he did not believe that the overall structure of this textbook was Standards-based. Matt’s assessment that the questions were not “horribly written” and “might make sense,” again, indicate that Matt did not have a great deal of curriculum trust.

Given that Matt clearly did not have a curriculum trust of his own textbook, one might ask: What did Matt say about how he used his textbook to plan a lesson, and how did he use his textbook as a tool to help him enact the curriculum? Interview data is used to explore the answer to this question. (Classroom observation data will be used in the next chapter to understand how Matt actually enacted the lessons.) I first explore how Matt said he used his textbook as a guide to help him plan his lessons. Next, data will be presented to illustrate the ways in which Matt tried to “connect the dots” for his students. Finally, a description of how Matt said he supplemented his textbook with additional proof will be provided. These descriptions provide evidence of how and why Matt said his teaching changed across time and how he developed a curriculum vision.

Living with Geometry: Developing a Curriculum Vision

Planning in Geometry

In the last example, Matt talked about how he rarely used his textbook with his students during class. The following interview data helps us understand what Matt said about how he used his textbook to help him plan a lesson.

Example 5-17
The book is really the beginning and the end and there's nothing in between. I take the topic from the book, and then I think, okay what can I do to have the kids understand this topic? And then I make a whole bunch of other stuff up that's not in the book. And then I say, okay, what problems in the book am I going to assign ‘cause that's the big thing for parents is, you have to have homework. And it has to be out of the textbook. And if you don't use the textbook any other time that's fine because they
didn't either. So pretty much it's what topic in the textbook am I gonna talk about and then at the end what problems out of the textbook am I gonna assign them for homework to check their understanding of whatever we just talked about. And that's pretty much it. (Interview D-3, 4/14/06)

So, according to Matt, he simply used his textbook as a guide for what to teach, but with the exception of the occasional activity (mentioned earlier in Example 5-16) and the homework exercises, he really did not use his textbook to help him think about how to teach the topic. Matt also mentioned that he believed that it was important to assign homework from the textbook because “that’s the big thing for parents.” This is not surprising given that research has shown that parents’ opinions can influence their children’s experiences in mathematics classrooms (see, e.g., Herbel-Eisenmann et al., 2006; Lubienski, 2004; Peressini, 1998).

In the above example, Matt also said, “I make a whole bunch of other stuff up that's not in the book.” This description of how Matt used his geometry textbook is in stark contrast to the way that Matt said he used MiC (see Example 5-5). In geometry, Matt felt that it was up to him to adapt the curriculum materials to engage students in the lesson and to tell a cohesive story. Using Geometry involved a lot more planning outside of class than using MiC did. Matt already mentioned (see Examples 5-1 and 5-11) that he “[stole]” from other textbooks (e.g., MiC, Serra’s Discovering Geometry, and in Example 5-26, Jurgensen et al., 1985). In the next chapter, data will be presented to illustrate that Matt explicitly tried to engage students with proof. In other words, we will learn more about the things that Matt did that were “not in the book” to engage students with “real math.” First, more data will be presented to help the reader understand how Matt said he used Geometry to enact the curriculum.
Developing a Story and Connecting the Dots

Matt was able to find some redeeming qualities about his textbook. These positive attributes were frequently coupled with criticism, however.

Example 5-18
The textbook that we have is actually readable, not that I want my students to read it, because again, it doesn't give reasons for very many things. It just says, this is what it is. But it's actually fairly readable for the students. If they're gone for a day, I can say look at the box, write down the formula….and I can, even though the story that the textbook tells is not the same story that I would tell, there's enough things in there that I can create what I think is a more sequential story or a better story for the students. So that's the good things about it…. [but] anything that any other textbook could be criticized on, it could be criticized on. (Interview D-3, 4/14/06)

Matt said here that the textbook was “fairly readable for students,” but he would not want his students to read it unless they were absent. In other words, Matt believed that his students would be able to pick up the textbook and read and learn something from it. However, due to the presentation style, in which finished results were presented in green boxes, Matt said that unless they were absent, he did not want his students to read the textbook. He preferred to engage students in an activity or a discussion that would assist them in understanding the mathematics. Matt also said that there were enough basic things in Geometry that allowed him to use it as a tool to tell “a more sequential story or a better story.” Matt’s description of his curricular adaptation represents a “shared” responsibility for curriculum design (Brown & Edelson, 2003). This responsibility is shared by teachers and the materials. Matt said that he was able to draw on the textbook to create a “story” for his students and that he was able to use the problems from the textbook in order to assign homework. He also pointed out, however, that the textbook could be criticized for “anything that any other textbook could be criticized on.” When Matt said, “any other textbook,” he seemed to mean any other conventional textbook. When pressed further to explain this statement, Matt said:
Example 5-19
There's no coherence, there's no explanation. It's just, here's all your formulas in the boxes now use them. They give you some nice examples of some places that you could use it in the real world that aren't horribly hokey. But it's not a situation where it really makes sense to the students when they read it. It's still very, presentation style...from the standpoint that I know what everything really is already and you don't, so bow at my feet you little peon of geometry 'cause I know what I'm doing already and you don't. (Interview D-3, 4/14/06)

Matt’s criticism of his geometry textbook presented here is similar to his criticism of all conventional textbooks that was provided earlier in this chapter (Example 5-7). Unlike the MiC materials, which he found to be very well-connected, Matt felt that there was no coherence or explanation in Geometry. Again, Matt mentioned the formulas in the [green] boxes. Like many conventional textbooks, in Geometry, examples followed the formulas, but what Matt objected to was the “presentation style.” He seemed to be alluding to the authoritative voice of the textbook again when he said, “that I know what everything really is already and you don’t, so bow at my feet you little peon of geometry.” Again, this objection seemed to be based in Matt’s belief that students can and should understand mathematics.

When Matt said, “I know what everything really is” and “I know what I’m doing already,” the “I” seemed to refer to the textbook authors, not Matt. On the other hand, it is possible that Matt was including himself in the “I,” meaning that he, as the teacher, was seen as the authority of the mathematics among those in the classroom. Matt also objected to some of the problems in the textbook.

Even the word problems (which Matt found interesting in MiC) were “not very interesting” in Geometry:

Example 5-20
It's pretty much a direct application, or do something else and then do an application of whatever this rule was. There are some that....are actually pretty good problems
that make people think. But… it's all about, here's your formula, plug these numbers into it and get the answer. (Interview D-3, 4/14/06)

Again, Matt commented on the procedural emphasis in Geometry. This comment might seem confusing or surprising given that the textbook is a geometry textbook, and the first six chapters were supposedly about proof. Looking through the textbook, however, it is easy to see what Matt was talking about. For example, in the Special Triangles section, there were 12 questions in “Practice and Applications” that involved algebraic applications of some of the theorems that were presented in the green boxes. Examples of two such applications are shown in Figure 5-2. So although there were some proofs at the end of this section, as was typical of all of the sections, there were many more applications than proofs.

![Figure 5-2: Exercises from Geometry (Larson et al., 2001, pp. 240-241)](image)

By Y2, Matt’s conceptions about the usefulness of his textbook seemed to have changed. In Example 5-18 (at the end of Y1), Matt said that although his textbook was readable for students, he would not want them to read it because “it doesn't give reasons for
very many things.” By Y2, however, Matt said that the Geometry textbook was a useful “reference” for his students:

Example 5-21
During class I totally reference the book all the time…constantly…because I really want [the students] to see that the book is something useful and isn’t just a collection of problems that, I mean, the one thing, the book is not great, but if what you want for your students is examples and practice, our book does a very good job, I think. Sort of setting out, ‘Here’s an example, here’s why it works’…[and] if what I want it to do is simply to teach skills I would have them read the book because it does that very well. (Interview D-5, 9/25/06)

In contrast to what Matt said in Example 5-18, here Matt said that he did want students to realize that their textbook was a useful reference rather than just a book of problems to be solved. He also said “if what you want for your students is examples and practice, our book does a very good job.” Although his department head and some of his students’ parents may have had this as a goal, clearly, having students read textbook examples and learn skills did not match Matt’s curriculum vision. Matt preferred to have more connections between topics, and he wanted his students to understand “why any of these things work.” So with a different vision for his students, how was Matt able to use Geometry as a tool to teach proof? He explained:

Example 5-22
So what I try to do then in class is a lot of times sort of, you know, sort of draw that together. Okay, I…try to kind of connect the dots and then, you know, just for the skill part let the book…fill that in. So I’d reference the book a lot ‘cause I want them to realize that the book is useful for things other than just a problem set. (Interview D-5, 9/25/06)

Matt explained that he helped make the material more cohesive for his students by connecting the dots for them. Also, rather than just emphasizing skills, it was his responsibility to “fill that in.” In other words, Matt did not follow the presentation style of his textbook, which simply gave the finished results in the green boxes. Matt said that he tried to
explain where the ideas came from and filled in the proofs for the theorems. These things should be done, according to Matt, rather than allowing the students to simply use the results of the theorems to complete simple algebraic applications. So it seemed that Matt not only wanted his students to see the textbook as a useful reference, but he was starting to see that he could use it to carry out his own curriculum vision.

Immediately following his claim that the textbook was a useful reference, however, Matt again spoke about reorganizing the order of presentation:

**Example 5-23**
Um, so eventually, what I’d really like to do is just totally reorganize the full first semester. And...probably I would like to just do it backwards. Probably put [Chapter] 2 at the end, put [Chapter] 3 before that put [Chapter] 4 before that and then do like chapters 1,...5,...[and] 6. (Interview D-5, 9/25/06)

Rather than talking about writing his own textbook (as he did in Example 5-14), by Y2, Matt spoke about using his current textbook but reorganizing the sequence of the six mandated chapters.

By Y3, there seemed to have been another small shift in the way that Matt talked about *Geometry*. He seemed to have developed a more harmonious relationship with his textbook. Matt said that he tried not to “undercut” his own textbook, and then he proceeded to name some of the positive and negative attributes of *Geometry*:

**Example 5-24**
What my book does really well is show examples and work through basic computational type problems. And what does my book not do very well? My book does not tell a cohesive story about math, and it doesn't tie these pieces together....In terms of the proofs, it does a very nice job of presenting all of the theorems in a consistent, if-then format. So what can I use that for? So we do a lot more things like...here's the textbook formulation of this theorem. Okay, how can we draw from that? Let's translate this into an if-then type of statement. What are our givens? What are we trying to prove? How can we fill in the actual proof of that as opposed to, you know, just taking the textbook for what it is. So you try to reach a point where you understand what the book does and let it do it. I mean if I were teaching MiC again,
the thing that I would do differently is… I would try to have more actual discussions at the end of the class to tie the different pieces (Interview P-6, 12/15/07)

In this interview, Matt seemed to have found a way of adapting his textbook so that he was able to live with it. He was no longer talking about writing a new textbook or reorganizing *Geometry*. Rather, he said that his textbook did “a very nice job of presenting all of the theorems in a consistent, if-then format.” As a result, he could take these theorems and use the conditional statements to figure out what the “givens” were and to figure out what it was that he and his students were “trying to prove.” Matt then mentioned “fill[ing] in” or doing the actual proof rather then “just taking the textbook for what it is.” This excerpt suggests that Matt began to adapt the textbook across time to more explicitly emphasize proof. The idea of explicitly emphasizing proof is an important finding that is discussed briefly in the next section and is then explored in more depth in the next chapter.

At the end of this passage, three years after coming to teach at the high school, Matt, again, returned to MiC when he was talking about his current textbook. He said, “If I were teaching MiC again, the thing that I would do differently is… I would try to have more actual discussions at the end of the class to tie the different pieces.” So it seemed that since Matt’s first year of teaching, he now placed more importance on the role of the teacher in helping students “connect the dots” and making sure that the students understood the point of the lesson. In the next section, I continue to explore the interview data to understand how and why Matt supplemented his textbook with additional proof.

**Supplementing *Geometry* with Additional Proof**

At the end of Y1, Matt talked about the proof-related goals that he had for his students in geometry for the following year:
Example 5-25
I think I like what I did with the proofs when I did it. What I don't like and this, it's been sort of discussed in our department, is that really after Chapter 3, well Chapter 4, you don't do anymore proofs at all after that…especially the whole second semester you don't do any proofs. Chapter 6, you really don't do any proofs. So…if I'm gonna do a lot of proofs at the beginning, then sort of move that through…. ‘cause the proofs as it stands now is really just…kind of this isolated, weird thing...not even a whole semester….and so I'd like to kind of draw that through at least until the end of first semester...so that they see that proofs don't just go away. (Interview P-1, 6/21/06)

Here, Matt said that he would like to move proof through the entire first semester so that students did not think that “proofs just [went] away.” Even though Matt said increasing the emphasis on proof had been discussed in his department, during the three years of the study, the common exams and the objectives within the department did not seem to change. If Matt was going to treat proof in a way that was not an “isolated, weird thing,” then he was going to have to do it on his own. As will be shown in the rest of this section and the next chapter, Matt did take on this challenge.

During Y2, Matt handed out a sheet with additional proofs. He explained his reasons for supplementing his textbook with these additional proofs:

Example 5-26
The proof sheet is new from last year….I found the, you know, the old Jurgensen\textsuperscript{33} or whatever, the orange book that everybody used...so I went through that just because...really I did it because the proofs in our book, some of them are interesting, but a lot of them are very semantical...like, you know, prove that angle A is congruent to angle A. Who cares? Like that's not very interesting to them.... So I tried to go through, and I just went through Chapter 2 or whatever their Chapter 2 equivalent was and try to come up with...like 10 proofs that are not super difficult and…they just were more interesting and actually had them doing some more things so it wasn't just logical semantical proof. That there was some more real actual proofs that they were doing. (Interview P-2, 9/26/06)

When pressed further, Matt explained that when he said that the proofs in his textbook were “very semantical,” he was referring to the use of the same simple properties and definitions

\textsuperscript{33} The textbook that Matt referenced here was Geometry (Jurgensen, Brown, & Jurgensen, 1985)
over and over again in the proofs. That is, many of the proofs involved moving from congruent to equals and back to congruent again. An example of the kind of proof that Matt referred to here is given in Appendix A. The proof shown in the appendix was one of the example proofs from the textbook, and Matt did this proof on the very first day that he introduced proofs to the students in Y1.

During the final interview in Y3, we revisited the addition of the proof sheet. Matt reiterated his distaste for the proofs in the beginning of Geometry, calling them “a dumb thing for them to start with” (Interview P-6, 12/15/07). He said that starting with proofs like that just makes proof seem “even more meaningless than they already think it is, which just makes it worse” (Interview P-6, 12/15/07). The sentiment that, for students, proof writing is often seen as an exercise in futility that is completed for the teacher has been noted in the literature (Alibert, 1988). By giving the students “more interesting” proofs, Matt was trying to help his students avoid the view that proof writing was “meaningless.”

In addition to Matt wanting to supplement the textbook with additional proofs, he also wanted to have proof last for more than just a few short chapters. Matt frequently spoke about pulling proof through the first semester. He continually claimed that “proof just [went] away” after Chapter 4. Further investigation into this claim was not fully substantiated by an analysis of the textbook. Although it did seem that proof was not emphasized as much after Chapter 4, Geometry still provided some proofs in the homework exercises. The bigger issue, it seemed, was that after Chapter 4 the departmental exams that Matt was using no longer included proofs. So even though Matt was no longer required to teach proof after Chapter 4, in Y1 he took on the challenge of having his students engage in proving for a longer period of time. As was already explored in Chapter 4 of this dissertation, Matt explained that it was
important to him that students continue to engage with proofs because proofs were “actual math” or “real math.”

**Using Geometry to Tell His Story**

After supplementing his textbook with additional proofs, Matt began to understand why the topics appeared in the order that they did:

**Example 5-27**

I had thought about drastically rewriting the order of all of it, but like when I started putting that proof stuff in [Chapters] 5 and 6, I kind of got the idea….I think I kind of saw where these different things were coming from, why these things were put in this order. So, I don't think, I mean I like the order. I think there's nice instances from [Chapters] 5 and 6 where things can be built on from [Chapter] 4. Um, and I'd like to continue that. (Interview P-3, 4/19/07)

Matt said that supplementing the textbook with additional proofs actually helped him to see that some of the ideas from textbook Chapters 5 and 6 built on the ideas from Chapter 4. So by Y3, Matt said, “I like the order.” Clearly this is a shift from the first year of the study. Not only did Matt move away from any discussion about writing his own materials, but now he seemed to be at ease with the sequential order of the topics in the textbook. He did not, however, retract any of his comments about the presentation style of the textbook.

**Adapting Geometry in the Absence of Curriculum Trust**

In the introduction to this chapter, I put forth the idea that Matt’s views and past experiences would likely impact his enactment of the geometry curriculum. In addition, research has shown that teachers can be changed by the curriculum programs they use (Lloyd, 2008; Philipp, 2007). In this chapter, data was presented to show that both Matt’s views about mathematics and proof, as well as his experience with MiC, influenced his enactment of Geometry. Because Matt’s curriculum vision was not aligned with the vision of
his textbook, Matt was not able to trust the textbook the way that he trusted MiC. In fact, Matt was influenced by MiC so much that he continued to mention it over and over during the three years of this study despite the fact that he no longer used it. One interesting point that can be made through this data is that, for Matt, because the curriculum vision of MiC was aligned with his own curriculum vision, the Standards-based curriculum materials were much easier to enact than the conventional textbook. Matt was able to see and appreciate the connected nature of MiC. In contrast, it seemed that the “presentation style” of Geometry was problematic for Matt. Because he wanted his students to understand and “see where these things [came] from,” using a conventional textbook created challenges and dilemmas. As Matt said, even though he was “already all about having kids think about things,” when Matt was handed a conventional textbook the first year, he stood in the front of the room “talking to [him]self.”

Matt was eventually able to adapt his textbook to create a “story” with which he was more comfortable. To be clear, I am not claiming that Matt was fully satisfied with his current teaching practices, but the nature of his comments about his textbook did shift during the three years of this study. During Y1, he spoke about reorganizing Geometry and even writing his own textbook. By Y3, Matt saw “why these things were put in this order” and decided that he liked the order. Although he still had criticisms of Geometry, Matt was able to find some redeeming qualities about it.

Since, in Matt’s view, Geometry did a “very nice job of presenting all of the theorems in a consistent if-then format,” Matt was able to use the textbook to teach proof in his own way. More specifically, Matt decided that he could translate the theorems into “givens” and “proves” to “fill in the actual proof.” In the next chapter, interview and classroom data are
used to explore the finding that across time, Matt more explicitly focused on proof in his geometry class.
CHAPTER 6: ENGAGING STUDENTS IN PROOF

Despite the fact that the curriculum vision provided by *Geometry* was not aligned with his own vision, Matt found a way to use the textbook as a tool to explicitly focus on proof. In this chapter, interview and classroom data are used to understand how and why Matt increasingly focused on proof from Y1 to Y3. First, data will be presented to show that rather than simply “covering” the theorems in the green boxes, more time was spent engaging the students in proof. Matt said that he was interested in helping the students understand why the theorems were true and in “filling in” the proofs rather than just presenting the theorems as finished results. Despite its shortcomings, Matt valued the fact that *Geometry* presented theorems in the if-then format which allowed him to use it as a tool to engage students with proof. More specifically, Matt said that over time he asked students to determine the “given” and the “prove” from the conditional statement and then set up the diagrams and the proof accordingly. In doing so, Matt moved closer to his goal of engaging students in “real math.”

In addition to showing that Matt increased the amount of time spent on proving, data from three lessons was analyzed and interpreted to show some of the specific changes that Matt made as he more explicitly focused on engaging students in the process of doing proofs. Also, data will be presented to illustrate the ways that Matt attempted to present proof practices that were more authentic than those that have been described as normative in school mathematics. Taken together, this evidence is presented to show that the changes from Y1 to Y3 were, for the most part, made *deliberately* to give students a more authentic experience.
with “real math” through proving. Prior to the qualitative descriptions of the lessons, I first present interview and quantitative data to illustrate an increased focus on proof.

**More Time Spent on Proof**

In Chapter 5, interview data were presented to document that after teaching proof for the first time, Matt set a goal to “drag proof through” the entire first semester. Even though he said that after Chapter 4 in *Geometry*, proofs “just [went] away” from the textbook and the departmental exams, Matt wanted his students to view proof as something other than an “isolated, weird thing.” As discussed in Chapter 5, Matt supplemented the textbook with additional proofs taken from another textbook. In addition, as will be shown in this section, Matt spent more class time engaging students with proof, rather than simply telling students the theorems. Data will be presented to show that more time was spent in the whole-class work activity and that the word “proof” was uttered more frequently in Y3 than in Y1. Before presenting this data, I first present additional interview data to show that Matt’s beliefs about the treatment of reasoning and proof in school mathematics were closely aligned with the call for change made in the Standards documents. Presentation of this data will help illustrate why Matt encountered challenges when he tried to enact a Standards-based pedagogy using a conventional textbook. In addition, understanding Matt’s beliefs about how proof should be taught can help the reader understand why Matt’s proof-related teaching practices evolved the way that they did.

**Proof throughout the Curriculum**

Matt was interested in extending the study of proof through the entire first semester, and maybe even beyond. During an interview at the end of Y2, Matt said:
Example 6-1
This year I did start to drag the proof concept through Chapter 5 and into Chapter 6….So that I was glad to do this year. I mean I think that needs to extend. I would eventually like to see that extend all the way through…first semester for sure and then…what would second semester proofs look like? Could I find some things other than like the Pythagorean Theorem which I already do? Can I find one or two other things in each chapter or unit that they could do to write a formal proof of so that we continue to drag this concept beyond just this particular set of things? (Interview P-3, 4/19/07)

In this example, Matt not only discussed extending proof through the first semester, but he also talked about wanting to extend proof into the second semester of the geometry course as well. His goal was to find “one or two other things” from each unit that students could formally prove. This example suggests that Matt believed that students should not experience proof as an isolated topic that is taught as part of the 10th grade geometry course. Matt also believed that students can and should start thinking about offering reasons and evidence for their conjectures prior to the 10th grade course. He said that students can start to think about reasoning and argumentation as early as first grade. In addition, students should see proofs after the 10th grade course as well:

Example 6-2
I think it's important for students to see proof….I wish that it was carried through, not only through geometry but….I wish that they were asked to do more proofs in Algebra II….and I wish they proved things in Precalculus and Calculus. I wish that they had to do more things like that. Not that they had to prove everything, but I think that there are certain things that are important enough that they use all the time like the quadratic formula for instance. They should have to prove the quadratic formula because that's a big cornerstone thing….but instead just get…hand-waved at. Here it is what it is. Accept it. Let's move on. Let's….not even discuss the reasons for it, let alone give the proof…just here's the formula. Have fun. (Interview P-3, 4/19/07)

Here we see that Matt would like for students to see proof throughout high school. As he said, there are some things like the Pythagorean Theorem that students use frequently, and they should understand that there are valid reasons that justify this theorem. Ideally, Matt
believed that students should discuss or construct proofs rather than simply be given the theorems or formulas.

Matt’s beliefs about the experiences that students should have with reasoning and proof are closely aligned with the Standards:

From children’s earliest experiences with mathematics, it is important to help them understand that assertions should always have reasons. Questions such as ‘Why do you think it is true?’ … help students see that statements need to be supported or refuted by evidence…. Part of the beauty of mathematics is that when interesting things happen, it is usually for a good reason. Mathematics students should understand this. (NCTM, 2000, p. 56)

Similar to this quotation from PSSM, Matt believed that students should understand that when something was true in mathematics, it was usually for a good reason. Data related to Matt’s vision of proof in school mathematics provides further evidence that Matt’s curriculum vision was not aligned with his written curriculum (i.e., the geometry textbook). As far as proof was concerned, Matt’s interest in carrying out a Standards-based practice was hindered by a textbook that presented theorems in green boxes. This was problematic for Matt since he believed that this type of presentation did not encourage students to think about why the theorem or the formula in the green box was true.

After teaching geometry for the first time and learning that the story that his textbook told was not the same story that he wanted to tell, Matt made some changes. More specifically, in our final interview during Y3, Matt talked about how he fulfilled his goal to “pull proof” through the entire first semester:

Example 6-3
You're trying to pull proof further… through the Chapter 5 and Chapter 6 so… you end up spending more of like the whole first semester doing proofs as opposed to 'we're done with Chapter 4 so, now we're finished.' So that's the biggest thing is that you're talking about it from a proof standpoint. (Interview P-6, 12/15/07)
As Matt said, by Y3, he extended proof through the entire first semester, teaching proof in Chapters 5 and 6, which he did not do in Y1. When he said, “you’re talking about it from a proof standpoint,” Matt was referring to the idea that when he talked about the Properties of Triangles (Chapter 5) and Quadrilaterals (Chapter 6), he said that he now discussed the proofs of the theorems. This shows that Matt was interested in making the geometry course about proof and not just about teaching theorems from green boxes that could be used to solve algebraic exercises like the ones in Figure 5-2.

Matt wanted students to have a more thorough experience with proof that went beyond a short treatment. After all, proof was what “real math” was all about, according to Matt. He believed that if he covered proof over only three chapters, then students “just try to survive” (Interview P-6, 12/15/07):

Example 6-4
Otherwise they don't actually try to prove anything. They don't try to understand what it is you're doing. Let's just, you know, if you only do it for Chapters 2, 3, and 4, it's like well, I can survive. Let me just get through these three chapters and then I'll never have to do it again. (Interview P-6, 12/15/07)

Clearly, Matt wanted his students to have a more substantial experience with proof beyond the three chapters that he was mandated to teach proof in. Again, Matt’s beliefs were aligned with the PSSM which say: “Reasoning and proof are not special activities reserved for special times or special topics in the curriculum but should be a natural, ongoing part of classroom discussions, no matter what topic is being studied” (p. 342). As the rest of this chapter will show, by Y3, Matt’s practices were more closely aligned with his beliefs about proof and “real math.”
More Time Spent on Whole-class Work

By Y3, Matt spent more time engaged in whole-class work activity. Table 6-1 shows that in the four focus lessons, Matt spent more than twice as much time engaged in whole-class work in Y3 than in Y1. During Y1, Matt spent a total of 58 minutes on whole-class work, but in Y3, he spent 123 minutes on whole-class work. There are two reasons that help explain this change. The first reason involves the fact that Matt said he was required by his department to give frequent quizzes to his students. After the first year of the study, Matt scheduled the quizzes on separate days so that students did not have to learn new material on quiz days. During Y1, students took quizzes on three of the four focus lesson days. The exception was that he did not give a quiz on the same day as the Bisectors lesson in Y1. During Y3, Matt did not schedule a new lesson and a quiz on the same day. A second reason that explains the time difference between Y1 to Y3 is that Matt scheduled two days for the First Proofs lesson in Y3 rather than just one day. The reason for this will be discussed later in this chapter.

Table 6-1: Time Spent in Whole-class Work

<table>
<thead>
<tr>
<th>Lesson</th>
<th>Y1</th>
<th>Y3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deductive Reasoning</td>
<td>10</td>
<td>15</td>
</tr>
<tr>
<td>First Proofs</td>
<td>30</td>
<td>50</td>
</tr>
<tr>
<td>Special Triangles</td>
<td>15</td>
<td>20</td>
</tr>
<tr>
<td>Bisectors</td>
<td>10</td>
<td>15</td>
</tr>
</tbody>
</table>
“Proof” Uttered More Often

More time spent in whole-class work provided Matt with time to actually work through and discuss the proofs of the theorems for these lessons. The words “proof” and “prove” were found six times more often during the whole-class work activity in the Y3 transcripts (of the focus lessons) than in the Y1 transcripts (30 times versus 5 times). Findings from the qualitative analyses will be provided in the upcoming sections of this chapter to show how the First Proofs, the Special Triangles, and the Bisectors lessons changed from Y1 to Y3. These comparisons show that Matt emphasized the proofs of the theorems more in Y3 and that Matt was more explicit about the process of proving. I begin by looking more closely at Matt’s introduction to proof in the First Proofs lessons.

Introducing Proofs

In this section, descriptions of the First Proofs lessons from Y1 and Y3 as well as interview data is presented to help understand the changes that were made and the reasons, from Matt’s perspective, that these changes were made.

First Proofs in Y1

During Y1, Matt began the First Proofs whole-class work by checking in with students about their definition sheets. Matt made this sheet to help students learn the definitions that they would be using in the upcoming proofs. He answered two student questions about definitions. Next, Matt explained the Segment Addition Postulate34 and the

34 The Segment Addition Postulate says: “If B is between A and C, then AB + BC = AC” (Larson et al., 2001, p. 18)
Angle Addition Postulate. After taking care of this initial business, Matt said: “Now here’s where the fun begins. Ready? Here’s your first proof, your first step into a larger world” (Y1 First Proofs, 9/30/05). Matt then proceeded to present two proofs which were both examples taken from Geometry. Matt referred to these proofs as very “semantical” (see Chapter 5). An example of one of these proofs appears in Appendix A.

While presenting the first proof, Matt told a story about Pokémon:

**Example 6-5**
Matt: If [the line segments] have the same length, then they have to be congruent. So, the definition of congruence, I choose you [inaudible]. Nobody in here watches Pokémon? Ever? Are you kidding me? Are you serious, nobody watches Pokémon? Nobody even has a sibling who watches Pokémon?
Ss: [inaudible]
Matt: We’re gonna have to rent it. Alright? Piccachu, I choose you. Right? That’s how you wanna think about this. I remember this. In college, my roommate one time, he was a good friend of mine [inaudible]. And we’re sitting there and one of his internet browsers wasn’t working, so he totally decides to switch his internet browser, and of a sudden we’re sitting there working and he goes “Minsky, I choose you.” [inaudible] That was really funny. But I remember that last night. That’s the way you want to think about this, right? Definition of congruence. Go, right? Symmetric property. Go. Definition of congruence. Go. Now I’m done, right? That’s how we proved this. Okay.
(Y1 First Proofs, 9/30/05)

In this example, Matt attempted to connect with the students by referencing Pokémon. Even after realizing that the students did not understand the reference, Matt continued to connect to Pokémon, saying “Symmetric property. Go. Definition of congruence. Go.” Matt seemed to be taking the primary responsibility for completing the proof when he said, “Now I’m done,” but then he switched to we, saying, “That’s how we proved this.” In fact, as will be shown in more detail in the next chapter, during Y1, the students were not given very many

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35 The Angle Addition Postulate says: “If P is in the interior of ∠RST, then m∠RSP + m∠PST = m∠RST” (Larson et al., 2001, p. 27).
36 Pokémon refers to the anime series and films from the U.S. and Japan that are based on the video game of the same name.
opportunities to participate in the construction of proofs. In Y1, a great deal more of the talk during the whole-class work was done by Matt, not the students. After he completed the two example proofs from the textbook, Matt started to go over one of the homework problems from a previous assignment. He did not complete it, however, because the bell rang and they ran out of time.

Matt wrote about how he thought this lesson went in his Daily Reflection:

**Example 6-6**

Today the students took a quiz…which took a decent amount of the class period, more than I had intended….The fact that the quiz ran a little long forced me to somewhat rush through the first few proofs we have done as a class. We worked through these proofs together as a class at the board….Today was fairly representative, these things aside, however. I don’t like to ever just tell students information blindly – I would rather do a limited call-and-response like I did today….I hope that I made the large mystique and psychological barrier of proofs seem more breachable by using the Pokémon example and terminology….I had to rush through a bit and just told the students what the postulates were that they would need to know, which was a little disappointing. I would also have liked to spend a little time talking about the difference between postulates, definitions, and theorems, but I didn’t have enough time. (*First Proofs* Daily Reflection, 9/30/05)

In this reflection, Matt explained that he felt that he was “forced” to “rush” through this *First Proofs* lesson because the quiz took more time than he had anticipated. Matt also claimed that he and his students worked through the proofs together. However, from my analysis of the discourse (which will be expanded on in the next chapter), it seemed as though Matt was not only doing most of the talking, but he was also the one doing most of the intellectual work. Matt also commented that he hoped the Pokémon example was helpful to students. He was disappointed that he “told the students what the postulates were.” Matt was disappointed about this because teaching by telling went against his teaching philosophy. He also wanted to talk about the difference between postulates, definitions, and theorems, but said that he did not have enough time for that either. Since postulates, definitions, and theorems are what
justify the statements in a proof, then being explicit about what these things were might have been helpful to the students. Matt was hopeful, however, that the Pokémon example was “psychologically” helpful to his students. Two years after this lesson took place, Matt provided a different evaluation of how it went.

After reading a transcript of this lesson during the final interview in Y3, Matt reacted to the *First Proofs* lesson that he taught in Y1:

**Example 6-7**
I'm glad you lost this tape...because I'm just, I'm talking to myself. The whole thing about Pokémon, I'm talking to myself. They don't understand it. [I should] just stop talking....The whole Pokémon thing that I thought was funny at the time, it just doesn't go anywhere. I mean I understand...what I was trying to do with it, but...get rid of all of that. (Interview P-6, 12/15/07)

Two years after teaching this lesson, Matt reacted to how much he talked in the lesson and to the irrelevance of the Pokémon example, saying that he was glad that I lost the tape of the lesson. Matt seemed to realize that since his students were not familiar with Pokémon, he was the only one who understood the example. The Pokémon example was one of several off-topic examples from Y1 that Matt seemed to use as fillers in his lessons. I rarely observed this kind of talk during the lessons in Y3. More evidence of this claim will be provided in Chapter 7. Stating that he was just talking to himself was a different perspective from the one that Matt had in the Y1 reflection when he said that he liked the call-and-response interaction that occurred between him and his students. One explanation for this shift in perspective could be his involvement in the larger Discourse Study. By the time the final interview was conducted, Matt had participated in the study/discussion groups focused on classroom

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37 This is the video tape that I referred to as missing data in Chapter 3. As I already mentioned, the lesson was transcribed from the audio-tape.
discourse. In the next section, I discuss some of the scheduling changes that Matt made in Y2.

**Making Changes to the First Proofs Lesson**

In an interview during Y2, Matt explained one of the changes that he deliberately made from Y1 to Y2:

**Example 6-8**
I deliberately scheduled more days to talk about Chapter 2. It got sort of squished last year, I thought. So I spent too much time on [Chapter] 1. So I made [Chapter] 1 a little shorter, and spent more time on [Chapter] 2. (Interview P-2, 9/26/06)

In this quotation, Matt was referring to Chapter 2 in *Geometry* which is the “Reasoning and Proof” chapter. In this chapter, students learned some basic logic laws, deductive reasoning, and then they began their study of proof. After changing the number of days spent on the chapter from 10 to 13 in Y2, Matt maintained the same pace during Y3. This change impacted how much time was spent on the First Proofs lesson, increasing the introduction of proof from one day to two days. This scheduling example combined with the issues related to scheduling quizzes on the same days that new material was presented suggest that novice teachers could benefit from having a mentor to help them with long range planning. These particular scheduling changes, which were made in Y2, remained stable in Y3. In the next section, I explore how Matt enacted the First Proofs lesson in Y3.

**First Proofs in Y3**

Two specific changes were observed in the way that Matt introduced proofs in Y3: (a) what Matt wanted students to do before they started writing their proofs; and (b) the flexibility Matt stressed related to the form of the proof. After presenting the classroom data,
interview data will be used to help explain, from Matt’s perspective, his reasons for making these changes. I begin by discussing what Matt wanted students to do before writing a proof and then explore the reasons that he made this change.

“Convince Yourself That It is True”

Rather than using the “semantical” example proofs from his textbook (as he did in Y1), in Y3, Matt wanted to start with a proof that was “more interesting.” He found a proof from the Jurgensen textbook (1985, p. 20) and included it on a proof sheet that he supplemented his geometry textbook with (see Chapter 5 for more about how he supplemented the curriculum materials). This proof appears in Appendix F. After distributing the proof sheet and telling the students which proof they would be working on, Matt began the lesson by talking about what the students should do before they write a proof:

Example 6-9
Before you ever write a proof, you want to make sure that you can convince yourself that it's true, okay? No one learns anything by writing a proof. They just write down what they already know has to be true. So let's look at number 12, here. Look at that problem for 25 seconds. See if you can convince yourself that it has to be true. (Y3 First Proofs, 9/21/07)

Instead of jumping right into the proof as he did in Y1, Matt gave the students time to think about the proof. Rather than giving students 25 seconds to look at this proof, Matt actually gave them about a minute before asking the students, “So how many people are convinced that this has to be true?” Even if a minute was not enough time for students to think through the proof, Matt attempted to involve students by giving them this time and then asking them if they were convinced of the truth of the proposition that they were supposed to prove.

Matt asked the students “why” the statement was true, and then he called on a student to provide an explanation. By soliciting a plan for the proof before writing anything down,
Matt actually breached a norm of the ‘typical’ proof process (in school mathematics). That is, in school mathematics, a proof is typically started “by listing all information that one knows about the figure involved” (Herbst & Brach, 2006, p.101). According to Herbst and Brach, writing down the “givens” is usually the first thing that one does when writing a proof, not thinking about the proof. It is also the case, however, that in school mathematics, students would typically only be asked to prove statements that are, in fact, true (ibid). For this reason, the exercise of convincing oneself that the statement is true might seem somewhat silly to the students. Proving statements that are already known to be true is one of the many ways that school mathematics and authentic mathematics are incongruent.

Matt’s addition of a first step that involved convincing oneself that the proof was true was not unique to this lesson. During Y3, Matt promoted this idea over the next few days. On the second day of the First Proofs lesson, the following exchange took place at the beginning of the whole-class work:

**Example 6-10**
Matt: Okay. Now, when you write a proof, what's the most important step?
   MS: The first step.
Matt: What is the first step?
   MS: The given.
Matt: No.
   MS: Well, I was wrong.
Matt: The most important thing when you write a proof happens before you ever write anything down.
   FS: The problem.
   FS: Convince yourself that it is true.
Matt: Convince yourself that it is in fact true.
   MS: Oh yeah.
(Y3 First Proofs, 9/24/07)

Before proceeding with the “givens,” Matt asked students to think about the proof or “convince yourself that it is in fact true.” After completing this first proof, Matt introduced a
second proof and, again, told students to: “Take 25 seconds while I erase the board. Convince yourself that it must be true” (Y3 First Proofs, 9/24/07). Even though the practice of proving statements that are already known to be true is not consistent with “real math,” by giving students time to think about the proof, Matt created space to include students in the construction of these proofs. In this sense, Matt introduced a different norm in his classroom – one that could potentially “involve students in proving” (Herbst, 2002a, p. 200). In the next section, interview data is provided to help us understand Matt’s reasons for making this change.

“Convince yourself that it has to be true” explained. After observing that Matt repeatedly provided students with time to think about the proofs before beginning them in Y3, I asked Matt about this change:

**Example 6-11**
Yeah that's a big thing...I mean I think I talked about that a little bit last year. But...I probably didn't make that explicit. I've been sort of harping on that...that you prove something to yourself first is true, and then once you know that, then you write it down...the book hints about it [too]. They talk about, like, you know, plan for proof, make a plan for your proof. And I don't want them to have a formal plan, but...you do have to have it mapped out before you ever write anything down. I mean you have to know where it's going before you can just write stuff down. (Interview P-4, 9/26/07)

Matt said that in Y2 he talked to his students a little bit about thinking through the proof before actually trying to write the proof. He also said that his textbook hinted at this idea when the authors wrote about constructing a plan for the proof. Matt now believed, however, that it was important to make this strategy “explicit” to his students. He elaborated on his reasons for bringing students through this process:

**Example 6-12**
I mean last year, I feel like I said a lot of just follow your nose. I mean like just write something down and then see where that leads you. This is just, you know, that anything is provable and all we gotta do is just write some stuff down and then
eventually we'll get to the end. And...I want them to think about what's going on and to visualize what's happening and then the formalization of that is the proof.

(Interview P-4, 9/26/07)

It seemed that through experience, Matt learned that telling students to “follow your nose” or “just write something down” was not successful. By Y3, Matt tried to help students participate in proving by being explicit about the need to think about the proof. He believed that students needed to create a mental and visual map of what was happening. This is similar to his idea of “seeing it” which was discussed in Chapter 4. At the end of Y1, Matt said that you either “see how you do [the proof] or you don’t” (Example 4-10). Here, however, Matt seemed to be saying that providing time for students to convince themselves that the proof was true might help them “see” a plan for the proof.

Otherwise, Matt found, his students gave up too easily:

**Example 6-13**

Because you get so tired of people writing down, you know, I've got the “givens.” I don't know what else to do. And, to me, it seems like those are the people who write down the “givens” before they've done anything else. They want a process. Step one, write down the “givens.” Step two, write down something else….when you write down the “given” before you've actually thought about what's going on, you shortcut that whole thinking procedure and that's why you end up getting stuck….If you thought about it beforehand, the “givens” is just like an afterthought…So I, I have made that a little more explicit, actually said...that's what I want them to do first is to figure out why it's true and then write it down. (Interview P-4, 9/26/07)

As Matt mentioned, students wanted to have a step-by-step process. He explained that when students thought about writing a proof as a procedure that involved steps, they ended up “getting stuck.” He hoped that by asking students to think about why something was true, students might have greater success with doing proofs. In this sense, Matt was asking students to think about proof as more of a problem solving process rather than something
procedural. By asking students to think about why the statement was true, Matt was asking them to engage in a more authentic mathematical process or, what he called, “real math.”

Even though, Matt seemed to reject the idea that proof should be thought of as a “process” or a procedure, he used the word “procedure” in this explanation. Matt said that thinking about writing down the “givens” as the first step shortcuts the whole “thinking procedure.” Perhaps, since his students were interested in following a procedure, he thought it best to include thinking about the proof as part of their procedure. In the next section, I discuss a second change that was observed in the Y3 First Proofs lesson.

**Flexible Proof Formats**

In *Geometry*, three proof forms were presented: the two-column proof, the flow proof, and the paragraph proof. Descriptions and illustrations of these three forms are provided in Appendix B. As was already mentioned in Chapter 5, the most commonly used proof form is the two-column proof. The 1999 *Standards*, however, called for the de-emphasis of two-column proof, and the 2000 *PSSM* stated: “The focus should be on producing logical arguments and presenting them effectively with careful explanation of the reasoning, rather than on the form of proof used (e.g., paragraph proof or two-column proof)” (NCTM, 2000, p. 310). So despite the fact that Matt’s textbook emphasized the two-column proof, Matt was interested in being flexible with proof forms as the *Standards* advocated. Evidence presented here suggests that from Y1 to Y3, Matt more explicitly emphasized various proof forms. After describing findings from the First Proofs lesson taught in Y3, interview data will be used to explain Matt’s reasons for making this change.
In *Geometry*, even though students first learned about proofs in Chapter 2, the paragraph proof and the flow proof were not emphasized until Chapter 3. When Matt taught proof in Y1, he introduced proof in the two-column form. Eventually, he also showed students how to write a flow proof. In Y3, however, Matt presented a two-column and a flow proof simultaneously. He explained the different forms to his students:

**Example 6-14**
Matt: Okay, so, the two major kinds of proof, on a two-column proof, you're going to write down a statement and then a reason, okay? So IT equals RU. Why?
Ss: [overlapping talk]
Matt: They told us so. It was given. Okay. On a flow proof, you're gonna write a statement, and then underneath it you're going to write the reason. And then you're gonna connect the different statements with arrows. Okay, so that you can see how these pieces fit together. Okay? So, we'll say IT equals RU. That's a given. Okay? Now what?
(Y3 First Proofs, 9/21/07)

During Y3, rather than privileging the two-column form over the flow proof, Matt presented the very first proof in both forms simultaneously. In his explanation of the “two major kinds” of proofs, however, Matt only mentioned the two-column and the flow proof, not the paragraph proof. The reason for this will be explained shortly through the interview data.

As Matt worked through the proof, he explicitly pointed out a difference between the flow and two-column proof:

**Example 6-15**
And this is where you'll see a bigger difference between the two-column and the flow. Okay? Is that on the flow proof, we're actually gonna show which pieces we put together to get that, whereas in the two-column proof, that's just sort of assumed…that we can see where that came from, okay? (Y3 First Proofs, 9/21/07)

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38 Although I am aware that IT and RU are both line segments, when representing a line segment in the spoken dialogue, I represent only what was said verbally, rather than placing bars over IT and RU. I do this because I do not wish to misrepresent what was said since Matt did not say *line segment* IT. When representing written text, I use the proper notation if that was what Matt did.
When Matt said that in the flow proof they could actually show how the different pieces were related to each other, he was referring to the fact that “the logical flow connecting the statements is indicated by arrows” (Larson, 2001, p. 138). This is not the case in the two-column proof in which the steps are listed sequentially. Matt then told the students that it did not matter to him which form they chose and that they could do their proofs “whichever way you want to do it.”

Matt was also explicit with the students about how *Geometry* presented proof and how they should deal with the textbook exercises:

**Example 6-16**
Okay. So, again, it doesn't matter to me which one you do. Some people really like the two-column, so go with that. Some people like to use the flow, because they like to see where the different pieces fit together. Whichever ways makes sense to you, okay. The book is gonna keep saying write a two-column proof, write a two-column proof. Just ignore that direction for the next three chapters. (Y3 First Proofs, 9/21/07)

Matt told students that they could choose either form, even if their book told them to write a two-column proof. He also said that the form did not matter to him. Matt emphasized a similar point during the second day of the *First Proofs* lesson in Y3 when students asked him about the number of steps that they needed for a particular proof:

**Example 6-17**
Okay? So I would consider that five steps. Don't worry about - it's not so much about step counting as it is about logically spelling out exactly what you're doing. How do you know that everything is true? Okay? (Y3 First Proofs, 9/24/07)

Here, Matt emphasized the process of being clear and explicit about their reasoning. Again, Matt’s practice seemed to be aligned with *PSSM* in the sense that he believed that students “should be able to produce logical arguments and present formal proofs that effectively
explain their reasoning, whether in paragraph, two-column, or some other form of proof” (NCTM, 2000, p. 345).

In Y3, Matt continued to emphasize both the two-column and the flow proof even after the first day. On the second day of the First Proofs lesson, before starting a proof, Matt said: “Okay, and I'm gonna do this… both ways again…not that you have to write down both ways, but just to kind of see what the difference would be between the two formats” (Y3 First Proofs, 9/24/07). Matt continued to model both of the forms so that students would understand that they were both valid. By emphasizing the reasoning process over the form, Matt tried to let his students know that their written communication of correct reasoning was more important than writing a proof in any one particular form. This practice runs counter to one that might produce the commonly held belief by students that: “One succeeds in school by performing the tasks, to the letter, as described by the teacher” (Schoenfeld, 1988, p. 151).

In the next section, interview data is used to explain why Matt placed more emphasis on the flow proof in Y3 than he did in Y1. I also explore the reason that Matt did not emphasize the paragraph proof.

More flexible proof practice explained. At the end of Y1, Matt talked about the value of having his students write proofs in various forms:

Example 6-18
I like that I had 'em do a lot of proofs, and…not very many other people have 'em do proofs in other formats….the book basically does everything in two-column proofs, and that's what everybody does. The book sort of mentions, in passing, um, paragraph proofs, and they do a little bit more with flow proofs….I made students do at least one of each way, I think, and then after that I said, you know, you can do it whatever way you want, um, and again, most of my students did two-column proof simply because that's what the book did, and that's what, you know, was seen in the homework or whatever….[but] the form is immaterial. (Interview P-1, 6/21/06)
During Y1, Matt introduced the three proof forms in the same order as the book. That is, flow proof and paragraph proof were not emphasized until Chapter 3 even though the *First Proofs* lesson was in Chapter 2. Matt pointed out that the other geometry teachers in his building typically did not have students use forms other than the two-column form. He seemed to understand that he breached a school mathematics norm by emphasizing other proof forms.

Matt also explained that even though *Geometry* included paragraph proofs “in passing,” the textbook authors seemed to emphasize the two-column form. Later, Matt said that being flexible about the proof form caused students to “scribble out the t-chart” (Interview P-1, 6/21/06) on their exams and write a proof in one of the other forms. This pleased Matt, and suggests that the students in his class understood that the proof form was flexible. During an interview in Y2, however, Matt explained why he would *not* emphasize the paragraph proof anymore:

**Example 6-19**

I probably then this year will just not have them write paragraph proofs, even though that's really how proofs are written, um, in math textbooks. [The students are] not used to being careful enough. The big problem with people who try to write a paragraph proof is that they leave everything out because...they make too many assumptions and they cram too many steps together...and they don't put the reasons...so I think I won't talk about paragraph proofs at all. I'll just have them do two-column or flow.  
(Interview P-2, 9/26/06)

Matt acknowledged that paragraph proof was “really how proofs are written,” in mathematics textbooks. By mathematics textbooks, I believe that he meant textbooks from higher level mathematics or “real math.”

The problem with emphasizing the paragraph form for Matt was that this proof form made it easy for his students to forget to write the reasons for each part of the proof. So it seemed that although Matt was willing to be flexible about proof forms with his students, he
was not comfortable with the way that the students wrote the paragraph proofs. This is interesting because if you read original proof documents written by Euclid, reasons are only given in proofs when they are not obvious (Weiss & Herbst, in review). So, as pointed out by Herbst (personal communication, January 11, 2008), even though paragraph proofs may be more “authentic” or closer to the mathematical norm, Matt may have become uncomfortable when the students’ proofs became too different from the two-column form. That is, students were leaving out the reasons, but unlike the mathematicians who follow this practice, my conjecture is that Matt’s students left out the reasons because they did not know them, not because they were obvious. The two-column form has a built-in structure that would not allow students to *forget* the reasons. This is less apparent with the paragraph form, which could explain why, by Y3, Matt was not comfortable with teaching the paragraph form to his students. Matt may have preferred the flow or two-column forms because they offered better support for student learning.

An interesting point regarding this issue is that the *PSSM* (2000) recommend that various methods of proof should be accepted. Also, when possible and reasonable, mathematics educators have called for a closer alignment between school mathematics and the discipline of mathematics (Ball, 1993; Lampert, 1992). Ball (1993), however, also pointed out that certain aspects of the discipline should not be replicated in mathematics classrooms. For example, according to Ball, the reputation that mathematicians have for being competitive loners is not a desirable quality that should be emphasized in school mathematics.

Another interesting point regarding Matt’s flexibility with different proof forms is that, in part, this flexibility was brought about by his interaction with his textbook:
Example 6-20
[The textbook] has also shown me that there are lots of different ways. You go through all these classes on proof and you kinda think there's only one way to prove anything. I mean the geometry proofs are nice in that there are multiple different ways….So that's been good too. (Interview P-3, 4/19/07)

Unlike in the previous chapter of this dissertation where many of Matt’s criticisms of Geometry were presented, here we find that Matt credited his textbook for promoting more than one way to write a proof. “All these classes on proof,” that Matt referred to were from his university mathematics courses which mainly required paragraph proofs, causing him to think paragraph proof was the only form he could use. Matt seemed to appreciate this aspect of his textbook.

Matt talked about his decision in Y3 to introduce flow proof alongside two-column proof:

Example 6-21
I just did them simultaneously. I made no conscious decision to do them at the same time.\(^{39}\) I don't know. I just did that. The increased importance in flow proof is…in every class there's a couple of people who say that they like flow proof and that that makes sense to them….but I think to emphasize that these pieces of things actually are interacting…showing them that…this is not just a random group of garbage. I'm actually connecting these things together in a meaningful way. I think it doesn't help a ton of people, but the people that it helps, I think it helps them significantly, and it makes something that was not gonna be as accessible be accessible to them. (Interview P-4, 9/26/07)

Matt said that he did not consciously decide to present both forms at the same time when he introduced proofs, but he acknowledged that flow proof had become more important. Matt seemed to like the idea that, through the connecting arrows, the flow proof emphasized that the pieces of the proof were actually interacting “in a meaningful way.” Another advantage to emphasizing this form was that when students were given options, Matt believed that

\(^{39}\) Earlier I said that the changes that Matt made were done so deliberately. This is the only time that Matt said he unconsciously made a change.
proof became more “accessible” to some of his students. The fact that Matt began to focus on students in more sophisticated ways across time (e.g., noticing that some students preferred the flow proof) is a finding that will be discussed in more detail in the next chapter.

So far, quantitative data was presented to demonstrate that Matt spent more time focused on proof in Y3. In large part, this was due to the fact that he no longer scheduled quizzes and new lessons on the same days. Matt spent an extra day on First Proofs in Y3. This gave him time to do some “more interesting” proofs that he found in another textbook. It also gave him time to bring students into the process by asking them to convince themselves that a statement was true. In addition, from day one of First Proofs, Matt presented the two-column and the flow proof at the same time to help make proofs more accessible to his students. Both of these changes brought the students closer to “real math” because: (a) students were now asked to think about and discuss a plan for the proof rather than watch Matt do the proof and (b) by giving students options related to the proof form, Matt did not present proof as something that needed to be completed in only one procedural way. Together, these descriptions were presented to build the case that across time, Matt more explicitly focused on engaging students with proof. Next, data from the Special Triangles lesson will be used to demonstrate that in Y3, Matt more explicitly focused on proving the theorems rather than simply presenting theorems as facts to be memorized. Again, establishing an expectation that theorems should be supported by proof gave the students an experience that was closer to “real math.”

A Focus on Proving the Theorems

Some issues and challenges from the Special Triangles lesson in the written curriculum were described in the previous chapter. The two main goals of this lesson were
“Using Properties of Isosceles Triangles” (e.g., the Base Angles Theorem) and “Using Properties of Right Triangles” (e.g., the Hypotenuse-Leg Congruence Theorem). More detail about the goals of this lesson is provided in Appendix D. As was mentioned in the previous chapter, Matt found it problematic that Side-Side-Angle (SSA) was not mentioned anywhere in *Geometry* because the Hypotenuse-Leg Congruence Theorem (HL) is a special case of SSA. As Matt developed his curriculum vision, however, he found ways to deal with this situation.

Interview and classroom data are reported in this section to illustrate that Matt’s enactment of the Special Triangles lesson changed dramatically from Y1 to Y3. I begin first by portraying the enactment of this lesson in Y1 through the use of the lesson transcript. In addition, interview and Daily Reflection data are also presented to help the reader understand Matt’s perspective on this lesson. Next, using the lesson transcript, I describe the enactment of this same lesson in Y3. By doing so, I show that, unlike the lesson in Y1, by Y3, Matt explicitly focused on the proofs in this lesson, rather than simply presenting the theorems as final results. This example also provides evidence that, as Matt claimed, after initially introducing the proofs from Chapter 2 in *Geometry*, by Y3, he made proving a primary focus throughout the entire first semester.

**Special Triangles in Y1**

In Y1, Matt began the Special Triangles lesson by going over homework. He also scheduled a quiz on this day. The whole-class work activity took place in between the homework review and the quiz. The following excerpt is the transcript of the whole-class work for this day:
Example 6-22
Matt: Okay. I wanna make sure to give you enough time to take the quiz, so I will tell you everything you need to know about…Section 4.6. Who's got a watch? Who's gonna time me? Jim, you want it?
Jim: Sure.
Matt: Okay. Ready? It's gonna take me thirty seconds. Got the timer start, got the timer set?
Jim: Yeah.
Matt: When can I go? [He draws an isosceles triangle on the board.] What kind of triangle is this?
Ss: Isosceles.
Matt: Isosceles triangle? What do we know about these two sides? [He points to the two sides with the hash marks (shown in Figure 6-1).]
Ss: They're equal.
Matt: They're the same length. In an isosceles triangle, the fact that the two sides are the same length means that you know what else? That
-------S: [inaudible]
Matt: Which angles? [He marks the two base angles congruent.]
Ss: [inaudible]
Matt: I'm done. How long did it take?
Jim: Uh, 30, uh, 26 seconds.
Matt: Yes! That's the whole thing. That's the whole point. In an isosceles triangle, if the sides are the same length, that means the base – and the book is gonna call these the base angles. That's the whole point.
(Y1 First Proofs, 9/30/05)

With the additional summary that Matt provided at the end, this entire exchange took place in just under two minutes. When Matt said, “I will tell you everything you need to know about…Section 4.6,” he positioned himself as an authority of the mathematics. Matt seemed
pleased to cover Section 4.6 in such a short period of time, as demonstrated by a gesture he made when he said, “Yes!” By presenting the lesson in this way, Matt only “covered” a small part of the stated lesson goals. He briefly talked about the Base Angles Theorem, but he did not mention the converse of the theorem, the corollaries, or the Hypotenuse-Leg Congruence Theorem (HL). In addition, he did not state the Base Angles Theorem in the if-then format. As a result, Matt did not “cover” most of the lesson goals of this section, and he did not establish the idea that the Base Angles Theorem was a theorem or a proposition that could be proved. In this sense, he emphasized a property of isosceles triangles, rather than engaging the students with proof. In the next paragraph, I provide a brief summary of the transcript from Example 6-22 and describe the drawing of the diagram that accompanied Matt’s dialogue. This explanation is provided to help the reader understand how Matt used the diagram to establish the theorem. I include this description to show that Matt presented a simplified version of the theorem that was based on what seemed obvious from the diagram. This kind of presentation could contribute to students’ beliefs that the “perceptual proof scheme” where students draw a conclusion from a diagram (Sowder & Harel, 1998) is sufficient justification for a mathematical theorem.

In this lesson, Matt first drew the triangle on the board (see Figure 6-1) and then asked, “What kind of triangle is this?” After the students said that the triangle was isosceles, Matt asked, “What do we know about these two sides?” The students answered, “They’re equal,” but Matt then made a repair by revoicing this answer so that it more mathematically precise, saying, “They’re the same length.” He then asked, “In an isosceles triangle, the fact

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40 This theorem says, “If two sides of a triangle are congruent, then the angles opposite them are congruent” (Larson, 2001, p. 236)
that the two sides are the same length means that you know what else?” Matt waited very briefly, and then marked the two base angles congruent (as shown in Figure 6-2). At this point, it was difficult to hear what the students said, but by drawing the congruent marks on the base angles of the triangle, he left little else for the students to conclude but that the angles were congruent. Matt then finished this lesson by saying, “In an isosceles triangle, if the sides are the same length, that means the base – and the book is gonna call these the base angles.” Although he marked the base angles congruent, neither Matt nor the students concluded (verbally or in writing) that the angles opposite the two sides were then congruent. A thematic map of the mathematics that was co-constructed by Matt and the students is rather simple (see Figure 6-2).

![Figure 6-2: Base Angles: Y1 Thematic Mapping and Boardwork 2](image)

Not only was the theorem not sanctioned (written in the if-then form), but a proof of the theorem was not even mentioned. After drawing the marks to indicate that the base angles
were congruent, Matt said, “That’s the whole thing. That’s the whole point.” He said this despite the fact that he left out most of the stated goals from the written curriculum.

In the daily reflection for this lesson, Matt wrote:

**Example 6-23**

I…noticed I was running out of time and so gave the students the 30 second version of the material from [Section] 4.6 – namely that the base angles of an isosceles triangle are congruent….Obviously I don’t want to spend 30 seconds presenting the new material every day, but in the interests of completing the quiz, which in my opinion is more important than (1) proving that the base angles in an isosceles triangle are congruent, or (2) looking at the Hypotenuse-Leg Theorem, which in my opinion is highly specialized and won’t ever really be used by the students, so I cut straight to the chase on that. *(Special Triangles Daily Reflection, 11/11/05)*

In this excerpt, Matt implied that the only thing from this section that he found worthy of spending time on was *telling* his students that “the base angles of an isosceles triangle are congruent.” This statement was really more of a property of isosceles triangles, or, at the very most, a statement of the theorem that was not in the if-then format. Matt did say that he “obviously” did not want to spend 30 seconds presenting new material, but completing the quiz was more important than the proof of the Base Angles Theorem or looking at HL. Matt called HL “highly specialized” and not useful to the students. Contrary to what Matt wrote, he seemed quite pleased during the lesson about spending only 26 seconds on it, and he said that “things went close to how I imagined them” *(Special Triangles Daily Reflection, 11/11/05).*

Matt did note that his discourse was *not* what he would typically want it to be:

**Example 6-24**

Today was another typical ‘call-and-response’ day; the students weren’t really talking about mathematics with each other. I was really talking about mathematics to them, which is sort of two steps removed from where I want my general classroom discourse to be. *(Special Triangles Daily Reflection, 11/11/05)*
Matt referred to the discourse as “call-and-response” when, in fact, he quickly led the students to the answers that he was looking for when he marked the diagram. Matt also commented that he was “really talking about mathematics to them” rather than having the students talk to each other about mathematics. In the next section, I use the classroom transcripts to describe how Matt taught the Special Triangles lesson in Y3. After describing the lesson, interview data are used to explain why these lessons looked so different.

**Special Triangles in Y3**

In Y3, Matt did not schedule a quiz on the same day that he taught the Special Triangles lesson. After spending about 20 minutes going over the homework, he began the whole-class work:

**Example 6-25**

Matt: Okay, so we’ve got about 20 minutes left. So, uh, the last section, Section 4.6 is about particular types of triangles so there’s a little bit about isosceles triangles. There’s a little bit on equilateral triangles, and there’s a little bit about right triangles. Um, we’re probably not gonna have enough time to talk about everything so I would just encourage you to look through that section. Some of the things are fairly straightforward. What I want to try to talk about are some of the things that aren’t quite as clear. For instance, how we can prove the Base Angles Theorem. Okay? So the Base Angle Theorem says we have an isosceles triangle because these two are supposed to be the same. And what is supposed to be true?

MS: The base angles are congruent.

Matt: The base angles are supposed to be congruent. So let’s give this, uh, A, B, C. Okay? So we are given that AB is congruent to BC. That’s a given. Okay? What we want is to prove that angle A is congruent to angle C. Okay. So I want you to think to yourself for 45 seconds, if we know that AB is congruent to BC, how can we show that angle A is congruent to angle C?

Matt: [whispers to student]: Don’t look at the book. Think about it.

(Y3 Special Triangles, 11/8/07)

Before diving into the content of the lesson, Matt gave his students an overview of the lesson explaining that in this section, there was a little bit of information about isosceles, equilateral, and right triangles. Matt also warned the students that they probably would not have enough
time to talk about everything. In light of the fact that Matt took less than two minutes to tell the students about this lesson in Y1, it is interesting that in Y3, Matt said that 20 minutes would probably not be “enough time to talk about everything.” In the next few paragraphs, I recap this example, both to illustrate the ways that Matt focused on more authentic proof practices (i.e., “real math”) by drawing a diagram and determining what was given and what was to be proved and to expand on the transcript by describing other aspects of the lesson that were not in the transcript.

After explaining that there was not enough time to talk about everything, Matt encouraged his students to “look through that section” in their textbook, providing further evidence of the finding (from Chapter 5) that by Y3, Matt encouraged his students to use their textbook as a reference tool. However, it is also interesting that at the end of the passage, Matt told a student, “Don’t look at the book. Think about it.” At that point in the lesson, Matt wanted the student to think for himself and try to make sense of the theorem rather than use the textbook as an authority.

Although he did not state the theorem in if-then format, Matt told the students, “So the Base Angles Theorem says we have an isosceles triangle because these two [sides] are supposed to be the same.” He then asked, “What is supposed to be true?” A student said that the base angles were congruent, and Matt revoiced, “The base angles are supposed to be congruent.” By using the phrase “supposed to be,” Matt implicitly pointed out that the congruence of the base angles was what they were trying to prove, and, at that point, they did not know this for certain.

Unlike the simple diagram that Matt constructed in Y1, this time, Matt drew the triangle and then said, “So let’s give this, uh, A, B, C.” He labeled and marked the diagram
as shown in Figure 6-3. Then Matt pointed out, “So we are given that AB is congruent to BC.

That’s a given.” Next, he said, “What we want is to prove that angle A is congruent to angle C.” Matt also wrote these things on the board (see Figure 6-4). Even though Matt (not the students) stated what was given and what was to be proved, he modeled his thinking by starting with the theorem, drawing and labeling a diagram, and then stating what the “given” and the “prove” were. Herbst & Brach (2006) pointed out that it is normative for the teacher to provide the diagram and spell out the “given” and “prove” for the students. According to Matt, however, in the next unit he actually gave students theorems in the if-then format and then expected them to construct the diagram and figure out the “givens” and the “prove.”

41 I was not present for this unit, but I did see the supplementary sheet of theorems that Matt gave to the students. He also discussed this in the final interview (Interview P-6, 12/15/07).
After completing the diagram, Matt asked, “So I want you to think to yourself for 45 seconds, if we know that AB is congruent to BC, how can we show that angle A is congruent to angle C?” Matt gave students about one minute to think quietly about this proof, and then he told the students to “turn and have a quick two-minute conversation with your neighbor about what you’re thinking about.” So before doing the proof, Matt actually gave students about three minutes to think and talk to each other about the proof. The amount of time that students were given to think about and discuss the proof was longer than the entire lesson was in Y1.

After giving students time to think about a plan for the proof, Matt asked the students, “Okay, what do we got?” A student suggested drawing a line through B and parallel to AC as shown in the diagram (see Figure 6-5). Matt briefly entertained this idea which led the students to conclude that the sum of three acute angles (with B as a vertex) was 180 degrees. Typically, proofs that require auxiliary lines are done by the teacher, and include hints to do so (Herbst & Brach, 2006). Eventually Matt did give a hint about where the line should go.

Because the construction of a line through B and parallel to $\overline{AC}$ was not helping them with the proof, Matt refocused students’ attention with this suggestion: “So let's try, if we try to use corresponding parts, we have to have two different triangles that we prove are
A student suggested drawing a new triangle. This suggestion was briefly entertained, and then Matt gave another hint:

**Example 6-26**
Matt: What do we have to add in here? We have to add something in. What do we have to add in?
MS: A line straight through the triangle.
Matt: A line straight through the triangle? Now the key question is how do we want to draw the line at B? What, what do we want this line to bisect?
Ss: [overlapping talk] angle B
Matt: Angle B. If this, if we draw this line specifically to bisect angle B, what do we know is true?
Ss: There's two right angles
Matt: Wait, we bisected angle B.
FS: [inaudible]
Matt: Okay. We don't, we don't know for sure exactly how this is gonna, it might, it might look like this, where it's not a not a right angle.
MS: [overlapping talk] well wouldn't that mean that [it’s bisecting the base]
Matt: How do you, how do you know that?
MS: Oh, but then it would be a [overlapping talk] perpendicular.
Matt: No, no, no. It's bisecting this angle. You haven't said it bisects this line.
(Y3 Special Triangles, 11/8/07)

In this transcript excerpt, Matt suggested, “We have to add something in.” When the student suggested drawing “a line straight through the triangle,” Matt drew a vertical line through B. After he drew the auxiliary line, they talked about whether the line would be the angle bisector or the perpendicular bisector of the base of the triangle (see Figure 6-6). Even

![Figure 6-6: Base Angles Boardwork 6](Image)
though Matt said, “if we draw this line specifically to bisect angle B…,” some of the students wanted to talk about right angles formed by the auxiliary line. After some debate over this idea, Matt eventually told the students:

**Example 6-27**

So when we draw this line what we want to say is that it's the angle bisector of angle B because then we are guaranteed that it goes through B. Right? So we don't know anything about what happens down here on AC. It might be perpendicular. It might not be. It might go through the middle. It might not. But we know that it's gonna, that this [points to angles 1 and 2] is gonna be congruent here. Now, can we show that these two triangles are congruent yet? (Y3 Special Triangles, 11/8/07)

Matt explained to the students that the auxiliary line had to be the angle bisector. As a result, the two angles (\(<1\) and \(<2\), see Figure 6-7) would be congruent. After this was established, a student offered a plan for the proof, and then the class verbally contributed to the written record of the proof on the board. This was done by Matt asking the students questions like, “Now if it’s the angle bisector, what can we say?” and “Now what’s our reason for that?” Through what Matt described as “call-and-response,” the students determined that the triangles were congruent by SAS, and then by corresponding parts, the base angles were
congruent. The thematic map of the discourse in Y3 lesson (see Figure 6-8) is much more complex than the thematic map from Y1 in that the proof of the theorem was explored.

During the final interview of the study, Matt watched a video clip of the Y1 lesson and the introduction to the lesson from Y3. He talked about a change that he observed:

**Example 6-28**

I mean the change is, I'm gonna talk about the proof....that's a small change, that then when you follow it all the way out then has large implic-....I mean to me it's a small change to talk about the proof...because I already know how to prove it....It's not like it takes more planning, right?...[but] from the students' standpoint, that is a giant change because now we have to talk about all this other stuff.

(Interview P-6, 12/15/07)

In this interview, Matt acknowledged that in Y3, as a class, they talked about the proof instead of having him simply state the theorem. As Matt said, this was a small change for him to make because he already knew the proof, but the implications for students were greater. From the students’ standpoint, he said, “we have to talk about all this other stuff.”

Here, Matt said “we” (which included the students this time) have to talk about this other stuff (the auxiliary line and proof itself). Recall that in Y1, Matt said: “I will tell you everything you need to know about…Section 4.6.” In Y3, however, Matt engaged the students with proof by: (a) asking the students to think about a plan for proving the theorem, (b) asking them to discuss their plans, and (c) using “call-and-response” to guide the students through the proof. Matt explored and addressed some of the students’ ideas about the proof such as the possibility that the auxiliary line was the perpendicular bisector of the base.
Figure 6-8: Base Angles: Y3 Thematic Mapping

M: If we try to use corresponding parts, we have to have two different triangles that we prove are congruent.

M: We have to add a line in.

St: A line straight through the triangle.

M: It has to bisect \( \angle B \).

Sts: \( \angle 1 \ cong \ \angle 2 \).

M: It's not the perpendicular bisector.

MS: The perpendicular bisector.

\( \triangle ADB \ cong \ \triangle CDB \).

\( \angle A \ cong \ \angle C \).

MS: The line they share [reflexive].
Addressing Hypotenuse-Leg

After proving the Base Angles Theorem, Matt also left enough time to briefly address HL. As was discussed in the previous chapter, Matt expressed frustration that there was a “whole back-story” about SSA that was completely left out of his textbook. During Y3, Matt did address HL. He began with a diagram of two triangles that looked like right triangles (see Figure 6-9) but were not marked as right triangles. (This is usually done by drawing a square in the vertex of the right angle.) Matt asked students if the triangles were congruent. Because Matt had already addressed SSA during an activity in a prior lesson, he expected students to know that SSA was not a valid method of proof. Matt referenced the previous lesson, and he constructed a diagram like the one in Figure 6-10 which showed an example of an ambiguous
case. Next, returning to the two triangles (Figure 6-11), Matt asked students, “If I tell you not just that the angles are congruent but that the angle that is congruent is 90 degrees, now can you say that these triangles are congruent?” What Matt said next was rather unusual in the Euclidean treatment of this theorem:

**Example 6-29**
Matt: I say right triangle, you say a word that starts with a "P."
  MS: Perp-
  MS: Pythagorean Theorem.
Matt: Pythagorean Theorem.
  Ss: Ohhh.
Matt: [pauses while writing, \(a^2 + b^2 = c^2\), see Figure 6-11] Okay? So if we know that this number [circles \(a\) on the left] is the same as this number [circles \(a\) on the right] because of this [points to Pythagorean Theorem] and if we know that this number [circles \(c\) on the left] is the same as this number [circles \(c\) on the right] because of this [points to the Pythagorean theorem], what has to be true about these two numbers [points to the two \(bs\)]?
  Ss: [overlapping talk]
Matt: They have to be equal.
  MS: Couldn't you just do that with the other triangles?
Matt: You don't know it's 90 degrees so you can't say. The Pythagorean Theorem doesn't work for a non-right triangle.
  Ss: [overlapping talk]
  MS: But we're talking about sides, not angles.
Matt: But, but if I, if this isn't a right, if this isn't a right triangle, then, then this doesn't work….So the last way that we can prove triangles congruent, so Hypotenuse-Leg is a specific case of Side-Side-Angle when it actually works out because of the Pythagorean Theorem.
*(Y3 Special Triangles, 11/8/07)*
By dealing with HL by connecting to students’ prior knowledge, Matt was able to explain why the “special case” of SSA (i.e., HL) was a valid theorem by using something that was already familiar to students. Here, Matt reminded students that by the Pythagorean Theorem, the third sides of the triangles were congruent. So by SSS, the triangles must be congruent. This is an interesting way to think about HL because, in my experience, many geometry teachers are unsure about how to explain when SSA “works” without using the Law of Sines, a topic that is usually saved for Advanced Algebra and Trigonometry or Precalculus. This points to a curricular issue in which the order of presentation in school mathematics can cause difficulties when proof or justification can not be easily provided (or constructed) for certain propositions.

Understanding that Matt thought about HL in this way helps explain why he viewed it as a trivial result and did not feel the need to spend time on it in Y1. On the other hand, Matt did not actually construct a proof of HL using the Pythagorean Theorem. Depending on which proof of the Pythagorean Theorem was done, it could be that the students had not yet progressed to the point where they could formally prove the Pythagorean Theorem. For example, in Euclid’s *Elements*, the proof of the Pythagorean Theorem did not appear until the end of Book I. In the *Elements*, the logical chain of propositions relied on the previous propositions. For this reason, it may not have been feasible for the students in Matt’s class to write a deductive “proof” using the Pythagorean Theorem, but because students were likely familiar with the result of the theorem, there is a good chance that it would be sensible to them.
Addressing Hypotenuse-Leg Explained

After watching the video clip from Y1 (shown during the final interview in Y3), Matt reread the part in the Daily Reflection where he referred to HL as “highly specialized and won’t ever really be used by the students.” Matt reacted to this statement:

Example 6-30
But now...I'm trying to pull the proof further along. Well...there are things in Chapter 5 that rely on Hypotenuse-Leg....‘cause the first year, I'm just looking at the test and there's one Hypotenuse-Leg question on the test and even that question you could answer with Side-Angle-Side....I mean you're in survival mode your first year. (Interview P-6, 12/15/07)

Matt’s opinion about the relevance of HL seemed to have shifted from Y1 to Y3. After acquiring curricular knowledge, Matt realized that HL was drawn on in a later chapter. He also indicated that teaching only what was on the test was part of his being in “survival mode” that first year.

Even though Matt seemed to indicate that his opinion about the relevance of HL had changed, he then said, “what you really want them to understand out of that [chapter] is Side-Side-Side, Side-Angle-Side, Angle-Side-Angle. I mean there's more stuff than this in this section. You know, we just leave out Hypotenuse-Leg ‘cause that's dumb” (Interview P-6, 12/15/07). Because this seemed like a contradiction, I pressed Matt on the claim that “we just leave out HL”:

Example 6-31
I: But you didn't [leave it out] this year?
Matt: I know I didn't this year....because you have to get comfortable enough with the first four [SAS, ASA, SSS, AAS] to be able to be able to move on, right?
I: You had to get comfortable enough?
Matt: Yeah, get comfortable enough with teaching. I mean, to me, Hypotenuse-Leg is like a joke, like why would you ever, yeah they're both right triangles so we know that [we need] the Pythagorean Theorem. So the third side has to be the same. So they're congruent by Side-Side-Side.
I: So what were you not comfortable with because that actually surprises me to hear
you say that?

Matt: Uh, why would you have to teach that? Like why would it be taught? Why would you have to take class time to explain it? To me it just sort of seems like, that it's just a big waste of time to talk about it [because it's obvious]....And...it wasn't connected to anything....Once I pushed the activity to talk about Angle-Angle-Side, then the discussion of Hypotenuse-Leg makes a lot more sense because it's tied to...Angle-Side-Side occurring in a specific case...when you push that further, then you've talked about Angle-Side-Side and that gives you a way to actually connect Hypotenuse-Leg to what you've been doing.

(Interview P-6, 12/15/07)

According to Matt, it took him some time to get comfortable with teaching HL. This surprised me because I knew that his content knowledge was very strong. When pressed further, Matt revealed that he viewed HL as a simple application of the Pythagorean Theorem and SSS, not a special case of SSA. This helps us understand why Matt thought it was a trivial result. Although Matt was comfortable with the content, in Y1 he did not always seem to have a grasp of why and how he should teach certain things. It is not uncommon for new teachers who are teaching subjects in which they have a strong content knowledge to long for guidance about how to help students understand concepts (Kauffman et al., 2002). Evidence was also provided in Chapter 5 that indicated that the ways that some of the ideas were connected in Geometry were inconsistent with the ways that Matt understood them. The data suggests that after teaching the course for a couple of years, Matt better understood the mathematical “story” that his textbook was trying to tell.

The “classroom activity” that Matt referred to in this interview was an activity that he called “The Blind Triangles Activity.” In this activity, Matt had students discover which pieces of information were necessary to construct a triangle that was described by their partner. For example, students could determine that they only needed three pieces of
information to construct a triangle that they could not see (i.e., SSS, SAS, ASA, or AAS).

Matt explained how doing this activity contributed to the discussion of SSA:

**Example 6-32**
The purpose of the activity is to have them talk about SSS, SAS, ASA, regular ones, right? And so out of that discussion, invariably you end up talking about, well can we do it in Angle-Side-Side? Either, you know, you say it as the teacher because you want to point out this doesn't work or somebody tries to do this, right?

(Interview P-6, 12/15/07)

Across time, Matt eventually realized that he could use this activity to discuss SSA, a topic which was not mentioned at all in his textbook. As Matt said, by doing the activity, invariably you end up talking about SSA. He said that either he would point it out to students or one of the students would try to construct a triangle using SSA. Matt described his progression of dealing with SSA:

**Example 6-33**
So I mean basically, first year, I didn't mess around with Angle-Side-Side [SSA] at all. And second year we talked about it a little bit in class and then...I went back and made a little construction to try to think about Angle-Side-Side, and I talked about it with like the four students who actually knew what I was talking about. This year, because we actually have the document camera, and I actually went out and bought a bunch of compasses....they had done more Side-Side-Side constructions and therefore it was very easy for me to demonstrate why Angle-Side-Side doesn't work....It didn't take me, like I didn't have to rewrite an activity. (Interview P-6, 12/15/07)

Through the supplemental activity, Matt found a way to address SSA. He also indicated that the use of the compasses contributed to the students’ understanding of SSA. To be clear, I am not claiming that after three years, Matt was completely satisfied with his enactment of this lesson. In the final interview, he mentioned his desire to improve it: “I need to come up with a better way...that's the part next year that needs to keep being pushed is how we talk about Angle-Side-Side and what do we do with it” (Interview P-5, 11/15/07). By Y3, Matt said that

42 A document camera is a device that projects any kind of document onto a screen or a wall.
he needed to continue to push or expand on the way that he handled SSA since it was not in his textbook and he was not altogether satisfied with the way that he handled it in the course.

This example suggests that during his first year of teaching proof, Matt was not provided with sufficient guidance from either his department head or his curriculum materials to determine which ideas he should emphasize and how he should enact them in the lesson. This issue has been noted in the literature: “Even when materials and resources are available, they may not provide sufficient guidance….Consumed by the mad scramble to prepare day to day….new teachers have to spend time planning what they would teach and how to teach it” (Kauffman et al., 2002, p. 282). On the other hand, one could argue that through the presentation-style of the theorems and the proofs, the textbook authors were implicitly suggesting that teachers should simply do each of the proofs for their students. Again, this points to a lack of alignment between the curriculum visions of the textbook authors and of Matt.

In this section, data were presented to explore the changes in the Special Triangles lesson. During Y3, Matt actually outlined a plan for the proof of the Base Angles Theorem. The thematic map showed a more complex treatment of the theorem in Y3. Also, in Y1, Matt did not even mention HL. In Y3, however, not only did Matt talk about HL, but he used the Pythagorean Theorem to help the students see why HL was valid. He also connected HL to SSA and an activity that they worked on in a previous lesson. Again, the rich treatment of the proof of the Base Angles Theorem, which included adding an auxiliary line, gave the students an experience that was more like “real math” than the two-minute lesson in Y1.

In the last section of this chapter, one more example of Matt’s explicit emphasis on proof is provided. This example comes from the last focus lesson, Bisectors. The difference
between Y1 and Y3 will be discussed briefly. This lesson from Chapter 5 in *Geometry* came at a point when, in Y1, Matt thought he was finished teaching proof. This example provides evidence that Matt maintained an emphasis on proof throughout the semester even though he was not accountable to do so and that he emphasized the proofs of the theorems rather than related definitions.

**Pulling Proof Through: Bisectors**

The goals of the *Bisectors* lesson were to use the properties of perpendicular bisectors and angle bisectors. The green boxes contained theorems (and their converses) related to both of these concepts (see Appendix D for more detail). During Y1, Matt asked students to write a definition of perpendicular bisector. He accepted two definitions from the students, “one based on how we make it, the second based on what it does” (Y1 *Bisectors*, 11/18/05). When Matt talked about the angle bisector, he said that the angle bisector was “a line that’s the same distance away from each side of an angle” (Y1 *Bisectors*, 11/18/05). When he talked about this concept, Matt mistakenly represented the theorem\(^{43}\) as the definition. It is important to note that during the three years that I spent in his geometry classroom, it was very rare that I observed Matt make a mathematical mistake. Towards the end of the lesson, Matt said:

**Example 6-34**

So your informal concept of angle bisector being the place where angle gets cut in half is good. Okay? And I want you to stay with that, but just know that the book is going to give us a more technical definition about points being the same distance away from lines. (Y1 *Bisectors*, 11/18/05)

\(^{43}\) The theorem said: “If a point is on the bisector of an angle, then it is equidistant from the two sides of the angle” (Larson et al., 2001, p. 266).
Again, when Matt talked about the Angle Bisector Theorem, he referred to the statement as a “technical definition.” With both the perpendicular bisector and the angle bisector, Matt did not talk about proofs of the theorems or their converses. He also told the students: “So in the book, you’re gonna see a whole lot of confusing diagrams that look like [the one on the board].” At the end of the lesson, Matt said, “Okay, that’s really all I want to blab about from [Section] 5.1” (Y1 Bisectors, 11/18/05).

In contrast, during Y3, Matt explained the Perpendicular Bisectors Theorem and then had students draw angle bisectors on their own papers. Next, he asked students, “What do we notice about this angle bisector?” Several students contributed their ideas about what they noticed before they talked through the proof of the theorem. In both cases, after stating the theorems related to these two bisectors, they devised a plan for the proofs of the theorems.

After watching a video clip from the Y1 lesson (in the final interview in Y3), Matt asked, “Why did I have them write their own definitions?...It seems awkward. I mean like it just went on for too long…I don’t know how much value that has” (Interview P-6, 12/15/07). After watching the clip from the Bisectors lesson in Y3, Matt reacted to the changes that he noticed:

**Example 6-35**
We actually did the proof....I mean…I’m not writing the steps on the board…but when we talked through it, I was convinced that they saw how it could be proved....I mean...in the first year, I’ve got this idea that Chapter 4 is over so we’re done with proofs now. So we don’t have to prove anything anymore. So here I am in Chapter 5 [this year] and...what’s the first thing I have them do? We talk about a CPCTC\(^44\) proof. (Interview P-6, 12/15/07)

When Matt watched the video clips, he noticed that he did not talk about the proofs of the theorems during Y1. He also questioned his choice to have students write definitions. In

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\(^{44}\) CPCTC is shorthand for Corresponding Parts of Congruent Triangles are Congruent.
addition, even if Matt had not confused the definitions with the theorems, after spending so much time having students write definitions, there would not have been enough time to address the proofs. The enactment of the *Bisectors* lesson in Y3 provides evidence that Matt did, in fact, extend the discussion of proof beyond Chapter 4 despite the fact that proof was no longer on the departmental exams. In addition, as Matt developed a curriculum vision, he found ways to use his textbook as a tool to maintain an emphasis on proof throughout the semester. Rather than mistakenly referring to the theorems as definitions, Matt was able to make use of the presentation style of his textbook and discuss proofs of the theorems that were given in the if-then format. When Matt reviewed the Y1 lesson in Y3, he questioned his decision to have students write their own definitions and noticed that he discussed the proofs in Y3.

**Summary**

I began this chapter by presenting quantitative evidence that Matt spent more time on the whole-class activity in Y3. The extra time created more space for Matt to explicitly focus on the proofs of the theorems rather than simply stating the theorems or talking about the definitions of the related concepts. In addition, classroom and interview data were presented to show that Matt emphasized the importance of thinking about a proof before trying to write one, and he more explicitly emphasized different proof forms to his students. Last, data was briefly presented to show that Matt extended the proof discussion beyond Chapter 4 of his textbook by addressing the proofs of the theorems in the *Bisectors* lesson. In Y1, he focused on definitions of the bisectors and did not mention proofs or even state the theorems from the lesson goals of that section.
Taken together, all of these pieces provide evidence of a more explicit focus on engaging students with proof in Y3. Because Matt believed that “real math” involves proof, it makes sense that Matt would want to place a greater emphasis on proof. It took time, however, for him to figure out how he could use his textbook as a tool to fulfill his curriculum vision of engaging students in proof. By Y3, Matt figured out that even though his textbook presented the theorems as final results in green boxes, he could talk about the *proofs* of the theorems in the green boxes. He was flexible with the proof forms and modeled more authentic proof practices by starting from the if-then format of the theorem and then determining what was given from the hypothesis of the conditional and what was to be proved from the conclusion. He also modeled the construction of the diagram. Eventually Matt said that he had students try to do these things on their own. As was already pointed out, some of these practices are not norms in school mathematics, but were done to help students have an experience that was closer to “real math.” To be clear, I am not trying to establish a new set of norms for school mathematics. I only claim that Matt is a teacher who was doing some things that were different from the norms of school mathematics. Matt found that having students write paragraph proofs (rather than two-column or flow proofs) was not a practice that was necessarily better for the students. However, encouraging students to think about the proof before writing down the “givens” is a practice that mathematics educators would likely deem favorable.

Related to the previous chapter of this dissertation, in some cases, Matt supplemented his textbook with additional proofs in order to provide this more authentic proof experience for his students. In addition, as was hinted at a few times in this chapter, as Matt more explicitly focused on proof, he also created more space to involve students in proving. For
example, much of the talk about of the Base Angles Theorem in Y3 was based on students’ ideas (e.g., drawing a line parallel to the base of the triangle). In the next chapter, I explore the ways that Matt focused on students in Y3 in ways that were more sophisticated than in Y1.
CHAPTER 7: CREATING SPACE FOR STUDENT PARTICIPATION

In Chapter 4, I claimed that Matt made many changes to his practice across time in order to help his students experience “real math.” Despite feeling frustrated with proof as an undergraduate mathematics major, Matt eventually came to believe that doing proofs was possible. It was therefore important to him that he communicate to his students that they too could do proofs. In Chapter 4, I also wrote about how Matt viewed the process of coming to know mathematics and what he thought his role should be as the teacher. Matt said that there were limitations to what he, as the teacher, could do to help students “see” the path to a proof. Saying that “math is not a spectator sport,” Matt also emphasized the fact that there were no steps that he could teach to help students prove. He articulated that his role was to help the students believe that it was possible and to act as a Sherpa guiding students up a mountain. What was not discussed in detail was the role that Matt believed that the students should play in class. In this chapter, I describe the ways in which Matt progressed toward opening up the classroom interactions in order to create space for student participation. I argue that he did so purposefully, and that his participation in the larger discourse study contributed to these changes. By providing students with more space to become involved in the lessons, Matt also created opportunities for students to participate more centrally in “real math.”

I begin this chapter by using interview data to describe one of the roles that Matt believed students should play in the process of learning mathematics: the role of active participants. I also use classroom data to highlight two lessons that illustrate the ways that Matt’s classroom discourse changed across time. Data from the Deductive Reasoning lesson
is presented to illustrate how supplementing the curriculum with an additional task created space for student participation. I also do a fine-grain analysis of the discourse from the *First Proofs* lesson to show how Matt’s use of particular discourse moves changed across time. Interview data from Y1 and Y2 are also used to understand some particular goals that Matt had for changing his practice. Finally, I end with Matt’s reflection on how and why his classroom discourse has changed.

**Student Participation**

Matt’s description of the expectations that he had regarding student participation were consistent throughout the study. During the first year of the larger discourse study when he was using the MiC program, Matt spoke about student engagement:

**Example 7-1**

I suppose my main philosophy is...students have to be engaged in what they’re learning. And they have to take responsibility for their own learning...Less from me is more from them...They know that I’m not gonna give them the answer...that I’m gonna help them but they’re gonna be the ones that are gonna...figure out what’s going on and they have to really understand it.

(Interview D-1, 3/9/05)

Matt said that students needed to be engaged and take responsibility for their own learning. The phrase “Less from me is more from them” is one that Matt used consistently throughout the study. Matt believed that the less he did *for* the students, the more active a role they would then take in their own learning. One of the things that he said he would give less of was answers. As Matt said, “I’m not gonna *give* them the answer.” It was up to students to be engaged with the material, and it was their responsibility to “really understand” the

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45 To be clear, Matt considered students to be participating if they were engaging with the mathematics. He acknowledged that this participation could be happening without it being evident to him.
mathematics. This did not mean, however, that Matt would sit back and do nothing while the students struggled to figure out the mathematics on their own.

In fact, there were specific things that Matt said he would do to engage students. At the end of Y1, when asked what kinds of mathematical goals he had for his students, Matt described some of the strategies that he used to encourage active participation in geometry:

**Example 7-2**

I try...to get them into a habit of talking to each other before they talk to me. I try to get them to show their work so that they can understand what they've done. I try to get them to explain how they got an answer and get used to being able to justify what you've said, or what you've claimed or what you've calculated. Those would be the big mathematical [goals].

(Interview D-2, 4/13/06)

It is interesting that Matt said here that he *tries* to get them to talk to each other, show their work, and explain how they got an answer. Matt’s experiences with two different sets of curriculum materials suggest that most of these goals (which are fundamental in many *Standards*-based curriculum programs such as MiC) require more planning to incorporate when using a more conventional textbook like *Geometry*. For example, Matt did not have ready-made activities and problems for students to engage with on a daily basis in geometry. This was similar to the experience of Herbel-Eisenmann et al.’s (2006) Jackie, who said: “If I wanted to teach Algebra as a *Standards*-based course, I’d have to design everything from the ground up” (p. 333). Like Jackie, if Matt wanted to encourage active participation, he was going to have to plan and think about how he could make that happen because the problems in *Geometry* did not specifically encourage this.

Matt also described the kind of learning disposition that he wanted his students to have:

**Example 7-3**

I'd like the students to become more autonomous….You get out of this what you put
in it. So if you come to school everyday and expect to sit there and have information be absorbed into your brain, you're not gonna get a whole lot out of it.

(Interview D-3, 4/14/06)

Saying that “you get out of [class] what you put in it,” Matt explained that he would like his students to be more “autonomous.” In his view, it did not work for students to come to school and expect to learn by sitting and absorbing information. One could take this to mean that Matt did not believe that lecturing to his students and teaching by telling was a good idea. In this chapter, however, the data will show that, as a relatively new teacher, Matt was not able to avoid this mode of teaching in Y1 when he was in “survival mode.” Despite the fact that he believed that students should be engaged and experience “real math,” the data suggest that Matt’s practice was not aligned with his professed beliefs about student engagement during Y1. By Y3, however, I observed a practice that was more aligned with his professed beliefs. In the next section, I describe a lesson that has not been discussed in previous chapters. This description will help the reader understand how Matt’s practice changed across time in ways that were more aligned with his goals for student engagement. More specifically, in this section, I highlight Matt’s use of an activity in Y3 that created space for student participation.

The analysis and interpretation of the data provided here suggest that the addition of a mathematical task that engaged students in Y3 and assisted Matt in providing opportunities that fostered participation that was not evident in the Y1 lesson. This is important because research suggests that tasks are central to students’ learning and can shape their view of mathematics (National Research Council, 2001). As is suggested by the Standards, Matt’s use of the task enacted in Y3 engaged his students and promoted communication about mathematics. The important relationship between instructional tasks and students’
engagement and opportunities to learn mathematics has been noted in the QUASAR project literature (see, e.g., Stein, Smith, Henningsen, & Silver, 1999).

**Deductive Reasoning across Time**

The *Deductive Reasoning* lesson was presented in Chapter 2 of *Geometry* and preceded the *First Proofs* lesson. The goals of this lesson were “Using the Laws of Logic” (i.e., the Laws of Detachment and Syllogism) and “Using Symbolic Notation” (see Appendix D). In this section, I use classroom transcripts to describe the enactment of this lesson in Y1 and then in Y3. Interview data helps shed light on why these lessons looked so different.

**Deductive Reasoning in Y1**

In the beginning of class, while Matt took attendance, he asked students to take out and look over the logic puzzle that they worked on during the previous class period (Appendix G). Matt then spent about 15 minutes on the whole-class work. As was the case with other lessons in Y1, Matt also scheduled a quiz on this day which students needed to take after the whole-class work. He used the logic puzzle as a launch for the lesson on deductive reasoning.

**Example 7-4**

Matt: Okay, cool. So we ended up solving the logic puzzle, I think everybody got the answer that they were looking for. Let's look at clue number three. Clue number three. If Maynard lives in Ravenna, then his favorite hobby is playing the guitar. How did that help you out later when you were trying to solve the rest of the puzzle? How did that clue help you? Yes?

MS: Because we find out later that Maynard does live in Ravenna.

Matt: Right. Later on because of clue number six and some other things, you found out that Maynard does live in Ravenna so what did that and clue three, what did that help you conclude? Chase?

Chase: That he plays the guitar.

Matt: That he plays the guitar, right? If Maynard lives in Ravenna, then his favorite hobby is playing the guitar. You knew that Maynard lived in Ravenna, and therefore
you concluded that Maynard likes playing, that his favorite hobby was playing the guitar, right? That makes sense. This [writes on the board, see Figure 7-1] is the kind of logic that you used here is a specific form. Okay? We say if this then this. We know this is true, therefore, I know that this other thing is true. Right? It makes sense. If all crows are black or if a bird is a crow, then it is black. Here is a crow, therefore, it is black. Right? Makes perfect sense, hopefully. This is called, the book's gonna use a fancy word for it, the Law of Detachment.

(Y1 Deductive Reasoning, 9/26/05)

| If \( p \) then \( q \)  
| \( p \)  
| Therefore \( q \) |

Figure 7-1: Deductive Reasoning Boardwork 1

Matt started the lesson by telling the students what they supposedly understood from the puzzle, saying “I think everybody got the answer that they were looking for.” Two students answered questions about how clue numbers three and six helped them solve the puzzle. Matt used the conditional statement from the puzzle, “If Maynard lives in Ravenna, then his favorite hobby is playing the guitar” paired with the statement, “Maynard lives in Ravenna” to conclude that Maynard’s favorite hobby was playing guitar. He then told students, “That makes sense.” What Matt wrote on the board next was a representation of these statements symbolized by \( p \) and \( q \) (Figure 7-1). He used vague pronouns as he pointed to the statements on the board when he told the students, “We say if this \([p]\) then this \([q]\). We know this \([p]\) is true, therefore, I know that this other thing \([q]\) is true.” Again, he told students, “It makes sense.” After citing another example about crows, Matt said, “Makes perfect sense, hopefully.” Matt explained to students that the book was going to use a “fancy word” for this law. He wrote “Law of Detachment” on the board.
Next, Matt reminded students that he had told them a story about a *Simpsons* episode\(^{46}\) the week before. He relayed this story, which involved Homer using the Law of Detachment incorrectly:

**Example 7-5**
Homer does this backwards. Homer says \(p\) implies \(q\), \(q\), therefore \(p\). He uses the existence, or the statement of his hypothesis, or his conclusion, to conclude that his hypotheses is correct, right? Which is totally backwards. That’s the whole joke, right? *(Y1 Deductive Reasoning, 9/26/05)*

After telling the *Simpsons* story and then explaining it symbolically, Matt continued explaining why the episode was funny. Then he told students that the kind of “bad logic” that Homer used happened a lot:

**Example 7-6**
But this kind of reverse or bad logic happens a lot, so this logic - good, makes sense. Okay? This logic - bad. Okay? So Law of Detachment, if \(p\) then \(q\), \(p\) therefore \(q\). You, we're gonna use this in math a lot. If two lines are perpendicular, then they intersect at 90-degree angles. Right? Here are two perpendicular lines, what can we conclude? [Pause, no answer] They have to intersect at 90-degree angles, right? It follows from our structure, okay? *(Y1 Deductive Reasoning, 9/26/05)*

When Matt said, “this logic – good,” he, again, for the fourth time, told the students that it “makes sense.” He wrote “good” next to the Law of Detachment and then wrote “bad” next to the “reverse or bad logic” (see Figure 7-2).

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\(^{46}\) Here, Matt was referring to the *Simpsons* cartoon, a popular cartoon in the United States. The episode that he referred to originally aired on May 5, 1996 and was called “Much Apu About Nothing.”
Matt also stated the conditional, “If two lines are perpendicular, then they intersect at 90-degree angles.” He then asked students what they could conclude if they had two perpendicular lines. At this point, however, students had not yet worked with definitions in the if-then format so Matt ended up answering his own question saying, “They have to intersect at 90-degree angles, right?” This occurred despite the fact that Matt said (in Example 7-1), “I’m not gonna give them the answer.” Also, Matt frequently “tagged” his questions with either “okay?” or “right?” as he did here. In Chapter 2, I described how, when used by credible sources, this discourse move is intended to persuade and cajole agreement (Blankenship & Craig, 2007).

After finishing the Law of Detachment, Matt told the students about the Law of Syllogism:

**Example 7-7**

Then we want to talk about the other law that the book is gonna use or state, is the Law of Syllogism. I was giving [teacher name], yesterday, a rough time. He's pronounced it, he's pronounced it wrong his entire life. I don't even want to try to pronounce it wrong, cause then you're all gonna pronounce and wrong, and I'm gonna end up messing you up. Syllogism, okay? The Law of Syllogism says if you have two, if statements that match up, then you can conclude that the resulting one is also true. I don't know if we used an example of this in our logic puzzle. Uh, [pause] well let's go back to what we looked at before. We said, clue number three said if Maynard lives in Ravenna, then his favorite hobby is playing the guitar. Right? No that's not gonna work. Never mind. Okay. [looks through textbook] Okay, so what's this, what, what the Law of Syllogism is gonna say is if we have two 'if' statements that sort of run together, then we can conclude that the last one, which we want to be true, is also true. Okay? Uh, let's see, let's come up with an example.

*(Y1 Deductive Reasoning, 9/26/05)*

During much of this monologue, Matt looked through his textbook. He did so, it seemed, to find the name of the second law. He also flipped through the pages of the textbook as he tried to come up with an example of the Law of Syllogism. In addition, Matt digressed with a story about one of his colleagues mispronouncing the word “syllogism.” This was another
example of the kind of space-filler that I described in Chapter 6. As I said, I did not observe Matt filling space in this way during Y3.

Matt had difficulty coming up with an example for the Law of Syllogism. He said, “I don’t know if we used an example of this in our logic puzzle.” Then he looked back through the textbook, and he said, “No that’s not gonna work. Never mind.” After Matt said, “let’s come up with an example,” he returned to the story of the *Simpsons*.

The last goal for the day involved using symbolic notation. When Matt asked students what symbol should be used to represent the negation of $p$, a student suggested a negative sign, rather than the tilda (~) symbol. The use of either the negative or the tilda symbol is an arbitrary component of the curriculum and an example of the kind of information that, according to Hewitt (1999), should not be given extensive attention. Continuing this monologue, Matt talked about an “on-going war” between mathematicians and logicians related to symbolism in mathematics. (see Cirillo & Herbel-Eisenmann (in review) for more detail related to this example). Again, this is another example of the kind of space-filler that occupied time during lessons in Y1.

The examples from this lesson showed a teacher-centered discourse with Matt doing almost all of the talking. Viewing the full transcript of this episode of whole-class work (Appendix H), one can see that student-turns were infrequent, and that Matt delivered long monologues. When Matt reflected on this lesson, he wrote about how much he talked:

**Example 7-8**

Today was a lot more “boring” than a typical day (I hope). I was thrown a little bit by being taped at the beginning, and I was planning on the students taking their quiz first so that it would give me a little time to go back over the logic puzzle and see more carefully what I would pull out to lend in discussion. Really I talked too much today. *(Deductive Reasoning* Daily Reflection, 9/26/05)*
This lesson occurred on the first day that Matt was video-taped for the larger study. He attributed the fact that he gave students a choice about when to take their quiz and being distracted by the video-camera as reasons that explained why the lesson was “boring.” Since students opted to take the quiz after the lesson, Matt said that he did not have time to plan for the lesson while students took the quiz. He had planned to look more carefully at the logic puzzle to find examples that he could use to explain the laws of logic.” Instead, he said, “I ended up just kind of winging it, which did not turn out good.” Matt seemed to view his choice to allow students to decide that they would take their quiz after the lesson as a mistake since he had not yet spent sufficient time planning the lesson. Appropriate planning of routine activities (such as giving quizzes) is one part of “enacting a beginning repertoire” (Feiman-Nemser, 2001, p. 1018), a central task that novice teachers face in the early stages of their careers. When students opted to take the quiz after the lesson, Matt said he was thrown off-course.

When Matt wrote his Daily Reflection, in the section where he was asked about providing opportunities to facilitate student understanding, he wrote:

**Example 7-9**
Unfortunately, because I talked the entire time and never really got a chance to let my students talk to me about the material, I really didn’t get a good chance to see if they understood or misunderstood anything today, which is rather unfortunate (“Learning is mostly telling, teaching is mostly listening” is running through my head)….Instead of a conversation, I just talked to myself, and hoped the students were listening.

(Daily Reflection, 9/26/05)

Clearly, Matt was not pleased with the enactment of this lesson. His remark about teaching being about “listening” indicates that he believed that he should have been doing less talking and more listening. Acknowledging that he did most of the talking in this lesson, Matt said that he was not able to gage whether or not the students understood the content of the lesson.
Before presenting data on how this lesson was enacted in Y3, I present data from an interview that occurred during Y2. This interview data provides insight into the kinds of things that Matt thought about in the interim through his involvement in the larger Discourse Study. This data will help explain why Matt made some of the changes to his classroom discourse that will be described in the Y3 lesson.

**Reflecting on His Classroom Discourse**

From May in Y1 to December in Y2, Matt and other participating teachers from the larger study spent about 54 hours engaged in a study group on classroom discourse. These teachers also watched video of their own teaching that was filmed during Y1 of this study. Matt reflected on some of the things that the discourse readings prompted him to notice in the video:

**Example 7-10**
I was still the one talking all the time….but just trying to be more aware about…who’s talking when, how long are people talking for and those kinds of things….I need to look up more. I spend a lot of time looking at the book. Not enough time looking at them….like I’ll look at the board. (Interview D-5, 9/25/06)

Even though the video that Matt viewed was not from the *Deductive Reasoning* lesson, he noticed some of the same kinds of things that were described in the analysis above. Matt said that he was the one talking “all the time” and that he wanted to be more aware of who and how often “people” were talking. By “people,” I assume Matt meant himself and his students. He also noticed that he spent more time looking at the textbook and the board than at the students.

In addition, Matt commented on how little the students talked during the lesson:

**Example 7-11**
But the thing that frustrates me is that so many students have one-word turns….I had
never really thought about that...how many student turns are only one word long?...How can you...create an environment to where the students know that...you’re not looking for a one-word answer? (Interview D-6, 1/13/07)

Matt was frustrated that when the students did speak, there were many one-word turns occurring. He said that he never thought about this aspect of classroom discourse and asked how he could create an environment where students understood that the teacher was looking for more than one-word answers. When the interviewer asked him about the possibility of students talking to each other, Matt responded, “Yeah, that’s very, very valuable.” So even though Matt believed that students should be engaged with the mathematics by taking longer talk turns and speaking with each other, he seemed unsure about how to make that happen. In addition, despite the fact that Matt said that his philosophy was that “students have to be engaged in what they’re learning” (Example 7-1), he also said that he had never thought about how many student turns were only one-word long. Matt did not seem to recognize that the kinds of activities that he asked students to engage in and the types of questions that he was posing may have facilitated a more closed, univocal interaction pattern. Or, it could be that giving one-word answers is a norm for students in mathematics and other classes. By Y3, some changes can be observed related to frequency and length of student turns. In the next section, classroom data is presented to illustrate this point and to describe an activity that Matt used to enact the Deductive Reasoning lesson in Y3, which seemed to increase student engagement.

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47 A quantitative turn-taking analysis was provided to participants from the Discourse Study. This analysis provided data about the length of Matt’s and his students’ talk turns. For example, if Matt said, “What do we do next?” this utterance would have counted as a turn-length of five. Frequency of student and teacher turns was also calculated across three weeks of lessons. This analysis was somewhat inaccurate, however, due to the lack of microphones that were sensitive enough to pick up accurate student contributions.

48 Because the turn-taking analysis was inaccurate in Y1, rather than do another turn-taking analysis for Y3, I encourage the reader to glance through the Appendices (H & I) to get a sense of the difference I describe here.
In Y3, Matt spent about eight minutes going over homework before beginning the whole-class work. Unlike in Y1, this time Matt did not schedule a quiz. He was then able to spend about 25 minutes on whole-class work. Matt explained to students that they would be watching a skit from Monty Python:49

Example 7-12
Matt: Okay. So we're gonna watch a skit from Monty Python. Okay. We're gonna watch it twice.
MS: I love this movie.
Matt: Okay, how many people have seen this movie before? [Some students raise their hands.]….So the setup is, for people who haven't seen the movie, the setup is that the guy standing on the platform, he's the knight in charge of the town and uh, the townspeople have brought someone that they think is a witch and they have a conversation where they try to determine whether or not she's a witch. Okay, cause, we're gonna watch it twice. So the first time just watch it 'cause it's funny so I wanna give people a chance to laugh. Just watch it and kinda think about what's happening. The second time we'll write some stuff down.
(Y3 Deductive Reasoning, 9/17/07)

Here, Matt first asked the students if they had seen the movie before. He provided background information for the clip that they were about to watch for those students who were not familiar with the movie. Then he told the students that they would watch the video clip twice. The first time they could watch and enjoy the clip, and the second time they would “write some stuff down.” In this sense, Matt was considering his students by letting them enjoy the humorous aspects of the clip prior to attending closely to the mathematics in the dialogue.

After they watched the video clip the first time, Matt explained what he wanted his students to do while they watched it the second time:

49 The video clip from this lesson was from Monty Python and the Holy Grail. This was a film starring the cast of the British comedy sketch show, Monty Python and the Flying Circus.
Example 7-13
Okay so we're gonna watch it again. And this time, I want you to write down as many if-then statements as you can find. Okay?...They use a lot of if-then logic chains to try to prove that she's a witch.\(^{50}\) I want you to write down as many as of them as you can find. (Y3 Deductive Reasoning, 9/17/07)

This time, instead of Matt supplying examples of if-then statements (as he did in Y1), he involved the students in the lesson by asking them to write down “as many of them as [they] could find” from the video clip. He was also explicit about the if-then format of the statements that they would be working with to prove things even though the previous lesson addressed the if-then format of conditionals.

After watching the video clip for the second time, Matt asked the students to speak to each other about the clip. He then asked them what they came up with:

Example 7-14
Matt: Okay. Check with your neighbor. Have a quick conversation with them. See what they come up with. See what you came up with. Let's take a look at the if-then statements.
[Students talk to each other for about two minutes.]
So…at the beginning of the sketch there's a group of if-then statements when, originally, when the townspeople are just trying to prove that she's a witch, right? Like statements about witchhood, right? What would be some of the things that, at the beginning, that the townspeople use to try to try to prove that she was a witch?

MS: If she looks like a witch.
Matt: If she looks like a witch. [writes on the board] What else?
MS: She's turning people into a newt.
Matt: Okay. If she turned that guy into a newt, [writes on the board], right, but he got better. Any other statements about what it means to be a witch or how you know if someone's a witch?

MS: If she has a huge nose.
Matt: If she has a big nose [writes on the board]

FS: Wart.

(Y3 Deductive Reasoning, 9/17/07)

\(^{50}\) Analysis of the *Monty Python* clip shows that this is not true per se. That is, the characters in the movie do not say “If..., then...” This is implied, however, through the questions that are asked and answered. For example, combining “How do you know that she is a witch?” with “She looks like one” can be written as “If she looks like a witch, then she is a witch.” Only one statement from the video clip came close to being stated as a conditional (i.e., “If she weighs the same as a duck, she’s made of wood). Matt was not explicit about this implied structure.
The lesson continued in this manner. As students suggested things that they noticed from the video clip, Matt revoiced what the students said and recorded their statements on the board. When Matt revoiced, he usually restated or rephrased what the student said and sometimes expanded on it. For example, after a student said, “She’s turning people into a newt,” Matt said, “Okay. If she turned that guy into newt, right, but she got better…” This exchange lasted about seven minutes, and there were 24 student turns that occurred during this part of the lesson. All of the students’ ideas were written on the board in the if-then format. For example, after the student said, “If she looks like a witch,” Matt wrote, “If she looks like a witch, then she is a witch.” Although Matt expanded on the students’ answers by writing them in the if-then format, the hypotheses (i.e., If she…) came from the students. This could be viewed as a different form of revoicing – a textual revoicing. Even though Matt expanded on students’ answers when he wrote them on the board, by making students’ answers part of the official record, he gave them a “bigger voice” (Cazden, 2001).

Matt then shifted the lesson to the more formal statements of the laws of logic:

**Example 7-15**
Matt: There’s two main logical moves that they try to make in this sketch. If \( p \) then \( q \), right? We figure out that \( p \) has to be true, therefore \( q \) has to be true. Okay? If she weighs the same as a duck, then she has to float. She weighs the same as a duck, therefore we think she floats. Okay?....So we have if \( p \) then \( q \), \( p \), therefore \( q \) has to be true. The book calls this the Law of Detachment. The other thing that they try to use in this sketch is to chain these logical things together. If \( p \) then \( q \), if \( q \) then \( r \), what should be true?

Ss: If \( p \) then \( r \).
Matt: If \( p \) then \( r \), right? If she's a witch, then she's made of wood. If she's made of wood, then she floats. If she floats, then she weighs the same as a duck. And then they try to chain all of those things together and say if she's a witch, then she weighs the same as a duck, right? By running this logic chain together. The book calls this the Law of Syllogism.

*(Y3 Deductive Reasoning, 9/17/07)*
With all of the if-then statements still on the board (see Appendix I), Matt was able to connect the statements noticed by the students to the mathematical concept that he was teaching. After asking, “If $p$ then $q$, if $q$ then $r$, what should be true?” students were able to answer this question and tell him what the logical conclusion would be. When they finished talking about the video clip, Matt had two long monologues which can be seen towards the end of the whole transcript (Appendix J). One of the things that Matt explained in the longer monologue was the way in which the townspeople were misusing the Law of Syllogism to conclude that the woman was a witch. When viewing the transcript, one can also see that there were many more student turns in Y3 (39 turns) than there were in Y1 (14 turns).

During Y3, Matt, again, connected to the logic puzzle, though he did so much more easily than in Y1 when he flipped through the textbook and said that it was not going to work. He also told the story about the Simpsons episode, but this time the students laughed at the punchline, which could be an indication that the students more easily followed Matt’s description of it or possibly that he had developed a better rapport with this group of students. At the end of the lesson, Matt asked his students, “Does this kind of make sense?” He looked at the students for feedback. This was different from Y1, when Matt told the students four times that it “[made] sense.”

Matt actually used the Monty Python video for the first time in Y2. At that time, I asked him about this decision:

Example 7-16
I: Could you tell me about your choice to use Monty Python this year?
Matt: I couldn't find a copy of the Simpsons and my wife wouldn't let me go out and buy a copy….so I'm in the shower...three days before I'm supposed to use this and I'm going, okay, what am I gonna do?....Well I could use Monty Python instead....and then I thought about it a little bit and I'm like yeah that'll probably
work, and then I watched it, and I'm like yeah this would work.
I: But since you didn't have [the *Simpsons* video] last year and you didn't have it this year...why did you feel compelled to have something?
Matt: Oh, 'cause just talking about...like you still lose something when you're just talking about it. Like, I wanted, I talked about [the *Simpsons*] still afterwards because it's sort of the same point. But...I really wanted to show them something this year as opposed to just talking about it....and it's more fun so....I think it made it more clear to them how this logical chain could happen in real life as opposed to just in my geometry book.
(Interview P-2, 9/26/06)

Initially, Matt wanted to show the *Simpsons* clip that he told the students about in Y1 and Y3. Since he did not have access to that video, however, he showed the *Monty Python* video instead. Matt said that he wanted to show the students something because it was fun. When he said that you lose something when “you’re just talking about it,” he was referring to the idea that students do not learn as much by simply listening to him talk about a concept. This is an example of Matt’s practice becoming more aligned with his belief that students should be engaged with the mathematics. By using the video and having the students pull out the conditional statements from the dialogue, Matt encouraged the students to participate in the mathematics both verbally and cognitively. Rather than telling the students what could be concluded through the Law of Syllogism, the students were able to tell him the conclusion. Matt also said that the video could help students see how the logical chain could happen in “real life.” When he said this, Matt meant that in real life, people sometimes reason incorrectly (personal communication, March 8, 2008).

**Reflecting on Y1 and Y3**

During the final interview of the study, Matt watched video clips from the *Deductive Reasoning* lessons from Y1 and Y3. As he watched the video clip from Y1, he often yelled at himself on the video. For example, he asked, “What are you doing?” when he watched
himself try to connect to the logic puzzle. After he used the example (in the Y1 video), “If two lines are perpendicular, then they intersect at 90-degree angles,” (Example 7-6), Matt questioned this move. He asked, “Why did I say that?...Had we already talked about the definitions?...I hope that I did.” In fact, he had not already talked about the definitions, which might explain why Matt answered his own question about what could be concluded. When he watched the video from Y1, Matt noticed that he sometimes answered his own questions:

Example 7-17
I mean…you're not actually asking a question…it's the pseudo, you were gonna ask a question and then you answer your own question….The frustration is you get in the cycle of they don't actually answer your questions and so that means you don't...ask any more questions ‘cause even when you try to get them to answer your questions, they just all sit there and look at you. Is it because they're dumb? No. It's because they're waiting for you to answer it. (Interview P-6, 12/15/07)

Matt talked here about the cycle that teachers can get into by answering their own questions. He explained that when teachers answer their own questions, then students do not even try to answer the teachers’ questions because students figure out that eventually the teachers will tell them the answer. He also said, “you were gonna ask a question and then you answer your own question.” When I asked Matt about his saying “you” instead of “I,” he laughed and said, “I’m trying to distance myself from my three-years-ago self’ (Interview P-6, 12/15/07).

As Matt watched himself flipping through the textbook on the video, he said, “Make it stop!” He laughed when he said this, but then he also said, “I mean it's just bad.....I'm talking to myself.” After watching the whole clip from Y1, Matt said:

Example 7-18
I mean it's all out of order. Why am I talking about the definition of perpendicular lines before they've actually talked about perpendicular line definition?...I mean I understand why I'm doing this at the time, but it's not, this is being done in an order to which the lesson makes sense to me. The lesson will not make sense to them. (Interview P-6, 12/15/07)
After realizing that he had not yet talked about the definition of perpendicular lines, Matt said that the lesson was “all out of order.” He said that even though he understood what he was thinking at the time, he now believed that the Y1 lesson would “not make sense to [students].” This statement suggests that as Matt developed pedagogical knowledge, he became more thoughtful about or aware of differences between the ways that the content made sense to him and the ways that it made sense to students. When I asked Matt why he laughed when he heard himself tell the students that it “makes perfect sense,” he said, “It doesn't make sense to them at all. Like that's just me making myself feel better, right?” (Interview P-6, 12/15/07). As a more experienced teacher, Matt was now able to critically analyze his teaching and explain why the lesson probably did not make sense to his students.

After viewing the Y1 clip of the lesson, Matt also watched a clip from the Y3 lesson. Matt compared how he interacted with students in these two lessons:

**Example 7-19**

I mean the only person who actually spoke [in Y1] other than me was Chase….so it's just me standing up in the front blabbing…I mean even in the [Y3] transcript, I still wind up talking in the end, right? But…they are talking in a particular context that allows them to participate in the discussion. So even though I end up blabbing a bunch at the end, which is still bad, at least…they've participated in the example part so then maybe my discussion of these different laws makes a little more sense to them because it's tied to something specific…as opposed to just being off in Neverland somewhere. (Interview P-6, 12/15/07)

Matt said that in Y1, with the exception of one student participating, he just stood “up in the front blabbing.” In contrast, in Y3, Matt noted that even though he ended up with two longer monologues at the end, students were at least able to participate in the discussion about the

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51 Topics that these monologues covered include: (a) the incorrect use of the Law of Detachment by the townsperson; (b) the formal establishment of the Laws of Detachment and Syllogism which involved naming them and illustrating them symbolically on the board; (c) a retelling of the story about the *Simpsons* episode that was told in Y1.
video. Matt said that even though the Y3 lesson probably made more sense to the students than the Y1 lesson, the amount of time that he spent talking at the end was “still bad.” These comments indicate that while Matt was able to recognize that he had made progress across time, he still believed that there was room for improvement.

Matt believed that, in part, the lesson made more sense to students because he set up the expectation for students that when he asked a question, he was not going to just wait for “five seconds to pass so that I can answer my own question.” Matt believed that if he did not answer his own questions, then students would think about an answer and, thus, the lesson would make more sense to them. Matt explained that this change, which was small for him, had large consequences for students:

**Example 7-20**
That is such a small change from a teaching standpoint…for us to make, but it’s such a giant shift for our students to make because it changes everything about how they approach the class. If every time a teacher asks a question, they actually have to answer it. (Interview P-6, 12/15/07)

Matt explained that, “from a teaching standpoint,” not answering his own questions and expecting the students to answer them was a “small change.” Similar to what Matt said about discussing the proof rather than just the theorem (Example 6-3), here Matt was saying that because he knew the answers already, it did not require extra work or additional planning for him to hold back from answering his own questions (Interview P-6, 12/15/07). However, this “small change” for him was actually a “giant shift” for students because there was now an expectation that the students should actively participate in the lesson by answering his questions. The discourse pattern in Y3 was more aligned with Matt’s claim that he did not want to give the students the answers (see Example 7-1).
Matt also spoke about another shift or “movement” related to how he approached the content of the lesson:

**Example 7-21**
So you have a movement to include the students in the pre-mathematical language…So like the next part is how do we include them in the precursor activity and include them more in the actual formulation of what it is that we're talking about? So…ideally, my only job at the end would be to say, ‘and that's called the Law of Detachment.’ Like…the only thing that I declare to the class is this is the name. (Interview P-6, 12/15/07)

Again, Matt expressed that although he recognized the growth in his practice, he did not think that he had progressed far enough. Ideally, he would like to be able to engage the students in the “actual formulation” of the topic or concept (similar to the “pre-mathematical language”) and then simply declare the name of the law or formula. When I suggested to Matt that he was talking about the “arbitrary” name of the concept (see Hewitt, 1999), he agreed. He also said that having students become “more involved in the creation of the mathematics” was “the next step” for him. By involving students in the creation of the mathematics, Matt hoped to bring them closer to experiencing “real math.”

In Y1, Matt’s enactment of the *Deductive Reasoning* lesson was teacher-centered. Matt described the lesson as “boring” and said that he “talked too much.” Through his participation in the discourse project, Matt noticed how much more he talked than his students. He also indicated that he was interested in increasing the frequency and length of student turns. In the Y3 lesson, Matt used the *Monty Python* skit to engage his students by having them write if-then statements. Analysis of this lesson showed a lesson that was less teacher-centered than the Y1 lesson. Although Matt said that there was a “movement to include the students in the pre-mathematical language,” he was still interested in moving further along the continuum toward a more student-centered lesson.
In the next section, I describe the enactment of the First Proofs lessons from Y1 and Y3. Although I previously included some data from this lesson in Chapter 6, the focus here will primarily be on the classroom discourse rather than the form of the proofs. This data will illustrate additional ways that Matt created space for students across time. I begin by describing the enactment of First Proofs from Y1.

**First Proofs across Time**

As was already explained, the First Proofs lesson involved the very first proofs that the students saw in the geometry class. In Geometry, these proofs were introduced in the section called “Proving Statements about Segments.” The lesson goals from this section were “Properties of Congruent Segments” (i.e., the Reflexive, Symmetric, and Transitive properties) and “Using Congruence of Segments” (see Appendix D for more detail). In this section, I follow the same format as with the Deductive Reasoning lesson. That is, I first describe the enactment of First Proofs in Y1. Next, I discuss some interview data to understand the ways that Matt thought about changing his practice. Then, I describe the enactment of the lesson in Y3. Finally, I present data from the final interview where Matt reflected on both of these lessons. Taken together, the descriptions of the changes across time in the First Proofs and Deductive Reasoning lessons demonstrate that by Y3 Matt started to create space for student involvement in the lessons, and he did so purposefully.

**First Proofs in Y1**

In Y1, after giving a quiz, Matt spent 20 minutes on whole-class work. He began whole-class work by doing a proof from Geometry on the board. This proof can be found in Appendix A. The transcript presented below is the entire transcript of this first proof. I
present the full transcript to allow the reader to compare Matt’s presentation of the first proof in Y1 and Y3. Because the transcript is longer than other examples, the turns are numbered to allow the reader to look back easily as I do a careful analysis of the text. In addition, in order to bring attention to the pronouns for analysis, they are emphasized as follows:

*you/your/you’re – italics; we/we’re/us – underlined; and I/my – bold.* I highlight the pronoun use because, as discussed in Chapter 2, through the use of “we,” teachers can signal solidarity with their students. I argue that the use of some other pronouns, such as “I” (used by the teacher), for example, can signal a lack of solidarity and a lack of collective responsibility.

**Example 7-22**

1. Matt: Here’s your first proof....PQ and XY are two line segments. If you are told that PQ is congruent to XY, if PQ is congruent to XY, is XY going to be congruent to PQ? Yes, right? It seems like a silly statement to have to prove. But how could we go about proving this statement?
2. MS: Measure both the line segments.
3. Matt: Measure both the line segments, right? What do we know about segments that are congruent? What do we know about them?
4. S: [inaudible]
5. Matt: They, what about them [is] equal each other?
7. Matt: The length, right? So the first step in my proof is going to be to say, ‘okay, I was told this.’ If I know that PQ is congruent to XY, what do I know about PQ and XY? Because they’re congruent.
8. FS: They have the same measure.
9. Matt: They have the same measure. Why do we know that? Because they’re congruent. That’s what it means to be congruent, right? In other words, to be congruent, they have to have the same measure. In order to be congruent, they have to have the same measure. Okay? Now I want to end up with the other way. What’s the first thing, what do I need to do in between in order to state it the other way? I know that PQ is the same measure as XY or PQ equals XY. What’s my third step gonna be?
10. MS: XY equals PQ.
11. Matt: XY equals PQ. How do I know that?
12. S: [inaudible]
13. Matt: Symmetric property, right? Symmetric Property. This comes from our algebra book, right? Symmetric property. Okay? Now that we know that XY is the same length as PQ, how do I know that XY is congruent to PQ? [long pause] Well this is a statement about length, right? And this is a statement about
segments being congruent. If two segments are the same length, how do I know that they are congruent?

S: [inaudible]

Matt: Because in order to be congruent, they have to have the same length. If they have the same length, then they have to be congruent. So, the definition of congruence.

(Y1 First Proofs, 9/30/05)

In the analysis that follows, I discuss four features of the discourse: revoicing, pronoun use, wait time and pacing, and questioning. I selected these four features both because they help illustrate the changes in Matt’s classroom discourse and because they have been shown to be important in the discourse literature in terms of opening up and closing down participation. Subsequently, I present classroom data from Y3 and compare these four discourse moves to highlight some of the changes that were noticed across time.

**Revoicing in Y1**

Although many of the student turns in this lesson were inaudible, when listening to the audiotape, I was able to tell that the student turns were brief (between two and five seconds). It seemed that the students responded to Matt’s questions, and then Matt revoiced or restated what the student said. For example, the student in turn 6 likely said, “the length,” because in turn 7, Matt said, “The length, right?” It was common for Matt to revoice what students said after they spoke. Matt ended this revoicing with a tag question (“right?”), however, which I argued (in Chapter 2) can have the effect of closing down the discourse by coercing agreement. Similarly, in turn 13, after the student’s response, Matt said, “Symmetric property, right?” Therefore, while this form of revoicing may have served the purpose of making student’s ideas available to others and giving students the opportunity to hear their

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52 As was noted in Chapter 3, I did not have video data for this lesson. Therefore, I relied on my field notes and the audio recording of this lesson for the analysis.
answers again (Chapin et al., 2003), the addition of the tag question discourages discussion by coercing agreement with the ‘correct’ answers.

**Pronoun Use in Y1**

In Matt’s first turn, he used the pronouns “you” and “your.” He told students, “Here’s *your* first proof.” He also communicated what was “given” when he said, “If *you* are told that PQ is congruent to XY.” By the end of his first turn, the pronoun changed to “we” when he asked, “How could *we* go about proving this statement?” In his next turn (3), Matt asked, “What do *we* know about segments that are congruent? What do *we* know about them?” So far, in turns 1 and 3, through his use of pronouns, Matt communicated a shared responsibility for doing the proof with the students.

As soon as Matt actually started doing the proof, his pronouns changed. Matt said, “So the first step in *my* proof is going to be to say, ‘okay, *I* was told this.’” In turn 9, Matt asked, “Why do *we* know that?” However, for the rest of this turn, he returned to using “I,” saying it three more times: “*I* want…, *I* need…, *I* know…” In addition, he asked, “What’s *my* third step gonna be?” Even after a student volunteered the correct answer (turn 10), Matt asked, “How do *I* know that?” In turn 13, Matt used both “we” and “I” when he asked, “Now that we know that XY is the same length as PQ, how do *I* know that they are congruent? When he said “Now that we know…,” Matt referred to a statement that was already written on the board. However, when he asked, “how do *I* know that they are congruent?” he was asking students for the reason that justified the statement. When Matt used “we” in turns 3 and 9, he asked a question about the definition of congruence, which students should have
already thought about in a previous lesson. In this sense, Matt was marking these ideas as common knowledge of the group.

So it seemed that Matt used “we” when the answer depended on prior knowledge, but “I” was frequently used for the construction of new knowledge. For example, in turn 11, Matt asked, “How do I know that?” The answer, symmetric property, was a property that students had just learned a few minutes earlier. When Matt switched from “you” and “we” to “I,” his language was less inclusive. More specifically, if teachers’ use of “we” can be viewed as an invitation that opens up the discourse to the students, then teachers’ use of “I” can be viewed as a discourse move that closes down the discourse because it does not imply a shared responsibility. Therefore, this pronominal choice closes down the discourse by not developing an atmosphere of cooperative interaction and consensus to achieve the common goal of writing the proof.

**Wait Time and Pacing in Y1**

After Matt switched back to “we” in turn 9 (“Why do we know that?”), he answered his own question, saying, “Because they’re congruent. That’s what it means to be congruent, right?” So, even though it seemed that he involved the students in the construction of proof by asking, “Why do we know that?” he paused for four seconds of wait time and then answered the question himself. In my field notes, I wrote that one male student answered most of the questions for this particular proof. Some things that might explain why only a single student participated include: the fast pace of the lesson, multiple questions being asked during any one of Matt’s turns, Matt’s tendency to answer his own questions, or perhaps the
students were not following Matt. He did not seem to make effective use of wait time to involve more students in the construction of the proof.

**Interaction Patterns in Y1**

While doing this proof, Matt asked several questions. Many of the questions were about *Matt’s proof* (e.g., “What’s my third step…” and “How do I know…”). In turn 9 when Matt did not get an answer to his question (“Why do we know that?”), he answered the question himself. He also answered his own question in turn 1 when he said: “…is XY going to be congruent to PQ? Yes, right?” Again, this tag (“right?”) serves the purpose of coercing agreement, rather than opening up discussion.

In many of his turns, Matt gave hints when he asked the questions. For example, in turn 9, Matt said: “Now I want to end up with the other way. What…do I need to do…in order to state it the other way? I know that…PQ equals XY. What’s my third step gonna be?” The answer to this question was simply XY equals PQ. Here, rather than answering his own question, Matt posed the question to the students, but because it was almost the same as the statement that was to be proved ($XY \cong PQ$), there is a good chance that Matt’s framing of this question made it relatively easy for many of his students. The answer that Matt was attempting to solicit from the students was the Symmetric Property of Equality, a very basic property in geometry. Also, in turn 13, students did not respond when Matt asked, “Now that we know that XY is the same length as PQ, how do I know that XY is congruent to PQ?” So he then said: “And this is a statement about segments being congruent. If two segments are the same length, how do I know that they are congruent?” When Matt said the

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53 The Symmetric Property of Equality says: “If AB = CD, then CD = AB” (Larson et al., 2001, p. 98)
last line about segments being the same length, he stated the definition of congruence, which was the answer to his question. This line of questioning is similar to “funneling” (Wood, 1998) in the sense that “the teacher [was] engaged in cognitive activity and the [students were] merely answering the questions to arrive at an answer, often without seeing the connection among the questions” (Herbel-Eisenmann & Breyfogle, 2005, p. 485). In some cases, however, the students did not answer the questions at all because Matt answered them himself. In Table 7-1, I highlight the ways in which Matt solicited reasons for each of the four statements in the proof.

<table>
<thead>
<tr>
<th>Turn Number</th>
<th>Matt’s Statement/Question</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>“So the first step in my proof is going to be to say, ‘okay, I was told this.’”</td>
</tr>
<tr>
<td>9</td>
<td>“They have the same measure. Why do we know that? Because they’re congruent.”</td>
</tr>
<tr>
<td>11</td>
<td>“XY equals PQ. How do I know that?”</td>
</tr>
<tr>
<td>13</td>
<td>“If two segments are the same length, how do I know that they are congruent?”</td>
</tr>
</tbody>
</table>

The way in which Matt proceeded through the proof is interesting when compared to the Y3 enactment of this lesson. In turn number 7, Matt said that the first step in his proof was going to be to say that he was told “this.” Although Matt did write “Given” as the reason for the first statement on the board, he never explicitly said it, nor did he explicitly discuss the process of writing a two-column proof. In turn 9, Matt supplied the reason for the second statement by answering his own question. In turn 11, after a student supplied the correct statement, Matt asked, “How do I know that?” Finally, in turn 13, after Matt told the students what the statement was (Now that we know that XY is the same length as PQ, how do I know that $XY$ is congruent to $PQ$?), he asked, “how do I know that they are congruent?”
providing the reasons for the statements in turns 7 and 9, and then asking students how Matt knew the reasons for the third and fourth statements, Matt seemed to assume responsibility for much of the reasoning done in this proof. As will be shown in a later section of this chapter, this was not how Matt handled the first proof in Y3.

In some ways it was understandable that Matt approached the proof in this way because it was the first proof he had ever taught and students were not yet familiar with the structure and process of proof writing. In addition, as Matt said, this particular proof seemed like a “silly statement to have to prove” and was very “semantical” (Example 5-26). I would consider this proof to be the kind of proof that Usiskin (1980) argued should be deleted from the curriculum because it is an “obvious” statement that should be “treated informally, as you would do with another adult” (p. 420). Later on, data will be presented to show that when Matt started off with a different first proof in Y3, one that was “more interesting” (Example 5-26), he did not funnel student responses in this way. In the next section, I present data from interviews that occurred between the Y1 and Y3 First Proofs lessons. This data will help the reader understand the kinds of things that Matt was thinking about related to his classroom discourse.

Reflecting on His Practice

In an interview at the end of Y1, Matt talked about his beliefs about communication. This prompted him to share something that he wanted to improve about his classroom discourse:

**Example 7-23**

I need to be doing something other than just waiting to talk while [students] speak [because] when you start responding…you might be responding to something that they didn't say because you've already made an assumption about what it is that
they're saying. I have to do a better job of not cutting people off and letting them answer questions for real….and the biggest thing that disrupts the sort of communication flow is the, in my opinion, is some kind of authority. You know, if I'm an authority figure over you then I don't really have to listen to what you say. (Interview D-4, 5/22/06)

Matt said that he needed to do something other than just wait for his students to finish speaking so that he could speak. He acknowledged that he sometimes interrupted students. As Matt indicated, this kind of interaction pattern did not allow students to “answer questions for real.” He said that these kinds of things disrupt the communication flow. Matt’s interruptions of students and his not providing wait time might explain why students sometimes did not answer Matt’s questions in Y1. Matt’s statements indicate that he wanted to do a better job of listening and responding to his students. He acknowledged that because he was the teacher in the classroom and more knowledgeable about mathematics, he had authority over his students. This, he believed, was not always conducive to authentic communication.

Matt also indicated that he wanted to create more space for students to actively engage with proofs. He said that doing a proof successfully involves “a certain amount of wait time after you've begun to think about something before it makes sense to you” (Interview P-2, 9/26/06). So even though in Chapter 4, I wrote that Matt said that there was very little that he could do to help his students “see” the path to a proof, here he indicated that providing students with wait time could help students make sense of the proof.

When asked about the kinds of things that he learned from teaching proof, Matt responded:

**Example 7-24**
I wouldn't say that there's been like an advancement of mathematical knowledge for me, but I feel like the, the mathematical knowledge that's there, I started to be able to
put reasons with all of those things. So instead of just being a random piece of knowledge, now I can get a student to understand where that piece of knowledge came from as opposed to just, here's the formula. (Interview P-3, 4/19/07)

So, according to Matt, rather than developing mathematical content knowledge, Matt seemed to think his pedagogical content knowledge became more sophisticated. He indicated that the mathematical knowledge was already in place but he started to be able to communicate the reasons for the concepts to his students. Matt indicated that he started to be able to help students with conceptual knowledge (“put reasons with all those things”) rather than simply providing them with procedural knowledge (“here’s the formula”). The data presented in Chapter 5 indicates that Matt’s curriculum materials did not contribute to his growth in this area. Because he was not provided with explicit support through his curriculum materials or his own district, Matt’s reflection on his practice through the discourse project likely contributed to this growth. Although data from the project meetings were not analyzed for this study, evidence that the time spent in the project meetings provided space for Matt to reflect on his practice can be found in Herbel-Eisenmann et al. (in review). As Castle and Aichele (1994) pointed out: “Through reflecting on lived experiences and having dialogue with others, teachers begin to reconstruct what it means to be a learner and teacher of mathematics” (p. 4). The reflection time and space that was afforded to Matt through the Discourse Study may have caused Matt to listen to his students more. In the data I provided in Example 7-9, I shared that Matt wrote in his Daily Reflection: “‘Learning is mostly telling, teaching is mostly listening’ is running through my head.” Perhaps by listening, Matt was able to consider how his students were thinking about the mathematics. By increasing the amount of time that Matt spent talking, and increasing the amount of time that he spent listening, Matt’s discourse shifted along the continuum away from serving a univocal
function toward a more dialogic one. The data suggests that by consciously trying not to dominate the classroom interactions, Matt created more space for student participation. In the next section, I present classroom data from the enactment of the First Proofs lesson in Y3. I then do an analysis of the same features of the discourse that I examined in Y1 for comparison.

**First Proofs in Y3**

In Y3, after going over the homework, Matt began the whole-class work. He spent about 18 minutes on whole-class work. As was already explained in Chapter 6, in Y3, rather than doing two short proofs from *Geometry*, Matt did one longer proof that he took from another textbook (Appendix F). A small amount of the data from the transcript presented below was already presented in Chapter 6. I use it again here because the focus of the analysis is different.

Matt began this lesson by passing out a supplementary sheet and asking the students to look at a particular proof. He gave the students about a minute to look at the proof and “convince yourself that it has to be true.” After the minute was up, Matt asked the students to explain why the proof had to be true:

**Example 7-25**

16 Matt: So how many people are convinced that this has to be true? [Students raise their hands.] You know for a fact that this must be true? Okay, why? Someone explain to me why this has to be true.
17 Sally: Because, uhm, since ST equals RN and IT equals RU, the difference from, from UN, from RN equals the same as the distance from SI to ST.
18 Matt: Does that make sense? What Sally said makes sense?
19 Ss: Yeah.
20 Matt: Right? If you start with equal lengths and you subtract off an equal amount, it should be equal to the same thing. Now how are we gonna say that so that we're sure completely, completely sure that we have to be correct? That's what we're trying to write the proof for. So where do we want to start?
Jim: Uh, RN minus RU equals ST minus.
Matt: Why? Where'd the minus come from?
Jim: Because take off the IT and the RU and it just goes.
Matt: But how can you, how can we justify that?
Jim: Because it's given that IT and RU is the same.
Matt: Okay, so let's start with that. So IT, okay, so, the two major kinds of proof, on a two-column proof, you're going to write down a statement and then a reason, okay? So IT equals RU. Why?
MS: Because it's given.
Matt: They told us so. It was given. Okay. On a flow proof, you're gonna write a statement, and then underneath it you're going to write the reason, and then you're gonna connect the different statements with arrows. Okay, so that you can see how these pieces fit together. Okay? So, we'll say IT equals RU. That's a given. Okay? Now what?
MS: ST equals RN.
Matt: Why does ST equal RN?
Ss: It's given.
Matt: 'Cause it's given. Okay, now what?
Ss: [inaudible, talking low to each other, Matt waits for 20 seconds before responding to something that he heard one student say]
Matt: Well we gotta do like four more steps before we can get there. We gotta be very clear about what we're doing here. Nate, what do you want to do?
Nate: I, wait, ST minus IT equals.
Matt: ST minus IT equals?
Nate: RN minus RU.
Matt: Why?
Nate: I don't know why. I just think it does.
Matt: So ST minus IT equals SI? That's the idea that we want to say, right? But we don't have any way to phrase this with a minus. What do we know how, let's phrase this positively so that we know it has to be true.
FS: SI plus IT.
Matt: Let's say SI plus IT equals ST. What's the reason for that?
MS: Because addition.
MS: Segment addition postulate.
Matt: Segment addition postulate. Okay. [writing] Okay? Now what?
MS: UN plus RU equals RN.
Matt: RU plus UN equals RN because?
MS: [inaudible]
Matt: Segment addition postulate. Okay, now what?
Nate: Ah, RU, no wait, SI plus IT equals RU plus UN.
Matt: Nate said SI plus IT equals RU plus UN. Why?
MS: Because ST and RN are equal.

54 This “I” is not highlighted because I am fairly certain that Nate was about to say IT, not the pronoun “I.”
The transcript presented here is not the entire transcript of the first proof, but it should be enough to help the reader see that the language choices in this lesson were different from the Y1 lesson.55 I now discuss the same features of the discourse that were analyzed in the Y1 lesson.

Revoicing in Y3

Matt did a great deal of revoicing in this lesson excerpt. In some cases (e.g., turn 45), Matt restated exactly what the student said. In other cases (e.g., turn 47) he rephrased what the student said slightly. Whether he restated or rephrased, he usually elaborated or followed up with a question. For example, in turn 32, Matt said, “‘Cause it’s given. Okay, now what?” Matt sometimes built on students’ answers by asking “why?” For example, responding to students’ answers, in turn 22, Matt asked, “Why? Where’d the minus come from?” and, in turn 30, he asked, “Why does ST equal RN?” After Jim mentioned that it was given that IT and RU were the same, Matt said, “Okay, let’s start with that.” Through the act of revoicing, Matt was able to work through the proof by building on students ideas. For example, Nate contributed the idea (in turns 35 and 37) that ST minus IT is equal to RN minus RU. Matt responded, “That’s the idea that we want to say, right? But we don’t have any way to phrase this as a minus…let’s phrase this positively” (turn 40). So here, rather than dismissing Nate’s answer as incorrect, Matt built on the idea by saying “let’s (i.e., let us) phrase this positively.”

Another thing that Matt did in this lesson was to give particular students credit for their ideas. For example, before revoicing Sally’s answer in turn 20, in turn 18, Matt asked,

55 Although discourse analysis was done with the entire transcript of the whole-class work from this lesson, this finding can be illustrated through use of this excerpt because the rest of the discourse had the same patterns.
“Does that make sense? What Sally said makes sense?” Another important point is that Matt asked students if the statements made sense rather than telling them that they made sense as he did in the Y1 Deductive Reasoning lesson (Example 7-4). When Sally explained her answer, Matt showed that he was listening to her by gesturing toward the diagram on the board as she spoke. More specifically, when Sally said, “ST,” Matt used his hands to highlight segment ST. When she said “RN,” Matt used his hands to point out line segment RN. After asking if what she said made sense, Matt rephrased her answer in turn 20. By asking students if Sally’s answer made sense, Matt placed students on a more “equal footing” (Brown & Renshaw, 2000, p. 65) by providing them with an opportunity to evaluate other students’ answers (Chapin et al., 2003).

Matt also credited Nate with an idea in turn 51 when he said, “Nate said SI plus IT…” In this case he restated exactly what Nate said. By doing so, Matt made Nate’s idea available to others, and he gave students time to hear it again. He then asked why what Nate said was true. Thus, through the discourse move of revoicing, Matt was able to explicitly give credit for and build on student ideas. This was not seen in the Y1 transcript.

**Wait Time and Pace in Y3**

Before they started the proof, Matt gave students time to look at the proof. He also asked students to “explain to me why this has to be true.” This was the only time that Matt said “me” (or “my”) in this transcript. Sally then provided the plan for the proof. It is unlikely that she would have been able to do this if Matt had not given students time to think about the proof in advance. Research has shown that students need at least three to five seconds of wait time in whole-class questioning (Rowe, 1986). Lee (2006) suggested that
students’ be given at least 30 seconds if they are supposed to speak to a partner. Considering that this was the very first proof of the course, it is significant that a student was able to outline a plan for the proof. I should also remind the reader that in Y3 Matt supplemented the textbook with a first proof that was less “semantical” than the ones from *Geometry*.

Even though Matt sometimes asked more than one question at a time (e.g., turns 20 and 22) his turns were shorter and there were many more student turns in this transcript. At one point, when the students did not have a ready answer (turn 32), rather than jumping right in and either giving the students a hint or the answer, Matt waited 20 seconds before responding to one of the things that he overheard a student say. He said that there were about four more steps that needed to be done before the one that the student suggested. Even still, Matt did not give the students the answer. By the time Matt called on Nate (turn 34), Nate had an answer.

**Pronouns in Y3**

The first time that Matt used a pronoun in this transcript, he asked, “*You* know for a fact that this must be true?” This was different from what he did in Y1 when he asked if the statement was true (turn 1) and then said, “*Yes, right?*” Even though both transcripts started off using the pronoun “you,” when Matt used it in Y3, the students verbally participated in the knowledge construction, rather than you being about the general “you” (e.g, in turn 1: “If you are told that PQ is congruent to XY…”). In Y1, however, Matt quickly shifted away from “you” (“your first proof”) to “we” and then to “I” as the proof got underway. In Y3, when Matt asked students to “explain to **me** why this has to be true,” that was the only time
that he said “me” or “I.” In fact, the only time that the pronoun “I” was used in this example was when a student said it twice in turn 39: “I don’t know why. I just think it does.”

Matt used the pronoun “you” when he revoiced what Sally said: “If you start with equal lengths and you subtract off an equal amount, it should be equal to the same thing” (turn 20). He also used “you” and “you’re” when he explicitly talked about the different proof forms (turns 26 and 28). For example, he said, “On a flow proof, you’re gonna write a statement and then underneath it, you’re going to write the reason, and then you’re gonna connect the different statements with arrows.” In this sense, Matt laid out the process that students were going to follow when they wrote a proof. Not only did Matt include the students in the process with his language, but he was also explicit about how they were “going to” participate in the process of writing proofs.

Matt also used the pronoun “we” in this lesson. He started off using “we” to explain to students why they were writing the proof. Matt said: “Now how are we gonna say that so that we’re sure completely, completely sure that we have to be correct? That’s what we’re trying to write the proof for. So where do we want to start?” (turn 20). Most of the “we’s” in this excerpt were about the inclusive “we” in the classroom (i.e., Matt + students, cf. Rounds, 1987). When Matt said, “That’s what we’re trying to write the proof for,” however, he could have been talking about the wider community of people who participate in mathematics (cf. Pimm, 1987). Matt also used “we” in turns 34 and 40 when he talked about the process of writing the proof. He said, “Well we gotta do like four more steps before we can get there” (turn 34). He also suggested, “Let’s phrase this positively so that we know it has to be true” (turn 40). When Matt said “Let’s,” he actually used another pronoun, “us” as part of the contraction “let us.” Again, this showed a collective effort that included Matt and his
students. Even though Matt made the suggestion, a student supplied the specific detail: “SI plus IT” (turn 41).

Matt’s use of pronouns in this transcript created a more inclusive space for students to be involved in the process of writing the proof than he did in Y1. Instead of saying “I” and “my proof” and assuming intellectual authority over the work done here, Matt actually used “you” and “we” to bring students into the mathematical community by being explicit about how students (“you”) would participate in the process of doing proofs. In doing so, Matt facilitated the students’ participation in “real math.”

Interaction Patterns in Y3

In the analysis of the Y1 lesson, I wrote about how Matt gave hints and asked questions that were similar to a funneling pattern. Matt was engaged in cognitive activity while the students merely answered questions to arrive at an answer. Because Matt did not explain the proof process (e.g., an explanation of the two-column form or that every statement needs to be supported by a reason), it is unlikely that the students were following along with the actual proof. In Y3, however, Matt simultaneously did the two-column and the flow proof after he explicitly spoke about their structures. Rather than giving hints and asking leading questions, Matt frequently asked, “Now what?” (turns 28, 32, 45, and 49). This open-ended question put the responsibility on the students to engage with the proof. It seemed that Matt had established a norm that students should answer his questions and that he was not going to answer the questions for them. In this sense, over the course of the first month of the school year, Matt had built this in as a routine in the classroom community (Wood, 1998). In fact, even though I frequently observed Matt answer his own questions in
Y1, I did not observe this behavior at all in Y3.

An analysis of the construction of the proof suggests that in Y3, Matt expected his students to participate in the construction of the proof from Day 1. After providing students with time to think through the proof, and discussing a plan for the proof, through the teacher-led recitation, statements for the proof were provided by the students in turns 26, 29, and 46). The only exception was in turns 41 and 42 when a student said “SI plus IT,” and Matt said, “Let’s say SI plus IT equals ST.” After these four statements were given by students, Matt asked the students to supply the reasons that these statements were true. The ways in which Matt did this are listed in Table 7-2.

**Table 7-2: Reasons for the First Proof in Y3**

<table>
<thead>
<tr>
<th>Turn Number</th>
<th>Matt’s Question</th>
</tr>
</thead>
<tbody>
<tr>
<td>26</td>
<td>“So IT equals RU. Why?”</td>
</tr>
<tr>
<td>29</td>
<td>“Why does ST equal RN?”</td>
</tr>
<tr>
<td>42</td>
<td>“Let’s say SI plus IT equals ST. What’s the reason for that?”</td>
</tr>
<tr>
<td>46</td>
<td>“RU plus UN equals RN because?”</td>
</tr>
</tbody>
</table>

In every case, for the four statements of the proof presented in this transcript, Matt expected the students to supply the reason.

Compared to Y1, Matt’s use of the tag question, “right?” decreased significantly. Matt used this tag after seven statements in Y1 (within his eight talk turns), but only twice (turns 20 and 40) in Y3 (within his 18 talk turns). Even more interesting is that one of these instances in Y3 occurred after the students agreed (in a choral answer) that what Sally said made sense to them (turn 20). Because the students said that they agreed with what Sally said, when Matt said “Right?” he may have been saying it more as an evaluation or affirmation of the correctness of what Sally said than as a tag to coerce agreement. In the
next section, data from the final interview is used to explain why, from his perspective, Matt made some of the changes that were observed.

Reflecting on Y1 and Y3

During the final interview of the study, Matt watched video clips from the *First Proofs* lessons. He said that he was interested to see how the students did on the end-of-semester exam in Y3:

**Example 7-26**

I'm interested to see this year though because I felt that this year went much worse than last year. In terms of the number of students I had by the end actually being able to write a legit, a legit proof...I thought last year I had more students by the end being able to write a legit proof, but maybe that's just because I'm remembering [wrong]. Right? First year I had three people, next year I had six people - 'oh my God, this is phenomenal.' This year there's only six people, it's just that now I realize that all the rest of 'em can't do it. (Interview P-6, 12/15/07)

At first Matt indicated that he believed that fewer students were actually able to write valid proofs [in Y3] than in past years. It was not that Matt believed that his teaching was worse in Y3, but rather his perspective changed when he realized how many students were *not* able to write a legitimate proof by the end of the year. As Matt said, the first year he thought that he was doing a good job when three students were able to write proofs. The next year a few more students seemed tuned in. By the third year, he was aware of how many of the students were either *not* engaged or *not* understanding the lessons. This suggests that Matt started to pay greater attention to students as learners and possibly his own practice by noticing some things that could be viewed as shortcomings.

In Example 7-10, I wrote about Matt’s concern over his looking at the board or the textbook, rather than the students. He indicated (at the beginning of Y2) that he did not spend
enough “time looking at [students]” (Interview D-5, 9/25/06). In the final interview, Matt noticed this again:

**Example 7-27**
I mean I still look at the board a lot and just looking at the clips, I look at the students more than I did in the first year…..If you actually broke it down…I bet I still spend more time looking at the board than I do in doing anything else, but the percentage of time that is actually looking at the students has gone up I hope.

(Interview P-6, 12/15/07)

Matt noticed that even though he still looked at the board more than he thought was appropriate, he believed that he did so less in Y3 than in Y1. By Y3, he did not need to look through *Geometry* to determine that his textbook referred to a certain logic law as the Law of Syllogism (rather than Hypothetical Syllogism or something else). Noticing how often he looked at the students also indicated an interest in connecting with the students and paying attention to the students as learners. Eye contact with students has been identified in the literature as an example of an “immediacy cue” that can have positive impacts in the classroom (Thweatt & McCroskey, 1996). Immediacy, defined as “those communication behaviors that reduce perceived distance between people,” has been said to decrease both the physical and psychological distance between teachers and their students (Thweatt & McCroskey, 1996, p. 198). In this sense, Matt seemed to believe that by decreasing the distance between himself and his students through eye contact, he could connect better to them.

One of the things that Matt did not articulate was whether or not he noticed any other changes to his classroom discourse. Therefore, I pointed out that I noticed changes related to the pace of his talk and more frequent and longer pauses and his pronoun use. When I asked him about these changes, he explained how they came about:
Example 7-28
If I'm sitting at home thinking about what I'm doing, I'm thinking about scope. I'm thinking about...what are some activities that I can make to get them to talk about this particular concept. I'm not thinking about the, the in-class decision elements....I mean I never come in and sit down in the morning and think, okay, the focus is to give my students more wait time. That doesn't happen. But what happens is you find yourself standing in front of the room wondering, why isn't anybody talking to me? Maybe it's because I just said ‘I’ when I wrote this equation on the board. So quick, say it again and say ‘we’ instead and see if something else happens.
(Interview P-6, 12/15/07)

Matt explained that he did not consider the specific language choices he would make during the daily planning of lessons. It seemed, however, that Matt was inspired to try using a different pronoun during a moment of instruction. Schon (1983) referred to this kind of ‘in-the-moment’ reflection as “reflection-in-action,” suggesting that “we can think about doing something while doing it” (p. 54). When a teacher reflects-in-action, he becomes a researcher in the practice context, constructing a new theory and technique (Schon, 1983) or performing an on-the-spot experiment to change things for the better (Schon, 1987).

Once Matt noticed success with this technique, he was interested in changing this discourse pattern. Matt explained that the discourse project influenced him to consider changing the way that he used pronouns. In fact, pronoun use was one of the topics that were discussed in the summer study group. According to Matt, his participation in the discourse project impacted his practice. This provides evidence that novice teachers can be influenced to consider students more extensively in the beginning of their practice. Matt was attending to his language choices as they related to his goal of wanting to engage more students.

Matt also reflected on the wait time that he gave students to think about a proof. He said he liked to give students time to think about and discuss a proof before beginning it as a whole class because “these things...don't happen just like that [snaps fingers]. You have to
give 'em time” (Interview P-6, 12/15/07). Unlike the way that he started talking about the proof right away in Y1, by Y3, Matt determined that he had to give his students time to think about the proof if he wanted them to actively engage with it. Although, one might criticize the fact that he only gave the students one minute to convince themselves that the proof was true, Matt did purposely attend to providing students with this wait time. He talked about why he decided to pay attention to wait time:

**Example 7-29**
I mean the wait time…and the pacing comes more just from actually watching yourself on [video-]tape and…seeing yourself from the other end of the room thinking about the fact that none of these other people in the room…have looked at the book the night before like you have. (Interview P-6, 12/15/07)

When Matt said “seeing yourself from the other end of the room,” he meant from the students’ perspective (which was the direction the video-camera was pointed). He also pointed out that, unlike what he did to plan for the lesson, his students had not already looked at the lesson in the textbook prior to actually engaging with the mathematics. Recognizing this helped him slow down his pace. Matt’s deliberate decision to slow down his pace in order to give his students time to think about a proof contradicts the claims that he made in Y1 and Y2 that there was “nothing” that he could do to help students “see” a path to the proof (Chapter 4). Here, Matt seemed to indicate that providing students with wait time was an important way to involve students in the process of proving.

Matt elaborated on why watching a video from his classroom for the discourse project at the beginning of Y2 was convincing him to make changes:

**Example 7-30**
Like just the one time watching the video…. [I waited after I asked a question and] I can remember teaching that lesson and feeling like it was a horrifically long time. And then you watch the tape and it's like four seconds…. The time lapse is not something that you normally pay attention to. And just having one video…that's
Matt explained that “time for me passes differently than the time for [students].” He said that when he watched himself on the video, what seemed like a long time in class was actually only a few seconds. He also realized that students needed a few seconds just to process that a question was just asked before they could even begin thinking about an answer. Matt’s statement is related to some things about deliberate silence in the literature. For example, Dillon (1983) noted that “the need for silence in a discussion comes from the fact that time is needed for sustained expression of student thought” (p. 38). Thus, silence can be used as a deliberate act by the teacher to enhance students’ responses and to encourage participation (Dillon, 1983).

Watching himself on video seemed to have been a powerful experience for Matt. This provides evidence that by paying attention to his classroom discourse, Matt was eventually able to think about his students as learners so that he could “help them develop understandings and participate in the learning community” (E. A. Davis et al., 2006, p. 620). Matt said that his goal was to make the discourse “actually more like a conversation and not a giant unscripted monologue from me standing in the front of the room” (Interview P-6, 12/15/07). Again, just because he was able to make some changes, it did not mean that he was completely satisfied with his progress.

Matt said that after three years, his classroom discourse looked, “Less different than I would like, but it looks different” (Interview P-6, 12/15/07). And while he acknowledged that
participation in the discourse project had helped him change his practice in positive ways, he said that knowing all that he now knows about discourse made his job “a lot harder.” He also said: “It's very easy to stand up in front of the room and just blab...it would be so easy - just stand up in the front, blah, blah, blah, blah” (Interview P-6, 12/15/07). Here, Matt seemed to be saying that teaching traditionally would have been much easier for him. However, clearly standing up in front of the room and telling the students about mathematics was not aligned with Matt’s professed beliefs about how people learn mathematics. He knew that if he wanted to engage his students with “real math,” monologues needed to be used minimally.

**Summary**

In this chapter, I presented evidence to support the finding that across time, Matt was able to consider students in more sophisticated ways. He decided that he needed to talk less and listen to his students more. One of the ways that he was able to do this was by supplementing the curriculum. In the *Deductive Reasoning* lesson, he added the *Monty Python* skit, and in the *First Proofs* lesson, he added a “more interesting” proof. These additions helped Matt create more space for student involvement in the lessons and reduced the frequency and length of his monologues. In Y3, he had the students talk to each other before telling him what they noticed in the skit and before talking about the proof. In the *First Proofs* lesson, he also gave students credit for their ideas when he revoiced. Matt was more explicit about the proof process in Y3 and he gave students choice about the form that they would write their proofs in. Last, Matt altered his pronoun use and increased his wait time.
Even though Matt said that participation in the discourse project made his job more difficult, it seemed to have had a positive impact on his practice in the sense that it helped him bring his practice closer to his professed beliefs about student participation. Matt’s discourse during Y1 could be characterized as univocal. That is, his discourse emphasized “the transmission of information from a sender to a receiver” (Wood, 1998, p. 168). In contrast, text that acts as a generator of new meanings has a “dialogic function” (Wertsch & Toma, 1995). Across time, Matt’s discourse became more dialogic, serving the purpose of “enabling students to generate new meanings for themselves” (Wood, 1998, p. 168). When students are given opportunities to explain their answers and support their reasons, they have experiences that are closer to “real math” than to “school math.”
CHAPTER 8: CONCLUSION

[Observation of] actual classroom practice indicates that the major emphasis is placed on a body of theorems to be learned rather than on the method by which these theorems are established. The pupil feels that these theorems are important in themselves and in [his/her] earnest effort to “know” them [s/he] resorts to memorization. (p. 1)

Harold P. Fawcett
The Nature of Proof (1938)

Eighty years ago Fawcett claimed that in the 10th grade geometry course, major emphasis was placed on teaching theorems rather than the method by which those theorems were established. Despite calls for change, namely the recommendation that students of all ages should have experiences that facilitate an understanding of reasoning and proof as fundamental aspects of mathematics (NCTM, 2000), research conducted more recently (e.g., Herbst & Brach, 2006; Herbst et al., in press; Newton, in press) indicates that changes proposed in the NCTM Standards related to the geometry curriculum and the teaching of proof have not occurred. Findings from this dissertation study suggest that there are challenges to carrying out the kinds of practices recommended by the Standards.

In this concluding chapter, I address three topics related to this study. First, I revisit the research questions and summarize the findings. Second, I return to my reflections on the fieldwork in order to discuss how my assumptions played out in the study and to discuss limitations of this work. Next, I address the significance of this work and implications for practice and for future research. I begin by revisiting the research questions.
Research Questions

My original research questions were broad. Because I was interested in understanding how a beginning geometry teacher developed in his teaching of proof across time, I designed a study that allowed me to investigate the changes that the teacher made and to understand why, from his perspective, he made these changes. As the study progressed, more specific questions arose. As a result, this study addressed the following questions:

- How did Matt’s teaching of geometry proof change across three years?
  - How did Matt develop pedagogically?
  - How did his teaching change as he acquired curricular knowledge?
- To what did Matt attribute these changes?
  - In what ways did Matt’s participation in the larger Discourse Study influence the ways in which he taught proof?

I presented evidence that suggests that three significant changes were observed across time. First, Matt’s relationship with his textbook changed across the three years of this study as he developed a curriculum vision that was different from the vision of the textbook. Because developing curriculum trust requires one’s curriculum vision to be aligned with the vision of the curriculum materials (Drake & Sherin, in press), Matt did not develop curriculum trust. Instead, as he developed curriculum knowledge, Matt found ways to adapt Geometry, in part, through supplementing it with other resources and activities in ways that helped him bring his practice closer to his professed beliefs.

Second, Matt’s explicit emphasis on proof increased across time. Rather than teaching theorems, Matt increasingly emphasized the process by which a theorem was proved. Matt supplemented the textbook with proofs that he claimed were “more interesting,”
and rather than emphasizing the final results of the theorems, he emphasized the proof process. Matt did so because, in his view, proof was what “real math” was all about. Third, through his attention to academic tasks and discourse, Matt created more space for student participation. I argued that Matt made these changes in an effort to provide his students with experiences that were more like “real math” than like school mathematics. At the same time, interview data suggested that the impetus for some of the discourse-related changes that were observed came from Matt’s participation in the larger Discourse Study. Taken together, Matt’s growing understanding of the nature of discourse, his increased attention to the students in his classroom, and his emphasis on mathematical processes provide evidence that Matt did, in fact, grow pedagogically across the three years of this study. These findings point to broader implications for research and for those who work with teachers. Before discussing the significance and implications of this study, I briefly return to my reflections on the fieldwork in order to discuss how my subjectivity impacted the study, how my assumptions played out, and some limitations of this work.

**Reflections on Fieldwork Revisited**

In Chapter 3, I reflected on the fieldwork and the assumptions that I had going into this study. Influenced by my own experiences as a high school geometry teacher, I anticipated that if I observed Matt’s practice over a couple of years, then I would see changes. I also discussed how my primary attention at the beginning of the study was on the mathematics. Because this was where most of my attention was drawn, it is likely that there were other changes that I did not attend to. For example, since I (and therefore the camera) was primarily focused on Matt, I did not attend as carefully to the students, their interactions,
and Matt’s interaction patterns with particular students. This became a dilemma for me as the study progressed. More specifically, one of the things that I noticed across time was that the ways in which Matt seemed to interact with female students were different from the ways in which he interacted with male students. The dilemma that I experienced was about how much attention I should or could give this particular issue at a point in the study when I started to think that it was problematic. In the paragraphs that follow, I discuss the kinds of things that I noticed and explain why this issue was not afforded greater attention in the findings.

Although the data suggest that Matt created space for his students in various ways across time, I could not help but notice that when I transcribed the classroom data, a majority of the questions were answered by male students. I found this troubling, but I did not have the right kind of data to document it thoroughly. In some cases, Matt merely followed through with one particular student in a teacher-student duolog\(^{56}\) (see, e.g., Matt’s duolog with Nate, turns 35-39 in Example 7-25). This may have occurred in other situations, but because the camera was focused on the teacher, rather than the students, it was usually difficult to know which student was speaking. In some cases, I was not able to determine the gender of the student at all, especially when the voice was inaudible.

What I would conclude, however, is that even though Matt was able to create more space for active participation of students across time, as an early career teacher, he may not have made this change equally for all students and, in particular, may not have opened the same kinds of spaces for girls as he did for boys. In Y3, the seating arrangement of the

\(^{56}\) Teacher-student duolog is a “prolonged series of exchanges between the teacher and one student” (Lemke, 1990, p. 217)
classroom also provided evidence that the female students were sometimes on the periphery of the classroom community. Most of the female students sat around the perimeter of the classroom while many of the male students sat in the middle of the classroom. Clearly this is problematic since much has been written about insufficient female participation in mathematics classrooms (Ambrose & Fennema, 2001; Walkerdine, 1998) and in the field of mathematics (Henrion, 1997).

In Chapter 3, I wrote about how I expected Matt’s teaching to change across time. In particular, I anticipated that Matt would become more skillful at making certain aspects of proof more explicit to his students. Certainly I did notice a change in Matt’s practice, however, it was difficult at first for me to pinpoint exactly what was changed. Some of the things that I initially investigated (e.g., the notion of explicitness (Bruna, Vann, & Escudero, 2007) and “stepping out” of classroom talk (Rittenhouse, 1998)) were not fruitful in helping me to articulate how Matt’s practice changed. Spending more time with the data eventually helped me to notice the kinds of things that were reported in this dissertation. The ideas that I reflected on in this section point toward limitations of this study. Before discussing other limitations of the study and implications for further research, I first discuss the significance of this study and some implications for practice.

**Significance and Implications**

There are three key points that I emphasize in this section. These three points highlight significant outcomes of this study. First, I discuss the (mis)alignment of a teacher’s curriculum vision with the vision that is put forth by the curriculum materials. Second, I discuss the challenges of bringing proof-related practices in school mathematics closer to
authentic mathematics. Finally, I discuss the implications for participating in a teacher education experience focused on classroom discourse. Throughout this discussion, I also address implications for practice.

(Mis)alignment of Curriculum Visions

Carrying out a *Standards*-based practice with a conventional textbook proved to be difficult for Matt. Because Matt’s curriculum vision did not match the vision put forth by the authors of *Geometry*, Matt felt constrained by the need to “cover” Chapters 1-6 during the first semester. This context prohibited him (at least in his view, especially due to his new teacher status) from trying other approaches. As was the case with Lloyd’s (2008) teacher, Anne, Matt’s prior experience with a different set of curriculum materials influenced the ways that he adapted *Geometry*. Matt’s previous experiences with MiC likely contributed to the frustration that he felt about having to use the conventional geometry textbook because, through his experience with MiC, he saw that there was “a different way” (Interview D-3, 4/14/06) to teach mathematics. Matt enjoyed using the MiC materials, both because his curriculum vision was aligned with that of MiC and because these materials provided a way for him to enact a practice that was aligned with his professed beliefs about how mathematics should be taught and how students should be engaged. In this sense, when Matt used MiC, there were two productive alignments: (a) Matt’s curriculum vision was aligned with the purported vision of the MiC developers, and (b) the alignment of these visions helped Matt carry out a practice that was more closely aligned with his professed beliefs.

In contrast, Matt found the presentation style of *Geometry* to be problematic because he wanted his students to discover the mathematics for themselves and to gain a conceptual
understanding of the material. The presentation of theorems and formulas in green boxes was not aligned with his curriculum vision. The fact that Matt found the need to supplement his geometry textbook with additional proofs may seem ironic given that practitioners and researchers have noted again and again that proof, while typically absent from the rest of the U.S. curriculum, is usually taught in this 10th grade course. Despite the claim that the geometry course is the primary place in the U.S. curriculum where proof is taught, Matt found that the conventional textbook that he used did not provide enough “interesting” proofs and that many of the textbook exercises were fill-in proofs or written in the “given/prove” style. In other words, even in the 10th grade course, in Matt’s view, his students were not getting an authentic experience with mathematical proof because the proof forms were inauthentic, and Geometry did not provide enough opportunities for students to engage with proof. Additionally, Matt was interested in finding ways that proof could be a primary emphasis throughout the course, rather than simply a topic that was addressed across three chapters. The proof experience provided in Geometry was, therefore, not close enough to “real math” to align with Matt’s curriculum vision.

Related to curriculum vision, there are three implications that extend beyond this study. First, any time that a teacher’s curriculum vision is not aligned with the curriculum materials, there is a tension that can be problematic. Lloyd (2008) pointed out that in order to address teachers’ use of curriculum materials, a growing number of teacher educators are engaging their preservice teachers in textbook analyses and adaptation. Much of this work, however, involves the analysis of Standards-based curriculum materials. Because teachers frequently work in schools where they may not have adopted Standards-based materials, attention needs to be given in mathematics education courses to the use and adaptation of
more conventional materials as well. Additionally, both beginning and experienced teachers may require support if their curriculum visions are not aligned with the visions of their curriculum materials, be they Standards-based or not. A misalignment of curriculum visions can be problematic because, as was pointed out by Matt and by Herbel-Eisenmann et al.’s (2006) Jackie, designing one’s own curriculum materials is extremely time consuming for a teacher at any stage in their careers. Discussions related to curriculum material use that take place in teacher education courses should, therefore, include issues related to curriculum vision alignment, beliefs about curriculum materials, and ways of adapting curriculum materials that require less time than redesigning new materials.

A third implication of this study is for school administrators and/or mathematics specialists who make important decisions about the curriculum and the policies related to teachers’ use of curriculum materials. As is the case with many teachers, the relationship that beginning teachers (who likely did not have any say in the textbook selection) have with their curriculum materials may resemble, at best, “an arranged marriage” (The El Barrio-Hunter College PDS Partnership Writing Collective, in press). More specifically, Matt might not have experienced the tensions reported here had he been given either more freedom to use materials that were better aligned with his curriculum vision or better support in using a textbook that was not aligned with his vision.

Another option (one that is even more ambitious than supporting teachers in this “arranged marriage”) is to attempt to find a suitable match between teachers and their curriculum visions with the curriculum materials that are adopted within particular districts. Matching up teaching candidates with teaching positions would begin at the hiring interview. For example, if a district is using a Standards-based set of curriculum materials, those who
interview prospective candidates should attempt to fill the position with a teacher who has a curriculum vision that is a good match for this set of materials.

School leaders must also recognize that many teachers are coming out of teacher-preparation programs where Standards–based practices are emphasized, yet curriculum materials that are aligned with the Standards are either not available (from curriculum developers) or are not provided in their particular contexts. If the goal of the school district or the department is to carry out a Standards–based practice, however, new teachers especially must be supported in doing so in a context where the provided curriculum materials do not help facilitate such a practice. This implication is related to the next area of significance that I discuss. That is, Matt experienced challenges in his attempts to provide his students with a more authentic experience of mathematics than the one that was supported through his textbook.

**Authentic Practices Related to Proof**

In Chapter 2, literature was reviewed related to the call to bring school mathematics closer to the discipline. Some researchers (e.g., Lampert, 1992; Weiss & Herbst, in review) have discussed the idea of “authentic mathematics,” which is similar to what Matt called “real math.” Related to proof, the Standards have called for a de-emphasis of the two-column form (NCTM, 1989) and a focus on the logical argument rather than the form of the proof (NCTM, 2000). Advocates of authentic mathematical proof do not have a high regard for the two-column form due to its artificial and limiting nature and because it is unlike the kind of proof that would be written by a practicing mathematician (Weiss & Herbst, in review).
Influenced by the presentation of various proof forms in *Geometry*, Matt allowed his students to write proofs in any form, and he emphasized that the argument, rather than the form of the proof, was important. Because Matt found that some of his students preferred the flow form, he increased its emphasis over time. Across time, however, Matt placed less emphasis on the paragraph proof because he found that his students did not provide reasons for their statements when they used this form of proof. Related to this, Herbst and colleagues have recently begun to address the various proof forms in their work.

In his earlier work, Herbst (2002a) wrote: “Whereas the appearance of acceptable proofs varied (students might write proofs in flow charts, paragraphs, or two columns), these were *just* graphical alterations of the two-column format” (p. 182, italics added). I would argue, however, that rather than being “*just* graphical alterations,” the two columns may provide a scaffold for students as they learn to write proofs. Unlike with the paragraph form, students using the two-column proof form could not easily “forget” to state their reasons as Matt claimed his students did. In a more recent study, Weiss and Herbst (in review) pointed out that when students are just learning to write proofs, the two-column form not only requires a reason, but points to the possible risk of not providing one. The findings of this dissertation study support Weiss and Herbst’s claim that proponents of including proof in school mathematics should consider the role that the two-column form (and, I would argue, all forms) can play in supporting or limiting authentic mathematical practices related to proof.

Through his work of establishing a system of norms for “doing proofs,” Herbst (2002a) noted: “The mandate to involve students in proving is likely to be met with the development of tools and norms that teachers can use to enable students to prove and to
demonstrate that they are indeed proving” (p. 200). Here, Herbst pointed to the call for change advocated by the Standards and other documents such as Adding it Up (National Research Council, 2001) that emphasize mathematical processes and thinking rather than procedures. More specifically, Herbst suggested that as teachers attend to this call, we may find that they develop unique tools and classroom norms (ones that are different from those described in his work) as they attempt to involve students in the proof process. Herbst also suggested that alternative ways of engaging students in proving must be found.

Building on this work and the work of Martin et al. (2005), data from this dissertation study suggest that attention to one’s discourse can be useful in engaging students in proving. For example, Matt’s use of wait time and questioning in Y3 supported interactions that moved his discourse patterns closer to discussion as the discourse became more dialogic. I argued that through various discourse moves (e.g., revoicing and wait time), Matt opened up the discourse, and, subsequently, engaged more students with proof. Additionally, despite the shortcomings of his textbook, Matt’s interest in developing a Standards-based pedagogy compelled him to engage in practices that, according to Herbst and colleagues’ model, were not normative. To be clear, I am not suggesting that the model developed by Herbst and colleagues is inaccurate. Nor am I trying to establish new norms by claiming that the norms of Matt’s practice are necessarily the new norms that we should be endorsing. Rather, I suggest that this study is a first step in thinking about alternative ways of engaging students in proving.

Some of the ways that Matt involved his students in proving were described in Chapters 6 and 7. For example, Matt supplemented his curriculum materials with proofs that were less “semantical” or, as suggested by Usiskin (1980), Matt de-emphasized the early
“rigorous” proofs of obvious statements (p. 74). Matt’s use of “more interesting” proofs combined with his explicit emphasis on the process involved more students in the construction of proofs. By Y3, it became a norm in Matt’s classroom that students were provided with time, both to convince themselves of the truth of a theorem and to think about how the theorem could be proved. By Y3, Matt also expected students to suggest the statements and the reasons for the proof. In addition, he modeled a more authentic practice by determining the “given” and the “prove” from the proposition. He also modeled the setting up of the diagram from the given information. Eventually, Matt said, he asked students to do this on their own. While not entirely authentic (mathematicians are not provided with the theorems that they intend to prove), this set of practices is more authentic than most of the practices suggested by the teachers and students in Herbst and colleagues’ studies.

The practices enacted by Matt, however, although more authentic and probably closer to “real math” than the experiences provided to many students in other classrooms, do not go far enough. Despite Matt’s efforts to carry out a Standards-based practice that was more authentic than the one provided through the presentation-style of his textbook, Matt was restricted by the “instructional reality” of his context as he attempted to accomplish certain instructional goals within the institutional setting in which he worked (Zhao, Visnovska, & McClain, 2004). This institutional setting that I refer to involves more than just one particular classroom in one particular school, but rather the institution of schooling which constrains educators as they attempt to teach a diverse population of students with a mandated curriculum in 42-minute periods in an era of high-stakes testing.
A final implication related to authentic proof practices involves the presentation style of conventional textbooks such as *Geometry*. This study points to some of the ways that the textbook “acted as a constraint” (Lloyd, 2008), rather than a resource for a teacher who was interested in a reform-oriented pedagogy. During Y1 of this study, I characterized Matt’s “coverage” of the first six chapters as being about teaching theorems, rather than teaching proofs. The textbook exercises that emphasized applications of the theorems, rather than the proofs (see Figure 5-2), seemed to encourage this practice. If we are serious about promoting more authentic practices in school mathematics classrooms, more attention must be given to the curriculum objectives and materials that teachers are using to teach proof. This claim leads to some important issues related to school mathematics curriculum in the United States.

First, this point reinforces the argument that students must have experiences with proof and reasoning prior to the expectation that they write formal proofs in the 10th grade course (see, e.g., National Research Council, 2001; NCTM, 2000). This is important if we believe that, as Wu (1996b) and others have argued, proof is the “guts of mathematics.” Second, rather than emphasizing quantity of proofs, quality should be emphasized. For example, instead of directing teachers to “cover” six chapters that are packed with theorems (as was the case with Matt), the *process* of proving should be emphasized throughout mathematics courses. This is not a new issue in the literature as many researchers have noted the dense curriculum in the U.S. where quantity rather than quality has been the focus (see, e.g., Haycock, 2002; Schmidt, McKnight, & Raizen, 1997).

Specifically related to proof, Usiskin (1980) argued that “doing proof after proof does not teach students to have either transferable skills or the positive feelings about proof that we want them to have” (p. 422). As a former teacher of geometry proof and a current
researcher in mathematics education, I echo this sentiment. Matt seemed to find it impossible to “cover” all six chapters during the first semester and emphasize the process of proving at the same time. As Bassler and Kolb (1971) argued more than 35 years ago, “The ultimate goal in teaching proof is for the student to be able to discover and write proofs” (p. 360). This brings me to my final point about teaching authentic mathematics.

An important aspect of Bassler and Kolb’s (1971) claim about the ultimate goal of teaching proof is that the student not only writes, but discovers proofs. This claim hints at the important work of having students conjecture about mathematical ideas before they even get to the work of actually attempting to write a proof. Both Lakatos (1976) and Lampert (1992) discussed the notion that authentic mathematical reasoning is not carried out in the linear fashion that it is presented in many mathematics textbooks:

The activity of developing a proof is not the straightforward series of logical steps that are portrayed to support assertions in textbooks but a “zigzag path” between conjectures and refutations. And the zigzag has much to do with trying to create a plausible argument and communicate it at the same time. (Lampert, 1992, p. 306)

Here, echoing the work of Lakatos (1976), Lampert explained that when engaging with authentic mathematics, the theorems to be proved are not necessarily known by the mathematician, and, in fact, mathematicians spend much of their time trying to find something to prove, rather than actually writing proofs.

Along these lines, promising work has been carried out by Cox (2004). She described the enactment of a self-designed unit where she collected students’ textbooks and, with particular goals in mind, asked her students to be involved in different mathematical activities such as: “conjecturing, testing conjectures, proving, critiquing proofs, and accepting conjectures as truths” (p. 49). While Cox found many advantages to this method,
she did point out that teaching the unit in this manner takes more time than teaching the unit through a more traditional method. Enacting this kind of practice, where teachers emphasize the process of proving rather than the theorems themselves, requires that teachers be given fewer curricular objectives related to “covering” a set number of textbook chapters so that they can place greater emphasis on the processes of conjecturing and proving. Carrying out such a practice relies heavily on productive discourse practices.

**Attending to Classroom Discourse**

The findings from studies of beginning teachers regarding their ability to attend to their students (rather than content or management issues) are inconsistent. For example, while some researchers have concluded that beginning teachers tend not to consider their students very extensively (e.g., E. A. Davis et al., 2006; Kagan, 1992), other researchers provided evidence that, through curriculum planning, some beginning teachers can focus strongly on their students (e.g., Brown, 1993; Wildman et al., 1989). The findings from this dissertation study support the claim that beginning teachers can attend to their students during the induction phase. Through his involvement in the larger Discourse Study, Matt was able to focus more explicitly on his classroom discourse and make conscious changes in his practices.

Data presented here suggest that attending to one’s discourse can help beginning teachers better align their professed beliefs with their practices. It was during Matt’s fourth year of teaching, however, that these changes were noted. This is not to say that change did not begin or occur during the second year of this study. Matt’s case might suggest that by their third or fourth year of teaching, as a teacher moves out of the induction phase,
involvement in a professional development experience such as the one described here is particularly appropriate. At that point in their careers, teachers have acquired curriculum knowledge (provided they are not teaching new courses) and they are out of the “survival stage.” There were at least three aspects of the Discourse Study that seemed to positively impact Matt’s practice and thus suggest broader implications for the design of professional development.

First, providing long-term support during the induction phase can be a productive way to support beginning teachers because learning to teach involves the ongoing attainment of professional expertise over a long period of time (Morine-Dershimer, 1992). In fact, reflective practitioners often view themselves as professionals who are “becoming teachers” throughout their careers (Mager, 1992). Despite the fact that research tells us that “short-term, fragmented professional development is ineffective for developing teaching proficiency” (National Research Council, 2001, p. 430), many school districts tend to put their teachers through short-term programs. Instead, as was the case with the professional development provided through the Discourse Study, professional development for mathematics teachers must be sustained over time that is measured in years, not weeks or months (National Research Council, 2001). It should also be noted that the participants in the Discourse Study were at various stages in their careers. The interactions between Matt and veteran mathematics teachers may have supported Matt in ways that interacting with other beginning teachers would not have.

A second important component of the Discourse Study was that Matt was provided with time, space, and resources to reflect on his practice. This is important because as was recommended by the NRC (2001), teachers should be provided with support that includes
stipends and release time for sustained professional development. In addition, support was provided through this project that allowed mathematics education researchers to collaborate with the teachers in the study to look at mathematics as it occurs in classrooms (NCTM, 2000). One key aspect related to this was that Matt’s reflection on his teaching through a combination of participation in the summer study group and his viewing of video of his own teaching seemed to influence his practice in positive ways. For example, after watching video-recordings of his teaching, Matt decided that he needed to slow down his pace and connect to his students by making eye contact with them.

Additionally, influenced by readings on pronoun use (e.g., Pimm, 1987), Matt reflected-in-action about his own pronoun use which caused him to shift from saying “we” instead of “I” (as described in Example 7-28). Data were presented that suggested that thinking carefully about various discourse moves such as revoicing and wait time helped Matt open up space for student participation and engagement with proof. Reflecting on lived experiences and having dialogue with other teachers seemed to help Matt better align his practices with his professed beliefs about teaching and learning mathematics.

In this section, I discussed three aspects of this study that I found to be significant. First, I discussed the issue of a (mis)alignment between a teacher’s curriculum vision and the curriculum vision of his/her curriculum materials. I suggested that being more thoughtful and cognizant of this issue during the hiring process may prevent such misalignments. Second, I described some challenges of enacting a proof-related practice that is more authentic. There is much work to be done with the school mathematics curriculum if we are truly to meet this goal. Last, I discussed how attending to one’s classroom discourse can better align professed beliefs with practice. By raising teachers’ awareness of particular discourse moves and
discussing how these moves can open or shut down discourse in the classroom, even teachers at the beginning of their careers may be able to attend to their students in more sophisticated ways than is typical. Additionally, I argued that Matt’s long-term engagement in this professional development experience supported his pedagogical growth. Despite the many significant aspects of this study, there were also limitations to the study that have not been discussed yet. In addition, this study raises further questions for research. In this final section of this dissertation, I discuss some limitations and future questions.

**Limitations and Questions for Future Research**

Like all studies, this study had a number of limitations. My choice of Matt as a participant, while purposeful, could also be viewed as problematic because I argued that he was not a ‘typical’ teacher. Matt’s laid-back approach to discipline combined with the school setting and his strong content knowledge likely contributed to his ability to focus on gaining other kinds of knowledge such as curricular knowledge and pedagogical content knowledge. Additional studies that investigate more typical teachers’ development as teachers of geometry proof are warranted if we are to better support beginning mathematics teachers. Also, there are likely additional issues that were not considered here that would arise in studies of various populations. For example, there may be unique challenges to teaching proof to English-language-learners since proof is more language-dependent than some other areas of school mathematics. Additionally, there may be other cultural ways of knowing that conflict with the logical, deductive nature of mathematical proof.

Like all case studies, this study is not generalizable. The choice of any participant for any study limits generalizability because each teacher brings his/her own beliefs and
knowledge to the particular teaching context. Additionally, the findings from this study may not be consistent with findings from another study of a similar beginning teacher using a different textbook. For example, a study that investigates beginning teachers using other kinds of materials (e.g., Serra’s (2004) *Discovering Geometry*) could shed light on whether or not other beginning teachers of proof struggle with issues similar to Matt. Generalizations to theory, however, are possible (Eisenhart, 1999). This study suggests that carrying out a *Standards*-based practice with a conventional textbook is challenging. More studies that examine different curriculum materials and the ways that they act as constraints or affordances for the teaching of proof are warranted. Additionally, this study points to the need for additional studies on teachers at every stage of their careers who are attempting to carry out a *Standards*-based practice in the geometry course. Research that builds on the work of Herbst and colleagues, Martin et al., and the study reported here is warranted if we are to better prepare teachers to teach formal proof.

As was already mentioned in Chapter 2, missing from the mathematics education literature are more studies of *beginning secondary* teachers’ enactment of *non-Standard-based* curriculum materials. As was previously mentioned, “understanding what teachers [at every level] do with [various kinds] of curriculum materials and why, as well as how their choices influence classroom activity is critical for informing ongoing work surrounding the development of new programs and their adoption in the world of practice” (Remillard et al., in press, p. 1). Such studies are important for helping teacher educators and professional developers support teachers as they use curriculum materials.

Last, this study did not consider the impact that Matt’s practices had (or not) on students. A variety of questions can be raised related to this issue. For example, did the
changes that Matt made impact students’ understandings of proof? Did (all) students in Matt’s classes feel as though his discourse practices in Y3 encouraged their participation? Clearly, more studies are needed that examine teachers in practice as they engage students with mathematics, in general, and specifically with proof.

If we really are to heed the call to make school mathematics more authentic, we need to think carefully about how we can prepare teachers to do so. For example, moving away from the two-column structure (as is advocated by the Standards) takes away one of the tools that was designed to help teachers teach proof, a tool that teachers have been inducted into through their own experiences as students of school mathematics. What then should replace this proof form? Do we want students to write paragraph proofs, leaving out their reasons, a practice that more closely resembles “real” mathematics? And, if, in fact, we are interested in having teachers teach proof rather than theorems, then we need to emphasize different kinds of curriculum objectives that are not about “covering” particular chapters in a textbook. We also need to move away from using curriculum materials that have theorems in green boxes in the “given/prove” format. Unfortunately, however, such curricular issues related to proof do not seem to be on the radar of the mathematics education field at this time. For example, it is interesting to note that, while there was some mention of reasoning, the topic of proof was literally absent from the recent publication of the National Mathematics Advisory Panel (2008).

Recognizing that teaching the content of the theorems and teaching proof are two very different things is critical to changing current practices. There is a need to help mathematics supervisors and teachers develop more sophisticated curriculum visions related to what teaching the process of proof might look like. Perhaps additional case studies such as
this one can provide starting points for the discussion of some new norms of *Standards*-based proof-practices. Clearly more studies are necessary in determining what kinds of problems will arise (e.g., students “forgetting” their reasons in a paragraph proof) when these new practices are carried out. In conceiving of new practices, what exactly is it that teachers would do every day when they go into their classrooms? Do we currently have curriculum materials that support this kind of teaching? How can we support new teachers to enact such practices if, in fact, curriculum materials are not available to support this kind of work? Additional studies that build on the work of Martin et al. (2005), Herbst (2002a), and the work reported here on the case of Matt are warranted that examine ways in which the decisions of the teacher impact the kinds of experiences that students have with proof.
APPENDIX A: FIRST PROOF FROM Y1

Given: $PQ \cong XY$

Prove: $XY \cong PQ$

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $PQ \cong XY$</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. $PQ = XY$</td>
<td>2. Definition of congruent segments</td>
</tr>
<tr>
<td>3. $XY = PQ$</td>
<td>3. Symmetric property of equality</td>
</tr>
<tr>
<td>4. $XY \cong PQ$</td>
<td>4. Definition of congruent segments</td>
</tr>
</tbody>
</table>

From Geometry (Larson et al., 2001, p. 102)
APPENDIX B: THREE TYPES OF PROOFS

<table>
<thead>
<tr>
<th>TYPES OF PROOFS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. <strong>TWO-COLUMN PROOF</strong> This is the most formal type of proof. It lists numbered statements in the left column and a reason for each statement in the right column.</td>
</tr>
<tr>
<td>2. <strong>PARAGRAPH PROOF</strong> This type of proof describes the logical argument with sentences. It is more conversational than a two-column proof.</td>
</tr>
<tr>
<td>3. <strong>FLOW PROOF</strong> This type of proof uses the same statements and reasons as a two-column proof, but the logical flow connecting the statements is indicated by arrows.</td>
</tr>
</tbody>
</table>

From *Geometry* (Larson et al., 2001, p. 138)

![Example of comparing types of proof](image_url)

**Method 1 Two-column Proof**

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ∠5 and ∠6 are a linear pair.</td>
<td>1. Given</td>
</tr>
<tr>
<td>∠6 and ∠7 are a linear pair.</td>
<td>2. Linear Pair Postulate</td>
</tr>
<tr>
<td>2. ∠5 and ∠6 are supplementary.</td>
<td>3. Congruent Supplements Theorem</td>
</tr>
<tr>
<td>∠6 and ∠7 are supplementary.</td>
<td></td>
</tr>
<tr>
<td>3. ∠5 ≡ ∠7</td>
<td></td>
</tr>
</tbody>
</table>

**Method 2 Paragraph Proof**

Because ∠5 and ∠6 are a linear pair, the Linear Pair Postulate says that ∠5 and ∠6 are supplementary. The same reasoning shows that ∠6 and ∠7 are supplementary. Because ∠5 and ∠7 are both supplementary to ∠6, the Congruent Supplements Theorem says that ∠5 ≡ ∠7.

**Method 3 Flow Proof**

From *Geometry* (Larson et al., 2001, p. 136)
APPENDIX C: TABLE OF SEPTEMBER OBSERVATIONS (3 YEARS)

<table>
<thead>
<tr>
<th>DATE</th>
<th>DAY</th>
<th>Section</th>
<th>Matt's &quot;Lesson Plan&quot;</th>
</tr>
</thead>
<tbody>
<tr>
<td>9/26/2005</td>
<td>M</td>
<td>2_3</td>
<td>Deductive Reasoning/Quiz 2.1 - 2.2</td>
</tr>
<tr>
<td>9/27/2005</td>
<td>T (BL)</td>
<td>2_4</td>
<td>Algebraic Reasoning/Alice Logic Puzzle</td>
</tr>
<tr>
<td>9/29/2005</td>
<td>Th</td>
<td></td>
<td>Defs, Posts, Theorems/ PB&amp;J</td>
</tr>
<tr>
<td>9/30/2005</td>
<td>F</td>
<td>2_5</td>
<td>Proving Statements about Segments/Quiz 2.3 - 2.4</td>
</tr>
<tr>
<td>9/14/2006</td>
<td>Th</td>
<td></td>
<td>Biconditionals/Definitions Sheet</td>
</tr>
<tr>
<td>9/15/2006</td>
<td>F</td>
<td></td>
<td>Logic Puzzle/PB&amp;J</td>
</tr>
<tr>
<td>9/18/2006</td>
<td>M</td>
<td>2_3</td>
<td>Deductive Reasoning/The Simpsons --&gt; Monty Python</td>
</tr>
<tr>
<td>9/19/2006</td>
<td>T</td>
<td>2_4</td>
<td>Algebraic Reasoning, Proof that 2 = 1</td>
</tr>
<tr>
<td>9/21/2006</td>
<td>Th</td>
<td>QUIZ</td>
<td>No Observation Today</td>
</tr>
<tr>
<td>9/22/2006</td>
<td>F</td>
<td></td>
<td>Begin Proofs</td>
</tr>
<tr>
<td>9/25/2006</td>
<td>M</td>
<td>2_5</td>
<td>Proving Statements about Segments</td>
</tr>
<tr>
<td>9/26/2006</td>
<td>T</td>
<td>2_6</td>
<td>Proving Statements about Angles</td>
</tr>
<tr>
<td>9/13/2007</td>
<td>Th</td>
<td></td>
<td>Biconditionals/Definitions Sheet</td>
</tr>
<tr>
<td>9/14/2007</td>
<td>F</td>
<td></td>
<td>Logic Puzzle/Getting a Drink of Water (formerly PB&amp;J)</td>
</tr>
<tr>
<td>9/17/2007</td>
<td>M</td>
<td>2_3</td>
<td>Deductive Reasoning/Monty Python - &quot;She turned me into a newt!&quot;</td>
</tr>
<tr>
<td>9/19/2007</td>
<td>W</td>
<td>2_4</td>
<td>Algebraic Reasoning</td>
</tr>
<tr>
<td>9/20/2007</td>
<td>Th</td>
<td>QUIZ</td>
<td>No Observation Today</td>
</tr>
<tr>
<td>9/21/2007</td>
<td>F</td>
<td></td>
<td>Begin Proofs</td>
</tr>
<tr>
<td>9/24/2007</td>
<td>M</td>
<td>2_5</td>
<td>Proving Statements about Segments</td>
</tr>
<tr>
<td>9/26/2007</td>
<td>W</td>
<td>2_6</td>
<td>Proving Statements about Angles</td>
</tr>
</tbody>
</table>
### APPENDIX D: LESSON GOALS FROM FOCUS LESSONS

<table>
<thead>
<tr>
<th>Section</th>
<th>Pages</th>
<th>Goals</th>
</tr>
</thead>
</table>
| 2.3 Deductive Reasoning | 87 – 95 | 1. Using Symbolic Notation  
2. Using the Laws of Logic  
   - Law of Detachment  
   - Law of Syllogism |
| 2.5 Proving Statements about Segments | 102 - 108 | 1. Properties of Congruent Segments |

#### THEOREM

**Properties of Segment Congruence**  
Segment congruence is reflexive, symmetric, and transitive. Here are some examples:

| Reflexive | For any segment $AB$, $AB \cong AB$. |
| Symmetric | If $AB \cong CD$, then $CD \cong AB$. |
| Transitive | If $AB \cong CD$ and $CD \cong EF$, then $AB \cong EF$. |

2. Using Congruence of Segments

| 4.6 Isosceles, Equilateral, and Right Triangles | 236 – 242 | 1. Using Properties of Isosceles Triangles  
   - **Base Angles Theorem**  
     If two sides of a triangle are congruent, then the angles opposite them are congruent.  
     **Corollary** – If the triangle is equilateral, then it is equiangular  
   - **Converse of the Base Angles Theorem**  
     If two angles of a triangle are congruent, then the sides opposite them are congruent.  
     **Corollary** – If a triangle is equiangular, then it is equilateral.  
2. Using Properties of Right Triangles  
   - **Hypotenuse-Leg (HL) Congruence Theorem**  
     If the hypotenuse and a leg of a right triangle are congruent to the hypotenuse and a leg of a second right triangle, then the two triangles are congruent. |
<table>
<thead>
<tr>
<th>Section</th>
<th>Pages</th>
<th>Goals</th>
</tr>
</thead>
</table>
| 5.1     | 264 -271 | 1. Using Properties of Perpendicular Bisectors  
- **Perpendicular Bisector Theorem**  
  If a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment.  
- **Converse of the Perpendicular Bisector Theorem**  
  If a point is equidistant from the endpoints of a segment, then it is on the perpendicular bisector of the segment.  
|         |        | 2. Using Properties of Angle Bisectors  
- **Angle Bisector Theorem**  
  If a point is on the bisector of an angle, then it is equidistant from the two sides of the angle.  
- **Converse of the Angle Bisector Theorem**  
  If a point is in the interior of an angle and is equidistant from the sides of the angle, then it lies on the bisector of the angle. |
APPENDIX E: EXAMPLE OF ‘CLEANED UP’ INTERVIEW TEXT

Original Example:57

Example 4-1
I was immediately asked to do all sorts of proofs, which, you know now, looking back at it, I can see as, you know, not being so bad, but at the time, I’m like, this is a joke. I’m like, this is impossible. You know, you can’t do this...How can people prove things about, you know, eigenspace values and all this other kind of stuff?...It was very much like what you thought was real is now no longer true.
(Interview P-1, 6/21/06)

Changed to:

Example 4-1
I was immediately asked to do all sorts of proofs, which, you know, now, looking back at it, I can see as not being so bad, but at the time, I’m like, this is a joke. I’m like, this is impossible. You know, you can’t do this...How can people prove things about eigenspace values and all this other kind of stuff?...It was very much like what you thought was real is now no longer true.
(Interview P-1, 6/21/06)

57 Bold words were ‘cleaned up.’
Given: ST = RN; IT = RU  
Prove: SI = UN

<table>
<thead>
<tr>
<th>Statement</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>IT = RU</td>
<td>Given</td>
</tr>
<tr>
<td>ST = RN</td>
<td>Given</td>
</tr>
<tr>
<td>SI + IT = ST</td>
<td>Segment Addition Postulate</td>
</tr>
<tr>
<td>RU + UN = RN</td>
<td>Segment Addition Postulate</td>
</tr>
<tr>
<td>SI + IT = RU + UN</td>
<td>Transitive</td>
</tr>
<tr>
<td>SI + RU = RU + UN</td>
<td>Substitution</td>
</tr>
<tr>
<td>SI = UN</td>
<td>Subtraction Property of Equality</td>
</tr>
</tbody>
</table>

This is a record of the boardwork for this proof from Jurgensen et al. (1985, p. 20).

Note: There is more than one way to do this proof.
APPENDIX G: LOGIC PUZZLE FROM DEDUCTIVE REASONING

From *Geometry* (Larson et al., 2001, p. 86)
Obs 1
9/26/05

Matt: So we ended up solving the logic puzzle, I think everybody got the answer that they were looking for. Let's look at clue number three. Clue number three - If Maynard lives in Ravenna, then his favorite hobby is playing the guitar. How did that help you out later when you were trying to solve the rest of the puzzle? How did that clue help you? Yes?

Ms: [inaudible] ...find out later... [inaudible]

Matt: Right. Later on because of clue number six and some other things, you found out that Maynard does live in Ravenna so what did that and clue three, what did that help you conclude? Cade?

Cade: That he plays the guitar

Matt: That he plays the guitar, right? If Maynard lives in Ravenna, then his favorite hobby is playing the guitar. You knew that Maynard lived in Ravenna, and therefore you concluded that Maynard likes playing, that his favorite hobby was playing the guitar, right? That makes sense. This is - the kind of logic that you used here is a specific form. Okay? We say if this then this. We know this is true, therefore, I know that this other thing is true. Right? It makes sense. If all crows are black or if a bird is a crow, then it is black. Here is a crow; therefore, it is black. Okay? Makes perfect sense, hopefully. This is called, the book's gonna use a fancy word for it, the Law of Detachment. De-tach-ment, okay? The book's just gonna use a fancy word. But it's the same basic idea. If p then q, we know that p is true, therefore q is true. Okay? Last week I told the story about the Simpsons, right? About Homer, okay, and the bear patrol. Alright, people remember this story? Homer is trying to use, Homer is trying to use the Law of Detachment, right? If the bear patrol is doing its job, then there won't be any bears around.

Is this a true statement? If the bear patrol is doing its job, then there won't be any bears around. That's true, right? Homer sees that there are no bears around, therefore he concludes, the bear patrol is doing its job, right? The marker's dying. Okay? Why is this wrong? Why do we all make fun of Homer for being an idiot because of this?

Ms: Because there can be no bears around while the bear patrol was out getting coffee.

Matt: Right. Maybe the bear patrol is out getting coffee, and the bears, why aren't there any bears there? Because they're hibernating. Because, Ms: They don't want to be there.

Matt: They won't want to be there. Because the bear patrol already killed them all. I mean there could be lots of reasons, right, for there not
being any bears, right? Homer does this backwards. Homer says, p implies q, q therefore p. He uses the existence or the statement of his hypothesis or his conclusion to conclude that his hypothesis is correct. Right? which is totally backwards, right? That's the whole joke with Lisa, right? I can conclude that this rock keeps tigers away, right? That's a funny, how does it, how does it work? Well I don't see any tigers around, do you? You know? Then he wants to buy Lisa's rock because he thinks that the rock is keeping the tigers away, when in fact, it might be something totally different other than the rock, in fact, hopefully, we don't think the rock actually keeps the tigers away. Alright? But this kind of reverse or bad logic happens a lot, so this logic – good, makes sense. Okay? This logic – bad. Okay? So Law of Detachment, if p then q, p therefore q. You, we're gonna use this in math a lot. If two lines are perpendicular, then they intersect at ninety degree angles. Right? Here are two perpendicular lines, then, what can we conclude? (Pause) They have to intersect at ninety degree angles, right? It follows from our structure, okay? Okay. Then we want to talk about the other law that the book is gonna use or state, is the Law of Syllogism. I was giving yesterday, a rough time. He's pronounced it, he's pronounced it wrong his entire life. I don't even want to pronounce it wrong, cause then you're all gonna pronounce and wrong, and I'm gonna end up messing you up. Syllogism, okay? The Law of Syllogism says if you have two if statements that match up, then you can conclude that the resulting one is also true. I don't know if we used an example of this in our logic puzzle. Uh, (pause) well let's go back to what we looked at before. We said clue number three said if Maynard lives in Ravenna, then his favorite hobby is playing the guitar. Right? No that's not gonna work. Never mind. Okay (pause) Okay, so what's this, what the Law of Syllogism is gonna say is if we have two 'if' statements that sort of run together, then we can conclude that the last one which we want to be true, is also true. Okay? Uh, let's see, let's come up with an example. If the bear patrol is doing its job, then there are no bears around, alright? If there are no bears around, then the citizens of Springfield are safer. Alright? That would be true. No bears makes people safer. From that we could conclude that if the bear patrol does its job, then the citizens of Springfield would be safer. Alright? That doesn't mean that just because the citizens are safer, the bear patrol is doing its job. Don't go backwards. Right? We always want to go forwards. If two lines intersect at right angles, then they form ninety degrees. If two lines form ninety degree angles, then we have supplementary pairs. Right? So therefore, if two lines intersect at right angles, then we form supplementary pairs. Okay? We just chain these things, it's like (daisy)-chaining these things together. If p then q, if q then r, from that we want to conclude - if q, if p then r. Okay? Cool. Is there anything else that we want to talk about? Okay. The other thing that the book is gonna do here is they're gonna start, they want to stop writing 'if, then, if'. That's really annoying, right? Takes too much time. Mathematicians want to shorten the amount of time everything takes. Okay? So instead of writing out 'if something something then something something something something' which takes too long, we're gonna start shortening down, this is my bad marker. So this is my original 'if-then' statement, we're gonna write it with an arrow, if p then q, okay? And remember that we also had the different statements last week - inverse, converse, contrapositive. If, if we write our 'if-then' statement as p arrow q, how are we gonna write the converse? What was the converse again?
Matt: The converse is where we did the, the switch. We wrote the opposite order. So if our if then is \( p \rightarrow q \), how are we gonna write the converse? (Name)

Ms: \( q \rightarrow p \)

Matt: \( q \rightarrow p \), right? \( q \rightarrow p \), right? If \( p \) then \( q \), the converse is if \( q \) then \( p \). What was the inverse? where we negated the statements, right? If two angles are acute, or if an angle is thirty degrees, then it's acute. If it's not acute, then it's not thirty degrees. How do you think we're gonna write that in symbolic our form?

Fs: Not (acute) then not [inaudible] .

Matt: Not \( p \) then not \( q \), okay. That will get the idea across, too many letters. But we're lazy. We want to be as lazy as possible. What are we gonna write instead?

Ms: [inaudible] cross out something

Matt: Good. Alright, like that? That could get kind of confusing, we can't read it very well. What do you think we're gonna do? Not is too long.

Fs: Switch into a equal sign and cross out?

Matt: Like this?

Fs: Yeah.

Matt: Okay. Bill, what were you gonna suggest?

Bill: You just put a line through the arrow?

Matt: Put a line through the arrow. Okay. Like that, but a line through the arrow, would imply that we're negating the 'if-then' right? I don't, I want to try and modify the \( q \) and \( p \). Okay? Any of these would be perfectly fine. Part of the problem in mathematics is it's the oldest contin - have I given you my mathematics is the oldest continuing discipline spiel yet? 

Matt: Mathematics is the oldest continuing discipline. So therefore things that are simple in other disciplines, mathematicians have had way too long to think about and come up with different terminology, okay. So then they end up with really weird symbols. It would make sense to us, if we're in math land to have minus signs there, right? Or negating \( p \) or negating \( q \), the opposite of \( p \), opposite of \( q \). Unfortunately, this particular branch of mathematics the la people in logic have started to try to take it over, okay? So the logical coup of about 700 years ago means that we now use this other random symbol, which has no place in mathematics that unfortunately our book has succumbed to the whim of the logicians.

Matt: Okay? Not \( p \), this, this this symbol the little tilda here, will indicate the negation of \( p \). Okay? So not \( p \), therefore not \( q \). You know? If
it were up to me we'd just use a minus sign because that makes more sense but, the logicians - there's this ongoing war in mathematics and logic. The mathematicians say math is real, logic is based on mathematics. The logicians say, logic is real, math is based on logic. And this is an ongoing battle - an intellectual war or kinds. It's been going on for many years. It's been going on for many years. Nobody cares except for me. That's alright. Trust me. It's an ongoing battle. So in this case, the logicians win. So the contrapositive we talked about last week, what was the contrapos - the inverse is when we kept the order the same but negated, the converse is when we switched. The contrapositive is when we (pause) did both. So how are we gonna write those in terms of our symbols? (pause)

Ms: Not \( p \)

Matt: squiggle

Ms: q

Matt: squiggle

Ms: I mean not q

Matt: [interrupting] not q therefore not p, right?

So this is a nice way, what I like about this way is that it makes it - it's a nice way to sort of put together the inverse, converse, and contrapositive and you can see really easily okay this one I switched, this one I negated, this one I negated and switched. Okay? So that's how we're, that's the symbols the book's gonna use - I think this is on excuse me page 88. They have the little thing about \( p \) then \( q \) and \( q \) then \( r \) and all that other kind of stuff. Okay? Cool. Questions on that? Anything people want to ask about? It's pretty much what we've been doing so far, just (hand gesture) written a little bit more formally. Okay? Questions going once, going twice.
APPENDIX I: IF-THEN STATEMENTS FROM DEDUCTIVE REASONING LESSON

If-then statements recorded on the board:

If she looks like a witch then she is a witch

If she turned that guy into a newt, then she is a witch

If she has a big nose, then she is a witch
    wart
    pointy hat

If witches burn then they are made of wood

If she floats then she is made of wood

If she floats then she weighs the same as a duck

If she weighs as much as a duck then she is made of wood

If she’s made of wood, then she is a duck.

She weighs the same as a duck.
APPENDIX J: DEDUCTIVE REASONING Y3 TRANSCRIPT

Sept. 17, 2007

WCW starts at 8:51

Matt: Okay. So we're gonna watch a skit from Monty Python. Okay. We're gonna watch it twice.

MS: I love this movie.

(A minute or so goes by. Students talking. Matt getting video ready.)

Matt: Okay, how many people have seen this movie before? Have seen this movie before? Okay. So the setup is for people who haven't seen the movie, the setup is that the guy standing on the platform, he's the knight in charge of the town and uh, the townspeople have brought someone that they think is a witch and they have a conversation where they try to determine whether or not she's a witch. Okay, cause, we're gonna watch it twice. So the first time just watch it 'cause it's funny so I wanna give people a chance to laugh. Just watch it and kinda think about what's happening. The second time we'll write some stuff down.

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STARTS MOVIE (about 10:10)

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Matt: Okay so we're gonna watch it again. And this time, I want you to write down as many if-then statements as you can find. Okay?

MS: Just on the back of our homework or where?

Matt: Sure on the back of your homework or somewhere...scratch paper. They use a lot of if-then logic chains to try to prove that she's a witch. I want you to write down as many as of them as you can find.

(pause)

(around 15) Begins movie again. This time starting with the "a witch, a witch, ..."

Ends movie around 19:39

Matt: That's my favorite line in the whole movie. Who are you who are so wise in the ways of science? Okay. Check with your neighbor. Have a quick conversation with them. See what they come up with. See what
you came up with. Let's take a look at the if-then statements.

Students talking for about 2 minutes. Matt says one or two things, but nothing significant. Ends around 21:30.

»<1293165> Matt: Okay, so uh, let's see (pause) so at the beginning, at the beginning of the sketch there's a group of if-then statements when, originally when the townspeople are just trying to prove that she's a witch, right? Like statements about witch-hood, right? What would be some of the things that, at the beginning, that the townspeople use to try to try to prove that she was a witch?

»<1316507> MS: [inaudible]

»<1320531> Matt: That she looks like a witch. (pause)

»<1329511> Matt: What else [inaudible]

»<1330437> Moe: She's turning people into a newt.

»<1332780> Matt: Okay. If she turned that guy into a newt, (pause), right, but he got better. Any other statements about what it means to be a witch or how you know if someone's a witch?

»<1357343> St: if she has a huge nose

»<1358368> Matt: If she has a big nose

»<1360526> St: wart

»<1362146> Matt: A wart. Right. Big nose [inaudible] wart, right? What else did they use to try to prove

»<1368636> MS: the witch's hat

»<1369243> Matt: The hat, right? (pause) So if she has a big nose then she's a witch. If she has a wart, she's a witch. If she has a pointy hat, right? Then she's a witch. Okay? So there's all these sort of original if-then statements about what it means to be a witch, right? They're trying to show that she must be a witch, right? But it turns out that, you know, they really gave her the nose, right, and the hat. The guy who she turned into a newt, is fine, so. Then, the, Sir Bedevere, the knight tries to lead them on a different logical chain to help prove that she's a witch. So what are the, when the knight starts talking to them

»<1416135> MS: If witches burn then they are made of wood.

»<1417897> Matt: Okay. If witches burn, then they are made of wood.

»<1428185> Moe: [inaudible] witches

»<1432749> Matt: So, if she floats, then she is made of wood, okay? (pause) okay.

»<1463516> MS: If she weighs as much as a duck, then she is made of wood.
Matt: Okay. So if she (pause - writing). Anything else?

Moe: If she weighs, if she's wood, then she's a witch.

Matt: So, we have all these different statements floating around here, right? If she's made of wood (pause) So let's see if we can kind of like put these in an order and see how their logical chain kind of works out. So if witches burn, then they are made of wood. Okay? And why is being made out of wood important?

MS: Cause if she's made of wood...

Matt: So how did they test if she's made of wood?

Students: [inaudible]

Matt: Well first they want to throw her in the pond, right? And then the knight says no, no, no. If she well, if she's made of wood then she's supposed to

Students: Float

Matt: Float, right? So they actually use the converse of this statement, right? If she's made of wood, then she should float. And so how do they test if she floats?

Students: overlapping talk [inaudible]

Matt: So if she floats, then she should weigh the same as a duck. Right? And then how do they test out?

Students: overlapping talk [inaudible]

Matt: They weigh her right? And what do they determine?

Students: overlapping talk [inaudible]

Matt: She weighs the same as a duck, right? (pause) So they figure out that she weighs the same as a duck, and so then what do they think that means?

MS: That she floats.

Matt: That she floats...and if she floats, she has to be made of wood

MS: made of wood

made of wood and if she's made of wood,

MS: she burns

Matt: She has to be, she must burn, right? We got another one in here, right? If, between the, or if she's made of wood, then she must burn, and witches burn obviously so, therefore, really she's a witch. So
there's two different things going on here. First of all, when they first reason this, right? They always, they start from the fact that she's a witch. They say if she's a witch then she's made of wood. If she's made of wood, then she must float. If she floats, then she weighs the same as a duck. Right? But then when they actually conclude that she's a witch, right? To actually conclude that she's a witch, which - Which witch, which way, right? They figure out that she weighs the same as a duck? Does that prove that she has to be a witch?

Students: No

Matt: Why not?

Students: [inaudible]

MS: Cause witches don't exist.

Matt: Alright, hold on. So that she weighs the same as a duck. If she floats, she weighs the same as a duck. And then they figure out that she weighs the same as a duck. So therefore they conclude

Students: She floats.

Matt: She floats. So what have they done to this statement?

MS: [inaudible]

Matt: They made the converse, right? Without thinking about it, they flipped all these if-then statements around and found the converse. If she floats then she weighs the same as a duck, but then they use, if she weighs the same as duck then she floats. And we already talked about the fact that just because an original statement is true, doesn't necessarily mean that it's converse is true, right? We looked at some different statements. That was the whole point of #39 on the first assignment is to find a statement that had a true converse, right? Not everything immediately has a true converse, okay? The other, well okay so that's why, I mean that's why it's supposed to be funny, right? Cause they flip all these statements around a large (logical policy?), right? But even if, even if all these statements are true, which is debatable, right? If she floats, then she weighs the same as a duck, is that a true statement? Right? I could come up with a lot of things that weigh more than a duck, that still float, right? Or, yeah, floating objects which weigh more than a duck. There's lots of them. I could think of a lot of things. Okay? But there's two, there's two main logical moves that they're trying to make here. Right? There's two main logical moves that they try to make in this sketch. If p then q, right? We figure out that p has to be true, therefore q has to be true. Okay? If she weighs the same as a duck, then she has to float. She weighs the same as a duck, therefore we think she floats, okay? Ah, if Maynard lives in Ravenna, then his favorite hobby is playing guitar. On its own that statement was not very helpful to us, but as soon as we knew that Maynard did in fact live in Ravenna, we could conclude that his favorite hobby was playing guitar. So we have if p then q, p, therefore q has to be true. The book calls this the law of detachment. (pause) The other thing that they try to use in
This sketch is to chain these logical things together. If p then q, if q then r, what should be true?

Students: If p then r.

Matt: If p then r, right? If she's a witch, then she's made of wood. If she's made of wood, then she floats. If she floats, then she weighs the same as a duck. And then they try to chain all of those things together and say if she's a witch, then she weighs the same as a duck, right? By running this logic chain together. The book calls this the law of syllogism. So the law of syllogism and the law of detachment. Those are the two main logical moves that we can make from a logic perspective, okay. If p then q, we know p is true, therefore q.

Or we can chain these things together, right? If p then q, if q then r, so if p then r. They try to do both of those in this sketch, right? Now the reason that this is funny is because they use this backwards. Okay? Okay? This is not, this is not a valid argument. This is a logical mistake that gets made a lot, right? And therefore, we want to talk about this specifically, right? If p then q, q, therefore p. Okay? You can't conclude this. The other sketch that I always think about showing for this day is uh, The Simpsons Bear Patrol episode. People watch the Simpsons, yes? Bear Patrol Episode? Anybody with me on that one? No Bear Patrol Episode, so at the beginning of Bear Patrol Episode a bear wanders into Springfield and, uh, like I think it likes dents up Homer's car or something. So the citizens decide that they want to create a bear patrol to handle the bear problem. Right? And they pay taxes, and then people come around and tranquilize the bears and take them all out of Springfield, right? So this happens for a while, and then Homer comes out of his house and says, There aren't any bears around. The bear patrol must be doing its job. And Lisa says, that's factitious reasoning Dad. By that logic, I can conclude that this rock keeps tigers away. And Homer says, what do you mean? How does it work? It doesn't work it's just a stupid rock. Well how does it work? Well I don't see any tigers around, do you? Homer thinks about this for a minute. Lisa I'd like to buy your rock. No, no, no. Okay. She gives up, and she sells him the rock for like 20 bucks, right?

Sts: laugh.

Matt: Why is that funny, right? If this rock keeps tigers away, then there will be no tigers here. Is that a true statement?

Sts: No

Matt: If this rock keeps tigers away

Sts: Yeah.

Matt: That's true, right? If the rock keeps tigers away, there would be no tigers around. That's a true statement, right? But just because there aren't any tigers around doesn't mean that the rock actually does anything, right? What, what might be another reason

MS: Springfield doesn't have any [inaudible]
Matt: Right? No tigers live in Springfield, right? Right? We're not in the middle of the Sahara, we're in Springfield, right?

Moe: [inaudible]

If the bear patrol

Moe: But no tigers live in Africa

Matt: If the bear patrol is doing it's job, then there will be no bears around. Is that a true statement?

Moe: Mr. [Teacher Name], tigers live in Asia, not Africa.

Matt: Okay. If, if the bear patrol is doing its job, then there will be no bears around. Is that true? If the bear patrol is doing its job, then there will be no bears around.

MS: yeah

Matt: That's a true statement. But just because there's no bears around doesn't mean that the bear patrol is doing it's job, right? The bears might be hibernating or they might have, you know, who knows what's happened to the bears. Just because, the converse, just because the condition of your statement is true doesn't mean that the first part must be (working?) Okay. There was something else I was going to say.

(Long pause while students are talking amongst themselves.)

Matt: Any questions about this?

MS: No

Matt: Does this kind of make sense? (pause) Um, we talked about uh, the other symbol that the book uses, we talked about finding the inverse and finding the contrapositive, having to negate statements. The book uses the tilda to denote negation. Okay? So, like when you go to a website, like tilda. So this is not p. Okay. This would be not q. Okay. So when you're trying to write the inverse, right, when we like, when we negate things, we're writing that, the book wants, talks about writing them in p,q format. Then the tilda means not. Then they use the arrow. So this would be if p, then q, not p, therefore, not q. [inaudible] So they use the arrow to denote the if then. Okay? That's all I want to talk about. So there's an assignment from page 91. Okay? Doing a little logical writing, doing a little if-then logical chains. Okay?

MS: How much longer are we doing if-then statements?

Matt: How much longer are we doing if-then statements. Specifically if-then statements or using if-then statements at all? Cause those are two different questions.

MS: [inaudible]

Matt: Two days.
MS: Two days? Alright.
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