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An analysis of marketing strategies for cattle producers who use direct marketing

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An analysis of marketing strategies for cattle producers who use direct marketing

by

Frances Ann Antonovitz

A Thesis Submitted to the Graduate Faculty in Partial Fulfillment of The Requirements for the Degree of

MASTER OF SCIENCE

Major: Economics

Signatures have been redacted for privacy

Iowa State University
Ames, Iowa

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CHAPTER I. INTRODUCTION

Present Method of Direct Cattle Marketing

An Iowa cattle producer marketing fed cattle has access to three alternative types of markets. He can use a terminal market, an auction market, or direct marketing. The results of a 1974 Iowa State University survey of eight Iowa packers indicate that about eighty-five percent of the finished cattle were sold by direct marketing, fourteen percent at terminal markets, and only one percent at auctions [12]. Thus, the majority of fed cattle are sold by direct marketing.

The term direct marketing is generally used to describe the movement of livestock from the producer to the slaughterer without the use of terminal or auction market facilities. In Iowa, direct marketing includes primarily country packer buyers, but also packer buying stations, local markets, and country dealers. In this study, the problems and decisions of the Iowa cattle producer selling his fed cattle to country packer buyers will be examined.

Most of the larger packers employ buyers who travel to various farms and feedlots to buy cattle. These buyers are usually well informed about current market conditions, plant needs, and relative availability of cattle as observed by other buyers and as indicated by other contacts. Many small producers are at a disadvantage relative to the commercial feedlots because they cannot offer the concentration of cattle and, consequently, do not have the bargaining power of the large operators. Small producers are also at a disadvantage because often
packer buyers are reluctant to visit the smaller yards, and thus the smaller producers will have fewer bids and less information about prices [9].

The cattle producer must make a number of decisions about marketing his cattle. First, the producer must decide when his cattle will be ready for market. This also involves deciding how long to feed the cattle to have them fall within a particular grade. When the producer feels that his cattle will be ready to market soon, he might begin to gather information about the market.

According to Skadberg [13], the cattle producer usually gathers information in the following way before selling his cattle. He will begin by reading the newspaper, listening to the radio and television, and talking to neighbors paying particular attention to the market situation. About seventy percent of the producers who sell their own cattle (and particularly those with under 200 head) will have only these forms of information. The more skillful sellers may look at livestock receipts or use marketing services such as Cattle-Fax, Grain and Livestock Service, or Pro-Farmer.

Once the producer knows approximately when he will market his cattle and has collected information about marketing conditions, he may begin to contact packers. Many times, the country packer buyers will initiate the marketing process by coming to the feedlot and giving a bid. If the buyer does not come to the feedlot on his own initiative, the producer can call the packing plant and ask that a buyer visit his feedlot and bid
on his cattle. The number of bids a producer checks before selling his cattle is usually a long-standing habit. Some producers will stay with one buyer, and others may check with two or three packers. But instead of contacting only certain packers by habit, a feeder in Iowa may be able to choose from packers in other states in addition to those in Iowa if the distances are not far enough to cause substantial shrinkage and death loss. In all, there may be as many as twenty-five to thirty such feasible packers worth considering for any producer in Iowa [13].

If a producer does not just contact a few packers out of habit, he must decide which packers to contact to obtain bids and in which order to contact them. Once a producer receives a bid, he must decide whether to sell to that packer immediately or to wait and obtain other bids during the day. At the end of the day, the producer must decide whether to sell his cattle to one of the packers he contacted that day or to wait until another day and sell later. Many times, the buyer may threaten that his bid will only be good for a specified period of time or until he leaves the farm [13]. However, bids and offers usually do not stand for more than one day [12].

When selling his cattle, the producer must also negotiate conditions of sale with the buyer. The producer can sell his cattle on a liveweight, carcassweight, or a carcass grade and weight basis. In Iowa, there are three categories of conditions of sale that are common on liveweight sales: (1) cattle are shipped in the morning, weighed at the packing plant, and a one to three percent pencil shrink is deducted depending on
distance from the feedlot to the packing plant; (2) cattle are shipped in
the morning, weighed at or near the feedlot and a three percent pencil
shrink is deducted; (3) cattle are held off feed and water the afternoon
before shipping, shipped in the morning, weighed at the packing plant, and
no pencil shrink is deducted. Over fifty percent of the fed cattle in
Iowa are sold on a liveweight basis [12].

Both carcassweight and carcass grade and weight sales also occur in
Iowa. Almost forty percent of the fed cattle in Iowa are sold on a
carcassweight basis. The carcassweight price is negotiated at the time
of the sale at the feedlot, and the producer then receives this price per
hundredweight of hot carcass. Generally, there are no feed and water
restrictions or pencil shrink associated with carcassweight sales. About
nine percent of the fed cattle marketed directly in Iowa were sold on a
carcass grade and weight basis. Using this basis of sale, separate prices
per hundredweight of hot carcass are negotiated for each grade or grade
and carcassweight category at the time of the sale at the feedlot. How-
ever, whatever the sales arrangement used, the Iowa producer will pay
the transportation costs to the packing plant and will deliver his
cattle nearly always within two business days after the sale agreement
[12].
Statement of the Problem

Although there are many decisions that a producer must make when marketing his cattle, only a few will be examined in this study. The producer's decisions that will be considered are the following: how many packers to contact each day, which packers to contact each day, in what order to contact the packers each day, and which day to sell cattle.

Literature Review

These decision problems faced by cattle producers have not been the focus of any earlier studies, but studies have been conducted on searching for the highest or lowest price in various marketing situations. Stigler [14] was the first to discuss the problem of ascertaining the most favorable price. He contends that dispersion of prices exists even for homogeneous goods, and thus, there will be a frequency distribution associated with the prices offered by various sellers (or buyers). Therefore, to obtain the most favorable price, a buyer (or seller) must canvass various sellers (or buyers). Suppose that the cumulative distribution function of sellers' asking prices, p, is F(p). Then, the buyer wishes to canvass various sellers to obtain the minimum price. The probability that the minimum of n independent observations will be less than or equal to p is 1 - [1-F(p)]^n. Also, the expected value of the minimum of a random sample of n observations is,

\[ E(n) = n \int_0^\infty p (1-F)^{n-1} F' \, dP. \]
For any buyer, the expected savings from an additional unit of search will be approximately

\[ \frac{\partial P_{\min}}{\partial n} \]

where \( P_{\min} = E(n) \) or the quantity, \( q \), he purchases times the expected reduction in price as a result of search. Thus, if the cost of search is equated to its expected marginal return, the optimum amount of search will be found.

Stigler [15] wrote a second article relating this problem to the labor market. That dispersion of prices exists even for homogeneous goods can be exemplified by the different wages offered to the same person. The person searching for work must canvass various potential employers to obtain the highest wage rate. To do this, the expected maximum wage offer in \( n \) searches, \( W_n \), must be found. Then, the worker should search for wage offers until the expected marginal wage rate increase from one additional search equals the marginal cost of search.

McCall [8] builds on Stigler's work by using a sequential approach to the problem. That is, after each potential employer offers a bid, the searcher must decide whether to wait and search more or whether to accept the bid. He considers the problem of the worker looking for a job assuming that the worker knows both the distribution of wages for his particular skill and the cost of generating a job offer. The job offers are independent random drawings from the distribution of wages. The worker receives only one job offer per period, and the job offer is only
good for the period in which it is offered. The producer will continue to receive job offers for an infinite length of time. McCall shows that under these conditions the optimal policy for the job searcher is to reject all offers below a single critical value and to accept any offer above this critical value. In deriving this result, the following symbols will be used:

\[ C = \text{cost per period of search} \]
\[ x = \text{a random variable denoting the utility gained by the job offer} \]
\[ \phi(x) = \text{the probability density function of } x \text{ (if more than one offer per period was permitted, } \max(x_k), \text{ over } k, \text{ would be used where } k \text{ is a random variable}) \]
\[ f(x) = \text{maximum return (utility less costs) obtainable when a job offer } x \text{ has just been observed.} \]

If employment begins after \( N \) job offers, then the return, \( f \), is

\[ f = x_N - CN. \]

If an \( x \) is observed during the first period and the search continues in an optimal manner, the return is given by

\[ f(x) = -C + \max[x, E(f(x))]. \]

Let \( \varepsilon = E(f(x)) \), then the optimal stopping rule has the following form:

- continue searching if \( x < \varepsilon \)
- accept employment if \( x \geq \varepsilon \).

Now, \( \varepsilon \) can be calculated. First, \( E(f|N) \), the expected value of the return given that searcher accepts the \( N \)th offer, is calculated.

\[ E(f|N) = E(x_N|N) - CN \]
then

\[ E(f) = \varepsilon = E(E(x_N | N)) - CE(N) \]

Note that

\[ E(x_N | N) = E(x_N | x_N \geq \varepsilon, x_{N-1} < \varepsilon, \ldots, x_1 < \varepsilon) \]

By the assumption that the offers are independent,

\[ E(x_N | x_N \geq \varepsilon, x_{N-1} < \varepsilon, \ldots, x_1 < \varepsilon) = E(x_N | x_N > \varepsilon) \]

Thus,

\[ E(x | x \geq \varepsilon) = \frac{\int_{\varepsilon}^{\infty} x \phi(x) dx}{P(x \geq \varepsilon)} \]

and

\[ \varepsilon = \frac{\int_{\varepsilon}^{\infty} x \phi(x) dx}{P(x \geq \varepsilon)} - CE(N). \]

\(E(N)\) is the expected waiting time until employment occurs. The appropriate random variable is the number of trials until \(x \geq \varepsilon\) or until employment occurs. This random variable may be assumed to follow a geometric distribution with parameter \(P = P(x \geq \varepsilon)\) and expected value

\[ E(N) = \frac{1}{P}, \ P > 0. \]

Combining these results,

\[ \varepsilon = \frac{\int_{\varepsilon}^{\infty} x \phi(x) dx}{P(x \geq \varepsilon)} - C \cdot \frac{1}{P} \]
The economic interpretation of this equation is the same as Stigler's conclusion. The marginal cost of generating another job offer, \( C \), is equated to the right hand side which is the expected marginal return from waiting another period.

McCall [8] also discusses the case where the job searcher has inadequate knowledge about the distribution of wages for his skills. It is assumed that the job searcher has imperfect information about the \( k \) parameters, \( \gamma = (\gamma_1, \ldots, \gamma_k) \), of the wage distribution, \( \phi(x) \). However, he does have a prior distribution, \( h(\gamma | \theta) \), over the unknown parameters, where \( \theta \) is a vector representing the parameters of the prior. The prior distribution summarizes the imperfect information that the searcher has about the mean and other moments of the true wage distribution. After each observation the prior distribution is revised in a Bayesian fashion, and a new value is calculated, say \( \theta' = T(\theta, x_1, x_2, \ldots, x_n) \) where \( n \) represents the number of offers observed. After a revision of the prior distribution, the searcher must decide whether to reject or accept the job offer. Let \( f_n(x, \theta) \) be the maximum expected return when
an offer $x$ has just been made where $n$ indicates that $n$ more offers will be forthcoming. Then

$$f_n(x, \theta) = -C + \max(x, \int f_{n-1}(x, \theta') \phi(x|\gamma) h(\gamma|\theta) dx dy) \cdot 1.$$ 

Let

$$e_{n-1} = \int f_{n-1}(x, \theta') \phi(x|\mu) h(\gamma|\theta) dx dy.$$ 

Then the optimal stopping rule is the same as before:

- accept employment if $x \geq e_{n-1}$
- continue searching if $x < e_{n-1}$.

Telser [16] discusses the problem of searching for the lowest price when the searcher knows the distribution of prices and when he does not know the distribution. Although Telser's approach and derivation are different than McCall's, the results of both studies are essentially the same. First, assume that the cumulative density function, $F(x)$, the minimum price, $M$, and the maximum price, $U$, are all known. Also, assume that the searcher can obtain the lowest price quoted through trial $t$ and denote this value by $x_{lt}$. Let $X$ be the random variable giving the outcome on trial $t + 1$. Now, the searcher must decide whether to stop on trial $t$ or to search once more. The loss function is defined as follows:

$$L(X) =
\begin{cases} 
X - M + C & \text{if } X < x_{lt} \\
x_{lt} - M + C & \text{if } X \geq x_{lt}.
\end{cases}$$ 

\footnote{This equation has been corrected for obvious errors in McCall's original paper.}
In other words, if at trial \( t + 1 \) the outcome is greater than the least observation drawn through trial \( t \), then the net loss rises by the marginal cost of search, \( C \). The expected loss is

\[
E[L(X)] = \int_{m}^{x_{lt}} (X-M-C) dF + \int_{x_{lt}}^{U} (x_{lt} - M + C) dF
\]

\[
= (C-M) + \int_{m}^{x_{lt}} [1-F(x_{lt})]x_{lt} + \int_{m}^{x_{lt}} X dF.
\]

The loss at trial \( t \) is \( L(x_{lt}) = x_{lt} - M \). Therefore, the optimal stopping rule is:

- If \( L(x_{lt}) \leq E[L(X)] \) stop,
- If \( L(x_{lt}) > E[L(X)] \) continue.

Simplifying this notation, the stopping rule becomes:

- If \( x_{lt} \leq C + [1-F(x_{lt})]x_{lt} + \int_{m}^{x_{lt}} X dF \) stop,
- If \( x_{lt} > C + [1-F(x_{lt})]x_{lt} + \int_{m}^{x_{lt}} X dF \) continue.

From the above stopping rule, a maximum acceptable price can be found. Let this critical value be denoted by \( a \). Then \( a \) can be found by solving the following equation:

\[
a = C + [1-F(a)]a + \int_{m}^{a} X dF.
\]

Therefore, the searcher will continue sampling until he receives a price less than \( a \).

Secondly, Telser assumes that the searcher does not know the
distribution of prices, but that he can specify a subset of admissible distributions. By using this subset, the optimal number of observations to pass in the learning state, \( p \), can be obtained. No decisions about whether to stop or continue are made until these \( p \) observations are obtained. Subject to the constraint that \( t \geq p+1 \), \( p \geq 0 \), the following stopping rule can be applied:

\[
\text{If } x_{1t} < x_{1,t-1} + C \text{ continue,}
\]

\[
\text{If } x_{1t} \geq x_{1,t-1} + C \text{ stop.}
\]

In the case where the distribution is unknown, the searcher's prior probabilities can be found for each distribution in the subset of admissible distributions. Then, loss functions and critical values can be formed by using these prior probabilities.

Objectives and Purposes

The objectives of this study are:

1) to model the situation faced by the producer who uses direct marketing in terms of economic, mathematical, and statistical theory and

2) to determine a procedure based on this model for producers to use for finding the optimal number of packers to contact.

One purpose of this study is to present alternative marketing strategies which are better than those presently used by cattle producers. Another purpose is to provide the theoretical framework on which to base
further studies of this currently unexplored topic.

Assumptions

To achieve the objectives and purposes stated, certain simplifying assumptions will be made. First, because only a very small number of country packer buyers will contact any one producer, assume that all contacts are initiated by the producer. The term "contact" will denote the act of the producer contacting a packer, generally by telephone, and asking that a buyer visit his feedlot and give a bid on his cattle. "Recontact" will denote the act of the producer contacting a packer for a second time to accept a bid. Recontacting and contacting costs will be those costs associated with the acts. "Transportation" costs will denote the costs of transporting the cattle from the feedlot to the packer. It will be assumed that the producer knows the contacting, recontacting, and transportation costs associated with each of his feasible packers.

The producer knows which day he will begin contacting packers and also knows the latest day he would sell his cattle. On any marketing day between and including these first and last days, the producer would consider selling his cattle. The producer's "marketing horizon" includes all days that he will consider selling his cattle. The producer will begin contacting packers only on this first day and will cease contacting packers on the day he markets his cattle. Once a packer makes a bid, it will be valid throughout only that particular day but will not be valid any following day. It is also assumed that the producer knows exactly
how many cattle he wishes to market and that this number remains constant.

The problem will be separated into two parts: a one-day horizon problem and a multi-day horizon problem. In the one-day horizon problem, the marketing horizon is only one day long. Thus, on the day the cattle will be marketed, the producer must decide how many packers to contact, which packers to contact, and in what order to contact them. In the multi-day horizon case, the producer's marketing horizon is longer than just one day. Therefore, the producer must decide which day to sell his cattle, how many packers to contact each day, and in what order to contact the packers each day.

The price offered by a packer is the price per hundredweight of fed cattle. This price can be based either on liveweight or carcassweight. The prices offered by packers on day $i$ are continuous and are independently and identically distributed with the continuous probability density function $f_i(p)$ where $p$ is price per hundredweight. The price offered by a particular packer on one day is independent of any of his subsequent or previous offers.

Outline of the Remaining Chapters

Two approaches will be used to solve the problem: sequential sampling and nonsequential sampling. In Chapter II these two approaches are used to solve the problem of finding the optimal number of packers assuming that $f_i(p)$ is known. The same approaches are used to address the problem in Chapter III, but there it is assumed that $f_i(p)$ is not known with
certainty. However, the producer does have some prior knowledge about the distribution of $p$. In Chapter IV, an empirical example is presented, and possible modifications of the approaches proposed in Chapter II are discussed. The summary and conclusions are presented in Chapter V.
CHAPTER II. PRODUCER MARKETING DECISIONS WHEN THE DISTRIBUTION OF PACKERS' PRICES IS KNOWN

In this chapter, it is assumed that the producer knows \( f_i(p) \), the distribution of prices offered by packers on day \( i \), for each day in his marketing horizon. Two methods of sampling will be considered: sequential sampling and nonsequential sampling. For each method of sampling, the producer's decisions will be examined for marketing horizons of two different lengths: one-day and multi-day.

Nonsequential Sampling

The term nonsequential sampling will be used to denote the following procedure used by the cattle producer. At the beginning of each marketing day before he has contacted any packers, the producer will decide how many packers he should contact on that particular day. After making this decision, the producer will then contact this specified number of packers. After all the packers have been contacted, the producer will decide whether or not to sell his cattle.

One-day horizon

Suppose that a producer knows that he has \( M \) feasible packers. To determine the order in which he should contact the packers, the costs associated with each packer should be considered. Note that contacting and recontacting costs will not depend on the quantity sold. However, transportation costs may change as the number of cattle transported changes. Contacting, recontacting, and transportation costs should be
found per hundredweight (cwt.) given the specific number of cattle the producer wishes to sell. Let total cost per cwt. be defined as the sum of contacting, recontacting, and transportation costs per cwt. Now, the producer should order his feasible packers from lowest to highest total cost per cwt. The producer should contact those packers with the lowest costs per cwt. first.

To put some of these concepts into symbolic notation, let $m$ denote a packer's position in the producer's list, where $m = 1, 2, \ldots, M$. Recall that if $m = 1$ for a particular packer, that packer has the lowest total cost per cwt. of all feasible packers. Let $CC(m)$ denote the cumulative contact costs per cwt. after contacting $m$ packers, or the sum of the contact costs for packer 1 through packer $m$. $RC(m)$ will be used to denote the recontacting costs per cwt. for the $m^{th}$ packer. The transportation cost per cwt. for the $m^{th}$ packer will be denoted by $TC(m)$.

Because there is only one day in the producer's marketing horizon, let the distribution of prices offered by packers be represented by simply $f(p)$. To simplify derivation of future results, let $p_i$ represent the price offered by the $i^{th}$ packer on the producer's list. Hence,

$$f(p) = f(p_i) \text{ for all } i = 1, 2, \ldots, M.$$  

Also, define the net price per cwt. offered by the $i^{th}$ packer to be:

$$NP(i) = p_i - RC(i) - TC(i).$$

To find the optimal number of packers to contact, the producer must consider both the expected increase in net price and the expected increase
in cumulative contact costs when more packers are contacted. If only one packer is contacted, the net gain will be the net price offered by the first packer minus the cost of contacting the first packer. If two packers are contacted, the producer would sell to the packer offering the highest net price; and the net gain would be the higher net price minus the cumulative contact costs. Similarly, if n packers are contacted, the net gain would be the highest net price offered minus the cumulative contact costs associated with the first n packers in the producer's list. Symbolically,

\[ g(n) = \max [NP(1), NP(2), \ldots, NP(n)] - CC(n) \]

Because the producer will not know which packer will offer the highest net price before he starts contacting packers, he must consider the expected net gain:

\[ E(g(n)) = E(\max_{i=1,\ldots,n} [NP(i)]) - CC(n) \]

To illustrate the expected gain function, consider the following example. The producer has M feasible packers listed in order of total cost per cwt. Let \( C(i) = RC(i) + TC(i) \). Then, the expected gain, if only one packer is contacted is:
If two packers are contacted, the expected gain is:
\[
E(g(2)) = \int_{-\infty}^{\infty} f(p_1)f(p_2)\max_{i=1,2} [NP(i)]dp_1 dp_2 - CC(2)
\]
\[
= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(p_1)f(p_2)\max\{p_1-C(1), p_2-C(2)\}dp_1 dp_2 - CC(2).
\]

The expected gain if \( n \) packers are contacted is:
\[
E(g(n)) = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} f(p_1)f(p_2)\cdots f(p_n)\max\{p_1-C(1), \ldots, p_n-C(n)\}dp_1 \cdots dp_n - CC(n).
\]

Thus, the expected gain for contacting \( i \) packers where \( i = 1, 2, \ldots M \) can be found.

To find the optimal number of packers to contact, it is necessary to find the number of packers associated with the largest expected gain. 

Or,
\[
\max_{n} \{E(g(n))\} = \max_{n} \{\max_{i=1 \ldots n} [NP(i)] - CC(n)\}. \tag{2.2}
\]

The properties of the function \( E(g_n) \) must be examined so that a method for determining \( \max_{n} \{E(g(n))\} \) can be found. Note that \( E(g(n)) \) is composed of two functions: \( E(\max_{i=1 \ldots n} [NP(i)]) \) and \( CC(n) \). Recall that \( CC(n) \) is an increasing function of \( n \) because it is the sum of positive contact costs.
However, although the cost of contacting packer $i$ would probably increase as $i$ increased, this assumption was not made and does not necessarily hold. Therefore, $CC(n)$ is not necessarily concave or convex, and it is not possible to determine if $E(g(n))$ is always increasing or decreasing. Thus, a local maximum will not necessarily be a global maximum. Hence, to determine $\max[E(g(n))]$, it is necessary to enumerate all solutions $n$ to $E(g(n))$ for $n = 1, 2, \ldots, M$ and then choose the $n$ which maximizes $E(g(n))$.

**Multi-day horizon**

In the multi-day horizon instance, the optimal numbers of packers to contact for each day in the marketing horizon must be determined. After the optimal number of packers for each particular day has been contacted, the producer must decide whether he wants to sell to the packer offering the highest net price or whether he wants to wait and sell on a later day in his marketing horizon.

The order in which to contact packers for each day is determined in the same manner as in the one-day horizon problem. If costs change on different marketing days, then the order of the producer's list may change. However, for simplicity, assume that on each day the producer has $M$ feasible packers and that the order of the list remains the same for each day in the marketing horizon.

The multi-day horizon problem is, therefore, a sequential decision problem or a sequence of decisions over the marketing days. Dynamic programming, the method of backwards induction, can be used to solve this
problem because it decomposes the optimization over the entire marketing period into smaller subsets. These subsets are the individual marketing days, and optimization should be carried out for each marketing day.

Recall that the producer knows how many days are in his marketing horizon, say J. Let \( j \) represent the number of marketing days remaining until the last day in the marketing horizon. Also, let \( g_j \) represent the gain realized on the marketing day \( j \) if the optimal number of packers has been contacted on that day. Thus, \( g_0 \) will be the gain realized on the last marketing day, \( g_1 \) the next-to-last marketing day, and so on. Let \( NP_j \) be the highest net price offered on marketing day \( j \) after the optimal number of packers has been contacted on that day. The optimal policy takes the form of a lower bound, \( X_j \), on the acceptable highest net price such that on the \( j \)th marketing day,

- if \( NP_j \geq X_j \), then sell to the packer who offers the highest net price and
- if \( NP_j < X_j \), then wait.

However, the producer will not know \( g_j \) and \( NP_j \) until the end of the \( j \)th marketing day and, therefore, must use \( E(g_j) \) to find the optimal number of packers to contact on each day in the marketing horizon. Dynamic programming will be used to find \( E(g_j) \) and \( X_j \).

To solve this problem using dynamic programming, first suppose that it is the last marketing day, day 0, and the producer has not yet sold his cattle. By the assumptions made previously, the producer must sell his cattle on day 0 if he has not already done so. Thus, the best action for the producer to follow is to contact the optimal number of packers and to
sell to the packer offering the highest net price. Recall from Equation (2.1) in the one-day horizon problem that the expected gain after contacting $n$ packers is

$$E(g_0(n)) = E(\max_{i=1,2,\ldots,n} [NP(i)]) - CC(n). \quad (2.3)$$

Let $n_j$ denote the optimal number of packers to contact on day $j$. By Equation (2.2), $n_0$ can be found

$$\max_n [E(g_0(n))] = \max_n [E(\max_{i=1,2,\ldots,n} [NP(i)]) - CC(n)]. \quad (2.4)$$

Note that this is the same problem as the one-day horizon problem. Hence, to determine $\max_n [E(g_0(n))]$, it is necessary to enumerate all solutions to $E(g_0(n))$ for $n = 1, 2, \ldots, M$ and then choose the $n$ which maximizes $E(g_0(n))$.

Next, consider the situation on day 1. If the producer does not sell his cattle on day 1, he will incur a waiting cost, $W_0$. This waiting cost can include the cost of feeding the cattle and any other cost associated with waiting from marketing day 1 to marketing day 0. Similarly, the waiting cost from day $j$ to day $j-1$ will be denoted by $W_{j-1}$.

Now, assume that the optimal number of packers has been contacted on day 1. If the highest net price offered on day 1 is greater than the expected gain on day 0 minus the cost of waiting, $W_0$, the producer should sell on day 1 because his actual gain on day 1 is greater than what he would expect to gain by waiting until day 0. To simplify notation, let

$$EG_0 = E(\max_{i=1,2,\ldots,n_0} [NP(i)]) - CC(n_0) - W_0. \quad (2.5)$$

Now, these ideas can be restated as the following stopping rule
if \( NP_1 \geq EG_0 \), then sell to the packer who offers the highest net price and

if \( NP_1 < EG_0 \), then wait.

If this stopping rule is observed, the producer will maximize his gain by selling to the packer who offers the highest net price if this net price is greater than \( EG_0 \), but if \( EG_0 \) is greater than the highest net price, the producer will maximize his gain by waiting until day 0. However, before any packers are contacted, the producer will not know the highest net price offered. Thus, the expected gain on day 1 given that \( n \) packers have been contacted and that stopping rule (2.6) is followed is:

\[
E(g_1(n)) = E(\max_{i=1,2,\ldots,n} [NP(i),EG_0]) - CC(n). \tag{2.7}
\]

Similar to finding \( n_0 \) for day 0 by using Equation (2.4), \( n_1 \) is found as follows:

\[
\max(E(g_1(n))) = \max\{E(\max_{i=1,\ldots,n} [NP(i),EG_0]) - CC(n)\}. \tag{2.8}
\]

Again, it is necessary to enumerate all the solutions to \( E(g_1(n)) \) for \( n = 0,1,2,\ldots,M \) to determine \( \max(E(g_1(n))) \).

Similarly, on day 2 if the producer does not sell his cattle, he will incur a waiting cost of \( W_1 \). Let

\[
EG_1 = E(\max_{i=1,2,\ldots,n_1} [NP(i),EG_0]) - CC(n_1) - W_1. \tag{2.9}
\]

Again, the producer should sell his cattle if the highest net price offered on day 2 after the optimal number of packers has been contacted is greater than \( EG_1 \). Or the optimal stopping rule can be stated
if NP \_2 \geq EG\_1, then sell to the packer who offers the highest net price and

\[ (2.10) \]

if NP \_2 < EG\_1, then wait.

If this stopping rule is observed, the expected gain on day 2 when n packers are contacted is

\[ E(g\_2(n)) = E(\max_{i=1,2,\ldots,n} [NP(i), EG\_1]) - CC(n). \]  

(2.11)

To find \( n\_2 \),

\[ \max_{n} [E(g\_2(n))] = \max_{n} [E(\max_{i=1,2,\ldots,n} [NP(i), EG\_1]) - CC(n)]. \]  

(2.12)

By an inductive argument, the optimal stopping rule for day j is given by

if NP \_j \geq EG\_j-1, then sell to the packer who offers the highest net price and

if NP \_j < EG\_j-1, then wait.

And EG\_j-1 is given by the following equation

\[ EG\_j-1 = E(\max_{i=1,2,\ldots,n\_j-1} [NP(i), EG\_j-2]) - CC(n\_j-1) - W\_j-1. \]

Also, the optimal number of packers to contact each day is given by

\[ \max_{n} [E(g\_j(n))] = \max_{n} [E(\max_{i=1,2,\ldots,n} [NP(i), EG\_j-1]) - CC(n)]. \]

To determine \( \max_{n} [E(g\_j(n))] \), it is necessary to enumerate all solutions to \( n \)
E\( (g\_j(n)) \) for \( n = 0,1,2,\ldots,M \) and then choose the \( n \) which maximizes
E\( (g\_j(n)) \). In this manner, the optimal number of packers to contact on
day \( j \) and the stopping rule for day \( j \) can be found for all marketing days in the producer's horizon or for \( j = 0,1,\ldots,J-1 \).

Sequential Sampling

The term sequential sampling will be used to denote the following procedure used by the cattle producer. In the one-day horizon problem, the producer must decide after contacting each packer whether to sell his cattle or whether to wait and contact another packer. In the multi-day problem, the producer must decide which day in the marketing horizon to begin contacting packers. Once he has begun contacting packers, the producer must decide after contacting each packer whether to sell his cattle, whether to wait and contact another packer on that marketing day, or whether to wait and contact packers on a later day in the marketing horizon.

One-day horizon

Suppose that the producer knows that he has \( M \) feasible packers. As in the one-day nonsequential sampling problem, the producer should contact those feasible packers associated with the lowest total cost per cwt. first. That is, the order in which to contact packers is determined in the same manner as in the one-day nonsequential sampling problem. Because the producer must sell on this single day in his marketing horizon, he must contact at least one packer. Thus, after contacting the first packer on the list, he must decide whether to sell to this packer or whether to contact the second packer on the list. If he decides to
contact the second packer, he must decide whether to sell to the packer offering the highest net price or whether he should wait and contact the third packer. Likewise, if the producer has contacted the $m^{th}$ packer, he must decide whether to contact the $(m+1)^{st}$ packer or whether to sell to the packer offering the highest net price.

The one-day sequential sampling problem is a sequence of decisions in time. Thus, once again, dynamic programming can be used to solve this problem. The optimization over the entire marketing day can be decomposed into $M$ subsets. Each subset corresponds to the decision, after a particular packer is contacted, of whether to contact another packer or whether to sell to the packer who has offered the highest net price.

To solve this problem by using dynamic programming, first consider the situation in which the producer has contacted $M$ packers, i.e., he has contacted all of the packers. In this situation, the producer will sell to the packer offering the highest net price. Let $NP^*_i$ represent the highest net price offered after contacting $i$ packers. Thus $NP^*_m$ will represent the highest net price offered after contacting $M$ packers. If $M$ packers have been contacted and the highest net price offered is $NP^*_M$, the expected gain under the optimal policy, or the policy yielding the greatest expected net gain, can be represented by

$$g(M,NP^*_M) = NP^*_M - CC(M).$$

In other words, the greatest gain will be the highest net price offered after contacting $M$ packers minus the costs of contacting all of the packers.
If (M-1) packers have been contacted, the producer must decide whether to sell to the packer offering the highest bid, \( NP^*_M \), or whether to wait and contact the last packer. If the producer sells to the packer offering the highest bid, \( NP^*_M \), the gain will be

\[
NP^*_M - CC(M-1).
\]

But if the producer decides to wait and contact the last packer, the expected gain is:

\[
E[max\{NP^*_M - CC(M)\}, \{NP(M) - CC(M)\}]^{M-1}.
\]

However, note that this expression is equivalent to

\[
E[g<M, max\{NP^*_M, NP(M)\}] = \int_{-\infty}^{\infty} f(p_M) \max[NP(M), NP^*_M] dp_M - CC(M).
\]

Thus, if (M-1) packers have been contacted, the expected gain under the optimal policy is

\[
g(M-1, NP^*_M) = \max\{NP^*_M - CC(M-1), E[g<M, max\{NP^*_M, NP(M)\}]\}.
\]

The optimal stopping rule can be derived from this expression.

If

\[
\{NP^*_M - CC(M-1)\} \geq E[g<M, max\{NP^*_M, NP(M)\}],
\]

then sell to the packer offering the highest net price.

---

1. The symbols "<" and ">" have been and will subsequently be used as left and right brackets.
If 
\[
\{NP_{M-1} - CC(M-1)\} < E[g<M, \max\{NP_{M-1}^*, NP(M)\}] 
\]
then wait and contact the next packer.

If \((M-2)\) packers have been contacted, the producer must decide whether to sell to the packer offering the highest bid, \(NP_{M-2}^*\), or whether to wait and contact the \((M-1)^{st}\) packer. If the producer sells to the packer offering the highest bid, \(NP_{M-2}^*\), the gain will be

\[
NP_{M-2}^* - CC(M-2). 
\]

But, if the producer decides to wait and contact the \((M-1)^{st}\) packer, the expected gain is

\[
E[\max\{E[\max\{NP_{M-2}^* - CC(M)\}, \{NP(M-1) - CC(M-1)\}\}, \{NP(M-1) - CC(M)\} \}]. \]

\[
= E[g<(M-1), \max \{NP_{M-2}^*, NP(M-1)\}] \]

where

\[
E[\max\{NP_{M-2}^* - CC(M-1)\}, \{NP(M-1) - CC(M-1)\}] 
= \int_{-\infty}^{\infty} f(p_{M-1}) \max[NP(M-1), NP_{M-2}^*] dp_{M-1} - CC(M-1)
\]

\(^1\) The symbol "\(\hat{\}}\) has been and will subsequently be used to represent brackets.
and
\[ E(\max\{NP_{M-2}^* - CC(M)\}, \{NP(M-1) - CC(M)\}, \{NP(M) - CC(M)\}) \]
\[ = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(p_{M-1})f(p_M) \max_{i=M-1,M} [NP(i), NP_{M-2}^*] dp_{M-1} dp_M - CC(M). \]

Thus, if (M-2) packers have been contacted, the expected gain under the optimal policy is
\[ g\{M-2, NP_{M-2}^*\} = \max\{NP_{M-2}^* - CC(M-2)\}, E[g\{M-1, \max\{NP_{M-2}^*, NP(M-1)\}\}] \]
The optimal stopping rule can be derived from this expression.
If
\[ \{NP_{M-2}^* - CC(M-2)\} > E[g\{M-1, \max\{NP_{M-2}^*, NP(M-1)\}\}] \]
then sell to the packer offering the highest net price.
If
\[ \{NP_{M-2}^* - CC(M-2)\} < E[g\{M-1, \max\{NP_{M-2}^*, NP(M-1)\}\}] \]
then wait and contact the next packer.

Consider the situation when m packers have been contacted. The producer must decide whether to sell to the packer offering the highest bid, NP_m^*, or whether to wait and contact the (m+1)st packer. If the producer sells his cattle to the packer offering the highest bid, NP_m^*, the gain will be
\[ NP_m^* - CC(m). \]
If the producer decides to wait and contact the next packer, the expected gain is
\[ E[g<(m+1), \max_{m} \{NP^*, NP(m+1)\}] \]

\[ = E[\max(E) \max_{m} \{NP^*-CC(m+1), \{NP(m+1)-CC(m+1)\}\}] \]

\[ E[\max(E) \max_{m} \{NP^*-CC(m+2), \{NP(m+1)-CC(m+2), \{NP(m+2)-CC(m+2)\}\}] \]

\[ \ldots, E[\max(E) \max_{m} \{NP^*-CC(M), \{NP(m+1)-CC(M), \ldots \{NP(M)-CC(M)\}] \] \]

where

\[ E[\max(E) \max_{m} \{NP^*-CC(m+1), \{NP(m+1)-CC(m+1)\}] \]

\[ = \int_{-\infty}^{\infty} f(p_{m+1}) \max_{m} [NP(m+1), NP^*] dp_{m+1} - CC(m+1) \]

and

\[ E[\max(E) \max_{m} \{NP^*-CC(m+2), \{NP(m+1)-CC(m+2), \{NP(m+2)-CC(m+2)\}] \]

\[ = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(p_{m+1}) f(p_{m+2}) \max_{i=m+1, m+2} [NP(i), NP^*] dp_{m+1} dp_{m+2} - CC(m+2) \]

and so forth until

\[ E[\max(E) \max_{m} \{NP^*-CC(M), \{NP(m+1)-CC(M), \ldots \{NP(M)-CC(M)\}] \]

\[ = \int_{-\infty}^{\infty} \ldots \int_{-\infty}^{\infty} f(p_{m+1}) \ldots f(p_{M}) \max_{i=m+1, \ldots, M} [NP(i), NP^*] dp_{m+1} \ldots dp_{M} - CC(M). \]

Thus, if \( m \) packers have been contacted, the expected gain under the optimal policy is

\[ g(m, NP^*) = \max[\{NP^*-CC(m)\}, E[g<(m+1), \max_{m} \{NP^*, NP(m+1)\}]]]. \]
The optimal stopping rule can be stated as follows.

If
\[
\{NP^*_m - CC(m)\} > E[g<(m+1), \max\{NP^*_m, NP(m+1)\} >]
\]
then sell to the packer offering the highest net price.

If
\[
\{NP^*_m - CC(m)\} < E[g<(m+1), \max\{NP^*_m, NP(m+1)\} >]
\]
then wait and contact the next packer.

**Multi-day horizon**

In the multi-day horizon problem, the producer must first decide which day in the marketing horizon to begin contacting packers. Then, after each packer is contacted, the producer must decide whether to sell his cattle, whether to wait and contact another packer on that marketing day, or whether to wait and contact packers on a later day in the marketing horizon.

To solve the multi-day horizon problem, some assumptions must be made. Again, assume that the producer has $M$ feasible packers. The producer should contact these packers in the same order as was determined in the one-day nonsequential sampling problem. That is, the producer should contact those feasible packers associated with the lowest total cost per cwt. first. The order of the list of feasible packers is assumed to be the same for each day in the marketing horizon. Also, assume that the producer knows how many days are in his marketing horizon, say $J$. Let $j$ represent the number of marketing days remaining until the last day in the marketing horizon. The distribution of prices offered by packers on
marketing day \( j \) will be represented by \( f_j(p) \). Let \( p_m \) represent the price offered by the \( m \)th packer on the producer's list. Hence, for each \( j \) where \( j = 0,1,\ldots,(J-1), \)

\[
 f_j(p) = f_j(p_m) \text{ for all } m = 1,2,\ldots,M.
\]

The multi-day horizon problem is a sequence of decisions which can again be solved by using dynamic programming. The solution to the entire problem can be found by decomposing it into individual marketing days. First, decisions faced by the producer on the last marketing day, day 0, will be examined, and the optimal policy for this marketing day will be found. Then, given the optimal policy on the last day, decisions faced by the producer on day 1 will be examined, and the optimal policy will be found. This procedure is continued until the optimal policy for the entire marketing horizon has been determined.

Suppose that it is the last marketing day, day 0, and the producer has contacted \( M \) packers. Let \( NP^*_{0,m} \) represent the highest net price offered after contacting \( m \) packers on day 0. Thus, \( NP^*_{0,M} \) is the highest net price offered after contacting \( M \) packers. The expected gain under the optimal policy on day 0 after \( M \) packers have been contacted, disregarding any costs incurred on previous days, is

\[
 g_0(M,NP^*_{0,M}) = NP^*_{0,M} - CC(M).
\]

If \((M-1)\) packers have been contacted on day 0, the producer must decide whether to sell to the packer offering the highest bid, \( NP^*_{0,M-1} \), or whether to wait and contact the last packer. If the producer sells to the
packer offering the highest bid, \( NP_{0,M-1}^* \) the gain will be

\[ NP_{0,M-1}^* - CC(M-1). \]

But, if the producer decides to wait and contact the last packer, the expected gain is

\[
E[\max<\{NP_{0,M-1}^*-CC(M)\}, \{NP(M)-CC(M)\}>] = \int_{-\infty}^{\infty} f_0(p_M) \max[NP(M), NP_{0,M-1}^*] dp_M - CC(M)\]

Thus, if \((M-1)\) packers have been contacted on day 0, the expected gain under the optimal policy disregarding any costs incurred on previous marketing days is

\[
g_0[(M-1), NP_{0,M-1}^*] = \max[\{NP_{0,M-1}^*-CC(M-1)\}, \]
\[
E[\max<\{NP_{0,M-1}^*-CC(M)\}, \{NP(M)-CC(M)\}>] = \max[\{NP_{0,M-1}^*-CC(M-1)\}, E[g_M, max\{NP_{0,M-1}^*, NP(M)\}>]\]

The optimal stopping rule can be derived from this expression.

If

\[
\{NP_{0,M-1}^*-CC(M-1)\} \geq E[g_M, max\{NP_{0,M-1}^*, NP(M)\}]\]

then sell to the packer offering the highest net price.

If

\[
\{NP_{0,M-1}^*-CC(M-1)\} < E[g_M, max\{NP_{0,M-1}^*, NP(M)\}]\]
then wait and contact the next packer.

Suppose it is day 0 and m packers have been contacted. The producer must decide whether to sell to the packer offering the highest bid, \( NP^\ast_{0,m} \), or whether to wait and contact the next packer. The expected gain under the optimal policy is

\[
g_0[y,m,NP^\ast_0,m] = \max \{ \{NP^\ast_{0,m} - CC(m)\}, E[g_0^<(m+1), \max \{NP^\ast_{0,m}, NP(m+1)\}] \}.\]

Note that

\[
E[g_0^<(m+1), \max \{NP^\ast_{0,m}, NP(m+1)\}]^\dagger = E[\max(E[\max<NP^\ast_{0,m} - CC(m+1), NP(m+1) - CC(m+1)>\}, NP(m+1) - CC(m+1)>\}],
\]

\[
E[\max(E[\max<NP^\ast_{0,m} - CC(m+2), NP(m+1) - CC(m+2), NP(m+2) - CC(m+2)>\}, NP(m+2) - CC(m+2)>\}],
\]

\[
\ldots, E[\max<NP^\ast_{0,m} - CC(M), NP(m+1) - CC(M), \ldots, NP(M) - CC(M)>\}
\]

where

\[
E[\max<NP^\ast_{0,m} - CC(m+1), NP(m+1) - CC(m+1)>\}
\]

\[
= \int_{-\infty}^{\infty} f_0(p_{m+1}) \max[NP(m+1), NP^\ast_{0,m}] dp_{m+1} - CC(m+1)
\]

and

\[
E[\max<NP^\ast_{0,m} - CC(m+2), NP(m+1) - CC(m+2), NP(m+2) - CC(m+2)>\}
\]

\[
= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_0(p_{m+1}) f_0(p_{m+2}) \max_{i=m+1, m+2} [NP(i), NP^\ast_{0,m}] dp_{m+1} dp_{m+2} - CC(m+2)
\]

and so forth until
The corresponding optimal stopping rule is:

If

\[ \{NP^*_0,CC(m)\} \geq \mathbb{E}\left[ g_0 <(m+1), \max\{NP^*_0,NP(m+1)\}\right] \]

then sell to the packer offering the highest net price.

If

\[ \{NP^*_0,CC(m)\} < \mathbb{E}\left[ g_0 <(m+1), \max\{NP^*_0,NP(m+1)\}\right] \]

then wait and contact the next packer.

Suppose it is the last marketing day, day 0, and the producer has not yet contacted any packers. Let the expected gain under the optimal policy when no packers have been contacted be represented by

\[ g_0(0,0) = \mathbb{E}\left[ g_0 <1, NP(1)\right] \]

\[ = \mathbb{E}\left[ \max\{E\{NP(1)-CC(1)\}, E\max\{E\{NP(1)-CC(2)\}, \{NP(2)-CC(2)\}\}, \ldots, E\max\{\{NP(1)-CC(M)\}, \{NP(2)-CC(M)\}, \ldots, \{NP(M)-CC(M)\}\}\right] \]

and where

\[ E\{NP(1)-CC(1)\} = \int_{-\infty}^{\infty} f(p_1)dp_1 - CC(1) \]
and

\[
E\{\max<\{NP(1)-CC(2)\}, \{NP(2)-CC(2)\}>\} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_0(p_1) f_0(p_2) \max_{i=1,2} [NP(i)] dp_1 dp_2 - CC(2)
\]

and so forth until

\[
E\{\max<\{NP(1)-CC(M)\}, \{NP(2)-CC(M)\}, \ldots, \{NP(M)-CC(M)\}>\} = \int_{-\infty}^{\infty} \ldots \int_{-\infty}^{\infty} f_0(p_1) \ldots f_0(p_M) \max_{i=1,2,\ldots,M} [NP(i)] dp_1 \ldots dp_M - CC(M).
\]

The producer has no decision to make in this situation because he must sell his cattle on the last marketing day if he has not sold them on a previous marketing day. Thus, the producer will contact the first packer and continue his search following the optimal stopping rules developed for day 0.

If it is day 1 and the producer has contacted M packers, the producer can either sell to the packer offering the highest price, \(NP_{1,M}^*\), or he can wait until the next day. The expected gain if the producer sells to the packer offering the highest price is

\[NP_{1,M}^* - CC(M).\]

The producer can also wait until day 0 to sell his cattle. If he does this, he will incur a waiting cost of \(W_0\). The expected gain if the producer waits until day 0 is

\[g_0(0,0) - CC(M) - W_0.\]
Thus, the expected gain under the optimal policy is

$$g_1[M, NP^*, \lambda, \mu] = \max\{NP^*, \lambda - CC(M)\}, \{g_0(0,0)-CC(M) - W_0\}.$$  

The corresponding optimal stopping rule can be stated as follows.

If

$$\{NP^*, \lambda - CC(M)\} \geq \{g_0(0,0)-CC(M) - W_0\},$$

then sell to the packer offering the highest price.

If

$$\{NP^*, \lambda - CC(M)\} < \{g_0(0,0)-CC(M) - W_0\}$$

then wait until the next day.

Suppose that it is day 1 and the producer has contacted (M-1) packers. The producer could sell to the packer offering the highest price, NP^*, M-1, he could wait and contact the M^th packer on day 1, or he could wait until day 0 and not contact the M^th packer on day 1. The expected gain if the producer sells to the packer offering the highest price is

$$NP^*, M-1 - CC(M-1).$$

If the producer waits and contacts the M^th packer on day 1, the expected gain is

$$E[g_1 < M, \max\{NP^*, M-1, NP(M)\}]$$

If the producer waits until day 0, the expected gain is

$$g_0(0,0) - CC(M-1) - W_0.$$  

Therefore, the expected gain if the optimal policy is followed is
Note that

\[
E\{g_1 < M, \max\{NP^*_{1, M-1}, NP(M)\}\}
\]

\[
= E\{\max(E\{\max\{NP^*_{1, M-1}-CC(M)\}, \{NP(M)-CC(M)\}\},
\{g_0(0,0)-CC(M)-W_0\})\}
\]

where

\[
E\{\max\{NP^*_{1, M-1}-CC(M)\}, \{NP(M)-CC(M)\}\}\]

\[
= \int_{-\infty}^{\infty} f_{\hat{p}}(\hat{p}) \max\{NP(M), NP^{*}_{1, M-1}\} \, dp_{\hat{p}} - CC(M).
\]

The stopping rule can be stated as follows.

If

\[
NP^{*}_{1, M-1}-CC(M) \geq E\{g_1 < M, \max\{NP^*_{1, M-1}, NP(M)\}\}
\]

and

\[
NP^{*}_{1, M-1}-CC(M) \geq \{g_0(0,0)-CC(M)-W_0\},
\]

then sell to the packer offering the highest net price.

If

\[
NP^{*}_{1, M-1}-CC(M) < E\{g_1 < M, \max\{NP^*_{1, M-1}, NP(M)\}\}\]

and

\[
g_0(0,0)-CC(M)-W_0 < E\{g_1 < M, \max\{NP^*_{1, M-1}, NP(M)\}\}\]

then contact the \(M^{th}\) packer.
If
\[ \{NP^*_1, M-1 - CC(M-1)\} < E\left\{ g_1 < M, \max\{NP^*_1, M-1, NP(M)\} \right\} \]
and
\[ E\left\{ g_1 < M, \max\{NP^*_1, M-1, NP(M)\} \right\} \leq \{g_0(0,0) - CC(M-1) - W_0\}, \]
then wait until day 0.

Consider the more general case on day 1 when the producer has contacted \( m \) packers. The producer could sell to the packer offering the highest bid, \( NP^*_1, m \), he could contact the \( (m+1) \) packer on day 1, or he could wait until day 0 and not contact anymore packers on day 1.

The expected gain under the optimal policy is
\[
g_1[m, NP^*_1, m] = \max\{NP^*_1, M - CC(m)\}, E[g_1 < (m+1), \max\{NP^*_1, m, NP(m+1)\}] \}
\[
\{g_0(0,0) - CC(m) - W_0\}. \]

Note that
\[
E[g_1 < (m+1), \max\{NP^*_1, m, NP(m+1)\}] = E[\max(E\left\{ g_1 < (m+2), \max\{NP^*_1, m, NP(m+1), NP(m+2)\} \right\} \}
\[
\{g_1(0,0) - CC(m) - W_0\})] \]

where
\[
E\left\{ g_1 < (m+1), \max\{NP^*_1, m, NP(m+1)\} \right\} \]
\[
= \int_{-\infty}^{\infty} f_{m+1}(p_{m+1}) \max\{NP(m+1), NP^*_1, m\} dp_{m+1} - CC(m+1). \]
The corresponding stopping rule can be stated as follows.

If
\[ \{NP^*_1, m - CC(m)\} \geq E[g_1^{<(m+1), \max\{NP^*_{1, m}, NP(M+1)\}}] \]
and
\[ \{NP^*_1, m - CC(m)\} \geq \{g_0(0,0) - CC(m) - W_0\}, \]
then sell to the packer offering the highest net price.

If
\[ \{NP^*_1, m - CC(m)\} < E[g_1^{<(m+1), \max\{NP^*_{1, m}, NP(M+1)\}}] \]
and
\[ \{g_0(0,0) - CC(m) - W_0\} < E[g_1^{<(m+1), \max\{NP^*_{1, m}, NP(M+1)\}}], \]
then contact the \((m+1)_{\text{st}}\) packer.

If
\[ \{NP^*_1, m - CC(m)\} < \{g_0(0,0) - CC(m) - W_0\} \]
and
\[ E[g_1^{<(m+1), \max\{NP^*_{1, m}, NP(M+1)\}}] < \{g_0(0,0) - CC(m) - W_0\}, \]
then wait until day 0.

Suppose it is day 1 and no packers have been contacted. The producer can either wait until day 0, or he can begin contacting packers on day 1. Thus, the expected gain under the optimal policy is
\[ g_1(0,0) = \max[E[g_{1, l}^{<l}, NP(l)>{\dagger}, \{g_0(0,0) - W_0\}]. \]

Note that
where
\[ E[g^l_{1,<1,NP(1)>}] = E \{ \max(E\{NP(1)-CC(1)\}, E[g^l_{1,<2,NP(1),NP(2)>}] \}, \]
\[ \{g_0(0,0)-CC(1)-W_0\})] \]

The corresponding stopping rule can be derived from this expression.

If
\[ E[g^l_{1,<1,NP(1)>}] \geq \{g_0(0,0)-W_0\}, \]
then contact the first packer on day 1.

If
\[ E[g^l_{1,<1,NP(1)>}] \leq \{g_0(0,0)-W_0\}, \]
then wait until the last day.

Consider the general case when it is marketing day j and the m th packer has been contacted. If 1\(\leq m\leq M-1\) and 1\(\leq j\leq J-1\), the producer must choose among three alternative actions. He could sell to the packer offering the highest net price, \(NP^*_j,m\), he could wait and contact the (m+1)st packer on day j, or he could cease contacting packers on day j and wait until day (j-1). Therefore, the expected gain under the optimal policy is
\[ g_j[m,NP^*_j,m] = \max\{NP^*_j,m-CC(m), E[g^l_j<(m+1), m], \max\{NP^*_j,m,NP(m+1)\}] \}, \]
\[ \{g_{j-1}(0,0)-CC(m)-W_{j-1}\})] \]
where
\[ g_{j-1}(0,0) = \max[E[g^l_{j-1,<1,NP(1)>}]}, \{g_j-2(0,0)-W_{j-2}\}] \].
The corresponding optimal stopping rule can be stated as follows. If
\[ \{NP^* - CC(m)\} < \mathbb{E}\{g_j < (m+1), \max\{NP^*, NP(m+1)\}\} \]
and
\[ \{g_{j-1}(0,0) - CC(m) - W_{j-1}\} < \mathbb{E}\{g_j < (m+1), \max\{NP^*, NP(m+1)\}\} \]
then contact the \((m+1)\)st packer on day \(j\).

If
\[ \{NP^* - CC(m)\} \geq \mathbb{E}\{g_j < (m+1), \max\{NP^*, NP(m+1)\}\} \]
and
\[ \{NP^* - CC(m)\} \geq \{g_{j-1}(0,0) - CC(m) - W_{j-1}\}, \tag{2.14} \]
then sell to the packer offering the highest net price on day \(j\).

If
\[ \{NP^* - CC(m)\} < \{g_{j-1}(0,0) - CC(m) - W_{j-1}\} \]
and
\[ \mathbb{E}\{g_j < (m+1), \max\{NP^*, NP(m+1)\}\} < \{g_{j-1}(0,0) - CC(m) - W_{j-1}\}, \]
then wait until day \((j-1)\). If \(m=0\) and \(1 \leq j < (J-1)\), the producer can choose between two alternative actions. He can either contact the first packer on day \(j\), or he can wait until day \((j-1)\). The expected gain under the optimal policy is

\[ g_j(0,0) = \max\{\mathbb{E}\{g_j < 1, NP(1)\}, \{g_{j-1}(0,0) - W_{j-1}\}\} \tag{2.15} \]
The corresponding stopping rule can be derived from this expression. If
\[ E[g_j < 1, NP(1)>] \geq \{g_{j-1}(0,0) - W_{j-1}\} \]
then contact the first packer on day j. If
\[ E[g_j < 1, NP(1)>] < \{g_{j-1}(0,0) - W_{j-1}\} \] (2.16)
then wait until day j-1.

The expected gain under the optimal policy and the corresponding
stopping rules when j=0 and 0\leq m\leq M have already been derived.

Summary

Regardless of the method of sampling used, the order in which the
producer should contact his feasible packers is the same. Given the
specific number of cattle the producer wishes to sell, contacting, re-
contacting, and transportation costs should be found per cwt. If
total cost per cwt. is defined to be the sum of contacting, recontacting,
and transportation costs per cwt., the producer should order his feasible
packers from lowest to highest total cost per cwt. The producer should
contact those packers with the lowest costs per cwt. first.

If the producer uses nonsequential sampling and has a one-day
marketing horizon, the optimal number of packers to contact can be found
by using Equation (2.2):

\[ \max[E(g(n))] = \max[E(\max_{n \geq 1,2,\ldots,n} [NP(1)]) - CC(n)]. \] (2.17)
Recall that

\[ NP(i) = p_i - TC(i) - RC(i) \]

or that net price offered by packer \( i \) is the price minus the transportation and recontacting costs associated with packer \( i \). Also,

\[
E(\max_{i=1,2,\ldots,n} [NP(i)])
\]

\[ = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} f(p_1)f(p_2)\cdots f(p_n) \max_{i=1,2,\ldots,n} [NP(i)] dp_1 dp_2 \cdots dp_n. \]

To determine the optimal number of packers to contact, it is necessary to enumerate all the solutions to \( E(g(n)) \) in Equation (2.17) for \( n = 1, 2, \ldots, M \) and then choose the \( n \) which maximizes \( E(g(n)) \).

To find the optimal number of packers to contact, \( n_j \), for each marketing day \( j \) in the horizon if the producer uses nonsequential sampling and has a multi-day horizon, dynamic programming is applied to the one-day horizon problem. The optimal stopping rule for day \( j \) in the marketing horizon is given by

- if \( NP_j \geq EG_{j-1} \), then sell to the packer who offers the highest net price and
- if \( NP_j < EG_{j-1} \), then wait.

Recall that \( NP_j \) is the highest net price offered on marketing day \( j \) after the optimal number of packers has been contacted on that day and that \( EG_{j-1} \) is the expected net gain if the producer waits until the next marketing day and follows the optimal policy for all the days following
day j. An expression for $E_{j-1}$ is given by the following equation

$$E_{j-1} = E(\max_{i=1,2,\ldots,n_j} [NP(i), E_{j-2}]) - CC(n_{j-1}) - w_{j-1}.$$ 

The optimal number of packers to contact on marketing day $j$ is given by

$$\max_{n}[E(g_j(n))] = \max_{n}[E(\max_{i=1,2,\ldots,n} [NP(i), E_{j-1}]) - CC(n)] \quad (2.18)$$

where

$$E(\max_{i=1,2,\ldots,n} [NP(i), E_{j-1}]) = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} f(p_1)f(p_2)\cdots f(p_n)_{\max_{i=1,2,\ldots,n} [NP(i), E_{j-1}]}dp_1dp_2\cdots dp_n.$$ 

To determine $n_j$, it is necessary to enumerate all the solutions to $E(g_j(n))$ in Equation (2.18) for $n = 0,1,2,\ldots,M$ and then choose the $n$ which maximizes $E(g_j(n))$.

If the producer uses sequential sampling and has a one-day marketing horizon, the following procedure should be used. Suppose that the producer has contacted $m$ packers and that the highest net price offered is $NP_m^*$. If the producer stops sampling and accepts the bid $NP_m^*$, the net gain will be

$$NP_m^* - CC(m).$$ 

If the producer decides to wait and contact the next packer, the expected gain can be represented by

$$E[g<(m+1), \max_{m}[NP_m^*, NP(m+1)]].$$
Thus, the expected gain under the optimal policy is
\[ g(m, NP^*_m) = \max \{ NP^*_m - CC(m), E[g(m+1), \max \{ NP^*_m, NP(m+1) \}] \}. \]
The corresponding optimal stopping rule is the following: if
\[ \{ NP^*_m - CC(m) \} \geq E[g(m+1), \max \{ NP^*_m, NP(m+1) \}], \]
then sell to the packer offering the highest net price and if
\[ \{ NP^*_m - CC(m) \} < E[g(m+1), \max \{ NP^*_m, NP(m+1) \}], \]
then wait and contact the next packer.

The following procedure should be used if the producer uses sequential sampling and has a multi-day marketing horizon. If it is marketing day \( j \) and the \( m \)th packer has been contacted where \( 1 \leq m \leq (M-1) \) and \( 1 \leq j \leq (J-1) \), the producer can choose among three alternative actions. He could sell to the packer offering the highest net price, \( NP^*_j, m \) and receive a net gain of
\[ \{ NP^*_j, m - CC(m) \}. \]
Or he could wait and contact the \((m+1)\)st packer on day \( j \) and receive an expected net gain of
\[ E[g_j(m+1), \max \{ NP^*_j, m, NP(m+1) \}] \].
If the producer ceases contacting packers on day \( j \) and waited until day \( j-1 \), the expected net gain under the optimal policy is
\[ \{ g_{j-1}(0,0) - CC(m) - W_{j-1} \}. \]
Therefore, the expected gain under the optimal policy is
\[ g_j[m, NP^*_j, m] = \max\{NP^*_j - CC(m), \mathbb{E}g_j<(m+1), \max\{NP^*_j, m, NP(m+1)\}\}; \]
\[ \{g_{j-1}(0,0)-CC(m)-W_{j-1}\}. \]

The corresponding optimal stopping rule can be stated as follows. If
\[ \{NP^*_j - CC(m)\} < \mathbb{E}g_j<(m+1), \max\{NP^*_j, m, NP(m+1)\}\]
and
\[ \{g_{j-1}(0,0)-CC(m)-W_{j-1}\} < \mathbb{E}g_j<(m+1), \max\{NP^*_j, m, NP(m+1)\}\]
then contact the \((m+1)^{st}\) packer on day \(j\). If
\[ \{NP^*_j - CC(m)\} \geq \mathbb{E}g_j<(m+1), \max\{NP^*_j, m, NP(m+1)\}\]
and
\[ \{NP^*_j - CC(m)\} \geq \{g_{j-1}(0,0)-CC(m)-W_{j-1}\}, \]
then sell to the packer offering the highest price on day \(j\). If
\[ \{NP^*_j - CC(m)\} < \{g_{j-1}(0,0)-CC(m)-W_{j-1}\} \]
and
\[ \mathbb{E}g_j<(m+1), \max\{NP^*_j, m, NP(m+1)\}\]
\[ < \{g_{j-1}(0,0)-CC(m)-W_{j-1}\}, \]
then wait until day \((j-1)\).

On the other hand, if \(m=0\) and \(1\leq j\leq (J-1)\), the producer can choose between two alternative actions. He can contact the first packer on day \(j\) and foresee an expected gain of
$E\{g_j<1,\text{NP}(1)>\}$,
or he can wait until day $j-1$ and foresee an expected gain of

$$\{g_{j-1}(0,0)-W_{j-1}\}.$$

Thus, the expected gain under the optimal policy is

$$g_j(0,0) = \max\{E\{g_j<1,\text{NP}(1)>\}, \{g_{j-1}(0,0)-W_{j-1}\}\}.$$

The optimal stopping rule can be derived from this expression.

If

$$E\{g_j<1,\text{NP}(1)>\} \geq \{g_{j-1}(0,0)-W_{j-1}\},$$

then contact the first packer on day $j$. If

$$E\{g_j<1,\text{NP}(1)>\} < \{g_{j-1}(0,0)-W_{j-1}\},$$

then wait until day $j-1$.

If it is the last day in the marketing horizon, or $j=0$, the producer should use the procedures developed for the one-day horizon problem when using sequential sampling.
CHAPTER III. PRODUCER MARKETING DECISIONS WHEN THE PARAMETERS OF
THE DISTRIBUTION OF PACKERS' PRICES ARE UNKNOWN

In this chapter it is assumed that the producer has imperfect knowledge about the k parameters, \( \hat{\theta} = (\hat{t}_{1i}, \hat{t}_{2i}, \ldots, \hat{t}_{ki}) \), of \( f_i(p) \), the distribution of prices offered by packers on marketing day i. However, because the producer has gathered various kinds of information, he is able to estimate the distributions of these k parameters for each of the i marketing days. The producer may wish to rely solely on these estimates to determine how many packers to contact on each day in his marketing horizon in a one-stage sampling approach. On the other hand, he may use a two-stage sampling approach by collecting some bids each day before he even considers selling so that he may be more certain about the distribution of prices. These two approaches will be discussed in this chapter.

As in Chapter II, both sequential sampling and nonsequential sampling will be considered. One-stage sampling and two-stage sampling will be discussed for both the nonsequential, one-day horizon problem and the nonsequential, multi-day horizon problem. Only one-stage sampling will be considered for the sequential, one-day and multi-day horizons.

Nonsequential Sampling

If the producer relies solely on his initial estimates of the distributions of the parameters, he will use these estimates at the beginning of each marketing day to decide how many packers he should contact on that particular day. After making this decision, the producer
will then contact this specified number of packers. After all the packers have been contacted, the producer will decide whether or not to sell his cattle. This procedure will be called one-stage sampling.

If the producer collects some bids each day before he considers selling, he must first decide how many packers to contact in this first stage. After he has contacted this specified number, he must use this information to determine how many additional packers to contact in the second stage. After packers have been contacted in both stages, the producer must decide whether or not to sell his cattle.

**One-day horizon**

The producer has imperfect knowledge about the k parameters, \( \hat{T} = (t_1, t_2, \ldots, t_k) \), of \( f(p) \), the distribution of prices offered by the packers on the only day in the marketing horizon. However, the producer can express his estimates of the unknown parameters in the form of a prior probability density function

\[ g(t_1, t_2, \ldots, t_k). \]

Since \( \hat{T} \) is unknown, let \( f(p|t_1, t_2, \ldots, t_k) \) denote the probability density function for the prices given the parameters. Again, it is assumed that the producer has M feasible packers listed in order of total costs. Carrying over the notation in Chapter II, let \( p_i \) represent the price offered by the \( i^{th} \) packer on the producer's list. Hence,

\[ f(p|t_1, t_2, \ldots, t_k) = f(p_i|t_1, t_2, \ldots, t_k) \text{ where } i = 1, 2, \ldots, M. \]

If a random sample of size n is taken from a distribution having p.d.f.

f(p | t_1, t_2, \ldots, t_k), the joint p.d.f. of p_1, p_2, \ldots, p_n is

\[ f(p_1, p_2, \ldots, p_n | t_1, t_2, \ldots, t_k) = f(p_1 | t_1, t_2, \ldots, t_k) f(p_2 | t_1, t_2, \ldots, t_k) \]
\[ \ldots f(p_n | t_1, t_2, \ldots, t_k). \]  

(3.1)

This function is also called the likelihood function. The joint prior p.d.f. of these n prices can be found as follows.

\[
\begin{align*}
  f(p_1, p_2, \ldots, p_n) & = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \ldots \int_{-\infty}^{\infty} g(t_1, t_2, \ldots, t_k) f(p_1, p_2, \ldots, p_n | t_1, t_2, \ldots, t_k) dt_1 \ldots dt_k.
\end{align*}
\]

(3.2)

In words, the joint prior p.d.f. of the n prices can be found by using the producer's estimates of the parameters or the prior p.d.f. of the parameters.

If the producer uses one-stage sampling, at the beginning of the marketing day before any packers have been contacted, the producer will decide how many packers he will contact based solely on the joint prior p.d.f. of the prices. The expected net gain after contacting n packers when sampling from a distribution with unknown parameters is very similar to finding the expected net gain when sampling from a known distribution. When sampling from a distribution with unknown parameters,

\[
E(g(n)) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \ldots \int_{-\infty}^{\infty} f(p_1, p_2, \ldots, p_n)^{\max_{i=1, 2, \ldots, n} [NP(i)]} dp_1 dp_2 \ldots dp_n
\]
\[
C(n)
\]

where \( f(p_1, p_2, \ldots, p_n) \) is the joint prior p.d.f. of a random sample of n
prices. To find the optimal number of packers to contact, it is necessary to find the number of packers associated with the largest expected gain. Or,

$$\max_{n}[E(g(n))] = \max_{n} \left[ \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} f(p_1, p_2, \ldots, p_n) \max_{i=1,2,\ldots,n} [NP(i)] dp_1 \cdots dp_n - CC(n)\right].$$

If the producer uses two-stage sampling, it is necessary to first find the optimal number of packers to contact in the first stage. Preposterior analysis will be used to determine this optimal number. First, suppose that a sample of size $n$ will be taken in the first stage. The joint prior p.d.f. of the $n$ prices is found in the same way as the one-stage sampling instance in Equations (3.1) and (3.2), or

$$f(p_1, p_2, \ldots, p_n) = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} g(t_1, t_2, \ldots, t_k) f(p_1, p_2, \ldots, p_n \mid t_1, t_2, \ldots, t_k) dt_1 dt_2 \ldots dt_k. \tag{3.3}$$

To find the maximum expected net gain if $n$ packers will be contacted in the first stage, it is necessary to find the expected net gain if $n$ packers will be contacted in the first stage and no packers will be contacted in the second. Then it is necessary to determine the expected net gain if $n$ packers will be contacted in the first stage and one packer will be contacted in the second stage, and so forth until the expected net gain if $(M-n)$ packers will be contacted in the second stage is found. If a sample of size $n$ will be taken in the first stage, the expected net gain if no
sampling will be done in the second stage is

\[ E_{n}(g(n)) = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} f(p_1, p_2, \ldots, p_n) \max_{i=1, 2, \ldots, n} [NP(i)] dp_1 dp_2 \cdots dp_n - CC(n) \]

where the subscript \( n \) on the left hand side of the equation denotes that a sample of \( n \) packers will be taken in the first stage and where the \( n \) in parentheses on left hand side of the equation denotes that a total of \( n \) packers will be sampled in both stages combined. The expected gain if only the \((n+1)^{st}\) packer will be contacted in the second stage is

\[ E_{n}(g(n+1)) = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} f(p_1, \ldots, p_n) \int_{-\infty}^{\infty} f(p_{n+1} | p_1, p_2, \ldots, p_n) \]

\[ \times \max_{i=1, 2, \ldots, n+1} [NP(i)] dp_{n+1} - CC(n+1)] dp_1 dp_2 \cdots dp_n \]

where

\[ f(p_{n+1} | p_1, p_2, \ldots, p_n) = \frac{\int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} g(t_1, t_2, \ldots, t_k) f(p_1, p_2, \ldots, p_{n+1} | t_1, \ldots, t_k) dt_1 \ldots dt_k}{\int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} g(t_1, t_2, \ldots, t_k) f(p_1, p_2, \ldots, p_n | t_1, \ldots, t_k) dt_1 \ldots dt_k} \]

The expected net gain if the \((n+1)^{st}\) and \((n+2)^{nd}\) packers will be contacted in the second stage is
\[
E_n(g(n+2)) = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} f(p_1, \ldots, p_n) \left[ \int_{-\infty}^{\infty} f(p_{n+1}, p_{n+2} | p_1, \ldots, p_n) \right] \times \max_{i=1,2, \ldots, n+2} \left[ \sum_{i} \right] \]

where

\[
f(p_{n+1}, p_{n+2} | p_1, \ldots, p_n) = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} g(t_1, t_2, \ldots, t_k) f(p_1, p_2, \ldots, p_{n+2} | t_1, \ldots, t_k) dt_1 \cdots dt_k
\]

In general, the expected net gain if \( n \) packers will be contacted in the first stage and the \( (n+1) \)\(^{st} \) through the \( (n+2) \)\(^{th} \) packers will be contacted in the second stage is

\[
E_n(g(n+\ell)) = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} f(p_1, \ldots, p_n) \left[ \int_{-\infty}^{\infty} f(p_{n+1}, \ldots, p_{n+\ell} | p_1, \ldots, p_n) \right] \times \max_{i=1,2, \ldots, n+\ell} \left[ \sum_{i} \right]
\]

where
The optimal expected net gain if \( n \) packers will be contacted in the first stage can be found as follows:

\[
\max_{s=1,2,\ldots,M-n} \left[ \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} f(p_{n+1},\ldots,p_{n+s} | p_1,\ldots,p_n) \times \max_{i=1,2,\ldots,n+s} \left[ NP(i) \right] dp_{n+1} \cdots dp_{n+s} - CC(n+s) \right].
\]

And thus, to determine the optimal number of packers to contact in the first stage:

\[
\max_{n=0,1,2,\ldots,M} \left[ \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} f(p_1,\ldots,p_n) \left( \max_{s=1,2,\ldots,M-n} \left[ \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} f(p_{n+1},\ldots,p_{n+s} | p_1,\ldots,p_n) \right] \right) \times \max_{i=1,2,\ldots,n+s} \left[ NP(i) \right] dp_{n+1} \cdots dp_{n+s} = E_{n_0,1} \left( g(n_0,1+S_{n_0,1}) \right) \right].
\]

where \( n_{0,1} \) denotes the optimal number of packers to contact in the first stage and \( S_{n_{0,1}} \) denotes the expected optimal number of packers to contact in the second stage before any sampling is done. Note that:

\[
E_{n_0,1} \left( g(n_0,1+S_{n_0,1}) \right) \]

represents the expected maximum net gain given that the expected optimal number of packers will be contacted in both stages.

The producer should then contact \( n_{0,1} \) packers. Let the bids obtained from the packers be denoted by \( p_1, p_2, \ldots, p_{n_{0,1}} \). These prices can be
used to revise the prior distribution of the parameters as follows
\[ g(t_1, \ldots, t_k | p_1, \ldots, p_{n0,1}) \]
\[ g(t_1, \ldots, t_k) f(p_1, p_2, \ldots, p_{n0,1} | t_1, \ldots, t_k) \]
\[ = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} g(t_1, \ldots, t_k) f(p_1, p_2, \ldots, p_{n0,1} | t_1, \ldots, t_k) dt_1 \cdots dt_k \]  
(3.5)

The distribution, \( g(t_1, \ldots, t_k | p_1, \ldots, p_{n0,1}) \), is called the posterior distribution of the parameters. The producer needs to determine the optimal number of packers to contact in the second stage. Thus, suppose that \( s \) packers will be contacted in the second stage. Recall that the joint p.d.f. or likelihood function of the prices \( p_{n0,1}^{n0,1} \) is
\[ f(p_{n0,1}^{n0,1} \cdots p_{n0,1}^{n0,1+s} | t_1, \ldots, t_k) \]
\[ = f(p_{n0,1}^{n0,1} | t_1, \ldots, t_k) f(p_{n0,1}^{n0,1+2} | t_1, \ldots, t_k) \cdots f(p_{n0,1}^{n0,1+s} | t_1, \ldots, t_k). \]  
(3.6)

The joint prior p.d.f. of \( p_{n0,1}^{n0,1} \cdots p_{n0,1}^{n0,1+s} \) can be found by using the posterior distribution of the parameters
\[ f(p_{n0,1}^{n0,1} \cdots p_{n0,1}^{n0,1+s} | p_1, \ldots, p_{n0,1}) \]
\[ = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} g(t_1, \ldots, t_k | p_1, \ldots, p_{n0,1}) f(p_{n0,1}^{n0,1} \cdots p_{n0,1}^{n0,1+s} | t_1, \ldots, t_k) dt_1 \cdots dt_k. \]  
(3.7)
Suppose that $NP^*$ is the maximum net price obtained after $n_{0,1}$ packers have been contacted in the first stage. If no packers will be contacted in the second stage, the net gain is

$$ E_{n_{0,1}}(g(n_{0,1})) = NP^* - CC(n_0). $$

If one packer will be contacted in the second stage, the expected net gain is

$$ E_{n_{0,1}}(g(n_{0,1}+1)) $$

$$ = \int_{-\infty}^{\infty} f(p_{n_{0,1}+1}|p_1, p_2, \ldots, p_{n_{0,1}}) \max[NP(n_{0,1}+1), NP^*] dp_{n_{0,1}+1} - CC(n_{0,1}+1). $$

If two packers will be contacted in the second stage, the expected net gain is

$$ E_{n_{0,1}}(g(n_{0,1}+2)) $$

$$ = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(p_{n_{0,1}+1}, p_{n_{0,1}+2}|p_1, \ldots, p_{n_{0,1}}) $$

$$ \times \max_{i=n_{0,1}+1, n_{0,1}+2} [NP(i), NP^*] dp_{n_{0,1}+1} dp_{n_{0,1}+2} - CC(n_{0,1}+2). $$

In general, if $s$ packers will be contacted in the second stage, the expected net gain is
\[ E_{n_{0,1}}(g(n_{0,1}+s)) \]
\[ = \int_{-\infty}^{\infty} \ldots \int_{-\infty}^{\infty} f(p_{n_{0,1}+1}, \ldots p_{n_{0,1}+s} | p_1, \ldots p_{n_{0,1}}) \]
\[ \times \max_{i=n_{0,1}+1, \ldots n_{0,1}+s} [NP(i), NP^*] dp_{n_{0,1}+1} \ldots dp_{n_{0,1}+s} - CC(n_{0,1}+s). \]  
(3.8)

By enumerating all solutions to \( E_{n_{0,1}}(g(n_{0,1}+i)) \), where \( i = 0,1, \ldots, M-n_{0,1} \), the optimal expected net gain and the optimal number of packers to contact in the second stage can be found. Mathematically,

\[ \max_{i=0,1, \ldots M-n_{0,1}} E_{n_{0,1}}(g(n_{0,1}+i)) = E_{n_{0,1}}(g(n_{0,1}+n_{0,2})) \]

where \( n_{0,2} \) denotes the optimal number of packers to contact in the second stage. Thus, if the producer has contacted \( n_{0,1} \) packers in the first stage, he should subsequently contact \( n_{0,2} \) packers in the second stage. After all packers have been contacted in both stages, the producer should sell to the packer offering the highest net gain.

**Multi-day horizon**

On marketing day \( j \), the producer has imperfect knowledge about the \( k \) parameters, \( \mathbf{\hat{r}} = (\hat{t}_{1j}, \ldots, \hat{t}_{kj}) \), of \( f_j(p) \), the distribution of prices offered by packers. Recall that there are \( J \) days in the producer's marketing horizon where the last day is day 0 and the first day is day \( (J-1) \). At the beginning of the first day in the marketing horizon, the producer can express his estimates of the unknown parameters for
each day in the marketing horizon in the form of a prior p.d.f.

\[ g(t_{1j}, t_{2j}, \ldots, t_{kj}) \text{ where } j = 0, 1, \ldots, (J-1) \]

and where \( j \) represents the day in the marketing horizon. Because it is assumed that the distributions of prices on each day are independent, the producer's estimates of the unknown parameters will not change from day to day. Recall from Equation (3.1) that if a random sample of size \( n \) is taken on day \( j \) from a distribution having p.d.f. \( f_j(p|t_{1j}, \ldots, t_{kj}) \) the joint p.d.f. of \( p_1, p_2, \ldots, p_n \) is

\[ f_j(p_1, \ldots, p_n|t_{1j}, \ldots, t_{kj}) = f_j(p_1|t_{1j}, \ldots, t_{kj}) f_j(p_2|t_{1j}, \ldots, t_{kj}) \]

\[ \ldots f_j(p_n|t_{1j}, \ldots, t_{kj}). \]

Thus, the joint p.d.f. of these \( n \) prices on day \( j \) is

\[ f_j(p_1, \ldots, p_n) = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(t_{1j}, \ldots, t_{kj}) f_j(p_1, \ldots, p_n|t_{1j}, \ldots, t_{kj}) \, dt_{1j} \ldots dt_{kj} \]

(3.9)

Suppose that the producer will use one-stage sampling. At the beginning of day \((J-1)\) before any packers have been contacted, the producer will decide how many packers to contact each day and will determine the optimal policy for each day. Recall that \( g_j \) represents the gain realized on marketing day \( j \) if the optimal number of packers have been contacted on that day and that \( NP_j \) is the corresponding highest net price. The optimal policy takes the form of a lower bound, \( X_j \), on the acceptable highest net price such that on the \( j^{th} \) marketing day,
if $NP_j \geq X_j$, then sell to the packer who offers the highest net price and

if $NP_j < X_j$, then wait.

Again, dynamic programming is used to solve this problem. Suppose that it is day 0 and the producer has not yet sold his cattle. The producer must sell his cattle by day 0, and thus the best action for the producer to follow is to contact the optimal number of packers and to sell to the packer offering the highest net price. As indicated in Equation (2.3), the expected net gain after contacting $n$ packers on day 0 is

$$E(g_0(n)) = E(\max_{i=1,2,\ldots,n} [NP(i)]) - CC(n).$$

The joint p.d.f. for day 0 can be obtained from Equation (3.3) and

$$E(g_0(n)) = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} f_0(p_1,\ldots,p_n) \max_{i=1,2,\ldots,n} [NP(i)] dp_1 \cdots dp_n - CC(n).$$

To find the optimal number of packers to contact on day 0, the producer must find the number of packers associated with the largest expected gain. Let $n_0$ denote the optimal number of packers to contact on day 0. Thus, $n_0$ can be found as follows

$$\max_n [E(g_0(n))] = \max_n [E(\max_{i=1,2,\ldots,n} [NP(i)]) - CC(n)].$$

Next, consider the situation on day 1. Recall that $W_{j-1}$ denotes the waiting cost from day $j$ to day $j-1$. To simplify notation, let

$$EG_0 = E(\max_{i=1,2,\ldots,n_0} [NP(i)]) - CC(n_0) - W_0.$$
Assume that the optimal number of packers have been contacted on day 1 and $NP_1$ has been observed. If $NP_1$ is greater than $EG_0$, the producer should sell on day 1 because his actual gain on day 1 is greater than what he would expect to gain by waiting until day 0. Thus, the following stopping rule applies

if $NP_1 \geq EG_0$, then sell to the packer who offers the highest net price and

if $NP_1 < EG_0$, then wait. (3.10)

However, the producer will not know the highest net price offered on day 1 before any packers are contacted. Thus, the expected gain on day 1 given that $n$ packers have been contacted and that the stopping rule (3.10) is followed is

$$E(g_1(n)) = E(\max_{i=1,2,...,n} [NP(i), EG_0]) - CC(n).$$

Thus, $n_1$ can be found as follows

$$\max(E(g_1(n))) = \max_{n} [E(\max_{i=1,2,...,n} [NP(i), EG_0]) - CC(n)].$$

By using stopping rule (2.10), Equations (2.9), (2.11), and (2.12), and the joint prior p.d.f. of the prices for day 2 given by Equation (3.9), the optimal number of packers to contact on day 2, $n_2$, can be determined. And similarly, the optimal stopping rule for day $j$ is given by

if $NP_j \geq EG_{j-1}$, then sell to the packer who offers the highest net price and

if $NP_j < EG_{j-1}$, then wait.
where $E_{j-1}$ is given by

$$E_{j-1} = E(\max[N_P(i), E_{j-2}]) - CC(n_{j-1}) - W_{j-1}$$

$$= \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} f_{j-1}(p_1, \ldots, p_{n_{j-1}}) \max[N_P(i), E_{j-2}] dp_1 \cdots dp_{n_{j-1}}$$

$$- CC(n_{j-1}) - W_{j-1}.$$ 

Also, the optimal number of packers to contact each day is given by

$$\max[E(g_j(n))] = \max[E(\max_{i=1,2,\ldots,n} [N_P(i), E_{j-1}]) - CC(n)]$$

$$= \max_{n} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} f_j(p_1, \ldots, p_n) \max[N_P(i), E_{j-1}] dp_1 \cdots dp_n - CC(n).$$

Thus, the same procedure is used whether the producer is uncertain about the exact values of the parameters or whether he knows the exact distribution of prices. However, if the producer is uncertain about the exact values of the parameters, he will use his estimate of the distribution of the parameters to form a joint prior distribution of prices. If the producer knows the exact distribution, he will use this exact form of the distribution of prices.

If the producer uses two-stage sampling, he will contact packers in two stages on each day in his marketing horizon. At the beginning of day $(J-1)$ before any packers have been contacted, the producer will decide how many packers to contact in the first stage for each day in the marketing horizon and will determine the optimal policy for each day. Let $g_j$ represent the gain realized on marketing day $j$ if the optimal
numbers of packers have been contacted in both stages. Also, let \( N_{P_j} \) be the highest net price after the optimal numbers of packers have been contacted in both stages. \( N_{P_j} \) will be used to denote the highest net price after the optimal number of packers has been contacted in the first stage. The symbol \( n_{i,j} \) will be used to represent the optimal number of packers to contact in stage \( j \) on day \( i \). Again, the optimal policy takes the form of a lower bound, \( X_j \), on the acceptable highest net price after contacting packers for both stages such that on the \( j \)th marketing day,

\[
\text{if } N_{P_j} > X_j, \text{ then sell to the packer who offers the highest net price and}
\]

\[
\text{if } N_{P_j} < X_j, \text{ then wait.}
\]

By beginning with day 0 and using dynamic programming, this problem will be solved. Because it is day 0, the producer must sell his cattle. Thus, the best action for the producer to follow is to contact the optimal number of packers for stage 1; and then using this information, the producer should contact the optimal numbers of packers for the second stage and sell to the packer offering the highest net price. To determine the optimal number of packers to contact in the first stage, preposterior analysis will be used. Recall from Equation (3.4) that if \( n \) packers are contacted in the first stage and \( l \) packers are contacted in the second stage, the expected net gain is
\[ E_n(g_0(n+2)) \]
\[ = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} f_0(p_1, \ldots, p_n) \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} f_0(p_{n+1}, \ldots, p_{n+2}|p_1, \ldots, p_n) \]
\[ \times \max_{i=1,2,\ldots,n+1} \left[ NP(i) \right] dp_{n+1} \cdots dp_{n+2} - CC(n+2) \]
\[ \times \max_{i=1,2,\ldots,n+1} \left[ NP(i) \right] dp_{n+1} \cdots dp_{n+2} - CC(n+2) \]

where
\[ f_0(p_{n+1}, \ldots, p_{n+2}|p_1, \ldots, p_n) \]
\[ = \frac{\int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} g_0(t_{10}, \ldots, t_{k0}) f_0(p_1, \ldots, p_{n+2}|t_{10}, \ldots, t_{k0}) dt_{10} \cdots dt_{k0}}{\int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} g_0(t_{10}, \ldots, t_{k0}) f_0(p_1, \ldots, p_{n}|t_{10}, \ldots, t_{k0}) dt_{10} \cdots dt_{k0}} \]

Again, the optimal expected net gain if \( n \) packers will be contacted in the first stage can be found as follows
\[ \max_{s=0,1,\ldots,M-n} \left[ \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} f_0(p_{n+1}, \ldots, p_{n+s}|p_1, \ldots, p_n) \right] \]
\[ \times \max_{i=1,2,\ldots,n+s} \left[ NP(i) \right] dp_{n+1} \cdots dp_{n+s} - CC(n+s) \]

And thus, to determine the optimal number of packers to contact in the first stage
\[ \max_{n=0,1,\ldots,M} \left[ \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} f_0(p_1, \ldots, p_n) \right] \]
\[ \times \left\{ \max_{s=0,1,\ldots,M-n} \left[ \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} f_0(p_{n+1}, \ldots, p_{n+s}|p_1, \ldots, p_n) \right] \right\} \]
\[ \times \max_{i=1,2,\ldots,n+s} \left[ NP(i) \right] dp_{n+1} \cdots dp_{n+s} - CC(n+s) \]
\[ = E_n^0 \left( g_0(n_0,1^n+2) \right) \]
where \( n_{0,1} \) denotes the optimal number of packers to contact in the first stage and \( S_{n_{0,1}} \) denotes the expected optimal number of packers to contact in the second stage before any sampling is done. Note that

\[
E_{n_{0,1}} (g_0(n_{0,1} + S_{n_{0,1}}))
\]

represents the expected maximum net gain at the beginning of day 0 given that the expected optimal numbers of packers will be contacted in both stages.

If \( n_{0,1} \) packers are contacted in the first stage of day 0, the producer must then determine how many packers to contact in the second stage. Recall from Equation (3.5) that the prices \( p_1, p_2, \ldots, p_{n_0} \), can be used to revise the prior distribution of parameters as follows

\[
g(t_{10}, \ldots, t_{k0} | p_1, \ldots, p_{n_0})
\]

\[
= \frac{g(t_{10}, \ldots, t_{k0}) f_0(p_1, p_2, \ldots, p_{n_0}, 1 | t_{10}, \ldots, t_{k0})}{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(t_{10}, \ldots, t_{k0}) f_0(p_1, \ldots, p_{n_0}, 1 | t_{10}, \ldots, t_{k0}) dt_{10} \ldots dt_{k0}} \quad (3.11)
\]

to form the posterior distribution of the parameters. Suppose that the producer will contact \( s \) packers in the second stage. The joint p.d.f. of \( p_{n_{0,1}+1}, \ldots, p_{n_{0,1}+S} \) can be found by using the likelihood function given in Equation (3.6) and Equation (3.11)
Let \( NP^* \) be the maximum net price obtained after contacting \( n^0,1 \) packers in the first stage. Recall from Equation (3.8) that if \( n^0,1 \) packers are contacted in the first stage and \( S \) packers will be contacted in the second stage, the expected net gain is

\[
E_{n^0,1}(g_0(n^0+S)) = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} f(p_{n^0,1+1}, \ldots, p_{n^0,1+S} | p_1, \ldots, p_{n^0,1}) \times \text{Dia}x [NP(i), NP^*] \cdot d_{n^0,1+S} \cdot \text{CC}(n^0,1+S). 
\]

The optimal expected gain and the optimal number of packers to contact in the second stage can be found by enumerating all solutions to

\[
E_{n^0,1}(g_0(n^0+1)) \quad \text{for} \quad i = 0,1, \ldots, M-n^0,1.
\]

Or

\[
\max_{i=0,1, \ldots, M-n^0,1} E_{n^0,1}(g_0(n^0,1+i)) = E_{n^0,1}(g_0(n^0,1+n^0,2))
\]

where \( n^0,2 \) denotes the optimal number of packers to contact in the second stage if \( n^0,1 \) packers have already been contacted in the first stage. After all packers have been contacted in both stages, the producer should sell to the packer offering the highest net gain.
Next, consider the situation on day 1. If the producer waits until
day 0 to sell his cattle, his expected net gain is

\[ EG_0 = E_{n_0,1} \left( g_0(n_0,1+S_{n_0,1}) \right) - W_0. \]

And by Equation (3.11),

\[ EG_0 = \max_{n=0,1,\ldots,M} \left[ \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} f_0(p_1, \ldots, p_n) \right. \]
\[ \times \left\{ \max_{s=0,1,\ldots,M-n} \left[ \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} f_0(p_{n+1}, \ldots, p_{n+s}, p_1, \ldots, p_n) \right. \]
\[ \times \max_{i=1,2,\ldots,n+s} \left[ NP(i) \right] dp_{n+1} \cdots dp_{n+s} - CC(n+s)] dp_1 \cdots dp_n \right\} - W_0. \]

If the optimal numbers of packers have been contacted in both stages, the
producer should take the following action. If \( NP_1 \), the highest net price
offered on day 1, is greater than the expected gain on day 0, \( EG_0 \), then
the producer should sell on day 1. If \( NP_1 \) is less than \( EG_0 \), the producer
should wait until day 0. These ideas can be restated in the following
stopping rule

\[ \text{if } NP_1 \geq EG_0, \text{ then sell to the packer who offers the highest net price and} \]
\[ \text{if } NP_1 < EG_0, \text{ then wait.} \]  

(3.12)

Thus, the expected net gain given that \( n \) packers have been contacted
in the first stage, \( \ell \) packers have been contacted in the second
stage, and stopping rule (3.12) is
The optimal expected net gain if \( n \) packers will be contacted in the first stage can be found as follows:

\[
\max_{i=1,2,\ldots,n+\ell} \left[ \prod_{s=0}^{n+n} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_1(p_{1}, \ldots, p_{n}) \right] \prod_{s=0}^{n+n} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_1(p_{n+1}, \ldots, p_{n+\ell} | p_{1}, \ldots, p_{n}) \]

To determine the optimal number of packers to contact in the first stage of day 1,

\[
\max_{n=0,1,\ldots,M} \left[ \prod_{s=0}^{n+n} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_1(p_{1}, \ldots, p_{n}) \right]
\]

\[
\prod_{s=0}^{n+n} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_1(p_{n+1}, \ldots, p_{n+s} | p_{1}, \ldots, p_{n}) \]

\[
x \max_{i=1,2,\ldots,n+s} \left[ \prod_{s=0}^{n+n} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_1(p_{1}, \ldots, p_{n}) \right] \prod_{s=0}^{n+n} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_1(p_{n+1}, \ldots, p_{n+s} | p_{1}, \ldots, p_{n}) \]

\[
x \max_{i=1,2,\ldots,n+s} \left[ \prod_{s=0}^{n+n} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_1(p_{1}, \ldots, p_{n}) \right] \prod_{s=0}^{n+n} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_1(p_{n+1}, \ldots, p_{n+s} | p_{1}, \ldots, p_{n}) \]

\[
= E_{n_1,1}^{n_1,1} (g_1(n_1,1+n_1,1)) \quad (3.14)
\]
where \( n_{1,1} \) denotes the optimal number of packers to contact in the first stage of day 1 and \( S \) denotes the expected optimal number of packers to contact in the second stage before any sampling is done. The expected maximum net gain given that the expected optimal number of packers will be contacted in both stages of day 1 and day 0 is given by

\[
E_{n_{1,1}, n_{1,1}} (g_1(n_{1,1} + S_{n_{1,1}})).
\]

If \( n_{1,1} \) packers are contacted in the first stage of day 1, the producer must determine how many packers to contact in the second stage. Recall from Equation (3.11) that the \( n_{1,1} \) prices, \( p_1, p_2, \ldots, p_{n_{1,1}} \), can be used to revise the prior distribution of the parameters for day 1 as follows

\[
g(t_{11}, \ldots, t_{k1} | p_1, \ldots, p_{n_{1,1}})
\]

\[
= \frac{g(t_{11}, \ldots, t_{k1})f_0(p_1, \ldots, p_{n_{1,1}} | t_{11}, \ldots, t_{k1})}{\int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} g(t_{11}, \ldots, t_{k1})f_0(p_1, \ldots, p_{n_{1,1}} | t_{11}, \ldots, t_{k1}) dt_{11} \cdots dt_{k1}}
\]

to form the posterior distribution of the parameters. Suppose that the producer will contact \( S \) packers in the second stage. The joint p.d.f. on \( p_{n_{1,1}+1}, \ldots, p_{n_{1,1}+S} \) is

\[
f_1(p_{n_{1,1}+1}, \ldots, p_{n_{1,1}+S} | p_1, \ldots, p_{n_{1,1}})
\]

\[
= \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} g(t_{11}, \ldots, t_{k1} | p_1, \ldots, p_{n_{1,1}})f_1(p_{n_{1,1}+1}, \ldots, p_{n_{1,1}+S} | t_{11}, \ldots, t_{k1}) dt_{11} \cdots dt_{k1}.
\]

Let \( NP^{*}_{n_{1,1}} \) be the maximum net price obtained after contacting \( n_{1,1} \) packers.
in the first stage. If the producer contacts S packers in the second stage, he will choose the maximum of \( NP_0 \), \( E_G_0 \), or the highest net price offered by packers contacted in the second stage. Mathematically, if S packers will be contacted in the second stage, the maximum expected gain is

\[
E_{n_{1,1}}(g_1(n_{1,1} + S)) = \int_{-\infty}^{+\infty} \cdots \int_{-\infty}^{+\infty} f(p_{n_{1,1} + 1}, \ldots, p_{n_{1,1} + S} | p_1, \ldots, p_{n_{1,1}}) \]

\[
\times \max_{i=n_{1,1} + 1, \ldots, n_{1,1} + S} [NP(i), NP_0, E_G_0] \cdot dp_{n_{1,1} + 1} \cdots dp_{n_{1,1} + S} - CC(n_{1,1} + S).
\]

To determine the S which maximizes the expected gain, it is necessary to enumerate the solutions to \( E_{n_{1,1}}(g_1(n_{1,1} + S)) \) for \( S = 0, 1, \ldots, M-n_{1,1} \) and choose the S which maximizes the expected gain. Or,

\[
\max_{S=0, 1, \ldots, M-n_{1,1}} E_{n_{1,1}}(g_1(n_{1,1} + S)) = E_{n_{1,1}}(g_1(n_{1,1} + n_{1,1,2}))
\]

where \( n_{1,2} \) denotes the optimal number of packers to contact in the second stage given that \( n_{1,1} \) packers have already been contacted in the first stage. Now, let \( NP_1 \) be the highest net price offered on day 1 after \( n_{1,1} \) packers have been contacted in the first stage and \( n_{1,2} \) packers have been contacted in the second stage. Stopping rule (3.12) can be used to decide whether to sell or wait.

Finally, consider the general situation on day j. If the producer waits until day \((j-1)\), his expected gain is

\[
E_{j-1} = E_{n_{j-1,1}}(g_{j-1}(n_{j-1,1} + S)) - W_{j-1}.
\]
And similar to Equation (3.14),

\[ E_{n_j-1,1}(g_{j-1}(n_j-1,1+S_{n_j-1,1})) \]

\[ = \max_{s=0,1,\ldots,M} \left[ \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} f_{j-1}(p_1,\ldots,p_n) \right. \]

\[ \times \left. \max_{s=0,1,\ldots,M-n} \left[ \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} f_{j-1}(p_{n+1},\ldots,p_{n+s} \mid p_1,\ldots,p_n) \right. \right. \]

\[ \left. \times \max_{i=1,2,\ldots,n+s} [NP(i),EG_{j-2}] dp_{n+1} \ldots dp_{n+s} - CC(n+s)] dp_1 \ldots dp_n \right] \]

If the optimal number of packers have been contacted in both stages on day \( j \), the producer should take the following action to maximize profits.

If \( NP_j \), the highest net price offered on day \( j \), is greater than the expected gain on day \( j-1 \), \( EG_{j-1} \), then the producer should sell on day \( j \).

If \( NP_j \) is less than \( EG_{j-1} \), the producer should wait until day \( j-1 \). Thus, the following stopping rule applies

\[
\text{if } NP_j \geq EG_{j-1}, \text{ then sell to the packer who offers the highest net price and} \\
\text{if } NP_j < EG_{j-1}, \text{ then wait.} \tag{3.15}
\]

The expected net gain given that \( n \) packers will be contacted in the first stage, \( l \) packers will be contacted in the second stage, and stopping rule (3.15) is

\[ E_n(g_j(n+l)) = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} f_j(p_1,\ldots,p_n) \left[ \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} f_j(p_{n+1},\ldots,p_{n+l} \mid p_1,\ldots,p_n) \right. \]

\[ \times \max_{i=1,2,\ldots,n+l} [NP(i),EG_{j-1}] dp_{n+1} \ldots dp_{n+l} - CC(n+l)] dp_1 \ldots dp_n. \tag{3.16} \]
The optimal expected net gain if \( n \) packers will be contacted in the first stage can be found as follows

\[
\max_{s=0,1,...,M-n} \left[ \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} f_j(p_{n+1} \ldots p_{n+s} \mid p_1 \ldots p_n) \right]
\]

\[
\times \max_{i=1,2,...,n+s} \left[ \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \left[ NP(i), EG_{j-1} \right] dp_{n+1} \ldots dp_{n+S} - CC(n+S) \right].
\]

To determine the optimal number of packers to contact in the first stage of day \( j \),

\[
\max_{n=0,1,...,M} \left[ \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} f_j(p_1 \ldots p_n) \right]
\]

\[
\times \left( \max_{s=0,1,...,M-n} \left[ \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} f_j(p_{n+1} \ldots p_{n+s} \mid p_1 \ldots p_n) \right] \right)
\]

\[
\times \max_{i=1,2,...,n+S} \left[ \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \left[ NP(i), EG_{j-1} \right] dp_{n+1} \ldots dp_{n+S} - CC(n+S) \right] \right] dp_1 \ldots dp_n
\]

\[= E_{n_{j,1}} \left( g_j(n_{j,1} + S_{n_{j,1}}) \right) \]

where \( n_{j,1} \) denotes the optimal number of packers to contact in the first stage of day \( j \) and \( S_{n_{j,1}} \) denotes the expected optimal number of packers to contact in the second stage before any sampling is done. The expected maximum net gain given that the expected optimal number of packers will be contacted in both stages of day \( j \) and all previous days is given by \( E_{n_{j,1}} \left( g_j(n_{j,1} + S_{n_{j,1}}) \right) \).

Suppose that the producer has already contacted \( n_{j,1} \) packers in the
first stage of day \( j \). He must determine how many packers to contact in
the second stage. By using Equation (3.11) and the observed prices,
\( p_1, \ldots, p_n \), the producer can revise the prior distribution of
parameters for day \( j \) to form the posterior distribution of the
parameters as follows

\[
g(t_{1j}, \ldots, t_{kj} | p_1, \ldots, p_n, l) \\
= \frac{g(t_{1j}, \ldots, t_{kj}) f_j(p_1, \ldots, p_n, l | t_{1j}, \ldots, t_{kj})}{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(t_{1j}, \ldots, t_{kj}) f_j(p_1, \ldots, p_n, l | t_{1j}, \ldots, t_{kj}) \, dt_{1j} \ldots dt_{kj}}.
\]

If the producer will contact \( S \) packers in the second stage, the
joint p.d.f. of \( p_{n_j, l+1}, \ldots, p_{n_j, l+S} \) is

\[
f_j(p_{n_j, l+1}, \ldots, p_{n_j, l+S} | p_1, \ldots, p_n, l) \\
= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(t_{1j}, \ldots, t_{kj} | p_1, \ldots, p_n, l) f_j(p_{n_j, l+1}, \ldots, p_{n_j, l+S} | t_{1j}, \ldots, t_{kj}) \, dt_{1j} \ldots dt_{kj}.
\]

Let \( NP^*_j \) be the maximum net price offered on day \( j \) after contacting \( n_{j, l} \)
packers in the first stage. Similar to the day 1 instance, the maximum
expected gain if \( S \) packers will be contacted in the second stage is
To determine the $S$ which maximizes expected gain,

$$
E_{n_{j,1}}^n (g_j(n_{j,1}+S))
= \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} f_j(p_{n_{j,1}+1}, \ldots, p_{n_{j,1}+S}) | p_1, \ldots, p_{n_{j,1}} \\
\times \max_{i=n_{j,1}+1, \ldots, n_{j,1}+S} [NP(i), NP^*, EC_{j-1}] dp_{n_{j,1}+1} \cdots dp_{n_{j,1}+S} - CC(n_{j,1}+S).
$$

(3.17)

Sequential Sampling

Recall that from various kinds of information the producer has gathered, he can form prior distributions of the $k$ parameters, \( \mathbf{T} = (t_{1j}, \ldots, t_{kj}) \) for each of the $i$ marketing days. With sequential sampling, no initial sampling will be done to obtain information about the distribution of prices. However, after each price is observed on day $i$, the distribution of the parameters on day $i$ will be revised in a Bayesian fashion.
One-day horizon

The producer can express his estimates of the unknown parameter of $f(p)$, the distribution of prices offered by packers on the only day in the marketing horizon, in the form of a prior probability density function $g(t_1, \ldots, t_k)$.

Because the producer must sell on this single day in his marketing horizon, he must contact at least one packer. Suppose that the producer contacts the first packer on his list. Using the price offered by this first packer to revise the prior p.d.f. of the parameters, the producer must decide whether to sell or whether to contact the second packer on his list. If the producer decides to contact the second packer, he must decide, using the prices obtained from the first and second packers to revise the prior distribution of parameters, whether to sell to the packer offering the highest net price or whether to wait and contact the third packer. Likewise, if the producer has contacted the $m^{th}$ packer, he must decide, using the prices obtained from the first through $m^{th}$ packers to revise the prior distribution of parameters, whether to sell to the packer offering the highest net price or whether to wait and contact the $(m+1)^{st}$ packer.

Once again, this problem can be solved by using dynamic programming. Thus, first consider the situation in which the producer has contacted all $M$ packers. In this situation, the producer will sell to the packer offering the highest net price. Again, $NP^*_i$ will be used to denote the highest net price offered after contacting $i$ packers. Therefore, the
expected net gain under the optimal policy after contacting M packers is

\[ g(M,NP^*_M) = NP^*_M - CC(M). \]

If (M-1) packers have been contacted, the producer must decide whether to sell to the packer offering the highest bid, NP^*_M-1, or whether to wait and contact the last packer. If the producer decides to sell for NP^*_M-1, the gain will be

\[ NP^*_M-1 - CC(M-1). \]

But if the producer decides to wait and contact the last packer, the expected gain is

\[
E\left[ \max\{NP^*_M - CC(M), NP(M) - CC(M)\} \right] \]

\[ = E[g(M,\max\{NP^*_M-1, NP(M)\})]. \tag{3.18} \]

Therefore, if (M-1) packers have been contacted, the expected net gain under the optimal policy is

\[ g((M-1),NP^*_M-1) = \max\{NP^*_M - CC(M-1), E[g(M,\max\{NP^*_M-1, NP(M)\})]\}. \tag{3.19} \]

To solve for this expression, recall from Equation (3.1) that if a random sample of size (M-1) is taken from a distribution having p.d.f. \( f(p|t_1,\ldots,t_k) \), the joint p.d.f. of \( p_1,p_2,\ldots,p_{M-1} \) is

\[ f(p_1,p_2,\ldots,p_{M-1}|t_1,\ldots,t_k) = f(p_1|t_1,\ldots,t_k)\cdots f(p_{M-1}|t_1,\ldots,t_k). \]

These (M-1) prices can be used to revise the prior distribution of the
parameters as follows

\[ g(t_1, \ldots, t_k | p_1, \ldots, p_{M-1}) \]

\[
\frac{g(t_1, \ldots, t_k) f(p_1, p_2, \ldots, p_{M-1} | t_1, \ldots, t_k)}{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(t_1, \ldots, t_k) f(p_1, \ldots, p_{M-1} | t_1, \ldots, t_k) dt_1 \ldots dt_k}
\]

The p.d.f. of the price quoted by the \( M \)th packer is

\[ f(p_M | t_1, \ldots, t_k) \]

and thus, the p.d.f. of \( p_M \) given that the prices \( p_1, \ldots, p_{M-1} \) having already been observed is

\[ f(p_M | p_1, \ldots, p_{M-1}) \]

\[
= \int_{-\infty}^{\infty} \ldots \int_{-\infty}^{\infty} g(t_1, \ldots, t_k | p_1, \ldots, p_{M-1}) f(p_M | t_1, \ldots, t_k) dt_1 \ldots dt_k.
\]

From Equations (3.18), the expression

\[ E\{ \max\{NP_{^*}^{M-1}CC(M), \{NP(M)-CC(M)\}\} \} \]

can be solved as follows

\[
E\{ \max\{NP_{^*}^{M-1}CC(M), \{NP(M)-CC(M)\}\} \}
\]

\[
= \int_{-\infty}^{\infty} f(p_M | p_1, \ldots, p_{M-1}) \max[NP(M), NP_{^*}^{M-1}] dP_M - CC(M).
\]

And therefore,
The optimal stopping rule can be derived from this expression. If
\[
\{NP^*_M - CC(M-1)\} > E[g<M, \max\{NP^*_M, NP(M)\}] 
\]
then sell to the packer offering the highest net price. If
\[
\{NP^*_M - CC(M-1)\} < E[g<M, \max\{NP^*_M, NP(M)\}] 
\]
then wait and contact the next packer.

If (M-2) packers have been contacted, the producer must decide whether to sell to the packer offering the highest net price, \(NP^*_{M-2}\), or whether to wait and contact the \((M-1)^{st}\) packer. If the producer sells to the packer offering the highest net price, the net gain will be
\[
NP^*_{M-2} - CC(M-2). 
\]
If the producer decides to wait and contact the \((M-1)^{st}\) packer, the expected gain is
\[
E[\max(\{NP^*_{M-2} - CC(M-1)\}, \{NP(M-1) - CC(M-1)\})], 
\]
\[
\{E(\max(\{NP^*_{M-2} - CC(M)\}, \{NP(M-1) - CC(M)\}, \{NP(M) - CC(M)\}))\} > 
\]
\[
= E[g<(M-1), \max\{NP^*_{M-2}, NP(M-1)\}]. 
\]
Thus, if (M-2) packers have been contacted, the expected net gain under
the optimal policy is

\[ g(M-2, NP^*_M) = \max\{NP^*_M - CC(M-2)\} \]

\[ E[g<(M-1), \max\{NP^*_M, NP(M-1)\}]]. \]

To solve for this expression, the joint p.d.f. of the \((M-2)\) observed prices, \(p_1, \ldots, p_{M-2}\), can be used to revise the prior distribution of the parameters to form the posterior distribution

\[ g(t_1, \ldots, t_k | p_1, \ldots, p_{M-2}) \]

\[ = \frac{g(t_1, \ldots, t_k) f(p_1, \ldots, p_{M-2} | t_1, \ldots, t_k)}{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(t_1, \ldots, t_k) f(p_1, \ldots, p_{M-2} | t_1, \ldots, t_k) dt_1 \ldots dt_k} \]

the p.d.f. of \(p_{M-1}\) given that \(p_1, \ldots, p_{M-2}\) have already been observed is

\[ f(p_{M-1} | p_1, \ldots, p_{M-2}) \]

\[ = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(t_1, \ldots, t_k | p_1, \ldots, p_{M-2}) f(p_{M-1} | t_1, \ldots, t_k) dt_1 \ldots dt_k \]

And the joint p.d.f. of \(p_M\) and \(p_{M-1}\) given that \(p_1, p_2, \ldots, p_{M-2}\) have already been observed is

\[ f(p_{M-1}, p_M | p_1, \ldots, p_{M-2}) \]

\[ = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(t_1, \ldots, t_k | p_1, \ldots, p_{M-2}) f(p_{M-1}, p_M | t_1, \ldots, t_k) dt_1 \ldots dt_k \].
Solving for part of Equation (3.20),

\[ E(\max\{NP_{M-2}^* - CC(M-1), \{NP(M-1) - CC(M-1)\}\}) \]

\[ = \int_{-\infty}^{\infty} f(p_{M-1}, \cdots p_{M-2}) \max[NP(M-1), NP_{M-2}^*] dp_{M-1} - CC(M-1) \]

and another part

\[ E(\max\{NP_{M-2}^* - CC(M), \{NP(M-1) - CC(M)\}, \{NP(M) - CC(M)\}\}) \]

\[ = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(p_{M-1}, p_M, \cdots p_{M-2}) \max_{i=M-1, M} [NP(i), NP_{M-2}^*] dp_{M-1} dp_M - CC(M). \]

Thus, the following stopping rule applies.

If

\[ \{NP_{M-2}^* - CC(M-2)\} > E[g<(M-1), \max\{NP_{M-2}^*, NP(M-1)\}>] \]

then sell to the packer offering the highest net price.

If

\[ \{NP_{M-2}^* - CC(M-2)\} < E[g<(M-1), \max\{NP_{M-2}^*, NP(M-1)\}>] \]

then wait and contact the next packer.

Finally, consider the situation if \( m \) packers have been contacted.
The producer must decide whether to sell to the packer offering the highest bid, \( NP_m^* \), or whether to wait and contact the \( (m+1) \text{'st} \) packer.

If the producer sells to the packer offering the highest bid, \( NP_m^* \),

the gain will be

\[ NP_m^* - CC(m). \]

If the producer decides to wait and contact the next packer, the expected net gain is
\[ E[g<(m+1), \max_{m}(\text{NP}_m^*, \text{NP}(m+1)) >] \]  

\[ = E[\max(E'\max_{m}(\text{NP}_m^*-\text{CC}(m+1)), \{\text{NP}(m+1)-\text{CC}(m+1)\} >], \]

\[ E<\max(E'\max_{m}(\text{NP}_m^*-\text{CC}(m+2)), \{\text{NP}(m+1)-\text{CC}(m+2)\}, \{\text{NP}(m+2)-\text{CC}(m+2)\} >], \]

\[ \ldots, E'\max_{m}(\text{NP}_m^*-\text{CC}(M)), \{\text{NP}(m+1)-\text{CC}(M)\}, \ldots\{\text{NP}(M)-\text{CC}(M)\}>]. \]

Thus, if \( m \) packers have been contacted, the expected gain under the optimal policy is

\[ g[m, \text{NP}_m^*] = \max\{\{\text{NP}_m^*-\text{CC}(m)\}, E[g<(m+1), \max_{m}(\text{NP}_m^*, \text{NP}(m+1)) >] \}. \]

The observed prices, \( p_1, \ldots, p_m \), can be used to derive the posterior distribution of prices as given by Equation (3.5). And similar to Equation (3.7), distributions of prices given \( p_1, p_2, \ldots, p_m \) can be found. Solving for part of Equation (3.21),

\[ E'\max_{m}(\text{NP}_m^*-\text{CC}(m+1)), \{\text{NP}(m+1)-\text{CC}(m+1)\} >] \]

\[ = \int_{-\infty}^{\infty} f(p_{m+1} | p_1, \ldots, p_m) \max_{m} [\text{NP}(m+1), \text{NP}_m^*] dp_{m+1} - \text{CC}(m+1) \]

and

\[ E'\max_{m}(\text{NP}_m^*-\text{CC}(m+2)), \{\text{NP}(m+1)-\text{CC}(m+2)\}, \{\text{NP}(m+2)-\text{CC}(m+2)\} >] \]

\[ = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(p_{m+1}, p_{m+2} | p_1, \ldots, p_m) \max_{i=m+1, m+2} [\text{NP}(i), \text{NP}_m^*] dp_{m+1} dp_{m+2} - \text{CC}(m+2) \]

and so forth until
Thus, the following stopping rule applies.

If
\[
\{NP^*-CC(m)\} \geq E[g<(m+1),\max\{NP^*,NP(m+1)\}>]
\]
then sell to the packer offering the highest net price.

If
\[
\{NP^*-CC(m)\} < E[g<(m+1),\max\{NP^*,NP(m+1)\}>]
\]
then wait and contact the next packer.

**Multi-day horizon**

Recall that in the multi-day horizon problem, the producer must first decide which day in the marketing horizon to begin contacting packers. After each packer is contacted, the producer must decide whether to sell his cattle, whether to wait and contact another packer on that marketing day, or whether to wait and contact packers on a later day in the marketing horizon. At the beginning of the first day in the marketing horizon, the producer can express his estimates of the unknown parameters for each day in the marketing horizon in the form of a prior p.d.f.

\[g(t_{1j}, t_{2j}, \ldots t_{kj}) \text{ where } j = 0, 1, \ldots, (J-1)\]

To solve this problem by using dynamic programming, consider the
general situation on day 0 when \( m \) packers have been contacted. The producer must decide whether to sell to the packer offering the highest net price on day 0, \( NP_0^* \), or whether to wait and contact the \((m+1)^{st}\) packer. The solution to the situation on day 0 is very similar to the one-day horizon problem with sequential sampling. Recall that if the producer sells to the packer offering the highest net price, \( NP_0^* \), the gain will be

\[
NP_0^* - CC(m).
\]

If the producer decides to wait and contact the next packer, the expected net gain is the same as the gain derived in Equation (3.21).

\[
E[g_0(m+1), \max\{NP_0^*, NP(m+1)\}] = E\{E\{\max\{NP_0^* - CC(m+1), NP(m+1) - CC(m+1)\}, \max\{NP(m+2) - CC(m+2), NP(m+1) - CC(m+2)\}, \ldots\}\}.
\]

The observed prices on day 0, \( p_1, \ldots, p_m \), can be used to derive the posterior distribution of prices for day 0 as given by Equation (3.5). And by using Equation (3.7), distributions of prices given \( p_1, p_2, \ldots, p_m \) can be found. Solving for part of Equation (3.23)

\[
E\{\max\{NP_0^* - CC(m+1), NP(m+1) - CC(m+1)\}\}
\]

\[
= \int_{-\infty}^{\infty} f_0(p_{m+1} | p_1, \ldots, p_m) \max\{NP(m+1), NP_0^*\} dp_{m+1} - CC(m+1)
\]
and

\[
E\{\max_{m}\{NP^*-CC(m+2), \{NP(m+2)-CC(m+2)\}, \{NP(m+2)-CC(m+2)\}\}\}
\]

\[
= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_{0}(p_{m+1}, p_{m+2} | p_{1}, \ldots, p_{m}) \max_{i=m+1, m+2} [NP(i), NP^*_{0}, NP^*_{m}] dp_{m+1} dp_{m+2} - CC(m+2)
\]

and so forth until

\[
E\{\max_{m}\{NP^*-CC(M), \{NP(m+2)-CC(M)\}, \ldots, \{NP(M)-CC(M)\}\}\}
\]

\[
= \int_{-\infty}^{\infty} \ldots \int_{-\infty}^{\infty} f_{0}(p_{m+1}, \ldots, p_{M} | p_{1}, \ldots, p_{m}) \max_{i=m+1, \ldots, M} [NP(i), NP^*_{0}, NP^*_{m}] dp_{m+1} \ldots dp_{M}
\]

- CC(M).

Thus, the following stopping rule for day 0 applies

If

\[
\{NP^*-CC(m)\} \geq E[g_0<(m+1), \max_{0, m} NP^*(m+1)>]
\]

then sell to the packer offering the highest net price.

If

\[
\{NP^*_{0, m}-CC(m)\} < E[g_0<(m+1), \max_{0, m} NP^*(m+1)>]
\]

then wait and contact the next packer.

Next, consider the general instance on day 1 when the producer has contacted m packers. The producer could sell to the packer offering the highest bid, NP^*_{1,m}, he could contact the (m+1)^{st} packer on day 1, or he could wait until day 0 and not contact anymore packers on day 1. According to Equation (2.13) and the derivation preceding it, the expected gain under the optimal policy is
\[ g_1[m, NP^*_1, m] = \max\{\{NP^*_1 - CC(m)\}, E[g_1<(m+1), \max\{NP^*_1, NP(m+1)\}]\} \]

\[ \{g_0(0,0) - CC(m) - W_0\} \] \hspace{1cm} (3.24)

where

\[ g_0(0,0) = E_i^i g_0<1, NP(1)>^\frac{3}{2} \]

\[ = E[\max(E\{NP(1)-CC(1)\}, \]

\[ E<\max(E_i^i \max<\{NP(1)-CC(2)\}, \{NP(2)-CC(2)\}>^\frac{3}{2}, \]

\[ \ldots, E_i^i \max<\{NP(1)-CC(M)\}, \{NP(2)-CC(M)\}, \ldots, \{NP(M)-CC(M)\}>^\frac{3}{2})] \]

and where

\[ E\{NP(1)-CC(1)\} = \int_{-\infty}^{\infty} f_0(p_1) dp_1 - CC(1) \]

and

\[ E_i^i \max<\{NP(1)-CC(2)\}, \{NP(2)-CC(2)\}>^\frac{3}{2} \]

\[ = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_0(p_1, p_2) \max_{i=1,2} [NP(i)] dp_1 dp_2 - CC(2) \]

and so forth until

\[ E_i^i \max<\{NP(1)-CC(M)\}, \{NP(2)-CC(M)\}, \ldots, \{NP(M)-CC(M)\}>^\frac{3}{2} \]

\[ = \int_{-\infty}^{\infty} \ldots \int_{-\infty}^{\infty} f_0(p_1, p_2, \ldots, p_M) \max_{i=1,2, \ldots M} [NP(i)] dp_1 \ldots dp_M - CC(M). \]

One part of Equation (2.24) will be expanded,
The observed prices on day 1, \( p_1, p_2, \ldots, p_m \), can be used to derive the posterior distribution of prices for day 1 as given by Equation (3.5). And by using Equation (3.7), distributions of prices given \( \tilde{P}_2, \tilde{P}_n \) can be found. Solving for part of Equation (3.26)

\[
E\left[ \max\{NP_{1,m}^{*} \text{CC}(m+1)\}, \{NP(m+1) - \text{CC}(m+1)\}\right],
\]

\[
E\left[ g_1(m+2), \max\{NP_{1,m}^{*}, NP(m+1), NP(m+2)\}\right],
\]

\[
\{g_0(0,0) - \text{CC}(m) - W_0\}).
\]

Now, the following stopping rule can be stated.

If

\[
\{NP_{1,m}^{*} - \text{CC}(m)\} > E[g_1(m+1), \max\{NP_{1,m}^{*}, NP(m+1)\}]
\]

and

\[
\{NP_{1,m}^{*} - \text{CC}(m)\} < \{g_0(0,0) - \text{CC}(m) - W_0\}
\]

then sell to the packer offering the highest net price.

If

\[
\{NP_{1,m}^{*} - \text{CC}(m)\} < E[g_1(m+1), \max\{NP_{1,m}^{*}, NP(m+1)\}]
\]

and

\[
\{g_0(0,0) - \text{CC}(m) - W_0\} < E[g_1(m+1), \max\{NP_{1,m}^{*}, NP(m+1)\}]
\]

then contact the \((m+1)^{st}\) packer.
If
$$\{NP^*_{1,m} - CC(m)\} < \{g_0(0,0) - CC(m) - W_0\}$$
and
$$E[g_1(m+1), \max\{NP^*_{1,m}, NP(m+1)\}] < \{g_0(0,0) - CC(m) - W_0\}$$
then wait until day 0.

Suppose that it is day 1 and no packers have been contacted. The producer could begin contacting packers on day 1 or he could wait until day 0. Thus, the expected gain under the optimal policy is
$$g_1(0,0) = \max[E[g_1<1,NP(1)>], \{g_0(0,0) - W_0\}] \quad (3.27)$$
where
$$E[g_1<1,NP(1)>] = E[\max(E[NP(1)-CC(1)], E[g_1<2,\max\{NP(1), NP(2)\}>], \{g_0(0,0) - CC(1) - W_0\})]. \quad (3.28)$$

Solving part of Equation (3.28),
$$E[NP(1)-CC(1)] = \int_{-\infty}^{\infty} f_1(p_1)(NP(1)) \ dq_1 - CC(1).$$

The corresponding stopping rule is
If
$$E[g_1<1,NP(1)>] \leq \{g_0(0,0) - W_0\}$$
then contact the first packer on day 1.

If
$$E[g_1<1,NP(1)>] < \{g_0(0,0) - W_0\}$$
then wait until the last day.
Now, consider the general case if it is marketing day \( j \) and the \( m \)th packer has been contacted. If \( 1 \leq m \leq (M-1) \) and \( 1 \leq j \leq (J-1) \), the producer must choose between three alternative actions. He could sell to the packer offering the highest net price, \( NP^*_{j,m} \), he could wait and contact the \((m+1)\)st packer on day \( j \), or he could cease contacting packers on day \( j \) and wait until day \((j-1)\). Therefore, the expected gain under the optimal policy is

\[
g_j[m, NP^*_{j,m}] = \max \left\{ \left\{ NP^*_{j,m} - CC(m) \right\}, \mathbb{E}\left\{ g_{j+1}(m+1) \right\}, \max\left\{ NP^*_{j,m}, NP_{m+1}\right\} \right\},
\]

\[
\{ g_{j-1}(0,0) - CC(m) - W_{j-1} \}\right\}
\]

(3.29)

where

\[
g_{j-1}(0,0) = \max \left\{ \mathbb{E}\{ g_{j-1} < 1, NP(1) \} \right\}, \mathbb{E}\left\{ g_{j-2}(0,0) - W_{j-2} \right\} \}
\]

The observed prices on day \( j \), \( p_1, \ldots, p_m \), can be used to derive the posterior distribution of prices for day \( j \) as given by Equation (3.5). And by using Equation (3.7), distributions of prices given \( p_1, p_2, \ldots, p_m \) can be found. The expansion of Equation (3.29) and the solutions can be found in the same manner as the day 1 and day 0 instances. The stopping rule is given by Equation (2.14). Similarly, the expected gain under the optimal policy and the corresponding stopping rule for the instance when \( m = 0 \) and \( 1 \leq j \leq (J-1) \) are given in Equations (2.15) and (2.16). Distributions of prices can be found by using the prior distributions of the parameters for the marketing days.
Summary

If the producer uses one-stage sampling in the nonsequential problem, the following procedure can be used to determine the optimal number of packers to contact for each day in the marketing horizon. At the beginning of the marketing horizon, the producer can express his estimates of the unknown parameters for each marketing day \( j \) in the form of a prior p.d.f.

\[ g(t_{1j}, t_{2j}, \ldots, t_{kj}) \]

where the \( k \) parameters of \( f_j(p) \) are \( \mathbf{t}_j = (t_{1j}, t_{2j}, \ldots, t_{kj}) \). The p.d.f. of the price offered by packer \( i \) on day \( j \) given the prior p.d.f. of the parameters is

\[ f_j(p_i | t_{1j}, t_{2j}, \ldots, t_{kj}) \]

If a random sample of size \( n \) is taken from a distribution having p.d.f. \( f_j(p | t_{1j}, t_{2j}, \ldots, t_{kj}) \), the joint p.d.f. of \( p_1, p_2, \ldots, p_n \) is

\[ f_j(p_1, p_2, \ldots, p_n | t_{1j}, \ldots, t_{kj}) \]

\[ = f_j(p_1 | t_{1j}, \ldots, t_{kj}) f_j(p_2 | t_{1j}, \ldots, t_{kj}) \ldots f_j(p_n | t_{1j}, \ldots, t_{kj}) \]

And thus the joint prior p.d.f. of these \( n \) prices is

\[ f_j(p_1, p_2, \ldots, p_n) \]

\[ = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} g(t_{1j}, t_{2j}, \ldots, t_{kj}) f_j(p_1, p_2, \ldots, p_n | t_{1j}, \ldots, t_{kj}) \, dt_{1j} \cdots dt_{kj}. \]

If the producer uses one-stage sampling in the nonsequential problem and the marketing horizon is only one day long, the optimal number of packers to contact is found as follows
If the producer has a multi-day horizon, the optimal number of packers to contact on day $j$ is

$$\max_{n}[E(g_j(n))] = \max_{n}[\max_{1,2,...,n}[\max\left\{NP(i), EG_{j-1}\right\}] - CC(n)]$$

where

$$EG_{j-1} = E[\max\left\{NP(i), EG_{j-2}\right\}] - CC(n_{j-1}) - W_{j-1}$$

The corresponding optimal stopping rule for day $j$ is given by

- if $NP_j \geq EG_{j-1}$, then sell to the packer who offers the highest net price and
- if $NP_j < EG_{j-1}$, then wait.

Thus, the producer uses the same procedure whether or not he knows the exact values of the parameters. However, if he is uncertain about the exact values of the parameters, he will use his estimate of the distribution of the parameters to form a joint prior distribution of prices instead of the exact form of the distribution.

To determine the optimal number of packers to contact if the producer uses two-stage sampling in the nonsequential one-day horizon problem, the producer can use the following procedure. If a sample of size $n$ will be taken in the first stage, the joint prior p.d.f. of
the n prices is found in the same way as the one-stage sampling instance

\[ f(p_1, p_2, \ldots, p_n) = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} g(t_1, \ldots, t_k) f(p_1, \ldots, p_k | t_1, \ldots, t_k) dt_1 \cdots dt_k. \]

The expected net gain if n packers will be contacted in the first stage and the \((n+1)\)st through the \((n+k)\)th packers will be contacted in the second stage is

\[ E_n (g(n+k)) = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} f(p_1, \ldots, p_n) \left[ \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} f(p_{n+1}, \ldots, p_{n+k} | p_1, \ldots, p_n) \right] \]

\[ \times \max_{i=1,2,\ldots,n+k} [NP(i)] dp_{n+1} \cdots dp_{n+k} - CC(n+k) ] dp_1 \cdots dp_n \]

where

\[ f(p_{n+1}, \ldots, p_{n+k} | p_1, \ldots, p_n) = \frac{\int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} g(t_1, \ldots, t_k) f(p_1, \ldots, p_{n+k} | t_1, \ldots, t_k) dt_1 \cdots dt_k}{\int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} g(t_1, \ldots, t_k) f(p_1, \ldots, p_n | t_1, \ldots, t_k) dt_1 \cdots dt_k} . \]

The optimal expected gain if n packers will be contacted in the first stage is

\[ \max_{s=1,2,\ldots,M-n} \left[ \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} f(p_{n+1}, \ldots, p_{n+s} | p_1, \ldots, p_n) \right] \]

\[ \times \max_{i=1,2,\ldots,n+s} [NP(i)] dp_{n+1} \cdots dp_{n+s} - CC(n+s) ] \]
To find the optimal number of packers to contact in the first stage,

\[
\max_{n=0,1,\ldots,M} \prod_{i=1}^{n} f(p_1, \ldots, p_n) \\
\times \left( \max_{x=1,2,\ldots,M-n} \prod_{i=x}^{\infty} f(p_{n+1}, \ldots, p_{n+S} | p_1, \ldots, p_n) \right) \\
\times \left( \prod_{i=1}^{n+1} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \sum_{g(i)} dp_{n+1} \ldots dp_{n+S} - CC(n+S) \right) dp_1 \ldots dp_n \\
= \sum_{n=0,1} \left( g(n_0,1 | n_{0,1}^+) \right)
\]

where \( n_{0,1} \) denotes the optimal number of packers to contact in the first stage. If the producer then contacts \( n_{0,1} \) packers, the prices received by the producer, \( p_1, p_2, \ldots, p_{n_{0,1}} \), can be used to revise the prior distribution of the parameters as follows

\[
g(t_1, \ldots, t_k | p_1, \ldots, p_0, 1) \\
= \frac{g(t_1, \ldots, t_k) f(p_1, \ldots, p_{n_{0,1}} | t_1, \ldots, t_k)}{\int \int \int \int g(t_1, \ldots, t_k) f(p_1, p_2, \ldots, p_{n_{0,1}} | t_1, \ldots, t_k) dt_1 \ldots dt_k}.
\]
Suppose that $S$ packers will be contacted in the second stage.

The joint prior p.d.f. of these $S$ prices, $p_{n_0,1}^{l+1}, \ldots, p_{n_0,1}^{l+S}$, is

$$f(p_{n_0,1}^{l+1}, \ldots, p_{n_0,1}^{l+S} | p_1, \ldots, p_{n_0,1}) = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} g(t_1, \ldots, t_k | p_1, \ldots, p_{n_0,1}) f(p_{n_0,1}^{l+1}, \ldots, p_{n_0,1}^{l+S} | t_1, \ldots, t_k) dt_1 \cdots dt_k.$$ 

If $N_{0,1}^*$ represents the maximum net price obtained after $n_{0,1}$ packers have been contacted in the first stage and if $S$ packers will be contacted in the second stage, the expected net gain is

$$E_{n_0,1}^l (g(n_{0,1}^{l+S})) = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} f(p_{n_0,1}^{l+1}, \ldots, p_{n_0,1}^{l+S} | p_1, \ldots, p_{n_0,1})$$

$$\times \max_{i=n_0,1^{l+1}, \ldots, n_0,1^{l+S}} [N_{0,1}^i, N_{0,1}^*] dp_{n_0,1}^{l+1} \cdots dp_{n_0,1}^{l+S} \cdot \text{CC}(n_0,1^{l+S}).$$

The optimal expected net gain and the optimal number of packers to contact in the second stage can be found as follows

$$\max_{i=0,1, \ldots, M-n_0,1} E_{n_0,1}^l (g(n_{0,1}^{i+1})) = E_{n_0,1}^l (g(n_{0,1}^{i+n_0,2}))$$

where $n_{0,2}$ denotes the optimal number of packers to contact in the second stage.
If the producer uses two-stage sampling in the nonsequential multi-day horizon, the optimal number of packers to contact for each day in the marketing horizon can be determined in the following manner. Suppose it is day $j$ in the marketing horizon. If the producer waits until day $(j-1)$, his expected gain is

$$E_{g_{j-1},l} = E_{g_{j-1},l}(g_{j-1}(n_{j-1},+S_{n_{j-1},l})) - W_{j-1}. $$

The following stopping rule applies:

- if $N_{j} \geq E_{g_{j-1},l}$, then sell to the packer who offers the highest net price and
- if $N_{j} < E_{g_{j-1},l}$, then wait

where $N_{j}$ is the highest net price offered on day $j$. The expected net gain given that $n$ packers will be contacted in the first stage, $l$ packers will be contacted in the second stage, and the preceding stopping rule is

$$E_{g_{j}(n+l)} = \int_{-\infty}^{\infty} \ldots \int_{-\infty}^{\infty} f_{j}(p_{1}, \ldots, p_{n}) \int_{-\infty}^{\infty} \ldots \int_{-\infty}^{\infty} f_{j}(p_{n+1}, \ldots, p_{n+l} | p_{1}, \ldots, p_{n})$$

$$x \max_{i=1,2,\ldots,n+l} [N_{j}(i), E_{g_{j-1},l}] dp_{n+1} \ldots dp_{n+l} - CC(n+l)].$$

The optimal expected gain if $n$ packers will be contacted in the first stage is

$$\max_{s=0,1,\ldots,M-n} [\int_{-\infty}^{\infty} \ldots \int_{-\infty}^{\infty} f_{j}(p_{n+1}, \ldots, p_{n+s} | p_{1}, \ldots, p_{n})$$

$$x \max_{i=1,2,\ldots,n+s} [N_{j}(i), E_{g_{j-1},l}] dp_{n+1} \ldots dp_{n+s} - CC(n+s)].$$

The optimal number of packers to contact in the first stage of day $j$, $n_{j,1}$, can be determined from
\[ \max_{n=0,1,...,M} \left[ \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} f_j(p_1, \ldots, p_n) \right] \]

\[ x \max_{s=0,1, \ldots, M-n} \left[ \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} f_j(p_{n+1}, \ldots, p_{n+s} | p_1, \ldots, p_n) \right] \]

\[ x \max_{i=1,2, \ldots, n+S} \left[ \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left( g_j(n_{j,1} + 1, \ldots, n_{j,1} + S) \right) \right] \]

If \( n_{j,1} \) packers have been contacted in the first stage of day \( j \), the prior distribution of the parameters can be revised to form the posterior distribution of the parameters as follows:

\[ g(t_{1j}, \ldots, t_{kj} | p_1, \ldots, p_{n_{j,1}}) \]

\[ \frac{g(t_{1j}, \ldots, t_{kj}) f_j(p_1, \ldots, p_{n_{j,1}} | t_{1j}, \ldots, t_{kj})}{\int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} g(t_{1j}, \ldots, t_{kj}) f_j(p_1, \ldots, p_{n_{j,1}} | t_{1j}, \ldots, t_{kj}) dt_{1j} \cdots dt_{kj}} \]

Let \( NP^*_{n_{j,1}} \) be the maximum net price offered on day \( j \) after contacting \( n_{j,1} \) packers in the first stage. The maximum expected gain if \( S \) packers will be contacted in the second stage is

\[ E_{n_{j,1}} (g_j(n_{j,1} + S)) = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} f_j(p_{n_{j,1} + 1}, \ldots, p_{n_{j,1} + S} | p_1, \ldots, p_{n_{j,1}}) \]

\[ x \max_{i=n_{j,1} + 1, \ldots, n_{j,1} + S} \left[ (NP(1), NP^*_{n_{j,1}}) d_{n_{j,1} + 1} \cdots d_{n_{j,1} + S} - CC(n_{j,1} + S) \right] \]

To determine the \( S \) which maximizes the expected gain,
where \( n_{j,2} \) denotes the optimal number of packers to contact in the second stage.

For the sequential problem, only one-stage sampling was discussed. If the producer has a one-day horizon and he has contacted the \( m^{th} \) packer, he can choose between two alternative actions. The producer can sell to the packer offering the highest bid, \( N_{m}^{*} \), or he can wait and contact the \((m+1)^{st}\) packer. Thus, if the optimal policy is followed, the expected gain is

\[
g(m, N_{m}^{*}) = \max\{N_{m}^{*} - CC(m), E[g(m+1), \max\{N_{m}^{*}, NP(m+1)\}]\}.
\]

Recall that the observed prices, \( p_{1}, \ldots, p_{m} \), can be used to derive the posterior distribution of prices. Thus, the corresponding optimal stopping rule is

if \( \{N_{m}^{*} - CC(m)\} \geq E[g(m+1), \max\{N_{m}^{*}, NP(m+1)\}] \)
then sell to the packer offering the highest net price and

if \( \{N_{m}^{*} - CC(m)\} < E[g(m+1), \max\{N_{m}^{*}, NP(m+1)\}] \)
then wait and contact the next packer.

For the sequential, multi-day problem when \( 1 \leq m \leq (M-1) \) and \( 1 \leq j \leq (J-1) \), the producer must choose between three alternative actions. If it is day \( j \) and the producer has contacted the \( m^{th} \) packer, he could sell to the packer offering the highest net price, \( N_{j,m}^{*} \), he could wait and contact the \((m+1)^{st}\) packer on day \( j \), or he could cease contacting packers on day
and wait until day \((j-1)\). Thus, the expected gain under the optimal policy is

\[
g_j[m, NP^{*}_{j,m}] = \max\{NP^*_{j,m} - CC(m)\}, \ E[g_{j}^{(m+1)}, \max\{NP^*_{j,m}, NP(m+1)\}\}
\]

\[
\{g_{j-1}(0,0) - CC(m) - W_{j-1}\}.
\]

The observed prices on day \(j\), \(p_1, \ldots, p_m\), can be used to derive the posterior distribution of prices for day \(j\). The corresponding stopping rule is

if \(\{NP_{j,m} - CC(m)\} < E[g_{j}^{(m+1)}, \max\{NP^*_{j,m}, NP(m+1)\}\]

and

\[
\{g_{j-1}(0,0) - CC(m) - W_{j-1}\} < E[g_{j}^{(m+1)}, \max\{NP^*_{j,m}, NP(m+1)\}\}
\]

then contact the \((m+1)^{st}\) packer on day \(j\) and

if \(\{NP^*_{j,m} - CC(m)\} \geq E[g_{j}^{(m+1)}, \max\{NP^*_{j,m}, NP(m+1)\}\]

and

\[
\{NP^*_{j,m} - CC(m)\} \geq \{g_{j-1}(0,0) - CC(m) - W_{j-1}\},
\]

then sell to the packer offering the highest net price on day \(j\) and

if \(\{NP^*_{j,m} - CC(m)\} < \{g_{j-1}(0,0) - CC(m) - W_{j-1}\}\)

and

\[
E[g_{j}^{(m+1)}, \max\{NP^*_{j,m}, NP(m+1)\}] < \{g_{j-1}(0,0) - CC(m) - W_{j-1}\},
\]

then wait until day \((j-1)\).
CHAPTER IV. AN ILLUSTRATIVE EMPIRICAL APPLICATION AND SIMPLIFIED FORMULATIONS OF THE ONE-DAY HORIZON PROBLEM

This chapter begins with a hypothetical example. It is assumed that the producer knows the distribution of packers' prices, uses nonsequential sampling, and has a one-day horizon. The producer's feedlot is located in Ames, Iowa, and there are five feasible packers. The following producer decisions are considered: in what order should the packers be contacted and how many packers should be contacted. In attempting to solve the problem of how many packers to contact, it was found that computer solutions were very costly. Therefore, in the second half of this chapter, some assumptions will be modified, and more operational procedures for the sequential and nonsequential, one-day horizon problems will be derived.

An Empirical Example

As already mentioned, the hypothetical feedlot is located in Ames, Iowa. To keep the example small, only five feasible packers were selected. Those packers chosen were the five closest to Ames located in the following Iowa cities: Des Moines, Marshalltown, Fort Dodge, Denison, and Cedar Rapids.

Before the order in which packers should be contacted can be determined, contact costs, recontact costs, and transportation costs must be estimated. Contact costs were estimated by the cost of telephoning the packer and by the value of the time spent by the producer
making the telephone call and talking to the buyer at the feedlot. It is assumed that the packer will talk on the telephone for three minutes and will spend a total of a half hour making each contact. The producer's time is assumed to be worth ten dollars per hour. The cost of the telephone calls were based on dial-direct, weekday rates. Recontact costs were estimated by the cost of a three minute telephone call and by the value of a quarter hour of the producer's time. The transportation costs were estimated by the cost of transporting cattle by a private trucking company from Ames to the various packers and by the value of the liveweight shrink resulting from transporting the cattle. The costs of transporting the cattle were obtained from Kennedy General Trucking in Kelley, Iowa. The estimates of liveweight shrink resulting from transporting cattle were obtained from unpublished results of an Iowa State University study and can be summarized as follows:

\[
\text{PERCENT LIVESTOCK SHRINK} = \begin{cases} 
0.000 & \text{if DISTANCE} = 0 \\
2.921 & \text{if DISTANCE} = 40 \\
3.501 & \text{if DISTANCE} = 80 \\
3.721 & \text{if DISTANCE} = 120.
\end{cases}
\]

The estimates of liveweight shrink for other distances were found by linear interpolation. Contact, recontact, and transportation costs are summarized in Table 1.

Other assumptions must be made so that costs can be stated in a more usable form. It will be assumed that the producer will sell 15,000 pounds of cattle. Then, costs per cwt. can be found. Also, packers' prices are
Table 1. Estimates of contact, recontact, and transportation costs

<table>
<thead>
<tr>
<th>Location of Packer</th>
<th>Distance from Ames</th>
<th>Telephone Costs</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>First Minute</td>
</tr>
<tr>
<td>Des Moines</td>
<td>27 miles</td>
<td>$.31</td>
</tr>
<tr>
<td>Marshalltown</td>
<td>38 miles</td>
<td>.35</td>
</tr>
<tr>
<td>Fort Dodge</td>
<td>62 miles</td>
<td>.39</td>
</tr>
<tr>
<td>Denison</td>
<td>93 miles</td>
<td>.43</td>
</tr>
<tr>
<td>Cedar Rapids</td>
<td>101 miles</td>
<td>.43</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Contact Costs</th>
<th>Recontact Costs</th>
<th>Transporting Costs</th>
<th>Percent Shrink</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5.63</td>
<td>$3.13</td>
<td>$.25/cwt.</td>
<td>1.972%</td>
</tr>
<tr>
<td>5.75</td>
<td>3.25</td>
<td>.30/cwt.</td>
<td>2.775%</td>
</tr>
<tr>
<td>5.87</td>
<td>3.37</td>
<td>.35/cwt.</td>
<td>3.240%</td>
</tr>
<tr>
<td>5.99</td>
<td>3.49</td>
<td>.40/cwt.</td>
<td>3.573%</td>
</tr>
<tr>
<td>5.99</td>
<td>3.49</td>
<td>.45/cwt.</td>
<td>3.617%</td>
</tr>
</tbody>
</table>
assumed to be normally distributed with a mean of $40.00 per cwt. and a standard deviation of $1.00 per cwt. Thus, the loss resulting from shrink can be estimated by determining the monetary loss resulting from obtaining the mean price of $40.00. The total costs associated with each packer are computed by summing the contact, recontact, and transportation costs per cwt. The order in which to contact packers can be found by listing the packers in order of total costs per cwt. These costs are summarized in Table 2.

Recall that the optimal number of packers to contact is given by

$$\max_{n=1,2,\ldots,5} \left[ E(g(n)) \right]$$

$$= \max_{n=1,2,\ldots,5} \left[ \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} f(p_1)f(p_2)\cdots f(p_n) \max_{i=1,2,\ldots,n} \left[ NP(i) \right] dp_1 \cdots dp_n - CC(n) \right].$$

Shrink may have been more accurately estimated by multiplying the bid price by the percent shrink. For example, the expected net gain if n=1 could be calculated by

$$\int_{36}^{44} \frac{1}{\sqrt{2\pi}} e^{-\frac{(p_1-40)^2}{2}} \left( p_1-.25-.789 p_1-.021 \right) dp_1-.038.$$  

And if n=2, the expected net gain is

$$\int_{36}^{44} \int_{36}^{44} \frac{1}{2\pi} e^{-\frac{(p_1-40)^2-(p_2-40)^2}{2}} \max \left[ (p_1-.25-.789 p_1-.021), (p_2-.30-1.110 p_2-.022) \right] dp_1 dp_2-.076$$

and so forth. Thus, liveweight shrink would be a function of the price offered by the packer.
Table 2. Estimates of contact, recontact, and transportation costs per cwt.

<table>
<thead>
<tr>
<th>Location of Packer</th>
<th>Distance from Ames (miles)</th>
<th>Contact Costs per cwt.</th>
<th>Recontact Costs per cwt.</th>
<th>Transporting Costs per cwt.</th>
<th>Shrink Costs per cwt.</th>
<th>Transportation Costs per cwt.</th>
<th>Total Costs per cwt.</th>
<th>Order in Producer's List</th>
<th>Cumulative Contact Costs per cwt.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Des Moines</td>
<td>27</td>
<td>$.038</td>
<td>$.021</td>
<td>$.25</td>
<td>$.789</td>
<td>$1.039</td>
<td>$1.098</td>
<td>1</td>
<td>$.038</td>
</tr>
<tr>
<td>Marshalltown</td>
<td>38</td>
<td>.038</td>
<td>.022</td>
<td>.30</td>
<td>1.110</td>
<td>1.410</td>
<td>1.470</td>
<td>2</td>
<td>.076</td>
</tr>
<tr>
<td>Fort Dodge</td>
<td>62</td>
<td>.039</td>
<td>.022</td>
<td>.35</td>
<td>1.296</td>
<td>1.646</td>
<td>1.707</td>
<td>3</td>
<td>.115</td>
</tr>
<tr>
<td>Denison</td>
<td>93</td>
<td>.040</td>
<td>.023</td>
<td>.40</td>
<td>1.429</td>
<td>1.829</td>
<td>1.892</td>
<td>4</td>
<td>.155</td>
</tr>
<tr>
<td>Cedar Rapids</td>
<td>101</td>
<td>.040</td>
<td>.023</td>
<td>.45</td>
<td>1.447</td>
<td>1.897</td>
<td>1.960</td>
<td>5</td>
<td>.195</td>
</tr>
</tbody>
</table>
Because the distribution of prices is \(N(40,1)\),

\[
f(p) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(p-\mu)^2}{2\sigma^2}} \quad \text{for } -\infty < p < \infty
\]

\[
= \frac{1}{\sqrt{2\pi}} e^{-\frac{(p-40)^2}{2}} \quad \text{for } -\infty < p < \infty
\]

The solution for \(E(g(n))\) for \(n = 1, 2, 3, 4\) were found by using a computer. For \(n=1\), the integral

\[
\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{(p-40)^2}{2}} (p - 1.098 - .021) dp = 0.38
\]

was determined by using the IMSL subroutine DCADRE. For \(n=2\), the expression

\[
\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{(p_1-40)^2}{2}} \frac{1}{\sqrt{2\pi}} e^{-\frac{(p_2-40)^2}{2}} \max[(p_1 - 1.098 - .021), (p_2 - 1.470 - .022)] dp_1 dp_2 = 0.076
\]

was solved by computing the inner integral by DCADRE and by computing the outer integral by SSP subroutine DQG32. By computing the inner integral by DCADRE, the middle integral by DQG32, and the outer integral by SSP subroutine DQG16, the triple integral expression
for $n=3$ was determined. The combined cost of computing the expected gains for $n=1,2,3$ was approximately $11.00$ using the FORTRAN language WATFIV. For $n=4$, computer costs for determining a quadruple integral rose sharply. To decrease these costs, the expression

$$\int_{36}^{44} \int_{36}^{44} \int_{36}^{44} \frac{1}{(2\pi)^{3/2}} e^{-\frac{1}{2} \max[(p_1-1.098-.021), (p_2-1.470-.022), (p_3-1.646-.022), (p_4-1.829-.023)] dp_1 dp_2 dp_3 dp_4} - .155$$

was solved by computing the inner integral by DQG16, the second most inner integral by the SSP subroutine DQG12, and the outer two integrals by the SSP subroutine DQSF. The cost of computing this quadruple integral was approximately $9.00$ using WATFIV. The expected net gains for $n=1,2,3,4$ are listed in Table 3. Attempts were made to determine the quintuple integral
Table 3. The expected net gain if 1, 2, 3, 4 or 5 packers are contacted

<table>
<thead>
<tr>
<th>Location of Packer</th>
<th>Order in the Producer's List</th>
<th>Expected Net Gain</th>
<th>An Approximate Expected Net Gain</th>
<th>E(y(n))</th>
<th>E(TC(i))</th>
<th>E(RC(i))</th>
<th>CC(n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Des Moines</td>
<td>1</td>
<td>$38.90</td>
<td>$38.90</td>
<td>$40.00</td>
<td>$1.039</td>
<td>$.021</td>
<td>$.038</td>
</tr>
<tr>
<td>Marshalltown</td>
<td>2</td>
<td>39.26</td>
<td>39.24</td>
<td>40.56</td>
<td>1.224</td>
<td>.022</td>
<td>.076</td>
</tr>
<tr>
<td>Fort Dodge</td>
<td>3</td>
<td>39.38</td>
<td>39.34</td>
<td>40.84</td>
<td>1.365</td>
<td>.022</td>
<td>.115</td>
</tr>
<tr>
<td>Denison</td>
<td>4</td>
<td>39.42</td>
<td>39.36</td>
<td>41.02</td>
<td>1.481</td>
<td>.022</td>
<td>.155</td>
</tr>
<tr>
<td>Cedar Rapids</td>
<td>5</td>
<td>39.38</td>
<td>41.16</td>
<td>1.564</td>
<td>.022</td>
<td>.195</td>
<td></td>
</tr>
</tbody>
</table>
By computing the inner integral by DQGL6, the second most inner integral by DQGL2, and the outer three integrals by DQSF, a very inaccurate solution which cost approximately $52.00 using WATFIV and FORTRAN G was computed. The solution was inaccurate by over $.30. The author feels that IMSL and SSP packaged subroutines for numerical quadrature are too costly and inaccurate to solve a quintuple integral. However, these results show that the producer should contact at least four packers before he sells his cattle. Thus, for this particular location and this specific distribution, the producer would not be maximizing his expected gain if he contacted only one, two, or three packers by habit.

A Simplified Formulation of the Nonsequential, One-Day Horizon Problem

Recall that for the nonsequential one-day horizon problem, the expected net gain if n packers are contacted is given by

\[ E(g(n)) = E(\max_{i=1,2,\ldots,n} [NP(i)] - CC(n)) \]

where

\[ NP(i) = p_i - TC(i) - RC(i). \]
However, a computationally easier but less accurate formulation can be developed. Instead of finding the expected value of the maximum net price, the expected maximum price minus the expected transportation and recontact costs could be calculated. Or mathematically,

$$E(g(n)) = E(\max_{i=1,2,...,n} (p_i)) - E(TC(i)) - E(RC(i)) - CC(n).$$

To determine $E(\max_{i=1,2,...,n} (p_i))$, the p.d.f. of $\max_{i=1,2,...,n} (p_i)$ must be found. Therefore, let $y(n) = \max_{i=1,2,...,n} (p_i)$. $y(n)$ is called the $n$th order statistic and its distribution can be derived as followed. If $F(p)$ is the cumulative distribution function (c.d.f.) of $p$, the c.d.f. of $y(n)$ can be derived by rewriting the event that defines it:

$$\{y(n) \leq p\} = \{p_1 \leq p, p_2 \leq p, \ldots, p_n \leq p\}.$$

The independence of the observations then permits factorization of the probability of the event on the right:

$$P(y(n) \leq p) = P(p_1 \leq p, \ldots, p_n \leq p)$$

$$= P(p_1 \leq p)P(p_2 \leq p) \ldots P(p_n \leq p) = [F(p)]^n.$$

Similarly, the c.d.f. of the $k$th smallest observation in terms of the population c.d.f. is

$$P[y(n) \leq p] = P(k \text{ or more of the } n \text{ observations are } \leq p)$$

$$= \sum_{j=k}^{n} \binom{n}{j} [F(p)]^j [1-F(p)]^{n-j},$$
the individual terms in this sum being probabilities that in $n$ independent trials precisely $j$ result in an observation that does not exceed $p$.
The individual trials are of the Bernoulli type with probability equal to $F(p)$. The density function of $y_{(k)}$, the $k^{th}$ order statistic, can be obtained from this c.d.f. by differentiating with respect to $p$:

$$f_{Y_{(k)}}(p) = \sum_{j=k}^{n} \binom{n}{j} [F(p)]^{j-1} f(p) [1-F(p)]^{n-j}$$

$$+ \sum_{j=k}^{n} \binom{n}{j}(n-j) [F(p)]^{j} [1-F(p)]^{n-j-1} [-f(p)]$$

$$= nf(p) \{ \sum_{j=k}^{n-1} \binom{n-1}{j-1} [F(p)]^{j-1} [1-F(p)]^{n-j}$$

$$- \sum_{j=k}^{n-1} \binom{n-1}{j} [F(p)]^{j} [1-F(p)]^{n-j-1} \}$$

Letting $j=m-1$ in the second sum results in terms identical with those in the first sum, but from $m=k+1$ to $n$. These then cancel except for the term $j=k$ in the first sum [7]:

$$f_{Y_{(k)}}(p) = nf(p) \binom{n-1}{k-1} [F(p)]^{k-1} [1-F(p)]^{n-k}.$$

For the $n^{th}$ order statistic, or for $k=n$

$$f_{Y_{(n)}}(p) = nf(p) \binom{n-1}{n-1} [F(p)]^{n-1} [1-F(p)]^{n-n} = nf(p) [F(p)]^{n-1}.$$

The expected value of the maximum price is given by

$$E(\max_{i=1,2,\ldots,n} (p_i)) = E(Y_{(n)}) = \int_{-\infty}^{\infty} p(nf(p) [F(p)]^{n-1}) dp.$$

Note that for any $n=1,2,\ldots,M$ only one integration is required. The
expected transportation and recontact costs can be determined by finding
the mean cost if n packers are contacted. Or,

\[ E(\text{TC}(i)) = \frac{\sum_{i=1}^{n} \text{TC}(i)}{n} \]

and

\[ E(\text{RC}(i)) = \frac{\sum_{i=1}^{n} \text{RC}(i)}{n} \]

Therefore,

\[ E(g(n)) = \int_{-\infty}^{\infty} p \cdot (n f(p) [F(p)]^{n-1}) dp - \frac{\sum_{i=1}^{n} \text{TC}(i)}{n} - \frac{\sum_{i=1}^{n} \text{RC}(i)}{n} - \text{CC}(n). \]

This approximate procedure can be applied to the empirical example
already developed. The expected gain for \( n = 1,2,\ldots,5 \) can be found by

\[ E(g(n)) = E(y_{(n)}) - E(\text{TC}(i)) - E(\text{RC}(i)) - \text{CC}(n) \]

\[ = \int_{-\infty}^{\infty} \frac{n p (p-40)^2}{\sqrt{2\pi}} e^{-\frac{(x-40)^2}{2}} [\int_{-\infty}^{p} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx]^{n-1} dp - \frac{\sum_{i=1}^{n} \text{TC}(i)}{n} - \frac{\sum_{i=1}^{n} \text{RC}(i)}{n} - \text{CC}(n). \]

The p.d.f.

\[ f(p) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(p-40)^2}{2}} \]

and

\[ F(p) = \int_{-\infty}^{p} \frac{1}{\sqrt{2\pi}} e^{-\frac{(x-40)^2}{2}} dx \]

can be found by using the SSP subroutine NDTR. The outer integral was
evaluated using three different SSP routines: DQG32, DQATR, and QATR.
The solutions obtained from these subroutines were identical to the
second decimal place. Each subroutine required less than $.50 to solve $E(y_{(n)})$ for all five solutions ($n = 1, 2, 3, 4, 5$). $E(y_{(n)})$, $E(TC(i))$, $E(RC(i))$, and the approximate $E(g(n))$ are listed in Table 3. Note that the approximate expected gains are all within $.06$ of expected net gains computed previously. However, it is more important that the approximate procedure gives the same (or nearly the same) optimal number of packers to contact. In the empirical example developed, it is not possible to determine whether both procedures will give the same optimal number of packers to contact because the optimal number cannot be determined for the original procedure. However, if the producer actually had only five feasible packers, the approximate procedure would suggest that all five packers be contacted. The author suggests that further work be done to compare the two procedures.

A Simplified Formulation of the Sequential, One-Day Horizon Problem

The empirical example has shown that the solution of multiple integrals by using the computer can become more costly and inaccurate as the number of integrals increases. Recall that the solution to the sequential, one-day horizon problem may require many multiple integrations. However, if the assumption that the number of packers is finite is relaxed and it is assumed that there is an infinite number of packers, a solution requiring fewer multiple integrations can be developed.

The alternative approach will be formulated as follows. Suppose that the producer has an infinite number of packers listed in order of
total costs per cwt. Because the producer must sell on the only day in his marketing horizon, he must contact at least one packer. Suppose that the producer contacts the first packer on his list and the net price obtained is $NP_1^\star$. The expected net gain if the producer does not obtain any more bids is

$$\{NP_1^\star - CC(1)\}$$

and the expected net gain if one more packer is contacted is

$$E(g(2)) = \int_{-\infty}^{\infty} f(p_2) \max[NP_1^\star, NP(2)] dp_2 - CC(2).$$

Because the producer wishes to maximize expected net gain, the optimal stopping rule is

- if $\{NP_1^\star - CC(1)\} \geq \int_{-\infty}^{\infty} f(p_2) \max[NP_1^\star, NP(2)] dp_2 - CC(2)$,
  then sell to the packer offering the highest net price and
- if $\{NP_1^\star - CC(1)\} < \int_{-\infty}^{\infty} f(p_2) \max[NP_1^\star, NP(2)] dp_2 - CC(2)$,
  then wait and contact the next packer.

Suppose that the producer decides to contact the second packer and that the higher net price obtained after contacting the first and second packer is $NP_2^\star$. If the producer does not obtain any more bids, his expected gain is

$$\{NP_2^\star - CC(2)\}.$$

But if one more packer is contacted, the expected gain is
\[ E(g(3)) = \int_{-\infty}^{\infty} f(p_3) \max[NP^*_2, NP(3)] dp_3 - CC(3). \]

Therefore, the optimal stopping rule is

if \( \{NP^*_2 - CC(2)\} \geq \int_{-\infty}^{\infty} f(p_3) \max[NP^*_2, NP(3)] dp_3 - CC(3), \)
then sell to the packer offering the highest net price and

if \( \{NP^*_2 - CC(2)\} \lt \int_{-\infty}^{\infty} f(p_3) \max[NP^*_2, NP(3)] dp_3 - CC(3), \)
then wait and contact the next packer.

Consider the general situation when the producer has contacted \( m \) packers. Let \( NP^*_m \) be the highest net price obtained. If the producer does not obtain any more bids, his expected net gain is

\[ \{NP^*_m - CC(m)\}. \]

If one more packer is contacted, the expected net gain is

\[ E(g(m+1)) = \int_{-\infty}^{\infty} f(p_{m+1}) \max[NP^*_m, NP(m+1)] dp_{m+1} - CC(m+1). \]

The corresponding stopping rule is

if \( \{NP^*_m - CC(m)\} \geq \int_{-\infty}^{\infty} f(p_{m+1}) \max[NP^*_m, NP(m+1)] dp_{m+1} - CC(m+1), \)
then sell to the packer offering the highest net price and

if \( \{NP^*_m - CC(m)\} \lt \int_{-\infty}^{\infty} f(p_{m+1}) \max[NP^*_m, NP(m+1)] dp_{m+1} - CC(m+1), \)
then wait and contact the next packer.

Although a list of twenty or thirty feasible packers may be considered infinite for all practical purposes, there is one major problem with this formulation of the problem. The producer may not be
able to contact as many packers within one day as may be suggested by the stopping rule. If the packer requires a half hour for each contact, he can only contact twenty-four packers within a twelve hour period. However, if this procedure is applied to actual empirical examples, this problem may not arise.
CHAPTER V. SUMMARY AND CONCLUSIONS

An overview of the work done in this study will be presented in this chapter. First, the reason why this study was conducted will be discussed. This will be followed by a general summary of the approaches used to solve the problem. Last, the limitations of this study and suggestions for further research will be presented.

Significance and Value of the Study

In this study, the following producer decisions were considered: how many packers should be contacted each day, which packers should be contacted each day, in what order should the packers be contacted each day, and on which day should cattle be sold. Generally, the producer will have only informal forms of information on which to base these decisions, and he will contact only a very few packers by habit. As an alternative to these ad hoc procedures, this study attempted to model the situations faced by the producer in terms of economic, mathematical and statistical theory. This model was then used to determine a procedure the producer could use to maximize his expected net gain.

Although no studies had previously focused on these particular decisions faced by cattle producers, studies had been conducted on "the economics of information" or searching for the highest or lowest
price in various marketing situations. However, the articles discussed in the literature review focused on the problem of obtaining a high or low price when these prices were identically distributed. But under the topic of "optimal stopping," numerous articles have been written about maximizing a reward based on observing a particular phenomena when these phenomena are not identically distributed.

The model of the decisions faced by the cattle producer is somewhat different from other studies of the economics of information in the following way. The producer wishes to maximize net gain which is a function of net price and cumulative contact costs. Net price is a function of the packer's bid price and transportation and recontact costs. And although the packers' prices are identically distributed, the net prices associated with different packers generally will not be identically distributed. This is because packing plants are not located the same distance from the producer, and consequently, transportation, contact, and recontact costs will not be the same for each packer. Therefore, the model derived for producer decisions is unique to models developed for other situations because of the following characteristic. The producer wishes to maximize net gain which is a function of net prices that are not identically distributed. However, net prices are a function of packers' bid prices which are identically distributed.

Although decisions similar to those faced by the cattle producer have been studied, there have been no attempts to model the particular
decisions of the cattle producer presented in this study. Thus, this study has provided the theoretical framework on which to base further empirical and theoretical studies of this topic.

Summary

To develop a procedure for the producer to use in making the decisions discussed in the previous section, certain simplifying assumptions were made. It was assumed that all contacts are initiated by the producer. The producer knows that he has M feasible packers, and he knows the contacting, recontacting, and transportation costs associated with each of these packers. It was assumed that the producer knows when his marketing horizon begins and ends and the exact number of days in the marketing horizon. The producer initiates contacts only during the marketing horizon. Once a packer makes a bid, it is valid throughout only that particular marketing day. It is also assumed that the producer knows exactly how many cattle he wishes to market and that this number remains constant throughout the marketing horizon. The price offered by a packer is price per cwt. of fed cattle. The prices offered by packers on day i are continuous and are identically and independently distributed. The price offered by a particular packer on one day is independent of any of his subsequent or previous offers.

The order in which the producer should contact his feasible packers is the same regardless of the method of sampling used and regardless of whether or not the parameters of the distributions of prices are known.
Contacting, recontacting, and transportation costs are found per cwt. Then, the producer should order his feasible packers from lowest to highest total cost per cwt. where total cost per cwt. is defined to be the sum of contacting, recontacting, and transportation costs per cwt. The producer should contact those packers with the lowest total costs per cwt. first.

In Chapter II, it was assumed that the producer knows the distributions of prices offered by the packers. Different procedures were developed for the producer to follow depending on whether he used sequential or nonsequential sampling. For each method of sampling, the producer's decisions were examined for both one-day and multi-day horizons.

In Chapter III, the parameters of the distribution of packers' prices are assumed to be unknown. Once again, procedures were developed for sequential and nonsequential sampling. For the nonsequential, one-day and multi-day horizon problems, both one-stage and two-stage sampling were discussed. Only one-stage sampling was discussed for the sequential, one-day and multi-day horizon problems.

A hypothetical example of the nonsequential, one-day horizon instance was presented in Chapter IV. Because it was found that computer solutions of multiple integrals can be costly and inaccurate, simplified formulations of the nonsequential and sequential, one-day horizon problems were also presented.
Limitations of the Study and Suggestions for Further Research

The major limitation of this study is the small amount of empirical work done. Computer programs could be developed for the other combinations of sampling methods and marketing horizon lengths. However, the empirical work that was done does suggest another limitation. It suggests that solutions to expressions involving more integrations than a quadruple integral may be too inaccurate and costly to be of any practical use. Thus, more work could be done on comparing the theoretical and empirical accuracy of the simplified formulations with the initial formulations.

It is suggested that the distribution of packer's prices be studied. Presently, only the range and sometimes the mode of the packers' bid prices are reported. Along with determining the distribution of packers' prices, some of the assumptions that were made about the distribution could be verified or discredited. For example, it was assumed that the prices offered by packers on day i are identically and independently distributed. Also, the price offered by a particular packer on one day is assumed to be independent of any of his subsequent or previous offers. These assumptions may seem unrealistic, but their validity could be studied.

Even though the small amount of empirical work done in this study suggested that computer solutions to the procedures developed may be too costly and inaccurate for practical use and although some of the assumptions may have been unrealistic, this study does provide a starting point
for future theoretical and empirical work examining the marketing decisions faced by cattle producers.
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BIBLIOGRAPHY


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