A new thermo-time domain reflectometry approach to quantify soil ice content at temperatures near the freezing point

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A new thermo-time domain reflectometry approach to quantify soil ice content at temperatures near the freezing point

Abstract
Soil ice content ($\theta_i$) is an important property for many studies associated with cold regions. In situ quantification of $\theta_i$ with thermo-time domain reflectometry (TDR) at temperatures near the freezing point has been difficult. The objective of this study is to propose and test a new thermo-TDR approach to determine $\theta_i$. First, the liquid water content ($\theta_l$) of a partially frozen soil is determined from a TDR waveform. Next, a pulse of heat is applied through the thermo-TDR sensor to melt the ice in the partially frozen soil. Then, a second TDR waveform is obtained after melting to determine the $\theta_l$, which is equivalent to the total water content ($\theta_t$) of the partially frozen soil. Finally, $\theta_i$ is calculated as the difference between $\theta_t$ and $\theta_l$. The performance of the new approach was evaluated in sand and loam soils at a variety of $\theta_t$ values. The new approach estimated $\theta_t$, $\theta_l$, and $\theta_i$ accurately. The root mean square errors (RMSE) of estimation were 0.013, 0.020, and 0.023 m$^3$m$^{-3}$ for sand, and 0.041, 0.026, and 0.031 m$^3$m$^{-3}$ for loam. These RMSE values are smaller than those reported in earlier thermo-TDR studies. Repeating the thermo-TDR measurements at the same location on the same soil sample caused decreased accuracy of estimated values, because of radial water transfer away from the heater tube of the thermo-TDR sensor. Further research is needed to determine if it is possible to obtain accurate repeated measurements. The use of a dielectric mixing model to convert the soil apparent dielectric constant to $\theta_l$ improved the accuracy of this approach. In our investigation, application of a small heat intensity until the partially frozen soil temperature became larger than about 1°C was favorable. The new method was shown to be suitable for estimating ice contents in soil at temperatures between 0°C and -2°C, and it could be combined with the volumetric heat capacity or thermal conductivity thermo-TDR based methods, which measured ice content at cooler temperatures. Thus, the thermo-TDR technique could measure $\theta_i$ at all temperatures.

Keywords
soil freezing and thawing, soil ice content, thermo-time domain reflectometry

Disciplines
Climate | Hydrology | Soil Science

Comments

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Title

A new thermo-time domain reflectometry approach to quantify soil ice content at temperatures near the freezing point

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**Keywords**

soil freezing and thawing, soil ice content, thermo-time domain reflectometry

**Abbreviations**

HPP (heat pulse probe); TDR (time domain reflectometry); RMSE (root mean square error)

**Highlights**

- A new thermo-TDR approach is described to quantify soil ice content at temperatures near the freezing point.
- The new approach determines soil liquid water and total water contents from TDR measurements made before and after a forced melting of a partially frozen soil.
- Ice content is determined from the difference between total water content and liquid water content.
- The new thermo-TDR approach provides improved estimations of ice content at temperatures near the freezing point compared with volumetric heat capacity based and thermal conductivity based thermo-TDR methods.
Abstract

Soil ice content ($\theta_i$) is an important property for many studies associated with cold regions. In situ quantification of $\theta_i$ with thermo-time domain reflectometry (TDR) at temperatures near the freezing point has been difficult. The objective of this study is to propose and test a new thermo-TDR approach to determine $\theta_i$. First, the liquid water content ($\theta_l$) of a partially frozen soil is determined from a TDR waveform. Next, a pulse of heat is applied through the thermo-TDR sensor to melt the ice in the partially frozen soil. Then, a second TDR waveform is obtained after melting to determine the $\theta_l$, which is equivalent to the total water content ($\theta_t$) of the partially frozen soil. Finally, $\theta_i$ is calculated as the difference between $\theta_t$ and $\theta_l$.

The performance of the new approach was evaluated in sand and loam soils at a variety of $\theta_t$ values. The new approach estimated $\theta_t$, $\theta_l$, and $\theta_i$ accurately. The root mean square errors (RMSE) of estimation were 0.013, 0.020, and 0.023 m$^3$ m$^{-3}$ for sand, and 0.041, 0.026, and 0.031 m$^3$ m$^{-3}$ for loam. These RMSE values are smaller than those reported in earlier thermo-TDR studies. Repeating the thermo-TDR measurements at the same location on the same soil sample caused decreased accuracy of estimated values, because of radial water transfer away from the heater tube of the thermo-TDR sensor. Further research is needed to determine if it is possible to obtain accurate repeated measurements. The use of a dielectric mixing model to convert the soil apparent dielectric constant to $\theta_l$ improved the accuracy of this approach. In our investigation, application of a small heat intensity until the partially frozen soil temperature...
became larger than about 1°C was favorable. The new method was shown to be suitable for estimating ice contents in soil at temperatures between 0°C and -2°C, and it could be combined with the volumetric heat capacity or thermal conductivity thermo-TDR based methods, which measured ice content at colder temperatures. Thus, the thermo-TDR technique could measure θ_i at all temperatures.

Introduction

Soil ice content, θ_i (m^3 m^{-3}), is an important property of partially frozen soils in many research fields associated with cold regions. It affects winter soil hydrology, because water permeability of partially frozen soil is in part controlled by θ_i (Kane and Stein, 1983; Andersland et al., 1996). Reduced water permeability due to large θ_i leads to rainfall-driven soil erosion in spring conditions (Cruse et al., 2001; Nishimura et al., 2011), impacting soil conservation. Frost heaving due to the local accumulation of ice in soil can damage surface construction (e.g., Fukuda et al., 1980; Peppin and Style, 2013). Therefore, quantifying in-situ θ_i is beneficial for many associated studies.

However, in situ measurement of θ_i has been difficult. Time domain reflectometry (TDR), which measures the soil apparent dielectric constant, K_a, is often used to quantify liquid water content, θ_l (m^3 m^{-3}) in partially frozen soil (e.g., Smith and Patterson, 1984; Spaans and Baker, 1995; Watanabe and Wake, 2009). Because the dielectric constant of water is much larger than those of other soil constituents, θ_l can be determined from K_a. Some researchers quantify θ_i
with a combination of TDR and neutron moderation, e.g., Hayhoe and Bailey (1985), Stähli et al. (1999) and Yi et al. (2014). TDR determines $\theta_l$ and neutron moderation determines total water content, $\theta_t$, which is a sum of $\theta_l$ and $\theta_i$, thus, $\theta_i$ is determined by subtracting $\theta_l$ from $\theta_t$. Although the combination of TDR and neutron moderation enables $\theta_i$ to be determined over a wide range of soil temperature, there are several weaknesses with this approach. For example, TDR and neutron moderation sensors must be separated in the soil to avoid interactions. Thus, the two methods do not sample the same soil volume. Although Cheng et al. (2013) developed a combination of dielectric constant measurement and neutron moderation with a common access tube inserted into ground to avoid different sampling locations, the method sacrifices automatic measurements. In recent years, researchers have tried to quantify $\theta_i$ from thermal property measurements in partially frozen soils. Thermal properties can be measured with a heat pulse probe (HPP) or thermo-TDR sensor, which is a sensor that combines HPP and TDR (Noborio et al., 1996; Ren et al., 1999). Liu and Si (2011), Zhang et al. (2011), He et al. (2015), and Tian et al. (2015) determined $\theta_i$ based on volumetric heat capacity of partially frozen soil. Volumetric heat capacity of the soil is expressed as the sum of the volume fraction multiplied by the volumetric heat capacity of each soil constituent, i.e., soil solids, liquid water, ice, and air. Therefore, $\theta_i$ could be calculated if other unknown parameters, i.e., liquid water content, were determined with TDR probes or models. The volumetric heat capacity-based approach successfully quantified $\theta_i$ at temperatures smaller than $-5^\circ$C. At temperatures larger than $-5^\circ$C, ice melting due to the heat applied by HPP or thermo-TDR into partially frozen soil caused large
errors. Tian et al. (2017) quantified $\theta_i$ from thermal conductivity instead of volumetric heat capacity. The thermal conductivity-based approach provided better estimates of $\theta_i$ than the volumetric heat capacity approach at temperatures near the freezing point, i.e., $-2^\circ\text{C}$ and $-1^\circ\text{C}$. However, the method still encountered errors, sometimes larger than 0.1 m$^3$ m$^{-3}$ (Tian et al., 2017). Moreover, the feasibility of using the thermal conductivity approach at temperatures between $-1^\circ\text{C}$ and $0^\circ\text{C}$ is unknown. Kojima et al. (2016) quantified dynamic changes in $\theta_i$ from sensible heat balance calculations based on soil temperature and thermal conductivity distributions measured with a HPP. Their method successfully quantified changes in $\theta_i$ at temperatures between $-5^\circ\text{C}$ and $0^\circ\text{C}$, but their experiments were performed only with simple freezing and thawing events, and the feasibility of the method for field use has yet to be elucidated (Kojima et al., 2014). Therefore, it is still challenging to measure $\theta_i$ at temperatures near the freezing point where $\theta_i$ fluctuates dynamically.

Therefore, the objectives of this study are to present and evaluate the performance of a new thermo-TDR approach to quantify $\theta_i$ near the freezing point. The new approach uses the heater in the thermo-TDR to completely melt soil ice around the sensor, and uses TDR waveforms to measure $\theta_i$ and $\theta_l$ before and after melting. The difference between $\theta_i$ and $\theta_l$ is equivalent to $\theta_i$, which is a concept similar to that used by the combination of TDR and neutron moderation, but in this new approach $\theta_i$ and $\theta_l$ are both measured by a single TDR sensor.

2. Materials and Methods
2.1 Principles of the new thermo-TDR approach

A schematic of the thermo-TDR probe used in this study is shown in Fig. 1. The thermo-TDR probe consists of three 0.077 m stainless steel tubes which have 0.7 mm and 1.6 mm inner and outer diameter. The center tube embeds a resistance heater wire and a type T thermocouple, and the side tubes embed a type T thermocouple only. The heater wire and thermocouples are normally used to measure soil temperature and soil thermal properties, however, in this study they were used only to heat the partially frozen soil and to monitor the temperature change during heating. The heater is a 75μm diameter Evanohm wire which is doubled over twice to produce a heater resistance of 1120 Ω m⁻¹. The three tubes are connected to a 75Ω coaxial cable, so that electromagnetic waves can propagate through the cable and tubes for TDR function. The thermo-TDR sensor measures soil $K_a$ based on the time that it takes for an electromagnetic wave to propagate down and back along the tubes, and $K_a$ is used to estimate liquid water content. Details of TDR measurements are described in Noborio (2001).

Thermo-TDR probes tend to be short, e.g., Ren et al. (1999) used 0.04 m long probes, and Tian et al. (2015) used 0.045 m probe length. Short probes are used to minimize the likelihood of probe deflection, because thermal property values are sensitive to the probe spacing (Kluitenberg et al., 1993, 1995). Longer probe lengths are preferred for the TDR measurements (Dalton and van Genuchten, 1986; Heimovaara, 1993; Kelly et al., 1995; Wen et al., 2018). Because accuracy of the TDR measurements is important in this study, we used longer probes, 0.077 m, than those used in other thermo-TDR studies.
The procedure of the new approach has four steps. First, the thermo-TDR measures $K_a$ of the partially frozen soil, which is used to determine $\theta_l$. Second, the sensor heater is used to apply a constant heat flux, $q$ (W m$^{-1}$), into the partially frozen soil. Temperatures at the two side tubes are monitored during the heating. When both of the side tubes reach a specified temperature (e.g., 1°C) larger than the freezing point of water (i.e., 0°C), the heating is stopped. Third, after heating ceases, a second $K_a$ measurement is performed in order to determine $\theta_t$ of the thawed soil. Finally $\theta_i$ is determined from the difference between $\theta_l$ and $\theta_t$ as

$$\theta_i = \frac{\rho_l}{\rho_i} (\theta_t - \theta_l)$$

where $\rho_l$ and $\rho_i$ are density of liquid water (1,000 kg m$^{-3}$) and ice (917 kg m$^{-3}$). Equation (1) accounts for volume change associated with phase change of water. The target final temperature of the side tubes during heating, $T_i$ (°C), must be larger than 0°C, because there is uncertainty in the temperature measurement and the TDR sampling volume is larger than the tube separation (e.g., Robinson et al., 2003). The TDR sampling volume is difficult to know exactly, because it depends on probe design and the $K_a$ of the surrounding soil (e.g., Ferré et al., 1998; Nissen et al., 2003). Therefore, it is important to choose $T_i$ large enough to ensure that ice in the sampling volume is completely melted.

### 2.2 Soil samples and their properties

The performance of the new method was evaluated on two soils, Toyoura sand and volcanic ash soil (Andisol), referred to as sand and loam. The volcanic ash soil samples are from
Hirosaki, Aomori prefecture, Japan which is a cold region and experiences annual soil freezing and thawing. Soil texture, organic matter content, and bulk density values of the soils are presented in Table 1, and soil water retention curves are shown in Fig. 2. The sand water retention curve was reproduced from Muto et al. (2015), and the loam water retention curve was measured with a hanging water column set-up (e.g., Bittelli, 2010) and a WP4 device (Meter Group, Inc. USA, Pullman, WA). The van Genuchten (1980) model was fitted to the water retention curve data;

\[ \theta_l = \theta_r + \left( \theta_s - \theta_r \right) \left[ \frac{1}{1 + |\alpha\psi|^n} \right]^{1-1/n} \]  \hspace{1cm} (2)

where \( \theta_r \) and \( \theta_s \) are residual and saturated water content (m\(^3\) m\(^{-3}\)), \( \psi \) is matric potential (m of water), and \( \alpha \) (m\(^{-1}\)) and \( n \) are fitting parameters. Equation (2) parameter values are presented in Table 1.

2.3 Relationships between apparent dielectric constant and liquid water content

\( K_a \) values measured with TDR are used to estimate \( \theta_l \) from empirical relationships, e.g., an equation proposed by Topp et al. (1980) has been widely used for a variety of soils. However, the Topp et al. (1980) equation is reported to not perform well for relatively short TDR probes, in particular for small \( K_a \) values associated with dry soil or partially frozen soils (Wang et al., 2014; He et al., 2015). In addition, it is known that volcanic ash soils have unique \( \theta_l-K_a \) relationships that differ from Topp et al. (1980) equation (Miyamoto et al., 2001). Therefore, prior to performing frozen soil measurements, we established the \( \theta_l - K_a \) relationships for the two
soils with the thermo-TDR sensor using unfrozen soil samples.

The thermo-TDR sensor was positioned horizontally inside a 0.077 m inner diameter and 0.025 m long acrylic column from the side wall of the column, i.e., the probes were located at 0.0125 m depth from top and bottom of the column. Individual soil samples at selected water contents (from 0.00 to 0.34 m$^3$ m$^{-3}$ for sand, and from 0.11 to 0.56 m$^3$ m$^{-3}$ for loam) were packed into the column. The $K_a$ of each soil sample was measured with a TDR100 (Campbell Scientific, Inc., Logan, UT), and TDR waveform information was collected by a CR1000 datalogger (Campbell Scientific, Inc.). Measurements were performed in a constant temperature chamber at a temperature of 5°C. The $\theta_t$-$K_a$ relationships were described by fitted cubic polynomial functions similar in form to the Topp et al. (1980) equation. Reported $\theta_t$-$K_a$ relationships have been expressed with dielectric mixing models. Although there have been various kinds of mixing model proposed (e.g., De Loor, 1964; Polder and van Santeen, 1946), we used the dielectric mixing model initially proposed by Birchak et al. (1974) and later modified for partially frozen soils (Seyfried and Murdock, 1996; Watanabe and Wake, 2009; Tian et al., 2019). The mixing model expressed the relationships as

$$\theta_t = \frac{(K_a - \varepsilon_a \beta - \varepsilon_s \beta - \varepsilon_l \beta)}{\varepsilon_l \alpha - \varepsilon_a \alpha}$$ \hspace{1cm} (T ≥ 0°C) \hspace{1cm} (3a)

$$\theta_t = \frac{(K_a - \varepsilon_a \beta - X_s (\varepsilon_s \beta - \varepsilon_a \beta)) - \frac{\rho_l}{\rho_i} \theta_t (\varepsilon_l \beta - \varepsilon_a \beta)}{\varepsilon_l \beta - \frac{\rho_l}{\rho_i} \varepsilon_l \beta + \left(\frac{\rho_l}{\rho_i}\right)^{-1} - 1\varepsilon_a \beta}$$ \hspace{1cm} (T < 0°C) \hspace{1cm} (3b)

where $\varepsilon_a$, $\varepsilon_s$, $\varepsilon_l$, and $\varepsilon_i$ were dielectric constants of air, soil solids, liquid water, and ice, $X_s$ was the volume fraction of soil solids (m$^3$ m$^{-3}$), and $\beta$ was a parameter for geometric arrangement of soil...
particles. The mixing model is separated into two forms, i.e., Eq. (3a) was for unfrozen soil, and Eq. (3b) was for partially frozen soil, i.e., Eq. (3b) accounted for the presence of ice. In unfrozen soil, $\theta_l$ was equal to $\theta_t$. The values of $\varepsilon_a$, and $\varepsilon_i$ were 0.99 and 3.27, and $\varepsilon_l$, a function of water temperature, was 85.9 at 5°C (Watanabe and Wake, 2009). The $\varepsilon_a$ depended on soil type. For example, Watanabe and Wake (2009) estimated the $\varepsilon_a$ values for Toyoura sand and Andisol 3.3 and 3.27 while Kameyama and Miyamoto (2008) reported that $\varepsilon_a$ of Andisol measured with the immersion method ranged from 5.6 to 6.1. The reason we chose this mixing model was its simple structure and geometrical fitting parameter $\beta$. The $\beta$ acted as a fitting parameter, thus, it was determined by fitting Eq. (3a) to the measured unfrozen soil values. This fitting geometrical parameter enabled the model to express the relationship accurately. The same parameter values were also used in Eq. (3b) when the soil was frozen. In order to use Eq. (3b), $\theta_t$ had to be known and, thus, it limited the applicability of the mixing model for partially frozen soils (Watanabe and Wake, 2009; Tian et al., 2019). However, that limitation was not a problem for our new approach, because it quantified $\theta_t$ and $\theta_l$ simultaneously.

2.4 Evaluating impacts of heating intensity and target heating temperature

For the new thermo-TDR approach, $q$ and $T_t$ were important parameters that could affect the accuracy of $\theta_t$ and $\theta_i$ estimates. Therefore, the impacts of different $q$ and $T_t$ were investigated. The experiments were performed with the same instruments described in section 2.3, i.e., TDR100 for $K_a$ measurements and CR1000 for controlling heater and monitoring.
temperature. Three different \( q \) values, 15, 30, and 80 W m\(^{-1}\) were examined. The 15, 30, and 80 W m\(^{-1}\) were heat intensities produced by 12V, 19V, and 30V DC power supplies. The accuracy of temperature measurement of our system relied on the built-in reference junction thermistor of datalogger CR1000, and it was ±0.3°C (Campbell Scientific, Inc., 2018). Since the \( T_i \) had to be larger than the accuracy, 0.3°C, four different \( T_i \) values, 0.5°C, 1.0°C, 2.0°C, and 4.0°C were examined. The measurements were performed on loam with \( \theta_t = 0.32 \) m\(^3\) m\(^{-3}\). This particular water content was used because it was approximately half of the saturated water content value. The loam was packed into the acrylic column holding the thermo-TDR sensor. The top and bottom of the column were sealed with acrylic plates and silicon adhesive to prevent water vapor loss during measurements. The column size was relatively small and may have had a minor influence on heat transfer in the soil, but we assumed that the effect was negligible in this study. The experiments were performed in a constant temperature chamber. Temperature of the chamber was initially set at −10°C to freeze the soil samples over a six-hour period, and then the temperature was changed to −2°C. The initial low temperature was used in order to induce freezing without supercooling of soil moisture. Once a soil sample equilibrates at the chamber temperature of −2°C, a TDR waveform was collected before starting soil sample heating. Heating and second TDR waveform collection followed. The measurements for each sample were repeated three times with a six-hour time interval. Following the three measurements, the \( \theta_t \) value of the soil sample was measured by oven-drying. Measured \( K_a \) values before and after heating were used to estimate \( \theta_l \) and \( \theta_t \) values with the cubic polynomial equations (Eq. (7))
described in 3.1). Although both $\theta_l$ and $\theta_t$ were determined, the impacts of $q$ and $T_t$ were evaluated by comparing thermo-TDR $\theta_t$ values with the reference $\theta_t$ value determined from mass measurements, because $\theta_l$ was not affected by the variation of $q$ and $T_a$.

2.5 Evaluation of the new thermo-TDR approach

The accuracies of the $\theta_l$, $\theta_t$, and $\theta_i$ estimates by the new thermo-TDR approach were evaluated in laboratory experiments. Sand and loam samples at fixed water contents (from 0.09 to 0.31 m$^3$ m$^{-3}$ for sand, and from 0.13 to 0.52 m$^3$ m$^{-3}$ for loam) were packed into the acrylic column holding the thermo-TDR sensor. The experimental procedures and equipment were the same as those described in 2.4. A $q$ value of 15 W m$^{-2}$ was used to heat the soil sample to minimize soil water migration (will be discussed in 3.3). Different values of $T_t$ were used for sand and loam samples, 1°C and 2°C (will be discussed in 3.3). Loam samples have larger $T_t$ values than sand, because loam tended to hold more water than sand (Fig. 2) and had smaller thermal conductivity than sand. Measured $K_a$ values before and after heating were used to estimate $\theta_l$ and $\theta_t$ values with either the cubic polynomial equations (Eqs. (6 and 7)) or the dielectric mixing model (Eq. (3)), and $\theta_i$ was determined from the difference between the $\theta_t$ and $\theta_l$ values with Eq. (1).

Estimated $\theta_l$, $\theta_t$, and $\theta_i$ values with the new thermo-TDR approach were compared to reference values, which were determined by change of mass measurements or by models. Reference values of $\theta_i$ were determined by mass changes due to oven-drying. Reference values
of \( \theta_i \) were estimated from the soil water retention curves (Fig. 2) using \( \psi \) value estimates by soil temperature. There existed a similarity between a soil water retention curve and a soil freezing characteristics curve, which was a relationship between \( \theta_i \) and temperature of partially frozen soil (e.g., Spaans and Baker, 1996). The \( \psi \) value of partially frozen soil was approximated from soil temperature \( T \) (°C) by the Clausius-Clapeyron equation (Kurylyk and Watanabe, 2013; Kojima et al., 2018):

\[
\psi = \frac{L_f}{g} \left( \frac{T}{273.15° C} \right)
\]

(4)

where \( L_f \) was the latent heat of fusion \((3.34 \times 10^5 \text{ J kg}^{-1} \text{ or equivalently m}^2 \text{ s}^{-2} \text{ at } 0°C)\), and \( g \) was the gravitational acceleration \((9.8 \text{ m s}^{-2})\). Equation (4) assumed that ice pressure and osmotic pressure were zero. This assumption was generally accepted when soils were unsaturated and solute-free (e.g., Fuchs et al., 1978). The \( \psi \) value of partially frozen soil at the initial soil sample temperature of −2°C was −250 m of water from Eq. (4), and the calculated Eq. (2) reference values of \( \theta_l \) were 0.03 and 0.18 m\(^3\) m\(^{-3}\) for sand and loam. The reference values of \( \theta_l \) were determined from the difference between reference values of \( \theta_l \) and \( \theta_i \) with Eq. (1). The estimation accuracy of the thermo-TDR approach in each soil was characterized as root mean square error (RMSE) values (Tian et al., 2017):

\[
\text{RMSE} = \sqrt{\frac{\sum [\theta_{\text{thermo-TDR}} - \theta_{\text{reference}}]^2}{m}}
\]

(5)

where \( m \) was number of data points, \( \theta \) was volume fraction of either liquid water, total water, or ice, i.e., \( \theta_l \), \( \theta_t \), or \( \theta_i \). The subscripts \text{thermo-TDR} and \text{reference} represented data obtained from thermo-TDR measurements and reference values.
3. Results and Discussion

3.1 Relationships between apparent dielectric constant and liquid water content

The $\theta_l$-$K_a$ relationships are shown in Fig. 3. The Topp et al. (1980) equation, the fitted cubic polynomial equation, and the fitted mixing model (Eq. (3a)) are presented. The determined cubic polynomial regression for sand and loam were

$$\theta_l = -2.63 \times 10^{-1} + 8.67 \times 10^{-2} K_a - 5.31 \times 10^{-5} K_a^2 + 1.26 \times 10^{-4} K_a^3$$  \hspace{1cm} (6)  

$$\theta_l = -1.03 \times 10^{-1} + 6.74 \times 10^{-2} K_a - 2.75 \times 10^{-3} K_a^2 + 3.97 \times 10^{-5} K_a^3$$  \hspace{1cm} (7)

The coefficients of determination, $R^2$, of sand for the Topp et al. (1980) equation and polynomial regression were 0.963 and 0.987. The fitted cubic polynomial equation had larger $R^2$ values than the Topp et al. (1980) equation. The Topp et al. (1980) equation only described the relationship well at $\theta_l$ values larger than 0.1 m$^3$m$^{-3}$. This was a result of the short TDR probe problem stated by Wang et al. (2014), where electromagnetic wave reflections at the beginning and the end of a probe interacted with each other when $K_a$ was small. This implied that the 0.077 m TDR probe might not have been long enough to avoid problems, and a calibration curve was necessary. The $\theta_l$-$K_a$ relationship of loam did not agree well with the Topp et al. (1980) equation, similar to earlier reports (e.g., Miyamoto et al., 2001). The relationship could be affected by both short probe length and unique properties of Andisols, such as low bulk density, large porosity and specific surface area, and aggregate structure. Thus, a calibration curve was required as well for the loam soil. The $R^2$ of loam for the polynomial regression was 0.978.
Fitting of the mixing model was first performed with $\varepsilon_s$ values of 3.3 and 5.8 for sand and loam. The value for sand was taken from Watanabe and Wake (2009), and the value for loam was an averaged value of measured $\varepsilon_s$ of Japanese Andisols reported by Kameyama and Miyamoto (2008). However, our fits resulted in poor descriptions of the relationships, and $R^2$ values were 0.956 for sand and 0.916 for loam (data not shown). For the sand relationship, the fitted mixing model overestimated $\theta_t$ at small $K_a$ soils, similar to the Topp et al. (1980) equation. This implied that the $K_a$ measurement errors associated with the short TDR probe resulted in $\theta_t$ estimation errors with the mixing model as well as those with the Topp et al. (1980) equation.

For the loam, the mixing model overestimated $\theta_t$ when $\theta_t$ was larger than 0.45 m$^3$ m$^{-3}$. Various studies stated that the use of a mixing model for Andisols was challenging because its unique properties affected the $\varepsilon_l$ value (Regaldo et al., 2003; Miyamoto et al., 2005). Bound water, which was water adsorbed on soil particle surface, usually showed smaller $\varepsilon_l$ than free water, and the amount and distribution of bound water significantly affected the $\theta_l$-$K_a$ relationships of Andisol. Miyamoto et al. (2003) stated that the $\theta_l$-$K_a$ relationships were partitioned into two different trends for a particular water content value due to aggregate structure, which resulted in a bimodal distribution of pore sizes. Our results showed that Eq. (3) overestimated $\theta_t$ at large water contents, possibly due to a bimodal pore size effect. Although several modified Andisol $\theta_l$-$K_a$ mixing models were proposed to account for bimodal pore-size distribution and bound water amount (Miyamoto et al., 2005; Dyck et al., 2019), the models were fairly complicated, and their application to partially frozen soils was not elucidated. Watanabe and Wake (2009)
proposed a mixing model to account for the change in $\varepsilon_1$ associated with water adsorption to the soil particle surface in partially frozen soils, but their model required additional information, such as specific surface and fitting parameters for the $\theta_l$-$K_a$ relationship in frozen conditions. He et al. (2014) also presented mixing models for $K_a$ of partially frozen soils, however, optimization of the model parameters required a complete dataset of $K_a$, $\theta_l$, and $\theta_i$. Therefore, Eq. (3) was still the most practical for determining $\theta_l$ in partially frozen soils. In order to obtain better fitting with Eq. (3), we used $\varepsilon_s$, in addition to $\beta$, as a fitting parameter for sand. For the loam, Kameyama and Miyamoto reported that $\varepsilon_s$ for Eq. 3 differed from its real value, and on average 9.35 was suitable for Andisol. Similar results could be found in Regalado et al. (2004). Therefore, we used 9.35 as $\varepsilon_s$ value for Eq. (3), and $\beta$ for loam was determined by fitting. Parameter values for Eq. (3) are presented in Table 2.

The $R^2$ values of the mixing model were 0.978 and 0.956 for sand and loam. The cubic polynomial fitted the measured data better than did the dielectric mixing model. In addition, the mixing model for loam slightly underestimated $\theta_1$ in the water content range between 0.2 and 0.4 $m^3 \, m^{-3}$. It should be noted that our mixing model parameters were specific to the sensor and soils used in this study, i.e., effects of sensor configuration and $\varepsilon_1$ variations were accommodated in the fitting parameter $\varepsilon_s$ and $\beta$. Even with this limitation, the dielectric mixing model was still beneficial in this study, because of its ability to account for ice in partially frozen soil. In addition, the practical use of the mixing model could be demonstrated with the advantage of our approach that both $\theta_1$ and $\theta_l$ were determined simultaneously. Therefore, both the mixing model


and the cubic polynomial were evaluated further.

3.2 Example of ice content determination with the new thermo-TDR approach

Figure 4 shows side tube temperature with time values during heating, and TDR waveforms before freezing (at 1°C), in a frozen state (−2°C), and after heating ($T \geq T_t$) for loam with $\theta_t$ of 0.32 m$^3$ m$^{-3}$. The values of $q$ and $T_t$ were 15 W m$^{-1}$ and 2°C, and the TDR waveform before freezing was collected only in this example. The temperature of side tube 1 increased faster than that of side tube 2 due in part to a slight difference in probe spacing. Because the temperatures of both side tubes were required to reach the $T_t$ value before heating was stopped, the temperature of side tube 1 reached 2.5°C. It took 37 minutes of heating for both side tubes to reach $T_t$. During the heating period, the center tube temperature reached 27.4°C (data not shown). The first and second reflection points were labelled in the example waveforms. The distance between the first and second reflection points was associated with the $K_a$ value, i.e., a long distance indicated a large $K_a$ value. The frozen state waveform had a short distance between the first and second reflection points compared to the before freezing waveform, which indicated that freezing decreased the $K_a$ value due to the phase change of water. The post-heating waveform was somewhat similar to the before freezing waveform, although it had slightly higher reflection coefficients than the before freezing waveform. The higher reflection coefficient implied a decreased soil bulk electrical conductivity (e.g., Noborio, 2001). The $K_a$ values derived from the waveforms were 9.7, 5.2, and 9.0, and $\theta_t$ values (or $\theta_l$ for the soil in frozen state)
calculated with Eq. (7) were 0.33, 0.18, and 0.31 m$^3$ m$^{-3}$ for the soils before freezing, in a frozen state, and after heating (thawing). The increase in reflection coefficient and decrease in $K_a$ could be associated with either (or both of) decreased $\theta_t$ or changed temperature in the TDR sampling volume. Because the heating period was 37 minutes long, which was much longer than the heating duration for general heat pulse probes, e.g., 8~60 seconds, if the applied heat pulse induced water transfer in soils, and it reduced water content in the TDR sampling volume. Wraith and Or (1999) and Or and Wraith (1999) showed that reflection coefficients and determined $K_a$ values were affected by soil temperature. The reflection coefficient and $K_a$ either increased or decreased by increased soil temperature depending on soil type and $\theta_t$. Thus, the difference could be due to the different temperatures, because the soil temperature after heating was larger than 1°C.

The difference between $\theta_t$ and $\theta_l$ could be converted to $\theta_i$ with Eq. (1), and $\theta_i$ estimated with the new approach was 0.14 m$^3$ m$^{-3}$. Although the $\theta_i$ of the soil after heating was slightly smaller than that before freezing, the $\theta_i$ determined from mass measurements was 0.32 m$^3$ m$^{-3}$, so that the estimations of $\theta_i$ were quite accurate (3% error). The determined value of $\theta_i$ matched with the reference value for loam (0.18 m$^3$ m$^{-3}$), and the determined $\theta_i$ (0.14 m$^3$ m$^{-3}$) was similar to the reference value (0.15 m$^3$ m$^{-3}$) in this example. The example indicated that the new approach provided accurate determinations of $\theta_t$, $\theta_l$, and $\theta_i$.

3.3 Influence of heat intensity and target heating temperature
Table 3 shows measured $\theta_t$ and $t_{\text{heat}}$ for a variety of $q$ and $T_t$. The values for three repeated measurements were presented along with reference values of $\theta_t$ determined from mass measurements. Increased $q$ caused decreased $\theta_t$ and $t_{\text{heat}}$. When $T_t$ was set at 2.0 °C, the measured $\theta_t$ with $q = 15$ W m$^{-1}$ was $0.34$ m$^3$ m$^{-3}$, which was equal to the reference values. However, measured $\theta_t$ with $q = 30$ W m$^{-1}$ was $0.28$ m$^3$ m$^{-3}$ which was $0.04$ m$^3$ m$^{-3}$ (13%) smaller than the reference value of $0.31$ m$^3$ m$^{-3}$, and measured $\theta_t$ with $q = 80$ W m$^{-1}$ was $0.23$ m$^3$ m$^{-3}$ which was $0.09$ m$^3$ m$^{-3}$ (28%) smaller than the reference value of $0.32$ m$^3$ m$^{-3}$. Only $\theta_t$ values measured with $q = 15$ W m$^{-1}$ were consistent with reference values. During the heating, ice melted by the heater tube was surrounded by frozen soil whose matric potential was small. The subsequent liquid water could move out of the TDR sampling volume due to the hydraulic gradients. There was also a dynamic temperature gradient between the center and side tubes, and it could induce liquid water and vapor transfer in the sampling volume. A short $t_{\text{heat}}$ value could reduce liquid water transfer due to hydraulic gradients. However, a small $q$ value with a long $t_{\text{heat}}$ treatment showed accurate determination of $\theta_t$ in our results. That indicated that the magnitude of water transfer due to hydraulic gradient was smaller than that due to thermal gradient, and the radial water transfer away from the center tube due to thermal gradient increased dynamically as $q$ increased. Thus, small $q$ was favorable even though the $t_{\text{heat}}$ tended to be large.

When $T_t$ was set at 0.5°C, the $\theta_t$ value measured with 15 W m$^{-1}$ $q$ was $0.29$ m$^3$ m$^{-3}$, which was $0.02$ m$^3$ m$^{-3}$ (6%) smaller than the reference $\theta_t$ value. Thus, there was a possibility that the ice in the TDR sampling volume did not completely melt when the side tube
temperatures reached 0.5°C. \( T_t \) values larger than 0.5°C provided accurate estimates of \( \theta_t \). Therefore, \( T_t \geq 1°C \) was preferred for the new approach with the current measurement setup. However, the optimal \( T_t \) value could vary with \( \theta_t \). This evaluation was performed with loam whose \( \theta_t \) ranged from 0.31 to 0.34 m\(^3\) m\(^{-3}\), however, the loam water content we tested in section 2.5 was at largest 0.52 m\(^3\) m\(^{-3}\). Thus, \( T_t \) of 2°C was chosen to evaluate the accuracy of the new approach with loam to ensure that the TDR sampling volume was completely melted when the water content (ice content) was large.

Repeated measurements over a 6-hour period caused decreases in obtained \( \theta_t \) values regardless of the \( q \) and \( T_t \) values. Even with the smallest \( q \), 15 W m\(^{-1}\), \( \theta_t \) values decreased for the second and third repeated measurements. The larger the applied \( q \) value, the greater the decrease in \( \theta_t \) for the second and third measurements. The \( \theta_t \) values determined in the third measurements were 15 - 19%, 19 - 26%, and 35 - 45% smaller than those of the reference values with \( q = 15 \) W m\(^{-1}\), 30 W m\(^{-1}\), and 80 W m\(^{-1}\). It could be a result of soil moisture moving out of the TDR sampling volume due to hydraulic gradient and thermal gradient during heating and also during the re-freezing period. The soil moisture that moved out of the TDR sampling volume re-froze before being able to move back into the sampling volume. Thus, repeated measurements induced decreased \( \theta_t \) values. Accurate measures of \( \theta_t \) values with this new thermo-TDR approach caused some re-distribution of soil water. Longer measurement intervals might improve repeated measurement values, due to the time required for water to move back into the TDR sampling zone.
3.4 Accuracy of estimating liquid water, total water, and ice contents

In total, 15 sand and 23 loam samples were measured. The heating duration ($t_{\text{heat}}$) differed for each sample, because of different soil type and water contents. The value of $t_{\text{heat}}$ varied from 27 to 131 minutes for sand and from 4 to 109 minutes for loam depending on water content. The longer $t_{\text{heat}}$ for sand than for loam may be attributed to thermal conductivity differences. Sand had larger thermal conductivity than loam, because of its larger bulk density and presumably higher quartz content. Heat applied to the center tube was transferred faster in sand than loam, due to its large thermal conductivity, rather than being consumed to increase soil temperature and to melt ice.

Comparisons between measured and reference values of $\theta_l$, $\theta_t$, and $\theta_i$ of sand are shown in Fig. 5. Estimations made with the fitted cubic polynomial equation (Eq. (6)) and the dielectric mixing model (Eq. (3)) were included. Reference values of $\theta_l$ were represented by a yellow line in the figure. Because the repeated measurements showed decreased in $\theta_t$, we only focused on the measurements after the first heating of each sample, i.e., Fig. 6 contains only first heating measurements. The measured sand $\theta_t$ values from the cubic polynomial equation and the dielectric mixing model were consistent with the reference values, although the mixing model had slightly better estimations of $\theta_t$ than did the cubic polynomial equation. The RMSEs of $\theta_t$ estimates were 0.015 and 0.013 m$^3$ m$^{-3}$ for the cubic polynomial equations and the mixing model. Maximum and minimum errors of the $\theta_t$ estimation with cubic polynomial equation were 0.027
and 0.001 m$^3$ m$^{-3}$, and those with mixing model were 0.026 and 0.003 m$^3$ m$^{-3}$. It was an important finding that we were able to determine $\theta_i$ accurately with a single thermo-TDR sensor.

While estimated and reference $\theta_i$ values agreed well, estimated $\theta_l$ values scattered and differed slightly from the reference values. Scattering of $\theta_l$ values could be attributed to errors in $K_a$ determination due to short probe length, because $K_a$ values in frozen sand were small. Equation (6) tended to overestimate $\theta_l$ values at $\theta_t$ values greater than 0.2 m$^3$ m$^{-3}$. When the cubic polynomial equation obtained for unfrozen soil was used to estimate $\theta_l$ of partially frozen soils, the influence of $\varepsilon_i$ on $K_a$ was assumed to be negligible. This assumption was only acceptable when $\theta_i$ was small, and overestimations of $\theta_l$ occurred due to increased $K_a$ associated with increased $\theta_i$. The dielectric mixing model (Eq. (3)) estimates of $\theta_l$ also differed slightly from the reference value, but differences were relatively small, because the mixing model accounted for the influence of $\varepsilon_i$ on $K_a$. Thus, the RMSE of $\theta_l$ with the mixing model (0.020 m$^3$ m$^{-3}$) was smaller than that for the cubic polynomial equation (0.029 m$^3$ m$^{-3}$). Maximum and minimum errors of the $\theta_l$ estimation with the cubic polynomial equation were 0.070 and <0.001 m$^3$ m$^{-3}$, and those with mixing model were 0.034 and <0.001 m$^3$ m$^{-3}$.

Both the cubic polynomial equation and the mixing model provided accurate estimates of $\theta_i$ (Fig. 5b). The mixing model showed slightly better estimates than did the cubic polynomial equation. The RMSE values were 0.039 and 0.023 m$^3$ m$^{-3}$ with Eq. (3) and Eq. (6). Maximum and minimum errors of the $\theta_l$ estimation with cubic polynomial equation were 0.105 and 0.002 m$^3$ m$^{-3}$, and those with mixing model were 0.052 and 0.002 m$^3$ m$^{-3}$. The overestimations of $\theta_l$
with Eq. (6) caused underestimations of $\theta_i$. The $\theta_i$ estimates with Eq. (3) and Eq. (6) had similar values when $\theta_i$ was less than 0.15 m$^3$ m$^{-3}$, because overestimations of $\theta_i$ occurred mainly when $\theta_i$ was large.

Comparisons between measured and reference values of $\theta_i$, $\theta_t$, and $\theta_l$ of loam are shown in Fig. 6. The reference value of $\theta_l$ was represented by the yellow line in the figure, and Fig. 6 also contained only first heating measurements. The thermo-TDR $\theta_l$ values agreed well with the reference values when estimated with the cubic polynomial equation (Eq. (7)), except at $\theta_l$ values greater than 0.45 m$^3$ m$^{-3}$. A similar trend was found for the dielectric mixing model (Eq. (3a)), although the mixing model slightly underestimated $\theta_t$ values at $\theta_l$ between 0.30 and 0.45 m$^3$ m$^{-3}$. The dielectric mixing model slightly underestimated $\theta_t$, because the fitted mixing model in Fig. 3(b) tended to underestimate $\theta_l$ over the middle range of water contents. There were a few possible reasons for the underestimation of $\theta_t$ values larger than 0.45 m$^3$ m$^{-3}$. The first possibility was that the relationship between $K_a$ and $\theta_l$ after heating and thawing could differ from that obtained prior to freezing and thawing. The equations (6 and 7) and mixing model parameters shown in Table 2 were obtained under isothermal condition at 5°C. However, the temperature within the TDR sampling volume was not isothermal after heating, i.e., temperature was large near center tube and small near the side tubes, and some portion of the soil could be larger or smaller than 5°C. Wraith and Or (1999) and Or and Wraith (1999) showed that $K_a$ was temperature dependent, because the amount of bound water and the dielectric permittivity of both bound water and free water were affected by temperature. Since Andisol was supposed to
have a large amount of bound water, the $K_a$ of loam could be affected by the soil temperature change more strongly than that of sand. The temperature within the TDR sampling volume was non-linearly distributed, so, it was difficult to account for the soil temperature effect on $K_a$. The second possibility was the liquid water transfer during heating. In large $\theta_t$ soils, a long heating period was necessary, because the heating front moving away from the center needle was severely delayed due to phase change associated with the large amount of ice. A long heating period could cause an accumulated effect of the liquid water transfer away from the TDR sampling volume. However, it should be noted that the underestimation of $\theta_t$ was not observed with sand even though sand with large $\theta_t$ required a long $t_{heat}$. The third reason could be that not all of the ice in the TDR sampling volume melted. Heat transfer takes time in large $\theta_t$ soils, and, thus, $T_i$ of 2°C which in part determined $t_{heat}$ might not have been large enough to melt all of the ice in the sampling volume when $\theta_t$ was larger than 0.45 m$^3$ m$^{-3}$. Thus, an extra large $T_i$ could be necessary for such a wet soil. The RMSEs for $\theta_t$ estimation with Eq. (7) and Eq. (3a) were 0.031 and 0.041 m$^3$ m$^{-3}$. The RMSE values for $\theta_t$ estimation with loam were larger than those with sand due to the underestimation of $\theta_t$ when $\theta_t$ was larger than 0.45 m$^3$ m$^{-3}$. Maximum and minimum errors of the $\theta_t$ estimation with cubic polynomial equation were 0.062 and 0.002 m$^3$ m$^{-3}$, and those with the mixing model were 0.089 and 0.008 m$^3$ m$^{-3}$. Thus, the cubic polynomial equation provided better estimates of $\theta_t$ for loam than did the mixing model.

The loam $\theta_t$ values were overestimated with Eq. (7) as were the sand values. The overestimations occurred at $\theta_t$ values greater than 0.35 m$^3$ m$^{-3}$, so the increased $K_a$ values due to
ice replacing air was significant at these water content. The $\theta_l$ values estimated with Eq. (3b) were most similar to the reference values (0.18 m$^3$ m$^{-3}$) even though they slightly underestimated $\theta_l$. The RMSEs were 0.031 and 0.026 m$^3$ m$^{-3}$ for $\theta_l$ estimated with Eq. (7) and Eq. (3b). Maximum and minimum errors of the $\theta_l$ estimations with the cubic polynomial equation were 0.064 and <0.001 m$^3$ m$^{-3}$, and those with the mixing model were 0.074 and <0.001 m$^3$ m$^{-3}$.

Both the cubic polynomial equation and the mixing model provided accurate estimates of $\theta_i$ (Fig. 6b) for loam. The $\theta_i$ values estimated with the dielectric mixing model (Eq. 3) agreed more closely with the reference values than did those with the cubic polynomial equation (Eq. (7)), although both Eq. (7) and Eq. (3) tended to slightly underestimate $\theta_i$ (Fig. 6b)). The RMSE values for $\theta_i$ estimates with Eq. (7) and Eq. (3) were 0.047 and 0.031 m$^3$ m$^{-3}$. Maximum and minimum errors of the $\theta_i$ estimation with the cubic polynomial equation were 0.064 and 0.005 m$^3$ m$^{-3}$, and those with the mixing model were 0.062 and 0.004 m$^3$ m$^{-3}$. Reasons for underestimating $\theta_i$ were overestimations of $\theta_l$ with Eq. (7) or underestimations of $\theta_l$ with Eq. (3a).

The proposed new thermo-TDR method provided simultaneous and accurate estimations of $\theta_t$, $\theta_l$, and $\theta_i$ (see Figs. 5 and 6). The new method estimated $\theta_t$ with a single thermo-TDR probe, so that $\theta_t$ and $\theta_l$, were measured in the same sampling volume. Tian et al. (2017) estimated $\theta_i$ values from thermal conductivity measurements with RMSE values of 0.041 to 0.070 m$^3$ m$^{-3}$ for a variety of soils over a range of temperatures. Our new thermo-TDR method had smaller RMSE values (0.023 m$^3$ m$^{-3}$ for sand and 0.031 m$^3$ m$^{-3}$ for loam) than those reported by Tian et al.
Thus, the new thermo-TDR method provided better estimates of \( \theta_i \) than the thermal conductivity based approach at temperatures near the freezing point. While the new method gave accurate estimates of \( \theta_t \), \( \theta_i \), and \( \theta_l \), long heating duration disturbed water and temperature distributions, and accuracy of the method decreased for a 6-hour interval between measurements. It may take a significant amount of time for the disturbed soil sample to return to its initial condition, or possibly the sample may not return to the initial condition in a laboratory experiment which is performed at a static condition. However, in the field, dynamic soil temperature changes and interactions between soil and atmosphere may result in much faster re-distribution of soil water. Therefore, further investigations are warranted on the measurement interval for in-situ applications. The use of the new method required a relatively large amount of power for the long heating duration, given that, field application with a power cable rather than batteries is preferred. The mixing model parameters, \( \varepsilon_s \) and \( \beta \), used in this study accommodated the \( K_a \) errors associated with the short TDR probes and the \( \varepsilon_t \) change of Andisol as mentioned earlier, so that the theoretical basis of the mixing model was weakened. Therefore, the good estimation of \( \theta_t \) with the mixing model was somewhat coincidental. The development of a thermo-TDR with longer probes and an extended mixing model for partially frozen Andisol were necessary. The polynomial equations provided accurate estimates of \( \theta_i \), and thus, they may be preferred because of their ease of use. Because the polynomial equations determined \( \theta_i \) accurately, combining them with Eq. 4 estimates of \( \theta_l \) and the water retention curve might result in improved \( \theta_i \) estimations.
4. Conclusions

A new thermo-TDR based method to simultaneously determine $\theta_t$, $\theta_l$, and $\theta_i$ of partially frozen soils at temperatures near freezing point was presented and tested. The new approach utilized heat inputs to melt ice in partially frozen soil. Compared to other thermo-TDR methods, e.g., heat capacity and thermal conductivity based approaches, the new method provided improved results. The best performance of the new approach occurred with the first heating measurement. A dielectric mixing model used to convert $K_a$ to $\theta_t$ or $\theta_l$ values provided the most accurate measurements. Small $q$ to melt ice in partially frozen soil provided the most accurate results, and setting $T_i$ values larger than 0°C was important. The measurements in this study began at an initial soil temperature of $-2\,^\circ$C. Thus, the new approach was able to accurately determine $\theta_t$, $\theta_l$, and $\theta_i$ at temperatures close to the freezing point. Temperatures close to the freezing point were favorable for this approach because the $t_{heat}$ values were short. Earlier studies reported that the thermo-TDR based thermal conductivity approach did not provide accurate estimates of $\theta_t$, $\theta_l$, and $\theta_i$ at soil temperatures greater than $-2\,^\circ$C. Thus, the newly proposed approach was intended to provide ice content measurements when soil temperatures were between 0°C and $-2\,^\circ$C. The new method demonstrated the possibility of contributing to a variety of studies associated with soil freezing and thawing.

Acknowledgements
This work was supported by Grants-in-Aid for Early-Career Scientist 18K13826 from Japan Society for the Promotion of Science, the National Science Foundation under grant 1623806, Army Research Office under grant W911NF-16-1-0287, USDA-NIFA, Iowa State University Department of Agronomy, the Hatch Act, and State of Iowa funds. The authors thank Dr. Yoshiko Muto for providing water retention curve data for Toyoura sand.

Citations


Dyck, M., Miyamoto, T., Iwata, Y., Kameyama, K., 2019. Bound water, phase configuration, and dielectric damping effects on TDR-measured apparent permittivity. Vadose Zone J. 18, 190027.


Analysis. 4(1).


Table 1. Particle size distribution, organic matter content, bulk density, and van Genuchten (1980) model hydraulic parameter values.

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<th>Textural fractions</th>
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<th>Hydraulic parameters</th>
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<td>kg kg⁻¹</td>
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Table 2. Parameters used for the dielectric mixing model (Eq. (3)).

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<th>$\varepsilon_l$</th>
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Table 3. Impact of heat intensity ($q$) and target temperature for outer probes during heating ($T_t$) on total water content ($\theta_t$) estimation and heating duration ($t_{\text{heat}}$) with the new thermo-TDR approach evaluated with loam (Andisol). Values obtained in each sequential repeated measurement and reference values measured from mass balance measurements are presented.

<table>
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<th>$T_t$</th>
<th>$q$ = 15 W m$^{-1}$</th>
<th>$q$ = 30 W m$^{-1}$</th>
<th>$q$ = 80 W m$^{-1}$</th>
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<td>$t_{\text{heat}}$ min</td>
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Figure captions

Fig. 1. Schematic of the thermo-time domain reflectometry sensor.

Fig. 2. Water retention curves for sand and loam. The solid and broken lines represent the van Genuchten (1980) model (Eq. 1) fitted to the sand and loam data.

Fig. 3. The relationships between liquid water content ($\theta_l$) and apparent dielectric constant ($K_a$) measured with time domain reflectometry at 5°C. The Topp et al. (1980) equation (Eq. (2)), the fitted cubic polynomial equation, and the fitted mixing model (Eq. (3a)) curves are labelled.

Fig. 4. Examples of (a) side tube temperature changes and (b) time domain reflectometry waveforms. The target temperature during heating ($T_t$) is presented by the broken line. The waveforms before freezing (1°C), in a partially frozen soil (−2°C), and after heating ($T \geq T_t$) are presented with first and second reflection points labelled.

Fig. 5. Comparison between reference and measured values of (a) total water content ($\theta_t$) and liquid water content ($\theta_l$), and (b) ice content ($\theta_i$) of sand. The x-axis of panel (a) only shows reference values for $\theta_t$, and the reference value of $\theta_l$ is presented as the yellow line. Equations referred to in the legends represent values determined by the polynomial equation (Eq. 6) or the
dielectric mixing model (Eq. 3).

Fig. 6 Comparison of reference and measured values of (a) total water content ($\theta_t$) and liquid water content ($\theta_l$), and (b) ice content ($\theta_i$) of the loam soil. The x-axis of panel (a) only shows reference values for $\theta_t$, and the reference value of $\theta_l$ is presented as the yellow line. Equations referred to in the legends represent values determined with the polynomial equation (Eq. (7)), the dielectric mixing model (Eq. (3)), or a combination of Eq. (7) and Eq. (3b).
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**Highlights**

- A new thermo-TDR approach is described to quantify soil ice content at temperatures near the freezing point.

- The new approach determines soil liquid water and total water contents from TDR measurements made before and after a forced melting of a partially frozen soil.

- Ice content is determined from the difference between total water content and liquid water content.

- The new thermo-TDR approach provides improved estimations of ice content at temperatures near the freezing point compared with volumetric heat capacity based and thermal conductivity based thermo-TDR methods.
Yuki Kojima: Drafting the manuscript, conception and design of study, Data acquisition
Yuta Nakano: Data acquisition
Chihiro Kato: Data acquisition
Kosuke Noborio: Sensor development, conception and design of study
Kohji Kamiya: Conception and design of study
Robert Horton: Conception and design of study, revising manuscript critically for important intellectual content.