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Vladimir G. Kogan
Iowa State University and Ames Laboratory

Masanori Ichioka
Okayama University

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Vortex Cores in Narrow Thin-Film Strips

Vladimir G. Kogan[1*] and Masanori Ichioka[2†]

[1]Ames Laboratory — US Department of Energy and Iowa State University, Ames, IA 50011, U.S.A.
[2]Department of Physics, RIIS, Okayama University, Okayama 700-8530, Japan

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We study vortex current distributions in narrow thin-film superconducting strips with the help of analytic expression of the vortex current derived within the London theory. Using definition of the vortex core "boundary" as a curve where the current reaches the depairing value, vortex core size and shape are estimated as a function of vortex position in the strip. We show that the core size near the strip edges is smaller than in the rest of the strip, indicating that the Bardeen–Stephen flux-flow resistivity should be reduced near the edges. Moreover, at elevated temperatures, when the depairing current is small, the vortex core may extend to the whole strip width, thus turning into an edge-to-edge phase-slip line.

1. Introduction

Long thin-film strips are essential elements of various superconducting circuits, the current carrying properties of which are determined by vortices residing there or crossing strips and causing non-zero voltages and energy dissipation, see, e.g., Ref. 1. Many vortex effects can be described by studying the supercurrent distributions, which away of vortex cores are well represented by the London theory, see, e.g., Refs. 2 and 3. However, within this theory, vortex cores are treated just as point singularities despite the fact that properties of cores and, in particular, their size and shape are relevant for evaluation the vortex self-energy and its dynamic properties, the flux-flow resistivity is just an example.

To describe properly the core structure is challenging even in uniform bulk materials far from sample boundaries. The question then whether one can extract some information about core shapes from the London current distribution. For a single vortex in isotropic bulk superconductors the current density distribution out of core is given by $j = (c\phi_0/32\pi^2\lambda^3)K_1(r/\lambda)$, where ϕ_0 is the flux quantum, λ is the penetration depth, and K_1 is the Modified Bessel function. At short distances $r \ll \lambda$, the London current $j = c\phi_0/32\pi^2\lambda^2 r$ reaches the depairing value (defined as $j_d = c\phi_0/16\pi^2\lambda^2\xi$ near the critical temperature T_c) at $r \sim \xi$ with ξ being the coherence length. In fact, this is one of popular ways to define the vortex core size. "Sweeping under the rug" complexities of the vortex core physics, the core model as the circular normal state region of radius ξ provides correct estimates of the core energy as the condensation energy within the core, $\pi\xi^2(H_c^2/8\pi)$, and of the flux-flow resistivity as the normal resistance within the core, $\rho_f \approx \rho_n 2\pi\xi^2 B/\phi_0$ (ρ_n is the normal resistivity, B is the magnetic induction).[2,4)]

In thin films, the vortex current distribution differs substantially from the bulk since the stray fields out of the film affect the currents in the film.[5)] Evaluation of these distributions are difficult in particular in finite samples, where the film edges may cause drastic modifications of currents for vortices situated at distances of the order of Pearl length $\Lambda = 2\lambda^2/d$ from the edges. Exceptions are the small samples, e.g., narrow thin-film bridges of a width $W \ll \Lambda$ where effects of self-fields can be disregarded.[6)] Following the above qualitative prescription for determination of the core shape and size, one may expect different from circular core shapes for vortices in such bridges, the subject of our discussion below.

We stress that the main advantage of the London approach is in its simplicity related to the linearity of London equations. The main limitation is its break down within the core. But if we are interested only in the core *shape*, the London theory provides qualitatively correct results, its simplicity notwithstanding. The unexpected core shapes we obtain within this approach may affect both static and dynamic properties of vortex systems in thin-film superconducting devices.

In the next section we derive an analytic expression of the vortex current in narrow thin-film superconducting strips within the London theory, and introduce definition of the vortex core boundary employed in the following work. In Sect. 3, intriguing features of the vortex core shape and size are studied as a function of vortex position in the strips at low and high temperatures. Discussion and summary conclude the paper.

2. Narrow Thin-Film Strips

Consider a thin-film strip in the plane (x, y), x axis is directed across the strip, $0 < x < W$, whereas y is along it. For a vortex at $x = a$, $y = 0$, the London equations for the film interior read

$$\boldsymbol{h} + 4\pi\lambda^2\,\mathrm{curl}\,\boldsymbol{j}/c = \phi_0\,\hat{\boldsymbol{z}}\,\delta(x-a, y), \tag{1}$$

$\boldsymbol{h}$ is the magnetic field and $\boldsymbol{j}$ is the supercurrent density. Averaging this over the thickness d, one obtains

$$h_z + 2\pi\Lambda\,\mathrm{curl}_z\mathbf{g}/c = \phi_0\delta(\mathbf{r}-\mathbf{a}), \tag{2}$$

where $\mathbf{g}(\mathbf{r})$ is the sheet current density, $\mathbf{r} = (x, y)$, $\mathbf{a} = (a, 0)$, and $\Lambda = 2\lambda^2/d$. Other components of Eq. (1) turn identities after averaging.

In strips of a width $W \ll \Lambda$, the self-field of the current $\boldsymbol{g}$, given by the Biot–Savart integral, is of the order g/c, whereas the second term on the left-hand side of Eq. (2) is of the order $g\Lambda/cW$. Hence, the self-field h_z can be disregarded, unlike the applied field if it exists.

It is convenient to introduce a scalar "stream function" $G(\boldsymbol{r})$ such that $\boldsymbol{g} = \mathrm{curl}\,G\hat{\boldsymbol{z}}$,[6)]

$$g_x = \partial_y G, \quad g_y = -\partial_x G. \tag{3}$$

Then, we obtain for G:

$$\nabla^2 G = -(c\phi_0/2\pi\Lambda)\delta(\mathbf{r}-\mathbf{a}). \tag{4}$$

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The boundary conditions $g_x = 0$ at the strip edges $x = 0, W$ give $G = 0$ at the edges. Thus, the problem is equivalent to that of two-dimensional electrostatic potential of a linear "charge" $q = c\phi_0/8\pi^2\Lambda$ situated at $\boldsymbol{r} = \boldsymbol{a}$ between two grounded metal plates parallel to zy at $x = 0$ and W. The solution of Eq. (4) is obtained by conformal mapping:[7)]

$$\tanh\frac{G}{2q} = \frac{\sin(\pi a/W)\sin(\pi x/W)}{\cosh(\pi y/W) - \cos(\pi a/W)\cos(\pi x/W)}. \quad (5)$$

The alternative way to present this result as due to an infinite sum over ± vortex images out of the strip, see, e.g., Ref. 8, is equivalent to Eq. (5), but having closed form, Eq. (5) is more convenient for numerical and analytic work.

Having the stream function G, one evaluates the sheet current components of Eq. (3) (Mathematica is helpful in this tedious calculation):

$$\mathcal{J}_x = g_x\frac{2\pi\Lambda W}{c\phi_0} = -\frac{\sin\pi a\sin\pi x\sinh\pi y}{\cos 2\pi a + \cos 2\pi x - 4\cos\pi a\cos\pi x\cosh\pi y + 2\cosh^2\pi y},$$
$$\mathcal{J}_y = g_y\frac{2\pi\Lambda W}{c\phi_0} = \frac{\sin\pi a(\cos\pi a - \cos\pi x\cosh\pi y)}{\cos 2\pi a + \cos 2\pi x - 4\cos\pi a\cos\pi x\cosh\pi y + 2\cosh^2\pi y}. \quad (6)$$

Here $\mathcal{J}$ is the dimensionless sheet current density in units of $c\phi_0/2\pi\Lambda W$ and x, y, a are measured in units of W.

Stream lines of the current coincide with contours of $G =$ const; indeed, $(\boldsymbol{ds}\times\boldsymbol{g})_z = g_y\,dx - g_x\,dy = dG = 0$, where $\boldsymbol{ds}$ is the line element. Example of current stream lines is shown in Fig. 1 for a vortex close to the left edge of the strip.

To have a better idea on the distribution of current values, one can plot contours of constant $\mathcal{J}(x, y) = \sqrt{\mathcal{J}_x^2 + \mathcal{J}_y^2}$. This distribution near the vortex core affects the core shape.

Using the standard estimate for the deparing current density one gets the sheet deparing density $g_d \approx c\phi_0 d/16\pi^2\lambda^2\xi = c\phi_0/8\pi^2\Lambda\xi$ and the dimensionless depairing current

$$\mathcal{J}_d \approx W/4\pi\xi. \quad (7)$$

It is worth noting that the depairing current value adopted here is not a universal quantity which may vary (slightly) with the sample geometry. This uncertainty may introduce an extra factor ~1 in Eq. (7). The vortex core "boundary" is reasonably defined as a curve where the current reaches the depairing value.

If one takes data from Ref. 9 for WSi 4-nm-thick narrow bridges with $T_c = 3.4$ K, the low temperature value $\xi_0 \approx 7.8$ nm, and $W = 2$ μm, one estimates the low temperature $\mathcal{J}_d(0) \approx 20.4$. Hereafter we use these data to compare with our model predictions.

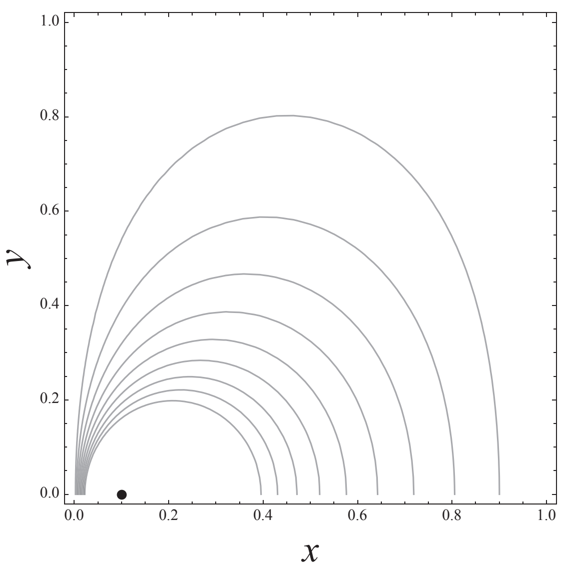

Fig. 1. Current lines for a vortex at $a/W = 0.1$. x, y are measured in units of W. The black dot marks the position of the vortex singularity.

On warming, the depairing current decreases according to empirical relation $\mathcal{J}_d = \mathcal{J}_d(0)(1 - t^2)^{3/2}$, $t = T/T_c$.[10,11)] For our example, $\mathcal{J}_d \approx 2$ at $T \approx 3$ K. The contour $\mathcal{J}(x, y) = 2$ is shown in Fig. 2 for a vortex at $a = 0.1W$. Hence, the core shape, defined by $\mathcal{J}(x, y) = \mathcal{J}_d$ for a vortex penetrating the edge $x = 0$, has the shape of a liquid droplet stuck to the film edge. This unusual shape can be attributed to enhanced current density between the edge and the position of the current singularity for a vortex close to the edge seen in Fig. 1. It is worth noting that since $\mathcal{J}_d$ decreases on warming, according to Fig. 2 the normal core expands on warming as it should.

3. Core Shape Dependence on Vortex Position

For the above example of thin and narrow bridge of WSi, $\mathcal{J}_d = 5$ corresponds to $T \approx 0.78T_c$. This value of $\mathcal{J}_d$ is chosen here to demonstrate changes of the core shape of a vortex at different positions. Figure 3 shows contours $\mathcal{J}(x, y) = \mathcal{J}_d = 5$ for a set of vortex positions near the edge.

One sees, that when $a \lesssim 0.03$, the core shape is close to semi-circles or ovals with a base at the edge, i.e., at the y axis, and with the size growing with increasing a. When the distance a from the edge increases further, the core acquires a shape reminiscent of a liquid droplet still attached to the edge, see curves for $a = 0.05, 0.062$. Eventually, the core droplet disengages from the edge and acquires a nearly circular shape, $a = 0.07, 0.10$. It is readily shown that the disengagement happens at

$$a = \frac{2}{\pi}\cot^{-1}(2\mathcal{J}_d). \quad (8)$$

One also observes that when the vortex proceeds up to $a \approx 0.03$, the core expands in both directions. For larger $a = 0.05, 0.062$, the increase stops in the y direction along the strip, but the width of the core in x direction expands. Therefore, the core size near the strip edges is smaller than in the rest of the strip. If so, the Bardeen–Stephen flux-flow resistivity should be reduced near the edges, since it is related to the vortex core size as noted in Sect. 1. The resistivity measurement of thin-film strips in Ref. 9 suggests that the flux-flow conductivity is enhanced near strip edges. While we can add a uniform drag current in Eq. (6) in our model, in order to properly describe vortex dynamics we have to

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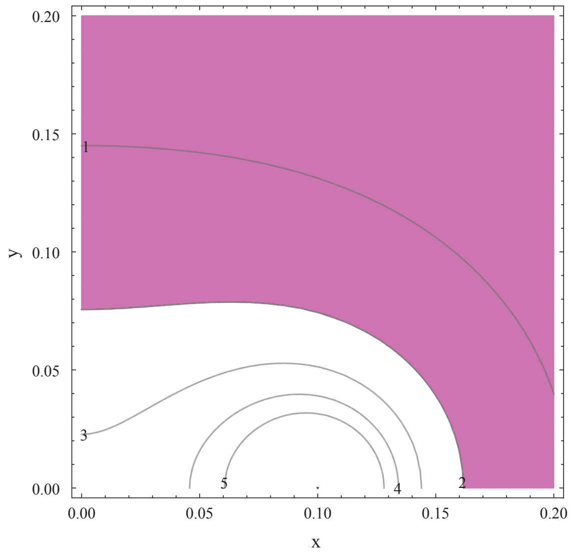

Fig. 2. (Color) Contours of constant current *values* for vortex at $a/W = 0.1$. The curve separating the white "normal core" from the magenta superconductor is the estimated "core boundary" for WSi thin-film strip at $T/T_c \approx 0.89$ with the depairing current value $\mathcal{J}_d = 2$. The numbers by the contours are $\mathcal{J}$ values.

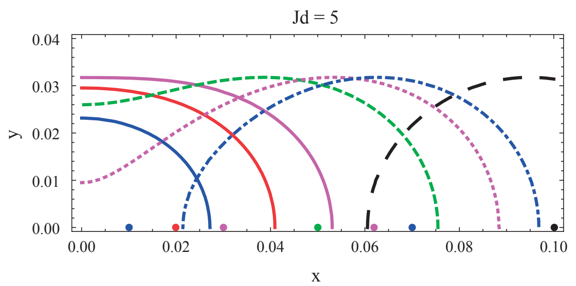

Fig. 3. (Color) Lines of constant current values $\mathcal{J}(x, y) = \mathcal{J}_d = 5$ for vortices at a set of positions a/W marked by dots: 0.01 (solid blue), 0.02 (solid red), 0.03 (solid magenta), 0.05 (dashed green), 0.062 (dotted magenta), 0.07 (dash-dotted blue), and 0.10 (dashed black). All curves are symmetric relative to $y = 0$, so that the part $y < 0$ is not shown.

assume certain models for the dissipation and local heating by moving vortex core. However, since these extensions become too far from our subject of the position dependence of the vortex core shape, here we just suggest that the vortex core size may be related to the flux-flow resistivity using a simple picture of the Bardeen–Stephen theory.[4]

The behavior of so defined core is even more intriguing at higher temperatures and lower $\mathcal{J}_d$. An example of $\mathcal{J}_d = 0.4$, shown in Fig. 4, corresponds to $T \approx 3.3\,\text{K} = 0.96 T_c$. One can see that up to $a = 0.4$ the core, being still attached to the left edge, ends up at some $x^* < 1$. One readily verifies that x^* satisfies

$$\mathcal{J}(x^*, 0, a) = \frac{\sin \pi a}{2|\cos \pi a - \cos \pi x^*|} = \mathcal{J}_d. \qquad (9)$$

However, for $a = 0.45$ and 0.5 the core turns into a normal state edge-to-edge channel. One may expect these channels to behave as line-type phase slips.[12] One must note, however, that since this situation is likely to occur close to T_c where ξ diverges, it might be difficult to distinguish between line-type and standard one-dimensional phase slips.

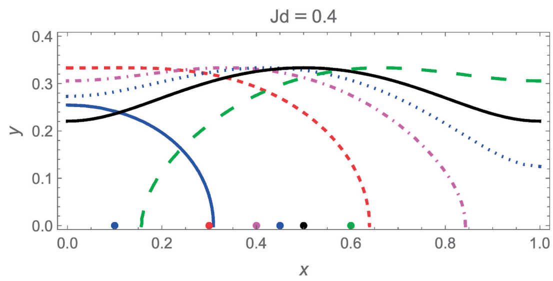

Fig. 4. (Color) Lines of constant current values $\mathcal{J}(x, y) = \mathcal{J}_d = 0.4$ for vortices at a set of vortex positions a marked by dots: 0.1 (solid blue), 0.3 (dashed red), 0.4 (dash-dot magenta), 0.45 (dotted blue), 0.5 (solid black), and 0.6 (dashed green).

Recall that one-dimensional phase slips are responsible for dissipation in wires thinner than ξ.[2] When a localized vortex crosses a superconducting strip of length $2L$ with the width $W \gg \xi$, the phase difference between edges $y = -L$ and L also "slips" by 2π as in thin wires, see, e.g., Ref. 13. The line phase slips were proposed as another mechanism of dissipation in wide strips.[12] Hence, one may expect a similar behavior in thin-film strips of interest here.

One can also look at the disengagement from the edge calculating the width $2y_1$ of the droplet base at the left edge at $x = 0$ where y_1 is found from $|\mathcal{J}_y(0, y_1, a)| = \mathcal{J}_d$. This gives

$$\cosh \pi y_1 = \frac{\sin \pi a}{2\mathcal{J}_d} + \cos \pi a. \qquad (10)$$

Similarly, for the edge at $x = W$, one obtains from $|\mathcal{J}_y(W, y_2, a)| = \mathcal{J}_d$:

$$\cosh \pi y_2 = \frac{\sin \pi a}{2\mathcal{J}_d} - \cos \pi a. \qquad (11)$$

Figure 5 shows $y_1(a)$ and $y_2(a)$ for $\mathcal{J}_d = 5$ (the upper panel) and $\mathcal{J}_d = 0.4$ (the lower panel). One sees that at low temperatures with large $\mathcal{J}_d$ the vortex cores are separated from the edges for most of vortex positions in the sample, except narrow belts near the edges. These belts expand on warming and at $\mathcal{J}_d = 0.5$, all vortices at $a < 0.5$ are stuck to the left edge, while those at $a > 0.5$ are attached to the right edge. With further warming $\mathcal{J}_d < 0.5$, the two domains overlap, as shown at the lower panel, in other words, in a finite interval of positions centered at the sample middle the cores are attached to both edges. That is the situation when the cores are expected to behave as line phase slips.

4. Discussion and Summary

A word of caution: the definition of the "core boundary" as a curve where the current *values* in the London approximation reach the depairing level is rather artificial, notwithstanding reasonable results it leads to in isotropic bulk situation. In fact, the London approach breaks down near these boundaries. Hence, a better theory should be employed in the core vicinity. To reaffirm the contours of $J(x, y) = J_d$ in Figs. 2–4 as representing, at least qualitatively, vortex core shapes, one has to see whether or not the order parameter modulus $|\Delta(x, y)|$ indeed decreases when one crosses these contours. This, of course, cannot be done within the London model where $|\Delta|$ is assumed constant. One should turn to a microscopic theory or, for temperatures close to T_c, to the Ginzburg–Landau (GL) theory.

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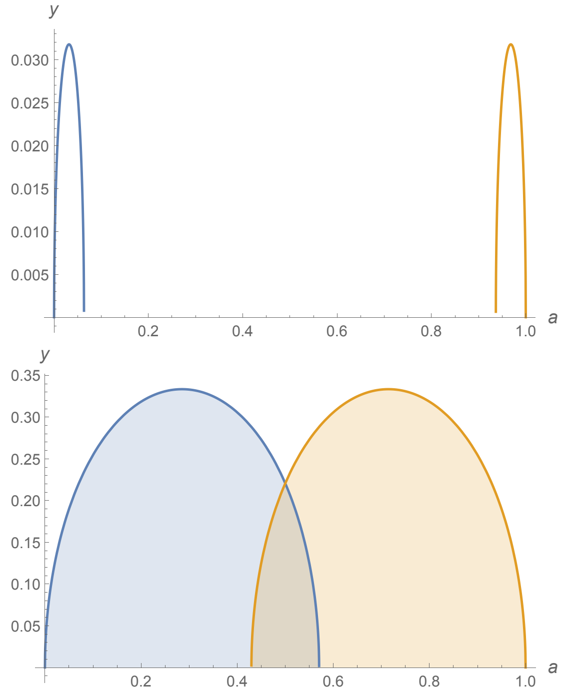

Fig. 5. (Color online) The upper panel: the width y of the droplet base at $x = 0$ as function of the vortex position a for $\mathcal{J}_d = 5$; the droplet base at the left edge is finite for $0 < a \lesssim 0.06$ and at the right edge if $0.94 \lesssim a < 1$. At other positions cores are disconnected from both edges. The lower panel: the same for $\mathcal{J}_d = 0.4$. For all a, the core droplet is connected to one of the edges; moreover, domains of nonzero droplet bases are overlapped if $0.43 \lesssim a \lesssim 0.57$, i.e., the core occupies an end-to-end belt as in line phase slips.

The problem of the order parameter distributions for vortices perpendicular to a thin film, is challenging because one has to take the stray fields into account. Given these difficulties, one could turn to other geometry, where the comparison between the current and order parameter distributions is easier to make. Such a case is a vortex parallel to faces of a thin superconducting slab.[8]

It is straightforward to show that the London current distribution for a vortex parallel to a slab thinner than the London λ are, in fact, the same as in narrow thin-film bridges discussed above, except now we have the length scale λ instead of Pearl's Λ and the slab thickness instead of the bridge width W. We are not aware of calculations describing the order parameter distribution within the core of such a vortex. Physically, however, such a distribution is similar to that of a vortex close to the surface of a bulk sample, the problem considered within the frame of GL theory,[14,15] and recently in the discussion of surface barriers near T_c.[16] Contours of constant order parameter $|\psi|$ obtained in these papers are indeed qualitatively similar to contours of our Fig. 2 of constant current values.

To conclude, we derived analytic expression of the vortex current in narrow thin-film superconducting strips, and employ definition of vortex core boundary via the depairing current value to study the vortex core shape and size in the strips. We have shown that along with the distorted vortex current distribution near the film edges, vortex cores are strongly affected as well. Close to the edge, the core is shaped as a liquid droplet attached to the edge, which grows when vortex singularity moves away from the edge. At some distance, depending on the depairing current value, the core disconnects from the edge, Eq. (8), and acquires a "normal" round shape. If temperatures are high, the depairing current is small, and the thin-film bridge is narrow, the core can be attached to both edges simultaneously thus forming a structure similar to line-type phase slips.

Our model is not dynamic, i.e., we do not consider equations of vortex motion, but focus on the core shape dependence on vortex position on a strip. A discussion of complicated vortex dynamics in the frame of time-dependent GL theory in given, e.g., in Ref. 17. Still, our discussion could be relevant for interpretation of data on resistivity transition from the normal to superconducting state in narrow thin-film strips reported in Ref. 9. In particular, these data were shown to be consistent with enhanced flux-flow conductivity near the strip side edges in applied fields on the Tesla order. The conductivity variation could be associated with changing core size of vortices crossing the strips.

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*kogan@ameslab.gov
†ichioka@cc.okayama-u.ac.jp

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