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Transfer costs in agricultural trade: implications for empirical research

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Transfer costs in agricultural trade:
Implications for empirical research

by

Sergio Horacio Lence

A Thesis Submitted to the
Graduate Faculty in Partial Fulfillment of the
Requirements for the Degree of
MASTER OF SCIENCE

Department: Economics
Major: Agricultural Economics

Signatures have been redacted for privacy

Iowa State University
Ames, Iowa
1988
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CHAPTER 1
INTRODUCTION

There is evidence supporting the hypothesis that the supply schedule of transfer services is upward sloping, at least in the short run. The price of these transfer services, commonly known as transfer costs or commercialization margin, is usually a significant proportion of the commodity price. Transfer costs establish a barrier to trade that is sometimes more important than that posed by taxes and tariffs. In relative terms, commercialization margins may be considerably more variable than commodity prices.

Despite these facts, most commodity trade models generally adopt either explicitly or implicitly the simplifying assumption that the supply of transfer services is infinitely price responsive. In general, these studies lack tests that justify the adoption of this assumption. Moreover, they do not discuss what the implications of this hypothesis are for the empirical results obtained.

The purpose of this research is to offer insight on how some of the results derived from trade models under perfectly elastic supply of transfer services are modified when this assumption is relaxed. The topics specifically addressed are the price transmission elasticities and the derived commodity elasticities. The approach used in the study is essentially theoretical, but some examples extracted from world agricultural trade are also presented to illustrate the major conclusions.
Definitions

For the purpose of the present study, transfer costs are defined as the difference in commodity prices between two successive stages of the commercialization channel with the exclusion of the taxes and/or subsidies applied to the commodity itself.

According to the above definition, transfer costs are the payments made to the transfer services industry for its contribution to the commercialization process. This industry provides handling, storage, transportation, processing, and sales.

Statement of the Problem

Transfer costs are an important and highly variable part of the price that consumers pay for agricultural commodities. These costs and their variability increase drastically when commodities are traded internationally, since in this particular case they must include payments to international and foreign transfer services in addition to the domestic ones. Charts 1 and 2 illustrate how substantial can be the transfer costs involved in the trade of grains. For the periods covered by the charts, the transfer costs between the farm and the importer's port ranged from 32% to 55% of the farm price for wheat, while the analogous range for corn was 24% to 45%. It must be noted that the CIF prices are still lower than the domestic price in the importer country (if this country does not subsidize consumption).

A look at Tables 1 and 2 gives a notion of the variability of transfer costs compared to commodity prices. They show that the
estimates of the standard deviation are higher for commodity prices than for transfer costs. However, a better measure to compare the relative variability of transfer costs and commodity prices is the coefficient of variation, defined as

\[
\text{Coefficient of Variation} = \frac{\text{Standard Deviation}}{\text{Mean}} \times 100
\]

It can be seen that, with the exception of the transfer costs between Kansas and Gulf for wheat, all of the other transfer costs have higher coefficients of variation than the respective commodity prices. This is especially so for the transfer costs corresponding to international trade, whose coefficients of variation double those of wheat and corn prices.

Studies by Finger and Yeats (1976), Sampson and Yeats (1977, 1978), and Yeats (1977), show that transportation costs (one component of transfer costs) may establish a barrier to international trade at least as important as tariffs. Despite this evidence, most international trade models have focused on the impact of tariffs on trade, while little attention has been paid to the effect of transfer costs.

George and King (1971) point out that poor knowledge about the behavior of the transfer services industry corresponding to a given commodity may severely restrict the understanding of the commodity market itself. Nevertheless, studies using derived commodity demand and supply functions have consistently overlooked the transfer services industry. These models have implicitly or explicitly assumed that either the transfer costs were constant or they were a function of the commodity

(a) Selected Transfer Costs

(b) Selected Price Ratios
Farm: U.S. farm price for Winter Wheat (Source: USDA).
Kansas: Kansas City price for Hard Winter Wheat, ordinary protein (Source: USDA).
Chart 2. Selected Transfer Costs and Price Ratios for Corn, 1977/78-1986/87 (October-September)

(a) Selected Transfer Costs
(b) Selected Price Ratios
Farm: U.S. farm price (Source: USDA).

Gulf: FOB Gulf price for No. 3 Yellow Corn (Source: USDA).

Rotterdam: Asking price for CIF Rotterdam 30 day delivery U.S. No. 3 Yellow Corn (Source: USDA).
Table 1. Estimated Statistics for Wheat Prices and Wheat Transfer Costs, 1977-1986

<table>
<thead>
<tr>
<th>Prices</th>
<th>Mean (US$/M.Ton)</th>
<th>Standard Deviation (US$/M.Ton)</th>
<th>Coefficient of Variation (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Farm</td>
<td>120.20</td>
<td>19.338</td>
<td>16.1</td>
</tr>
<tr>
<td>Kansas</td>
<td>134.00</td>
<td>22.435</td>
<td>16.7</td>
</tr>
<tr>
<td>Gulf</td>
<td>147.60</td>
<td>24.213</td>
<td>16.4</td>
</tr>
<tr>
<td>Japan</td>
<td>176.00</td>
<td>32.010</td>
<td>18.2</td>
</tr>
<tr>
<td>Kansas - Farm</td>
<td>13.80</td>
<td>3.938</td>
<td>28.5</td>
</tr>
<tr>
<td>Gulf - Kansas</td>
<td>13.60</td>
<td>2.221</td>
<td>16.3</td>
</tr>
<tr>
<td>Japan - Gulf</td>
<td>28.40</td>
<td>11.276</td>
<td>39.7</td>
</tr>
</tbody>
</table>

Farm: U.S. farm price for Winter Wheat (Source: USDA).

Kansas: Kansas City price for Hard Winter Wheat, ordinary protein (Source: USDA).


Table 2. Estimated Statistics for Corn Prices and Corn Transfer Costs, 1977/78-1986/87 (Oct.-Sept.)

<table>
<thead>
<tr>
<th></th>
<th>Mean (US$/M.Ton)</th>
<th>Standard Deviation (US$/M.Ton)</th>
<th>Coefficient of Variation (%)</th>
</tr>
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<tbody>
<tr>
<td><strong>Prices</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Farm</td>
<td>96.80</td>
<td>19.606</td>
<td>20.3</td>
</tr>
<tr>
<td>Gulf</td>
<td>112.40</td>
<td>21.125</td>
<td>18.8</td>
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<tr>
<td>Rotterdam</td>
<td>128.70</td>
<td>22.381</td>
<td>17.4</td>
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<td><strong>Transfer Costs</strong></td>
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<tr>
<td>Gulf - Farm</td>
<td>15.60</td>
<td>4.169</td>
<td>26.7</td>
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<tr>
<td>Rotterdam - Gulf</td>
<td>16.30</td>
<td>6.567</td>
<td>40.3</td>
</tr>
</tbody>
</table>

Farm: U.S. farm price (Source: USDA).
Gulf: FOB Gulf price for No. 3 Yellow Corn (Source: USDA).
Rotterdam: Asking price for CIF Rotterdam 30 day delivery U.S. No. 3 Yellow Corn (Source: USDA).
price. These studies neither tested the validity of these simplifying assumptions nor presented their implications for the analysis of the results obtained.

At a theoretical level Gardner (1975) demonstrated that neither a fixed percentage of price nor a fixed absolute spread between farm and retail prices are pricing rules consistent with a competitive food industry.

There are studies by Zannetos (1966), USDA (1968), Binkley and Revelt (1981), Binkley (1983), Gallagher (1983), and Meilke and Moschini (1987) that give empirical support for the hypothesis that the supply of transfer services is upward sloping, at least in the short run. This evidence should not be neglected, because in some cases it may change dramatically the results and/or inferences attained from trade models (for empirical examples of this see Gallagher, 1983, and Meilke and Moschini, 1987).

With these precedents it would be of interest to further develop a theory of trade with a supply of transfer services that is not perfectly elastic. This should allow a better understanding of the limitations of the empirical results obtained under the standard assumption of an infinitely price-responsive supply of transfer services.

Goals of the Study

The goal of the study is to reveal some of the implications for trade models of relaxing the standard assumption about transfer services supply, namely, that it is infinitely price responsive. The research
is mainly theoretical, and it deals specifically with price transmission elasticities and with derived commodity demand and supply elasticities.

The manner in which transfer services in less than perfectly elastic supply affect the standard result that there is only one price transmission elasticity is examined.

The findings of the price transmission elasticities study are then used to analyze derived commodity demand and supply elasticities. In this case the objective is to assess how the commodity elasticity estimates obtained from standard trade models compare to "true" commodity elasticities.
Studies by Finger and Yeats (1976), Sampson and Yeats (1977, 1978), and Yeats (1977) point out that transport costs are at least as important as tariffs in restricting international trade.

Zannetos (1966) found that the short-run supply of tankers was very inelastic at volumes close to industry capacity. He estimated that at 95% of industry capacity, the elasticity of supply was 0.02.

A report of the USDA (1968) asserts that "although distance is a factor in determining rates, the major determinant appears to be the short-run relationships between the supply of shipping available for grain cargoes and the demand for such shipping."

A paper by Binkley and Revelt (1981) supports the hypothesis that the supply of ocean transportation services is very elastic at low volumes of trade and quite inelastic when it is close to full capacity. One of its main conclusions is that assuming that transport costs do not affect trade models may lead to incorrect results. Also, it suggests increased efforts to endogenize transportation costs in non-spatial trade models.

According to Binkley (1983), the supply of marketing services is inelastic in the short-run, mainly due to high investment costs in this industry. He examined monthly data for U.S. Gulf and Rotterdam prices of wheat, corn, and soybeans and concluded that freight rates (measured by a
world index comprising all dry bulk commodities) were a major cause of the fluctuations in the price spread between both locations.

Gallagher (1983) found evidence that the international transfer costs affected the U.S.-Japan lumber trade, and also that the international marketing services, mainly transportation, were in inelastic supply. According to his results, the price transmission elasticity between both countries was much higher than in the presence of constant international transfer costs.

Meilke and Moschini (1987) studied the quarterly spatial price differentials between the U.S. and Canadian livestock industries. They found that quantity traded was a very important explanatory variable for the price spreads analyzed, supporting the hypothesis that transfer services are in inelastic supply. It is worth mentioning that transportation costs taken in isolation had little significance in the price linkage relations and even less when the volume of trade was simultaneously included in these equations.

By means of a three-sector model Gardner (1975) showed that fixed-percentage and/or fixed-absolute-transfer-cost rules between the farm and retail levels are incompatible with perfect competition. He also demonstrated that in general there is not a unique price transmission elasticity between the farm and retail levels. His model is specifically designed for the case in which commodity and transfer costs are substitutes for each other, since it breaks down when this kind of substitution is not allowed.
Tweeten (1967) measured the elasticity of foreign demand for U.S. farm output. He considered aggregate supply and demand elasticities for farm products in foreign countries and assumed all price transmission elasticities equal to one.

Another measure of the elasticity of U.S. foreign demand for farm products was obtained by Johnson (1977), who first calculated the elasticities of foreign demand for individual commodities, and then aggregated them. He also assumed that the price transmission elasticities equaled one.

Bredahl, Meyers, and Collins (1979) recognized the fact that the insulating policies of several countries involved in the international trade of grains made it hard to sustain the assumption that the price transmission elasticities for them were one. Working with the hypothesis of a zero price transmission elasticity for these countries, they showed that the elasticities of U.S. foreign demand were substantially lower than those obtained by Johnson and Tweeten.

In a review of 45 studies on major U.S. agricultural commodities, Gardiner and Dixit (1986) found that their estimates of the elasticities of foreign demand for each commodity lay in a very wide range. In none of those studies were transfer costs endogenized.

Using a logarithmic model, Collins (1980) calculated the price transmission elasticities between the U.S. and numerous countries for wheat, corn, and soybeans. It was assumed that the transfer costs between the U.S. Gulf ports and the countries' internal markets were
constant percentages of the U.S. Gulf price. The calculated values for the price transmission elasticities were between almost zero and one.

Meyers, Helmar, and Devadoss (1986a, 1986b) and Bahrenian, Devadoss, and Meyers (1986) obtained price transmission elasticities between prices in the U.S. and in several of the major countries involved in the world grain trade. They used linear equations to link the prices, and they assumed that transfer costs were a linear function of prices.

In most of the reviewed studies there was an implicit assumption of transfer costs behavior, but no explicit recognition or treatment of this issue. The large size and variability of transfer costs during past years (refer to Charts 1 and 2, and Tables 1 and 2) is evidence that the transfer services industry needs to be treated more explicitly in trade studies than it has been in the past.
CHAPTER 3
CONCEPTUAL PRESENTATION OF THE MODEL

Adapting the analysis of marketing margins made by George and King (1971) to the present context, some interesting assumptions regarding price relationships underlying most partial equilibrium trade models can be easily discovered. It is important to note that these assumptions are implicit and not always recognized by the users of those models.

From now on "seller" and "buyer" will mean any two groups of economic agents interacting through the market. The seller supplies a specific commodity, while the buyer demands this same commodity or a processed product requiring a fixed proportion of this commodity. It must be clearly understood that the "seller-buyer" linkage is representing here any relationship between successive sellers and buyers through the commercialization channel; it may be describing linkages such as "exporter-importer," "farmer-retailer," "wholesaler-processor," "wholesaler-exporter," and so on.

Variables may be defined as follows

\[ Q = \text{amount of commodity} \]
\[ A_Q = \text{amount of commodity supplied by the seller} \]
\[ B_Q = \text{amount of commodity (or fixed-proportion processed product in commodity units) demanded by the buyer} \]
\[ P = \text{price per unit of commodity} \]
\[ A_P = \text{price received by the seller} \]
\( B_P \) = price paid by the buyer

\[
E \left[ B_P \mid I_A \right] = \text{expected buyer's price given the information available at the time the purchase from the seller is made}
\]

\( M = E \left[ B_P \mid I_A \right] - A_P \) = expected transfer costs between buyer and seller levels at the time of purchase

\( A_0, B_0 \) = other variables affecting the behavioral relationships at the buyer and seller levels

Using the definitions above, some behavioral relationships regarding commodity demand and supply can be formulated implicitly in a very general way.

(buyer demand) \( B(B_Q, B_P, B_0) = 0 \) \hspace{1cm} (1)

(supply of transfer services) \( T^S(B_Q, B_P, M) = 0 \) \hspace{1cm} (2)

(demand for transfer services) \( T^D(A_Q, A_P, M) = 0 \) \hspace{1cm} (3)

(seller supply) \( A(A_Q, A_P, A_0) = 0 \) \hspace{1cm} (4)

Since \( M = E \left[ B_P \mid I_A \right] - A_P \), the supply of and demand for transfer services (expressions 2 and 3) can be stated also as

\( T^S(B_Q, E \left[ B_P \mid I_A \right], A_P, B_P) = 0 \) \hspace{1cm} (5)

\( T^D(A_Q, E \left[ B_P \mid I_A \right], A_P) = 0 \) \hspace{1cm} (6)
Under certain assumptions these implicit functions can be simplified substantially. This becomes very important for research, especially that related to empirical applications. In this paper the most important ones are

1. Perfect competition
2. \( B_Q = A_Q \)
3. \( E \left[ B_P \mid I_A \right] = B_P \)

To assume \( B_Q = A_Q \) means that it is not possible to substitute transfer services for commodity, which may be realistic for many commodities. But it also means that the market is always in equilibrium, which is a strong assumption. This is especially so when very short periods of time are involved because of the possibility of market disequilibrium (see Kinnucan and Forker, 1987).

To consider \( E \left[ B_P \mid I_A \right] = B_P \) is not always innocuous, as noted by George and King (1971). It may prove to be important according to trade modalities, and it is hard to justify it when there is a long interval between the effective purchase to the seller and the effective sale to the buyer.

In explicit form a very simple special case of expressions (1) and (4) can be stated as

(buyer demand) \( B_Q = \alpha - \beta B_P; \quad \alpha > 0, \beta > 0 \) \hspace{1cm} (7)

(seller supply) \( A_Q = \gamma + \delta A_P; \quad \delta > 0 \) \hspace{1cm} (8)
To obtain the derived demand at the seller level ($B_Q^d$) the buyer price must be expressed in terms of the seller price:

\[
(\text{derived seller demand}) \quad B_Q^d = \alpha - \beta [A_p + M] \tag{9}
\]

Finally, in equilibrium, $B_Q^d = B_Q = A_Q \tag{10}$

Equations (7) through (10) form a system that allows one to solve for the values of $B_Q^d$, $A_Q$, $B_Q$, $A_p$, and $B_p$ in equilibrium. This is so if $M$ is either an exogenous constant or an endogenous variable. In this latter case one more equation should be added to the system above, specifying the functional form of $M$.

It is important to point out here that the equilibrium amount of commodity in this system, $B_Q^d = B_Q = A_Q$, represents simultaneously the equilibrium quantity of transfer services expressed in units of commodity. This is so because it is assumed that the commodity is always combined with a fixed proportion of transfer services, that is, substitution of transfer services for commodity is not allowed in this system. Therefore, the quantity of transfer services will be expressed by $T$ or $Q$ interchangeably from now on.

At the same time, relations (7) and (8) also provide the demand for transfer services. Rearranging them:

\[
B_p = \frac{\alpha}{\beta} - \frac{1}{\beta} B_Q \tag{11}
\]

\[
A_p = -\frac{\gamma}{\delta} + \frac{1}{\delta} A_Q \tag{12}
\]
By definition, in equilibrium, \( M = M^S = M^D \) \hspace{1cm} (13)

where \( M^D = B_p - A_p \).

By the equilibrium condition, \( B_Q = A_Q = T^D = T^S \). \hspace{1cm} (14)

Now, substituting (11), (12), and (14) into (13) and solving for \( T^D \):

\[
T^D = \left( \frac{\alpha \delta + B_T}{\beta + \delta} \right) - \left( \frac{\beta \delta}{\beta + \delta} \right) M^D
\] \hspace{1cm} (15)

This is the demand schedule for transfer services, which is negatively sloped:

\[
\frac{\partial T^D}{\partial M^D} = - \left( \frac{\beta \delta}{\beta + \delta} \right)
\]

On the other hand, the supply schedule of transfer services adopts a very simple form in most non-spatial partial equilibrium models (N-SPEM) and spatial partial equilibrium models (SPEM) as well.

In the case of N-SPEM most attention is devoted to the estimation of the wholesale demand and farm supply functions for the commodity, while the supply schedule of transfer services is overlooked. In N-SPEM the price linkage equations are generally formulated as

\[
B_p = \theta + (1+\phi) A_p + u
\] \hspace{1cm} (16)

where \( u = \) random disturbance, and \( \theta, \phi = \) constants.
To adopt this kind of linkage equation is to implicitly postulate a supply of transfer services of the form

\[ M^S = \theta + \phi A_P + u \]  

(17)

This is so because to attain equilibrium in the transfer services market, \( M^S = M^D \) must hold. Then, from expressions (13) and (17),

\[ B_P - A_P = \theta + \phi A_P + u \]

Now, solving this expression for \( B_P \) yields (16).

In SPEM the supply of transfer services adopts a more elementary form, since most of these models postulate constant transfer costs:

\[ M^S = \rho \]  

(18)

where \( \rho \) = positive constant.

To provide a better understanding of what has been said thus far, the same analysis is made now with graphical tools. Figure 1 depicts the seller supply of and buyer demand for the commodity. The intersection of these two curves would provide a market equilibrium at \( \tilde{Q} \) and \( \tilde{P} \) if transfer costs were zero. As long as transfer costs are positive, the equilibrium amount of commodity is smaller than \( \tilde{Q} \), the equilibrium seller price is lower than \( \tilde{P} \), and the equilibrium buyer price is higher than \( \tilde{P} \), the difference between them being the equilibrium transfer cost. One such equilibrium is provided by \( Q^*, A_P^*, B_P^* \), and \( M^* = B_P^* - A_P^* \).

Moreover, if transfer costs exceed \( \tilde{M} = B_P - A_P \), the equilibrium quantity of commodity is zero.
Figure 1. Commodity Supply and Demand

\[ M = B_p - A_p \]

Figure 2. Demand for Transfer Services

\[ \tilde{M} = B_p - A_p^{\prime} \]

\[ \tilde{M} = B_p^{\prime} - A_p^{\prime} = 0 \]
The information provided by Figure 1 is put another way in Figure 2. The difference between both figures is that the second one has the difference between the buyer and seller prices on the vertical axis. The schedule shown in Figure 2 is the vertical difference between the buyer demand and the seller supply. This curve is actually a demand function for transfer services: as long as \( M = BP - AP \) is smaller than \( BP - AP \), there is a positive quantity of transfer services demanded, and this increases as \( M = BP - AP \) decreases.

It also follows that if \( Q^* \) is an equilibrium quantity of commodity, it must also be an equilibrium level of transfer services. This means that the supply schedule of transfer services passes through the point with coordinates \((Q^*, M^*)\). If this supply function is as stated in equation (17), its shape is as shown in Figure 3, which reproduces Figure 2 with the addition of the supply of transfer services.

For empirical modeling the supply of transfer services is generally restricted to adopt the form of equation (17) in N-SPEM, through price linkages like (16), or equation (18) in SPEM. The best reason for this is that it simplifies matters drastically, as it will be shown in the next chapter. But it is surprising that trade models generally present no tests along with the results to validate the hypothesized supply of transfer services.

If expression (17) is analyzed in more detail, it will soon become apparent that it is a very restrictive formulation for a supply of transfer services. First, it assumes a perfectly elastic supply of transfer services (actually, there would be a one-to-one relationship
Figure 3. Supply of and Demand for Transfer Services

Figure 4. Supply of Transfer Services of Standard N-SPEM
Figure 5. Supply of Transfer Services of Standard SPEM.

Figure 6. Supply Functions of Transfer Services Supported by Empirical Evidence
between $A_p$ and $Q$ if the changes in prices were due only to shifts in the demand schedule for the commodity). Second, it does not take into account the fixed capacity of the industry of transfer services. Represented graphically, equation (17) is of the form shown in Figure 4.

On the other hand, the supply of transfer services used in SPEM, that is, equation (18), is more restrictive still. Transfer costs are constant as depicted in Figure 5.

The works by Binkley and Revelt (1981), Binkley (1983), and Gallagher (1983) support the hypothesis of a supply of transfer services positively related to the amount of commodity traded. The shapes of the functions found by them are shown in Figures 6a (Binkley and Revelt, 1981; Binkley, 1983) and 6b and 6c (Gallagher, 1983).

If there is a positive relationship between $M^S$ and $Q$ as shown in those studies, the price transmission elasticities and the derived supply and demand elasticities for the commodity must be adjusted. This is so because their values will in general be different from those calculated under the assumption of a perfectly elastic supply of transfer services. The analysis of these points, which constitutes the principal purpose of this thesis, is made in the following chapter.
CHAPTER 4
ANALYTICAL RESULTS

Price Transmission Elasticities

The price transmission elasticity found in the literature (Horner, 1952; Cronin, 1979; Tweeten, 1967; Johnson, 1977; Bredahl et al., 1979; Collins, 1980; George and King, 1971, among others) is defined by the expression

\[ \eta_{AB} = \frac{\frac{\partial A}{\partial P} \frac{B}{A}}{\frac{\partial B}{\partial P} \frac{A}{P}} \]  

where \( \eta_{AB} \) = price transmission elasticity between the seller and buyer levels.

The price transmission elasticity is used as a device to estimate derived commodity demand and/or supply elasticities. For instance, suppose that the elasticity of supply at the seller level \((S)\) is known, and one wants to attain the value of the elasticity of supply at the buyer level, that is, the derived elasticity of supply \((B_S)\). Then

\[ B_S = S \eta_{AB} \]  

since

\[ \frac{\partial A_Q}{\partial B_P} \frac{B_P}{A_Q} = \left( \frac{\partial A_Q}{\partial A_P} \right) \left( \frac{\partial A_P}{\partial B_P} \right) \frac{B_P}{A_Q}. \]

Some studies state that in order to obtain a simple expression for the price transmission elasticity it is necessary to adopt a simple
structure of the supply of transfer services (namely, expressions 17 or 18). But, with the exception of Gardner (1975), it is not made clear that if the supply of transfer services is not perfectly elastic, it is not possible to have only one price transmission elasticity relating two successive stages in the commercialization channel. Gardner uses a model different from the present one in that it collapses when substitution of commodity for transfer services is not allowed. He concludes that under perfect competition the relative change in the price ratio \( \frac{A_p}{B_p} \) caused by a commodity supply shock is different from the relative change due to a commodity demand shock.

As shown below, when the supply of transfer services is of the form

\[
M^S = M^S(Q); \quad \frac{\partial M^S}{\partial Q} > 0
\]  

(21)

one must define one price transmission elasticity for a commodity demand shock and another for a commodity supply shock.¹ The expressions for the redefined price transmission elasticities are

\[
\eta^S_{AB} = \left( \frac{\partial B_p}{\partial A_p} \right)^S \frac{B_p}{A_p}
\]

(22)

\[
\eta^D_{BA} = \left( \frac{\partial B_p}{\partial A_p} \right)^D \frac{A_p}{B_p}
\]

(23)

¹This result is true whenever \( \frac{\partial M^S}{\partial Q} \neq 0 \), but it is assumed throughout this paper that \( \frac{\partial M^S}{\partial Q} > 0 \).
where \( n^S_{AB} \) = price transmission elasticity for supply and \( n^D_{BA} \) = price transmission elasticity for demand.

The somewhat arbitrary names chosen to label the price transmission elasticities just defined are due to the fact that these elasticities are mainly used to calculate derived commodity elasticities of demand and/or supply as shown in expression (20). Expression (20) now becomes

\[
B^S_S = S n^S_{AB} \tag{24}
\]

If the elasticity of demand at the buyer level (D) is known, and it is necessary to estimate the elasticity of demand at the seller level or derived elasticity of demand \((A_D)\), the estimation can be done in a similar way:

\[
A^D_D = D n^D_{BA} \tag{25}
\]

where

\[
D = \frac{B_P}{B_Q} \left| \frac{\partial B_P}{\partial B_Q} \right|.
\]

Returning to definitions (22) and (23), it will now be shown how the expressions for \( \frac{\partial A_P}{\partial B_P} \) and \( \frac{\partial B_P}{\partial A_P} \) are derived. To achieve this, graphical tools will be used. Figure 7 depicts the necessary elements to illustrate the estimation of an expression for \( \frac{\partial A_P}{\partial B_P} \).

The assumptions underlying Figure 7 are
and linear commodity demand and supply. The linearities in commodity demand and supply and in the supply of transfer services are not required to demonstrate the results but facilitate the understanding of the problem.

Suppose that the initial equilibrium in the commodity market is at point \((0)\), where the buyer demand \((B_0)\) intersects the derived supply \((B_A)\). The derived supply is obtained by adding the supply of transfer services to the seller supply \((A)\). With the given derived supply schedule a positive shock in demand causes the demand to shift from \(B_0\) to \(B_1\) with a new equilibrium at point \((1)\). The increase in demand, therefore, produces the following changes:

\[
\Delta B_P = B_{P_1} - B_{P_0} = 31 \tag{28}
\]

\[
\Delta A_P = A_{P_1} - A_{P_0} = 32 \tag{29}
\]

\[
\Delta Q = Q_1 - Q_0 = 30 \tag{30}
\]

\[
\Delta M = (B_{P_1} - A_{P_1}) - (B_{P_0} - A_{P_0}) = (B_{P_1} - B_{P_0}) - (A_{P_1} - A_{P_0})
= 31 - 32 = 21 \tag{31}
\]

Using expressions (28) and (29)
The result stated by expression (32) can alternatively be derived using expressions (26) and (27):

\[
\Delta B_P = \Delta A_P + \Delta M \\
= \Delta A_P + \beta \Delta Q
\]

Dividing through equation (33) by \((\Delta A_P)\):

\[
\left( \frac{\Delta B_P}{\Delta A_P} \right)^S = 1 + \beta \frac{\Delta Q}{\Delta A_P}
\]

\[
= 1 + \beta \frac{30}{32}
\]

\[
= 1 + \frac{31}{32}
\]

\[
= \frac{32 + 31}{32}
\]

\[
= \frac{31}{32}
\]

Therefore,

\[
\left( \frac{\Delta A_P}{\Delta B_P} \right)^S = \frac{32}{31} < 1
\]
Figure 7. Market Equilibrium with a Commodity Demand Shock
This means that for infinitesimal changes, equation (34) becomes

$$\left( \frac{\partial B_p}{\partial A_p} \right)^S = 1 + \beta \frac{\partial A_q}{\partial A_p}$$  \hspace{1cm} (35)

To estimate $$\left( \frac{\partial A_p}{\partial B_p} \right)^S$$, calculate the reciprocal of $$\left( \frac{\partial B_p}{\partial A_p} \right)^S$$; that is,

$$\left( \frac{\partial A_p}{\partial B_p} \right)^S = \frac{1}{1 + \beta \frac{\partial A_q}{\partial A_p}} < 1 = \left( \frac{\partial A_p}{\partial B_p} \right)^S \bigg|_{\beta=0}$$  \hspace{1cm} (36)

The inequality stated in (36) holds because $$\beta>0$$, and supply is "normal" in the sense that $$\frac{\partial A_q}{\partial A_p} > 0$$.

Multiplying both sides of (36) by the ratio $$\left( \frac{B_p}{A_p} \right)^S$$:

$$\frac{B_p}{A_p} \left( \frac{\partial A_p}{\partial B_p} \right)^S < \frac{B_p}{A_p} \left( \frac{\partial A_p}{\partial B_p} \right)^S \bigg|_{\beta=0}$$

$$\eta_{AB}^S < \eta_{AB}^S \bigg|_{\beta=0} = \eta_{AB}$$  \hspace{1cm} (37)

Therefore, when the supply of transfer services is less than perfectly elastic, the price transmission elasticity for supply is smaller than the price transmission elasticity generally used; that is, than the one estimated assuming an infinitely price-responsive supply of transfer services.
On the other hand, the expression for \( \frac{\partial B_p}{\partial A_p} \) can be worked out in a similar manner by making use of Figure 8.

The assumptions underlying Figure 8 are the same as for Figure 7. Suppose that the initial equilibrium occurs at point (0), where the derived demand \( \bar{A} \bar{B} \) crosses the seller supply schedule \( A_0 \). If there is a positive supply shock the seller supply shifts from \( A_0 \) to \( A_1 \), and the new equilibrium is point (1). The changes caused by this increase in supply are

\[
\Delta B_p = B_{p1} - B_{p0} = -32
\]

(38)

\[
\Delta A_p = A_{p1} - A_{p0} = -31
\]

(39)

\[
\Delta Q = Q_1 - Q_0 = 36
\]

(40)

\[
\Delta M = (B_{p1} - A_{p1}) - (B_{p0} - A_{p0}) = (B_{p1} - B_{p0}) - (A_{p1} - A_{p0})
\]

\[
= -32 - (-31) = 31 - 32 = -1
\]

(41)

From (38) and (39):

\[
\frac{\Delta B_p}{\Delta A_p} = \frac{32}{31} < 1
\]

(42)

A more general solution can be derived from expressions (26) and (27) as follows:
\[ \Delta^A_p = \Delta^B_p - \Delta M \]

\[ = \Delta^B_p - \beta \Delta Q \]  \hspace{1cm} (43)

Dividing through equation (43) by \((\Delta^B_p)\):

\[ \left( \frac{\Delta^A_p}{\Delta^B_p} \right)^D = 1 - \beta \frac{\Delta^B Q}{\Delta^B_p} \]

\[ = 1 - \beta \frac{30}{(-32)} \]

\[ = 1 + \beta \frac{30}{32} \]

\[ = 1 + \frac{21}{32} \]

\[ = \frac{32 + 21}{32} \]

\[ = \frac{31}{32} \]  \hspace{1cm} (44)

And the inverse of expression (44) is equation (42). Therefore, for infinitesimal changes, a general expression for \(\left( \frac{\partial B_p}{\partial A_p} \right)^D\) is

\[ \left( \frac{\partial B_p}{\partial A_p} \right)^D = \left. \frac{1}{1 - \beta \frac{\partial^B Q}{\partial^B p}} \right|_{\beta=0} < 1 = \left( \frac{\partial^A p}{\partial^B p} \right)^D \]  \hspace{1cm} (45)
Figure 8. Market Equilibrium with a Commodity Supply Shock
Again, the inequality stated in (45) holds because $\beta > 0$, and the buyer demand is "normal"; that is, $\frac{\partial B}{\partial P} < 0$.

Now, multiplying both sides of the inequality (45) by $\left(\frac{A_p}{B_p}\right)$:

$$\frac{A_p}{B_p} \left(\frac{\partial B}{\partial P}\right) < \frac{A_p}{B_p} \left(\frac{\partial A}{\partial P}\right) \bigg|_{\beta=0}$$

$$\eta^D_{BA} < \eta^D_{BA} \bigg|_{\beta=0} = \eta^D_{BA}$$  (46)

This means that the price transmission elasticity for demand is smaller when the supply of transfer services is not perfectly elastic.

In summary, the price transmission elasticity for supply ($\eta^S_{AB}$) is the relative change in seller and buyer prices when the system is affected by a demand shock. Conversely, the price transmission elasticity for demand ($\eta^D_{BA}$) is the relative change in prices caused by a supply shock. The inequalities (37) and (46) can be used to derive the following general relationships among the different kinds of price transmission elasticities:

$$\eta^S_{AB} < \eta^D_{AB} < \eta^D_{AB}$$

$$\eta^S_{BA} > \eta^D_{BA}$$  (47)

Since the price transmission elasticities for demand and supply are different when the supply of transfer services is not perfectly elastic ($\beta \neq 0$), it follows that in this case the relationship between seller and
buyer prices is sensitive to the source of change in the quantity of commodity traded. In fact, it is more sensitive the more inelastic the supply of transfer services.

Derived Supply and Demand Elasticities

The results just derived concerning price transmission elasticities can be readily applied to the analysis of derived supply and demand elasticities.

Take for instance the derived elasticity of supply at the buyer level \((B_S)\). Using equation (24), which is reproduced below for convenience, it is straightforward to derive the inequality (48) applying the results from inequality (37):

\[
B_S = S \eta^S_{AB} \tag{24}
\]

\[
B_S = S \eta^S_{AB} < S \eta^S_{AB} \bigg|_{\beta=0} = B_S \bigg|_{\beta=0} \tag{48}
\]

The inequality above says that the derived buyer elasticity of supply assuming perfectly elastic supply of transfer services \((B_S \bigg|_{\beta=0})\) provides an upper limit estimate of the "true" buyer elasticity of supply.

Similarly, if the direct elasticity of demand \((D)\) is known, and it is desired to calculate the derived elasticity of demand at the seller level \((A_D)\), this can be done using equation (25):

\[
A_D = D \eta^D_{BA} \tag{25}
\]
From the inequality (46), it follows that

$$A_O = D \eta^D_{BA} < D \eta^D_{BA} \bigg| \beta=0 = A_D \bigg| \beta=0$$

(49)

In words, expression (49) states that the derived seller elasticity of demand assuming perfectly elastic supply of transfer services ($A_D \bigg| \beta=0$) gives an upper limit estimate of the "true" seller elasticity of demand.

Although fairly obvious, the two conclusions just mentioned contain important implications for assessing the bias of empirical estimates of derived elasticities calculated assuming transfer services in perfectly elastic supply. Nevertheless, the framework employed so far is very restrictive, and at this point some interesting questions arise. For instance:

- Do these propositions hold when there is more than one aggregate commodity supply and/or demand?
- Can they be derived from more general frameworks?
- What happens if the supply of transfer services is non-linear?

Answers to the above questions are given in the remainder of this section, which is devoted only to the analysis of derived commodity demand elasticities. A similar development can be made to explore derived commodity supply elasticities, with completely analogous results.

The explicit treatment of the transfer services industries greatly increases the complexity of trade models, as it will soon become apparent. This is why two models are used here instead of a single general analytical model including several aggregate commodity demand and
supply schedules. Model 1 consists of one aggregate commodity supply and two aggregate demands, and it is specially designed to uncover the effect of the characteristics of the transfer services industries serving the trade flows originating in each seller. Model 2 is composed of one aggregate commodity demand and two aggregate supplies, and it shows the consequences of the nature of the transfer services industries attending the trade flows arriving at each buyer.

Model 1

A simple model to study the derived commodity demand elasticity consists of two buyers ("B" and "b") that buy a homogeneous commodity from one seller ("A"), that does not discriminate between buyers. The scheme is depicted by Figure 9, in which the arrows indicate commodity flows:

![Diagram of commodity flows](image)

**Figure 9. Commodity Flows in a Trade System with Two Aggregate Demands and One Aggregate Supply**
This can be thought of as two importing countries buying grains from an importer, in which case the demands are excess demands, and the supply is an excess supply.

The set of equations representing this model is as follows:

\[ A_Q = A_Q(A_P); \quad A_{QA} = \frac{\partial A_Q}{\partial A_P} > 0 \]  
(50)

\[ B_Q = B_Q(B_P); \quad B_{QB} = \frac{\partial B_Q}{\partial B_P} < 0 \]  
(51)

\[ b_Q = b_Q(b_P); \quad b_{Qb} = \frac{\partial b_Q}{\partial b_P} < 0 \]  
(52)

\[ B_P = A_P + B_M \]  
(53)

\[ b_P = A_P + b_M \]  
(54)

\[ B_T = B_Q \]  
(55)

\[ b_T = b_Q \]  
(56)

\[ B_M = B_M(B_T, b_T); \]

\[ B_{MB} = \frac{\partial B_M}{\partial B_T} > 0, \quad B_{Mb} = \frac{\partial B_M}{\partial b_T} > 0 \]  
(57)
(Supply of transfer services for trade "b")

\[ b_M = b_M(B_T, b_T); \]
\[ b_M = \frac{\partial b_M}{\partial b_T} > 0; \]
\[ b_M = \frac{\partial b_M}{\partial B_T} > 0 \quad (58) \]

(Equilibrium)

\[ A_Q = B_Q + b_Q \quad (59) \]

The symbols correspond to

- \( A_Q \) = quantity of commodity supplied by seller "A"
- \( B_Q, b_Q \) = quantity of commodity demanded by buyers "B" and "b", respectively
- \( B_T, b_T \) = amount of commodity traded in trades "B" and "b", respectively
- \( A_P, B_P, b_P \) = price received by seller "A", and paid by buyers "B" and "b", respectively
- \( B_M, b_M \) = transfer costs for trades "B" and "b", respectively

Equations (57) and (58) deserve more comment. They are intended to be the most general formulation of the supply of transfer services applicable to this system. Examples of particular situations follow.

1. One transfer services industry serving both trades, its supply is less than perfectly elastic:

\[ B_M = b_M = \frac{\partial b_M}{\partial (B_T + b_T)} > 0; \]
\[ b_M = b_M = \frac{\partial b_M}{\partial (B_T + b_T)} > 0 \]
2. There is one transfer services industry specialized in each trade, but they are related; their supplies are less than perfectly elastic:

\[ B_M > B_M > 0; b_M > b_M > 0 \]

3. There is one transfer services industry specialized in each trade, and they are completely unrelated; their supplies are less than perfectly elastic:

\[ B_M > B_M = 0; b_M > b_M = 0 \]

4. The standard assumption made in most N-SPEM and SPEM is that both transfer services industries have a perfectly elastic supply:

\[ B_M = B_M = b_M = b_M = 0 \]

These different specifications for the supply of transfer services will lead to particular solutions of the model with different implications, as will be seen after the system is solved for a general solution.

The first step is to find the formula for the derived commodity demand elasticity that seller "A" faces \((A_D)\). This will allow us to explore analytically the consequences for \(A_D\) of different assumptions regarding the transfer services industries.

\[ A_D = \left| \frac{\partial A_D}{\partial A} \right| \frac{A_P}{A_Q} \]  

(60)

where \( A_D = B_Q + b_Q \) = aggregate demand that seller "A" faces.
From the definition for $A_Q^D$ the value of $\frac{\partial A_Q^D}{\partial A^P}$ is obtained as

$$\frac{\partial A_Q^D}{\partial A^P} = B_Q^D \left( \frac{\partial B^D}{\partial A^P} \right) + b_Q^D \left( \frac{\partial b^D}{\partial A^P} \right)$$  \hspace{1cm} (61)

The expressions for $\left( \frac{\partial B^D}{\partial A^P} \right)$ and $\left( \frac{\partial b^D}{\partial A^P} \right)$ can be derived simultaneously from equations (53), (54), (57), and (58).

$$\left( \frac{\partial B^D}{\partial A^P} \right) = 1 + B_M^B B_Q^B \left( \frac{\partial B^D}{\partial A^P} \right) + B_M^b b_Q^b \left( \frac{\partial b^D}{\partial A^P} \right)$$  \hspace{1cm} (62)

$$\left( \frac{\partial b^D}{\partial A^P} \right) = 1 + b_M^B B_Q^B \left( \frac{\partial B^D}{\partial A^P} \right) + b_M^b b_Q^b \left( \frac{\partial b^D}{\partial A^P} \right)$$  \hspace{1cm} (63)

Expressions (62) and (63) can be solved for $\left( \frac{\partial B^D}{\partial A^P} \right)$ and $\left( \frac{\partial b^D}{\partial A^P} \right)$:

$$\left( \frac{\partial B^D}{\partial A^P} \right) = \frac{1 - (b_M^B B_Q^B - b_M^b b_Q^b)^{-1}}{1 - B_M^B B_Q^B - b_M^b b_Q^b + (B_M^B b_M^b - b_M^b B_M^b)^{-1} B_Q^B b_Q^b}$$  \hspace{1cm} (64)

$$\left( \frac{\partial b^D}{\partial A^P} \right) = \frac{1 - (B_M^B - b_M^b)^{-1} B_Q^B}{1 - B_M^B B_Q^B - b_M^b b_Q^b + (B_M^B b_M^b - b_M^b B_M^b)^{-1} B_Q^B b_Q^b}$$  \hspace{1cm} (65)
Substituting (65) and (64) into (61) yields

\[
\frac{\partial A_Q^D}{\partial A_p} = \frac{B_Q B + b_Q b - (B_M B + b_M b - B_M B - b_M B)B_Q b + b_Q b}{1 - B_M B - b_M b + (B_M b - B_M b)B_Q b + b_Q b}
\]  

(66)

Finally, replacing expression (66) in (60), and after rearrangement, the solution for the derived commodity demand elasticity is obtained:

\[
A_D = \frac{Dg + dr(1-f) + [\tau_{BB} r(l-g)(1-f) + \tau_{bb} g(l-r)f - \tau_{BB} g(l-r)(1-f)]Dd}{1 + \tau_{BB} D(l-g) + \tau_{bb} d(l-r) + (\tau_{BB} \tau_{bb} - \tau_{BB} \tau_{bb})Dd(l-g)(1-r)}
\]  

(67)

where

\[
D = - B_Q B_P
\]

\[
d = - b_Q b_P
\]

\[
\tau_{ij} = \frac{\partial i}{\partial j} M_{ij} Q_{ij}
\]

\[
i = B, b; \quad j = B, b
\]

\[
f = \frac{B_Q}{A_Q} \leq 1
\]

\[
g = \frac{A_P}{B_P} \leq 1
\]

\[
r = \frac{A_P}{B_P} \leq 1
\]
and the restrictions \( \tau_{BB} \geq \tau_{Bb} \left( \frac{f}{1-f} \right) > 0 \)
\( \tau_{bb} \geq \tau_{bB} \left( \frac{1-f}{f} \right) > 0 \)

Here \( D \) and \( d \) are the direct elasticities of demand of buyers "B" and "b", respectively. The inverse of the own elasticity of supply of transfer services "i" is \( \tau_{ii} \), and \( \tau_{ij} \) is the inverse of the cross elasticity of supply of transfer services "j" with respect to transfer costs "i". The proportion of total consumption demanded by buyer "B" is \( f \), while \( g \) and \( r \) are the seller price relative to the buyer prices.

Expressions (66) and (67) are the basis for the analysis that follows, which consists of several cases regarding different assumptions about the transfer services industry. These exercises are attempted to help assess the bias of the derived demand elasticities estimated by means of SPEM and N-SPEM that assume perfectly elastic supply of transfer services.

**Case 1:** Expression (25) is just a special case of (66) and (67); it corresponds to the situation where buyer "b" does not demand commodity. Therefore, setting \( b_Q = 0 \), (66) becomes

\[
\left. \frac{\partial A_Q}{\partial P} \right|_{b_Q = 0} = \frac{B_{QB}}{1 - B_{MB} B_{QB}}
\]

Or, in elasticities

\[
\left. A_D \right|_{b_Q = 0} = \frac{Dg}{1 + \tau_{BB} D(1-g)}
\]
If the supply of transfer services is perfectly elastic, \( \tau_{BB} = 0 \), and (69) becomes

\[
A_D \bigg|_{b_Q = \tau_{BB} = 0} = Dg
\] (70)

Comparing (69) and (70) it is clear that

\[
A_D \bigg|_{b_Q = 0} < A_D \bigg|_{b_Q = \tau_{BB} = 0}
\] (71)

There are two noteworthy observations concerning the relation between the two derived elasticities. First, the difference between them increases as the supply of transfer services becomes more inelastic, since

\[
\underbrace{\frac{\partial}{\partial \tau_{BB}} \left( A_D \bigg|_{b_Q = \tau_{BB} = 0} - A_D \bigg|_{b_Q = 0} \right)}_{\partial D \tau_{BB}} = \left[ \frac{D}{1 + \tau_{BB} D(1-g)} \right]^2 g(1-g) > 0
\] (72)

Second, the gap between the derived elasticities is bigger the more elastic the (direct) buyer demand (D):

\[
\underbrace{\frac{\partial}{\partial D} \left( A_D \bigg|_{b_Q = \tau_{BB} = 0} - A_D \bigg|_{b_Q = 0} \right)}_{\partial D} = g - \frac{g}{\left[1 + \tau_{BB} D(1-g)\right]^2} > 0
\] (73)

A graphical representation of the mentioned facts for the simplest example (1 seller and 1 buyer) is given below.
In Figure 10 the direct buyer demand is represented by $\overline{AA'}$. The lines $\overline{BB'}$, $\overline{CC'}$, and $\overline{AD'}$ are the derived demands at the seller level; $\overline{BB'}$ is drawn assuming $\tau_{BB'} = 0$, while for $\overline{CC'}$ and $\overline{AD'}$ it is postulated that $\tau_{BB'} > 0$. Moreover, for $\overline{AD'}$ the supply of transfer services is more inelastic than for $\overline{CC'}$ ($\tau_{BB'} > \tau_{BB'}$). The derived elasticities of demand for the three curves can be measured by

\[ A_D \bigg| _{\tau_{BB'} = 0} = \frac{Q'B'}{OQ*} \]  

\[ A_D \bigg| _{\tau_{BB'} = 0} = \frac{Q'C'}{OQ*} \]  

\[ A_D \bigg| _{\tau_{BB'} = 0} = \frac{Q'D'}{OQ*} \]

Then:

\[ A_D \bigg| _{\tau_{BB'} = 0} - A_D \bigg| _{\tau_{BB'} = 0} = \frac{Q'B' - Q'C'}{OQ*} = \frac{CT'B'}{OQ*} \]  

\[ A_D \bigg| _{\tau_{BB'} = 0} - A_D \bigg| _{\tau_{BB'} = 0} = \frac{Q'B' - Q'D'}{OQ*} = \frac{D'B'}{OQ*} \]  

Therefore:

\[ \left( A_D \bigg| _{\tau_{BB'} = 0} - A_D \bigg| _{\tau_{BB'} = 0} \right) - \left( A_D \bigg| _{\tau_{BB'} = 0} - A_D \bigg| _{\tau_{BB'} = 0} \right) = \frac{D'B' - C'B'}{OQ*} = \frac{D'C'}{OQ*} > 0 \]
Figure 10. Direct and Derived Commodity Demands Under Different Supply Schedules of Transfer Services
The size of the bias in the estimated derived elasticity of demand due to not considering that transfer services are in less than perfectly elastic supply is bigger the more inelastic the supply of transfer services.

In Figure 11, $\overline{AA}'$ and $\overline{CC}'$ are buyer demands with $\overline{AB}$ and $\overline{CE}$ as their corresponding derived demands at the seller level, assuming $\tau_{BB} > 0$. The picture is drawn such that the elasticity of supply of transfer services is the same for both derived demands. The direct elasticity of demand for $\overline{AA}'$ is smaller than for $\overline{CC}'$ ($D_A < D_C$), since

$$D_A - D_C = \frac{Q^*A'}{OQ^*} - \frac{Q^*C'}{OQ^*} = -\frac{A'C'}{OQ^*} < 0 \quad (80)$$

The respective differences between derived elasticities with and without perfectly elastic supply of transfer services are

$$A_D |_{\tau_{BB}=0} - A_D = \frac{Q^*A''}{OQ^*} - \frac{Q^*B}{OQ^*} = \frac{BA''}{OQ^*} \quad (81)$$

$$A_D |_{\tau_{BB}=0} - A_D = \frac{Q^*C''}{OQ^*} - \frac{Q^*E}{OQ^*} = \frac{EC''}{OQ^*} \quad (82)$$

The comparison between these differences yields

$$\left( A_D |_{\tau_{BB}=0} - A_D \right) - \left( A_D |_{\tau_{BB}=0} - A_D \right) = \frac{EC''}{OQ^*} - \frac{BA''}{OQ^*} > 0 \quad (83)$$
Figure 11. Different Direct and Derived Commodity Demands Under the Same Supply Schedules of Transfer Services
In words, expression (80) together with (83) says that the bias of the estimated derived elasticity of demand that results from assuming a perfectly elastic supply of transfer services becomes bigger the more elastic is the direct buyer demand.

These two observations are very important and intimately related to each other for empirical research. Their relevance has to do with the period of adjustment of the supply of transfer services and the demand for commodity.

There is some evidence (Binkley and Revelt, 1981; Binkley, 1983) that the longer the time period, the more elastic is the supply of transfer services. Taking this alone into consideration, one can conclude that in order to increase the accuracy of the estimate of the derived demand, it is more important to specify a price-responsive supply of transfer services in the short run. This is so because it is then that 
\[ \left( A_D \mid b_{Q=0}^{BB} = 0 - A_D \right) \]

would become bigger.

On the other hand, it constitutes a stylized fact that, at least within certain limits, commodity demands are more inelastic the shorter the run.\(^1\) From this knowledge only one can infer that the support for considering explicitly a less than perfectly elastic supply of transfer services is weaker the shorter the run, since 
\[ \left( A_D \mid b_{Q=0}^{BB} = 0 - A_D \right) \]

would be smaller in the short run.

\(^1\)For a detailed study of this see Pasour and Schrimper (1965).
It follows from the two paragraphs above that it is not possible to make a general recommendation as for which length of run it is more important to specify carefully the supply of transfer services in the one-seller/one-buyer case. It would be of interest to conduct empirical research to shed more light on this point.

Case 2: The standard simplification made in SPEM and N-SPEM consists of making the supply schedules of transfer services infinitely price responsive, that is,

\[ B_{M_B} = B_{M_b} = b_{M_B} = b_{M_b} = 0 \] (84)

or alternatively \( \tau_{BB} = \tau_{Bb} = \tau_{bb} = \tau_{bB} = \tau = 0 \) (85)

Therefore, expressions (66) and (67) collapse to

\[ \frac{\partial A_Q}{\partial A_P}_\tau=0 = B_{Q_B} + b_{Q_b} \] (86)

\[ A_D|_{\tau=0} = D_{gf} + d_r(1-f) \] (87)

This particular case is of crucial importance, because it represents most of the empirical estimates of derived demand elasticities found in the literature.

From the comparison with the unrestricted situation of equation (67), it follows that\(^1\)

\(^1\)The proof of this result is in Appendix 1.
This means that for this kind of model the standard derived demand elasticity overestimates the "true" derived elasticity of demand.

Case 3: If both trades "B" and "b" are attended by the same transfer services industry, and this has a supply schedule that is not infinitely price responsive, the restrictions upon the model are

\[ B_{MB} = B_{Mb} = \frac{2^{BM}}{a(T+T^b)} > 0 \]

\[ b_{Mb} = b_{MB} = \frac{2^{bM}}{a(T+T^b)} > 0 \]

And the expressions (66) and (67) become

\[ \frac{\partial Q}{\partial A_P} \bigg|_{B_{MB}=B_{Mb}; b_{Mb}=b_{MB}} = \frac{B_{QB} + b_{Qb}}{1 - B_{MB}B_{QB} - b_{Mb}b_{Qb}} \]

\[ A_D \bigg|_{B_{MB}=B_{Mb}; b_{Mb}=b_{MB}} = \frac{D_{Q} + dr(1-f)}{1 + \tau_{BB}B(1-g) + \tau_{bb}d(1-r)} \]

This derived commodity demand elasticity may be either bigger or smaller than the unrestricted one, but it is unambiguously smaller than \( A_D \bigg|_{\tau=0} \). Therefore, if the actual situation is such that there is only one transfer services industry serving both trades, the standard derived commodity demand elasticity \( A_D \bigg|_{\tau=0} \) is biased upward.
Case 4: When there is no linkage at all between the transfer services industries attending trades "B" and "b", and their supplies are not perfectly elastic, the restrictions on equations (57) and (58) are

\[
B_M B > B_M b = 0 \quad \text{and} \quad b_M b > b_M B = 0 \tag{92}
\]

or alternatively

\[
\tau_{BB} > \tau_{Bb} = 0 \quad \text{and} \quad \tau_{bb} > \tau_{bB} = 0 \tag{93}
\]

In this case expressions (66) and (67) are reduced to

\[
\frac{\partial \Delta}{\partial \Delta} \bigg|_{\tau_{BB}=\tau_{Bb}=0} = \frac{Q_B B + b_Q b - (B_M B + b_M b) B Q_b b Q_b}{1 - B_M B Q_B b - B_M b Q_b b + B_M b B Q_b b Q_b} \tag{94}
\]

\[
A_D \bigg|_{\tau_{BB}=\tau_{Bb}=0} = \frac{D g f + d r (1-f) + [\tau_{BB} r (1-g) (1-f) + \tau_{bb} g (1-r) f] D d}{1 + \tau_{BB} B (1-g) + \tau_{bb} d (1-r) + \tau_{BB} \tau_{bb} D d (1-g) (1-r)} \tag{95}
\]

The comparison with the unrestricted situation and Cases 2 and 3 yields

\[
A_D \bigg|_{\tau_{BB}=\tau_{Bb}=0} > A_D \tag{96}
\]

\[
A_D \bigg|_{\tau_{BB}=\tau_{Bb}=0} < A_D \bigg|_{\tau=0} \tag{97}
\]

\[
A_D \bigg|_{\tau_{BB}=\tau_{Bb}=0} > A_D \bigg|_{B_M B = B_M b; b_M b = b_M B} \tag{98}
\]

\[\text{1The proof of these results is in Appendix 1.}\]
This set of results is very important, particularly the inequalities (96) and (97): transfer services industries unrelated to each other and with upward-sloping supply schedules lead to a derived commodity demand elasticity smaller than the standard estimate \( \left( A_D \right| \tau_{bb} = 0 < A_D \right| \tau = 0 \). Moreover, if the transfer services industries are related to each other, the derived commodity demand becomes even more inelastic \( A_D < A_D \right| \tau_{bb} = 0 \). The intuition behind these concepts is given by means of Figures 12, 13, and 14 in which sets (a) and (b) depict the individual direct and derived commodity demand curves of buyers "B" and "b", respectively, and set (c) shows the aggregate direct and derived commodity demand curves that seller "A" faces.

Figures 12, 13, and 14 are exactly the same with the exception of their derived commodity demands. In Figure 12 it is assumed that transfer services are in perfectly elastic supply; in Figure 13 it is hypothesized that there are two transfer services industries completely unrelated to each other and with upward-sloping supply schedules; and in Figure 14 it is assumed that there is only one transfer services industry and it has a less than infinitely price-responsive supply. It can be seen that at \( (Q^*, P^*) \) the derived demand of Figure 12c is more elastic than the derived demands of either Figure 13c or 14c; that is, the derived commodity demand is more inelastic when the transfer services industries have upward-sloping supply than when they have infinitely price-responsive supply. On the other hand, the derived commodity demand of Figure 13c is more elastic than that of Figure 14c. This means that
(a) Direct and derived commodity demands of Buyer "B".
(b) Direct and derived commodity demands of Buyer "b".
(c) Direct and derived aggregate commodity demands.

Figure 12. Direct and derived commodity demands for Model 1, under perfectly elastic supply of transfer services
(a) Direct and derived commodity demands of Buyer "B".

(b) Direct and derived commodity demands of Buyer "b".

(c) Direct and derived aggregate commodity demands.

Figure 13. Direct and derived commodity demands for Model 1, under completely unrelated transfer services industries with less than perfectly elastic supply of transfer services.
(a) Direct and derived commodity demands of Buyer "B".

(b) Direct and derived commodity demands of Buyer "b".

(c) Direct and derived aggregate commodity demands.

Figure 14. Direct and derived commodity demands for Model 1, under a single transfer services industry with less than perfectly elastic supply of transfer services.
the derived commodity demand elasticity is smaller when the transfer services industries are related to each other than when they are completely independent.

The synthesis of the main results obtained with Model 1 is presented in Table 3, which contains the analytical relationships among the derived commodity demand elasticities under different restrictions regarding the supply of transfer services.

The bottom line of these observations is that, as long as the transfer services industries have less than perfectly elastic supply schedules, the standard estimate of the derived commodity demand elasticity is biased upward, since \( A_D | \tau=0 > A_D \). The existence of interdependence among the transfer services industries attending the trade flows emanating from each seller makes the derived commodity demand more inelastic, as \( A_D | \tau_{bb}=0 > A_D \). However, since

\[
A_D \geq A_D | \tau_{bb}=0 \quad b_M=b_M
\]

it does not necessarily follow that the derived commodity demand elasticity is smallest when the trade flows originating in each seller are attended by a single industry.

Model 2

This consists of two sellers ("A" and "a") supplying a certain commodity to a buyer ("B"). The commodity is homogeneous, and the buyer is completely indifferent to the origin of the commodity. The schematic
Table 3. Relationships Among Derived Commodity Demand Elasticities for Model 1

<table>
<thead>
<tr>
<th></th>
<th>Perfectly Elastic Supply of Transfer Services</th>
<th>One Transfer Services Industry</th>
<th>Two Transfer Services Industries Completely Unrelated to Each Other</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$A_D</td>
<td>\tau=0$</td>
<td>$A_D</td>
</tr>
<tr>
<td>Unrestricted Supply of Transfer Services</td>
<td>$A_D$</td>
<td>$+$</td>
<td>$+$</td>
</tr>
<tr>
<td>Two Transfer Services Industries Completely Unrelated to Each Other</td>
<td>$A_D</td>
<td>\tau_B=\tau_{bB}=0$</td>
<td>$+$</td>
</tr>
<tr>
<td>One Transfer Services Industry</td>
<td>$A_D</td>
<td>B_M=M_B; b_M=b_M_B$</td>
<td>$+$</td>
</tr>
</tbody>
</table>

Note: Each cell of the table contains the sign of the value obtained by subtracting the elasticity in the row from the elasticity in the column. For instance, the fact that $A_D | \tau_B=\tau_{bB}=0 - A_D > 0$ is represented by (+) in the respective cell.
The mathematical representation of this model is as follows:

(Seller Supply "A") \[ A_Q = A_Q (A_P); \quad A_{QA} = \frac{\partial A_Q}{\partial A_P} > 0 \] (99)

(Seller Supply "a") \[ a_Q = a_Q (a_P); \quad a_{QA} = \frac{\partial a_Q}{\partial a_P} > 0 \] (100)

(Buyer Demand) \[ B_Q = B_Q (B_P); \quad B_{QB} = \frac{\partial B_Q}{\partial B_P} < 0 \] (101)

(Price linkage equation "A") \[ A_P = B_P - A_M \] (102)

(Price linkage equation "a") \[ a_P = B_P - a_M \] (103)
(Trade "A") \[ A_T = A_Q \] (104)

(Trade "a") \[ a_T = a_Q \] (105)

(Supply of transfer services for trade "A")
\[
A_M = A_M(A_T, a_T);
\] (106)
\[
A_{M a} = \frac{\partial A_M}{\partial A_T} \geq A_{M a} = \frac{\partial A_M}{\partial a_T} \geq 0
\]

(Supply of transfer services for trade "a")
\[
a_M = a_M(A_T, a_T);
\] (107)
\[
a_{M a} = \frac{\partial a_M}{\partial a_T} \geq a_{M a} = \frac{\partial a_M}{\partial A_T} \geq 0
\]

(Equilibrium) \[ A_Q + a_Q = B_Q \] (108)

The symbols are

\[ A_Q, a_Q = \text{quantity supplied by sellers "A" and "a", respectively} \]
\[ B_Q = \text{quantity demanded by buyer "B"} \]
\[ A_T, a_T = \text{amount of commodity traded in trades "A" and "a", respectively} \]
\[ A_P, a_P, B_P = \text{price received by sellers "A" and "a", and paid by buyer "B", respectively} \]
\[ A_M, a_M = \text{transfer costs for trades "A" and "a", respectively} \]

The goal is to make some inferences with respect to the derived elasticity of commodity demand that one seller faces when there is at
least one more seller in the market. The analysis is done here using the derived demand for production of seller "A" ($A_D$). The value for this elasticity can be obtained by making use of a version of Yntema's formula (1932):

$$A_D = \left| \frac{\partial A_Q}{\partial A_p} \right| \frac{A_p}{A_Q} = \left| B_{Q_B} \frac{\partial B_p}{\partial A_p} - a_{Q_a} \frac{\partial a_p}{\partial B_p} \frac{\partial B_p}{\partial A_p} \right| \frac{A_p}{A_Q}$$

(109)

where $A_Q^D = B_Q - a_{Q_a} = \text{derived demand for production of seller "A".}$

Yntema's formula is widely used. One of its most popular applications has been to estimate the price elasticity of export demand for U.S. agricultural products (for a survey on this see Gardiner and Dixit, 1986).

Equation (109) is correct as long as the supply of transfer services is perfectly elastic. When this is not the case, as shown in previous pages, a more careful specification must be used.

$$A_D = \left| \frac{\partial A_Q}{\partial A_p} \right| \frac{A_p}{A_Q} = \left| B_{Q_B} \left( \frac{\partial B_p}{\partial A_p} \right)^D - a_{Q_a} \left( \frac{\partial a_p}{\partial B_p} \right)^S \left( \frac{\partial B_p}{\partial A_p} \right)^D \right| \frac{A_p}{A_Q}$$

(110)

The term between bars of equation (110), namely, expression (111), is central to the discussion and its solution is derived in the next few pages.

$$\frac{\partial A_Q}{\partial A_p} = \left[ B_{Q_B} - a_{Q_a} \left( \frac{\partial a_p}{\partial B_p} \right)^S \left( \frac{\partial B_p}{\partial A_p} \right)^D \right]$$

(111)
The first step will be to obtain a solution for $\left(\frac{\partial a_p}{\partial B_p}\right)^S$. From equations (103) and (107):

$$\left(\frac{\partial a_p}{\partial B_p}\right)^S = 1 - a_{M_A} \frac{\partial a_T}{\partial B_p} - a_{M_A} \frac{\partial a_T}{\partial B_p}$$  \hspace{1cm} (112)

Equations (104) and (108) allow us to arrive at an expression for $\left(\frac{\partial a_T}{\partial B_p}\right)$:

$$\frac{\partial a_T}{\partial B_p} = \frac{\partial a_Q}{\partial B_p}$$

$$= \frac{\partial (B_Q - a_Q)}{\partial B_p}$$

$$= B_{Q_B} - a_{Q_A} \left(\frac{\partial a_p}{\partial B_p}\right)^S$$  \hspace{1cm} (113)

Also, from (105):

$$\frac{\partial a_T}{\partial B_p} = a_{Q_A} \left(\frac{\partial a_p}{\partial B_p}\right)^S$$  \hspace{1cm} (114)

Plugging (113) and (114) back into (112) and solving for $\left(\frac{\partial a_p}{\partial B_p}\right)^S$:

$$\left(\frac{\partial a_p}{\partial B_p}\right)^S = \frac{1 - a_{M_A} B_{Q_B}}{1 + (a_{M_A} - a_{M_A}) a_{Q_A}}$$  \hspace{1cm} (115)

On the other hand, an expression for $\left(\frac{\partial B_p}{\partial a_p}\right)^D$ can be attained from (102) and (106):
\[ 1 = \left( \frac{\partial B_P}{\partial A_P} \right)^D - A_M \frac{\partial A_D}{\partial A_P} - A_M a_Q \left( \frac{\partial a_P}{\partial B_P} \right)^S \left( \frac{\partial B_P}{\partial A_P} \right)^D \]  

(116)

Solving (116) for \( \frac{\partial B_P}{\partial A_P}^D \):

\[ \left( \frac{\partial B_P}{\partial A_P} \right)^D = \frac{1 + \frac{A_M}{\partial A_P} \frac{\partial A_D}{\partial A_P}}{1 - A_M a_Q \left( \frac{\partial a_P}{\partial B_P} \right)^S} \]  

(117)

After substitution of (115) and (117) into expression (111), and solving for \( \frac{\partial A_Q}{\partial A_P} \):

\[ \frac{\partial A_Q}{\partial A_P}^D = \frac{B_Q + B_Q a_Q - a_Q}{1 + (A_M a_M - A_M a_M) a_Q - (A_M a_M - A_M a_M) B_Q a_Q} \]  

(118)

In elasticities, expression (118) can be stated as:

\[ A_D^D = \frac{D + \tau_{aa} D_s(n-1) + s_n(1-p)}{[\text{denominator}]} \]  

(119)

where \([\text{denominator}] = mp + [\tau_{AA} n(m-1)(1-p) + \tau_{aa} m(n-1)p - \tau_{AA} n(m-1)p\]

- \(\tau_{aA} m(n-1)(1-p)]s + (\tau_{AA} \tau_{aa} - \tau_{AA} \tau_{aa}) D_s(m-1)(n-1) + \tau_{AA} D(m-1)\)

\[^1\text{The derivation of (119) is in Appendix 2.}\]
\[
D = -BQB \frac{BP}{BQ}
\]
\[
s = a_Q \frac{a_p}{a_Q}
\]
\[
\tau_{ij} = \frac{a^iM^jQ}{a^iT^jM}; \quad i=A,a; \; j=A,a
\]
\[
p = \frac{A_Q}{B_Q} \leq 1; \quad m = \frac{BP}{A_P} \geq 1; \quad n = \frac{BP}{a_P} \geq 1
\]

Expression (119) is subject to the restrictions \( \tau_{AA} \geq \tau_{Aa} \left( \frac{1-p}{1-p} \right) \geq 0; \)
\( \tau_{aa} \geq \tau_{aA} \left( \frac{1-p}{p} \right) \geq 0, \) which correspond to those stated in equations (106)
and (107), but in terms of elasticities.

The meaning of \( D, \tau_{ii}, \) and \( \tau_{ij} \) is the same than in Model 1,
while \( s \) is the direct elasticity of supply of seller "a". The proportion
of total trade that seller "A" supplies is \( p, \) and \( m \) and \( n \) are relative
prices.

Some important observations\(^1\) regarding the formula (119) are that
the derived demand becomes more inelastic when

1. The direct elasticities of supply of transfer services are
   smaller (i.e., \( \tau_{AA} \) and \( \tau_{aa} \) are bigger).
2. The cross elasticities of supply of transfer services are bigger
   (i.e., \( \tau_{Aa} \) and \( \tau_{aA} \) are smaller).

\(^1\)The mathematical proof of the following assertions is in Appendix 3.
3. The elasticity of supply of seller "a" is smaller (i.e., s is smaller).

4. The relative margin costs of trade "A" are higher (i.e., m is bigger).

Given the assumptions of the system, it does not necessarily follow that the derived demand \( (A_D) \) becomes more inelastic when the direct demand of buyer "B" (D) is more inelastic. A sufficient condition for a more inelastic direct demand being translated into a more inelastic derived demand is that \( [\tau_{aa} m(n-1) > \tau_{An} n(m-1)] \) or equivalently that \( a_M > A_m \). This condition means that a change in trade "a" produces a bigger absolute change in transfer costs "a" than in transfer costs "A".

In a similar way, a smaller share of seller "A" in the total trade (p) does not imply that its derived elasticity of demand \( (A_D) \) is bigger. Again, the condition \( [\tau_{aa} m(n-1) > \tau_{Ad} n(m-1)] \) is sufficient to establish unambiguously that a smaller share is corresponded by a more elastic derived demand.

On the other hand, the effect of a higher relative margin cost of trade "a" (n) on the derived demand of seller "A" \( (A_D) \) is ambiguous.

In a manner similar to Model 1, expressions (118) and (119) will now be used to obtain the relationships between the derived commodity demand elasticities under different assumptions about the transfer services industries.

Case 1: Equation (25) is a special case of Model 2. If seller "a" does not supply commodity at all, then \( a_Q = 0 \) and expressions (118) and (119) are
If the supply of transfer services is infinitely price-responsive, \( \tau_{AA} = 0 \), and (121) becomes
\[
A_D \bigg|_{\tau_{AA} = 0} = \frac{D}{m + \tau_{AA}D(m-1)}
\]
Comparing expressions (120), (121), and (122) pairwise with (68), (69), and (70), it is clear that they are the same except for notation. Therefore, whatever was said for Case 1 of Model 1 applies here also.

Case 2: When the transfer services industry has an infinitely elastic supply:

\[
A_M = A_m = a_M = a_m = 0
\]

or
\[
\tau_{AA} = \tau_{Aa} = \tau_{aa} = \tau_{aA} = \tau = 0
\]
Under these restrictions, equations (118) and (119) collapse to
\[
\frac{\partial A_Q}{\partial A_P} \bigg|_{\tau=0} = B_{QB} - a_Qa
\]
\[
A_D \bigg|_{\tau=0} = \frac{D + sn(1-p)}{mp}
\]
This simplification of Model 2 is very important for empirical work because it characterizes most N-SPEM and SPEM. Because of this it will
be used as the basis for comparison with the solutions obtained when the supply of transfer services is less than infinitely elastic.

**Case 3:** If there is only one transfer services industry serving both trades "A" and "a" and its supply is less than perfectly elastic, the situation can be characterized by

\[
A_M = A_a = \frac{\partial A}{\partial (A+\partial_T)} > 0
\]

**With these constraints,** expressions (118) and (119) become

\[
\begin{align*}
A_M &= A_a = \frac{\partial A}{\partial (A+\partial_T)} > 0 \\
A_a &= A_M = \frac{\partial A}{\partial (A+\partial_T)} > 0
\end{align*}
\]

(127)

Although it is ambiguous if \( A_D \) is smaller or bigger than \( A_D \) \( \tau = 0 \), it is readily observable that the derived elasticity of demand obtained by using equation (129) is always bigger, ceteris paribus, than the unconstrained one. This is because the denominator of expression (119) is always bigger than that of equation (129), while both expressions have the same numerator.
Case 4: When the transfer services industries serving trades "A" and "a" are different and completely independent of each other, and their supplies are less than perfectly elastic, the derivatives of margin costs with respect to quantity traded are

\[ A_{M_A} > A_{M_a} = 0 \quad \text{and} \quad a_{M_a} > a_{M_A} = 0 \]  \hspace{1cm} (130)

Alternatively, in elasticity form:

\[ \tau_{AA} > \tau_{Aa} = 0 \quad \text{and} \quad \tau_{aa} > \tau_{aA} = 0 \]  \hspace{1cm} (131)

Therefore, equations (118) and (119) collapse to

\[
\frac{\partial A_D}{\partial A_p} \bigg|_{\tau_{Aa} = \tau_{aa} = 0} = \frac{B_{Q_B} + B_{Q_B} a_{M_a} a_{M_a} - a_{Q_a}}{1 + (A_{M_A} + a_{M_a}) a_{Q_a} - A_{M_A} a_{M_a} B_{Q_B} a_{Q_a} - A_{M_A} B_{Q_B} a_{Q_a}} \]  \hspace{1cm} (132)

\[ A_D \bigg|_{\tau_{Aa} = \tau_{aa} = 0} = D + \tau_{aa} D s (n-1) + s n (1-p) \left[ \tau_{AA} a_{M_A} a_{M_a} + \tau_{AA} a_{M_a} a_{M_A} D s (m-1)(n-1) + \tau_{AA} D (m-1) \right] \]  \hspace{1cm} (133)

Other things equal, \( A_D \bigg|_{\tau_{Aa} = \tau_{aa} = 0} \) is smaller than the unrestricted \( A_D \).

This comes as a consequence of the fact that the right-hand side denominator of equation (119) is always smaller than its counterpart of (133), while both expressions bear the same numerator.

Since \( A_D \bigg|_{A_{M_A} = a_{M_a}; \; a_{M_a} = a_{M_A}} > A_D \) (as seen in Case 3), it follows by transitivity that the derived commodity demand obtained under the assumption of one transfer services industry is always more elastic, ceteris paribus, than the demand estimated under the hypothesis of two transfer services industries completely unrelated to each other.
Therefore, for any given $\tau_{AA}$ and $\tau_{aa}$, the values that the unrestricted derived demand elasticity can take are bounded below by $A_D \mid \tau_{AA} = \tau_{aa} = 0$ and above by $A_D \mid A_{M_A} = A_{M_a}; a_{M_A} = a_{M_a}$. It can also be shown that the standard estimate of the derived commodity demand elasticity, that is, $A_D \mid \tau = 0$, is bigger than $A_D \mid \tau_{AA} = \tau_{aa} = 0$.

As a summary of what has been said about Model 2, Table 4 presents the main analytical results concerning the relationships among derived commodity demand elasticities estimated under different assumptions about the transfer services industries. To provide a better understanding of the logic behind some of these results, especially the counterintuitive

\[
(A_D \mid A_{M_A} = A_{M_a}; a_{M_A} = a_{M_a} > A_D \mid \tau_{AA} = \tau_{aa} = 0)
\]

and

\[
(A_D \mid A_{M_A} = A_{M_a}; a_{M_A} = a_{M_a} \geq A_D \mid \tau = 0)
\]

a graphical representation is provided in Figures 16, 17, and 18. To simplify the exposition, all three figures are drawn under the assumption that $A_M = 0$. Set (a) depicts the direct commodity demand of buyer "B" as well as the direct and derived commodity supply schedules of seller "a". On the other hand, set (b) presents the derived demand curve that seller "A" faces, which is obtained by subtraction of the derived supply of seller "a" from the direct demand of buyer "B". The direct commodity demand of buyer "B" and the direct commodity supply of seller "a" are the same in the

\[1\] For a demonstration of this result see Appendix 4.
Table 4. Relationships Among Derived Commodity Demand Elasticities for Model 2

<table>
<thead>
<tr>
<th></th>
<th>Perfectly Elastic Supply of Transfer Services</th>
<th>One Transfer Services Industry</th>
<th>Two Transfer Services Industries Completely Unrelated to Each Other</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>$A_D</td>
<td>\tau = 0$</td>
<td>$A_D</td>
</tr>
<tr>
<td>Unrestricted Supply of Transfer Services</td>
<td>$A_D$</td>
<td>+/−</td>
<td>+</td>
</tr>
<tr>
<td>Two Transfer Services Industries Completely Unrelated to Each Other</td>
<td>$A_D</td>
<td>\tau_a = \tau_a = 0$</td>
<td>+</td>
</tr>
<tr>
<td>One Transfer Services Industry</td>
<td>$A_D</td>
<td>A_{M_A} = a_{M_A}, a_{M_a} = a_{M_A}$</td>
<td>+/−</td>
</tr>
</tbody>
</table>

Note: Each cell of the table contains the sign of the value obtained by subtracting the elasticity in the row from the elasticity in the column. For instance, the fact that $A_D | \tau_a = \tau_a = 0 - A_D < 0$ is represented by (−) in the respective cell.
three pictures. The differences in the figures are due to the assumptions about the transfer costs of trade "a": Figure 16 is drawn for $a_M = a_A = 0$, Figure 17 for $a_M > a_A = 0$, and Figure 18 for $a_M = a_A > 0$.

In other words, the value of the derived commodity demand elasticity that seller "A" faces at point $(Q^*, P^*)$ is just a particular case of $A_D | \tau = 0$ in Figure 16b, of $A_D | \tau_A = \tau = 0$ in Figure 17b, and of $A_D | A_M = a_M; a_M = a_M$ in Figure 18b. The particular example chosen is such $A_M = a_M$; $a_M = a_M$

that $\left( A_D | A_M = a_M; a_M = a_M > A_D | \tau = 0 \right)$ holds, but this inequality can be reversed by making either $A_M$ or both $A_M$ and $a_M$ positive enough.

The graphs confirm that $\left( A_D | \tau = 0 > A_D | \tau_A = \tau = 0 \right)$ and also that $\left( A_D | A_M = a_M; a_M = a_M > A_D | \tau_A = \tau = 0 \right)$. The explanation of this last seemingly odd inequality is that, given the transfer services supply functions and the direct commodity demand and supply schedules, the only way in which seller "A" can sell a higher amount of commodity to buyer "B" is through a lower commodity price. But this simultaneously leads to a smaller quantity supplied by seller "a", and to a larger total amount of commodity bought by buyer "B", that is, as trade "A" increases, trade "a" decreases but total trade gets bigger. Therefore, since the transfer cost of trade "a" responds positively to total trade $\left( \frac{\partial a^2 M}{\partial \left( a^2 T + a^2 T \right)} > 0 \right)$, $a_M$
Figure 16. Direct and derived commodity demands and supplies for Model 2 under perfectly elastic supply of transfer services to trade "a".
Figure 17. Direct and derived commodity demands and supplies for Model 2 under completely unrelated transfer services industries and less than perfectly elastic supply of transfer services to trade "a".

(a) Direct commodity demand of Buyer "B" and direct and derived commodity supplies of Seller "a".

(b) Derived commodity demand that Seller "A" faces.
(a) Direct commodity demand of Buyer "B" and direct and derived commodity supplies of Seller "a".

(b) Derived commodity demand that Seller "A" faces.

Figure 18. Direct and derived commodity demands and supplies for Model 2 under a single transfer services industry with less than perfectly elastic supply of transfer services to trade "a".
actually becomes smaller when more commodity units are supplied by seller "a" over the relevant quantity range. This is just the opposite of what happens when $a_M^a > a_M^A = 0$, and it is the cause for \[ A_D | A_M^a = a_M^A; a_M^a = a_M^A > A_D | \tau_{AA} = \tau_{aA} = 0 \] or, more generally, \[ A_D > A_D | \tau_{AA} = \tau_{aA} = 0 \]. In words, if the transfer services industries attending the trade flows arriving at each buyer are interdependent, the derived commodity demand is more elastic than if these industries are completely unrelated to each other.

The comparative study of the results obtained by means of Models 1 and 2 is very illuminating. Figure 19 is drawn to help with the analysis: it represents a general trade model composed of 2 sellers ("A" and "a") that trade simultaneously with 2 buyers ("B" and "b").

Model 1 is a simplification of this general system, in which seller "a" does not supply commodity at all, and in which trades "B" and "b" stand for trades "AB" and "Ab", respectively. In a similar fashion, Model 2 is obtained by eliminating buyer "b" from the system depicted in Figure 19, and by labeling trades "AB" and "aB" as trades "A" and "a", respectively.¹

Inequalities (97) and (134) were derived from Models 1 and 2:

Model 1: \[ A_D | \tau = 0 > A_D | \tau_{BB} = \tau_{bB} = 0 \]  

¹An alternative interpretation of Model 2 is given in the presentation of the numerical examples at the end of this chapter.
Figure 19. Commodity Flows in a Trade System with Two Aggregate Supplies and Two Aggregate Demands
This means that when the transfer services industries serving each trade flow are completely unrelated to each other, the standard estimates of $A_D$ obtained through N-SPEM and SPEM ($A_D | \tau = 0$) are unambiguously biased upward.

When the transfer services industries serving the trade flows "AB", "Ab", "aB", and "ab" are interrelated, the situation is much more complex. The results obtained by means of Models 1 and 2 are

Model 1: $A_D | \tau = 0 > A_D | \tau _{bb} = \tau _{BB} = 0 > A_D$  \hspace{1cm} (135)

Model 2: $A_D > A_D | \tau _{aa} = \tau _{aA} = 0 ; A_D | \tau = 0 > A_D$  \hspace{1cm} (136)

In words, the inequality (135) says that if there is interdependence between the transfer services industries attending trade flows "AB" and "Ab" the derived commodity demand is more inelastic than if they are completely unrelated. In general, if the actual situation is characterized by interdependent transfer services industries serving the trade flows originating in each seller, the standard estimate of the derived demand elasticity is more positively biased than if these transfer services industries are completely unrelated:

Model 1: $\left( A_D | \tau = 0 - A_D \right) > \left( A_D | \tau = 0 - A_D | \tau _{bb} = \tau _{BB} = 0 \right)$  \hspace{1cm} (137)
On the other hand, expression (136) means that when the transfer services industries attending the trade flows "AB" and "ab" are related to each other, the derived commodity demand elasticity is bigger than when that relationship does not exist. Therefore, if the existing circumstance is such that the transfer services industries serving the trade flows arriving at each buyer are interdependent, the usual derived demand elasticity estimate is less positively biased than if these transfer services industries are completely unrelated to each other:

\[ \text{Model 2: } \left( A_D \mid \tau = 0 \right) < \left( A_D \mid \tau_a = 0, \tau_{AA} = 0 \right) \] (138)

Moreover, the right-hand side inequality in expression (136) states that with interrelated transfer services industries attending the trade flows reaching at each buyer, the sign of the bias of \( A_D \mid \tau = 0 \) is unknown a priori.

Models 1 and 2 do not allow elucidation of what happens when the transfer services industries serving the cross trade flows (i.e., trades "AB" and "ab" and trades "Ab" and "aB" in Figure 19) are related to each other. This analysis is not presented here, but it is easy to show graphically that for this situation it cannot be determined if the derived commodity demand elasticity is bigger or smaller than \( A_D \mid \tau = 0 \).

In summary, the standard derived commodity demand elasticity \( \left( A_D \mid \tau = 0 \right) \) is unambiguously overestimating the "true" \( A_D \), unless the transfer services industries attending each buyer are interrelated, and/or the transfer services industries serving the cross trade flows are interdependent.
In either of these last situations the sign of the bias of \( A_D|_{\tau=0} \) may be either positive or negative. Therefore, it seems more likely that \( A_D|_{\tau=0} \) is biased upward in most of the cases, although one should not dismiss the possibility of a downward bias without further knowledge of the transfer services industries.

The recognition that transfer services are in less than perfectly elastic supply introduces the problem of greater complexity for practical modeling. Moreover, if the transfer services supply is allowed to respond to different trade flows (in the present models this means allowing \( \tau_{aA}>0, \tau_{aA}>0, \tau_{bb}>0, \tau_{bb}>0 \)), the system may easily become intractable for empirical work.

For this reason, it would be very helpful to have an analytical tool to make educated guesses about the importance of increasing the sophistication of the model at hand by explicitly including in it the supply schedules of transfer services. Equation (119) is this tool despite the simplicity of its underlying model. By plugging into it values for the direct demand elasticity (D), supply elasticity of the competitor seller (s), market share (p), and price ratios (m and n), the range of estimates of the derived demand elasticity (\( A_D \)) under different behavioral assumptions regarding the transfer services industries can be obtained. In this way not only the likely sign of the bias of the calculated derived demand elasticity may be obtained, but also a rough estimate of its magnitude as compared to the situation of transfer services in less than perfectly elastic supply.
To make the point more explicit, three examples are developed below. They are concerned with the estimation of the elasticity of foreign demand for U.S. soybeans, U.S. wheat, and Brazilian soybeans. Seller "A" is identified therefore with the U.S. in the first two examples and with Brazil in the third one. Buyer "B" is the group of importer countries, while seller "a" is the group of exporting countries other than that of interest.

The focus of these exercises is to illustrate the effect of different parameters characterizing the supply of transfer services (\( \tau \)) on the estimated derived elasticity of demand (\( A_D \)). Because of this, the direct demand and supply commodity elasticities (\( D \) and \( s \)) are not estimated. Instead, following Johnson (1977), it is assumed that domestic demand elasticities (\( D \)) are 0.2 for wheat and 0.4 for soybeans. The supply elasticity (\( s \)) is assumed to be 0.2 for both wheat and soybeans.

The market shares (\( p \)) and the price ratios (\( m \) and \( n \)) used for the calculations are four-year averages, and they appear in Table 5.

All of these parameters, together with the behavioral parameters of the transfer services industries attending the trade flows arriving at buyer "B" (\( \tau_{AA}, \tau_{Aa}, \tau_{aa}, \tau_{AA} \)) were plugged into equation (119), and the results obtained for \( A_D \) are reported in Tables 6 through 8.

The set of \( \tau \) specifically used is arbitrary. However, it is chosen so that it includes the whole spectrum of values that the derived commodity demand elasticity can allowedly adopt. With respect to the nature of the industries supplying transfer services to buyer "B", two
Table 5. Market Shares and Price Ratios Used to Estimate the Elasticities of Foreign Demand for U.S. Soybeans and Wheat, and Brazilian Soybeans

<table>
<thead>
<tr>
<th>Example</th>
<th>Period</th>
<th>Market Share (p)</th>
<th>Price Ratios</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. U.S. Soybeans(^a)</td>
<td>1983/84 - 1986/87</td>
<td>0.73</td>
<td>1.05(^b) 1.11(^c)</td>
</tr>
<tr>
<td>2. U.S. Wheat(^d)</td>
<td>1982/83 - 1985/86</td>
<td>0.36</td>
<td>1.24(^e) 1.21(^f)</td>
</tr>
<tr>
<td>3. Brazilian Soybeans(^a)</td>
<td>1983/84 - 1986/87</td>
<td>0.09</td>
<td>1.11(^c) 1.05(^b)</td>
</tr>
</tbody>
</table>

\(^a\)Source: USDA.
\(^b\)CIF Rotterdam/FOB Gulf Ports.
\(^c\)CIF Rotterdam/FOB Rio Grande, Brazil.
\(^d\)Source: International Wheat Council.
\(^e\)CIF Japan/FOB Gulf.
\(^f\)CIF Japan/FOB Australia.
limiting situations are addressed: one with two industries completely unrelated to each other and the other with only one industry. The reason for this is that these two cases provide respectively the lower and upper limit estimates for $A_D$, given a pair $\tau_{AA}$ and $\tau_{aa}$. For instance, the intersection of $\tau_{AA}=1$ and $\tau_{aa}=10$ of Table 6a shows $A_D=0.57$, while its counterpart of Table 6b presents $A_D=0.70$. This means that, ceteris paribus and under the assumptions of the model, $A_D$ may be anywhere between 0.57 and 0.70 depending on the "true" values of $\tau_{AA}$ and $\tau_{aa}$. On the other hand, $\tau_{AA}$ and $\tau_{aa}$ are selected so that they cover the whole range allowed, from an infinitely elastic supply of transfer services ($\tau_{AA}=0$, $\tau_{aa}=0$) up to a completely inelastic supply ($\tau_{AA}=\infty$, $\tau_{aa}=\infty$) including the unitary elasticity case ($\tau_{AA}=1$, $\tau_{aa}=1$).

Once more, it should be noted that $\tau_{AA}=\tau_{aa}=0$ corresponds to the standard estimate of derived commodity elasticity found in SPEM and N-SPEM (i.e., $A_D\big|_{\tau=0}$), and because of this it is used as the basis for comparison.

The justification for this kind of use of equation (119) is that Model 2 can be thought of as a trade system consisting of many buyers and sellers in which all buyers are aggregated into a single buyer "B", and all sellers except the one of interest into a single seller "a". In Figure 19 this is equivalent to adding up buyers "B" and "b" into buyer "B", to aggregate trades "AB" and "Ab" into trade "A", and to join trades "aB" and "ab" in trade "a". In this situation it is impossible to distinguish either trade "AB" from "Ab" or trade "aB" from "ab", what implicitly amounts to
\[ AB_M = Ab_M; \quad a^B_M = ab_M \] (139)

and

\[ \frac{\partial AB_M}{\partial (AB_T + Ab_T)} = \frac{\partial Ab_M}{\partial (AB_T + Ab_T)} \] (140)

\[ \frac{\partial a^B_M}{\partial (a^B_T + ab_T)} = \frac{\partial ab_M}{\partial (a^B_T + ab_T)} \]

where \( ij_M \) = transfer cost of trade "ij", for \( i=A,a; j=B,b \), and %

\( ij_T \) = trade "ij", for \( i=A,a; j=B,b \).

Equation (140) is just a special case of a single transfer services industry attending the trade flows originating in each seller. Taking this observation together with the right-hand side of inequality (135) it can be concluded that except for \( A_D \bar{\tau} = 0 \) the values contained in Tables 6, 7, and 8 underestimate \( A_D \) if the transfer services industries serving the trade flows emanating from each seller are completely independent. However, even in this situation of no relationship at all, the numbers reported in Tables 6a, 7a, and 8a cannot exceed their respective \( A_D \bar{\tau} = 0 \), as it follows from the left-hand side inequality of (135) taken jointly with inequality (134):

\[ Model 1: \quad A_D \bar{\tau} = 0 > A_D \bar{\tau}_{bB} = \tau_{bb} > A_D \] (135)

\[ Model 2: \quad A_D \bar{\tau} = 0 > A_D \bar{\tau}_{aA} = \tau_{aA} = 0 \] (134)
In summary, the values reported in Tables 6 through 8 correspond to a single transfer services industry attending all trade flows originating in each seller. In the opposite case (no relationship at all) the values for the derived commodity demand elasticity of Tables 6, 7, and 8 would be bigger with the exception of $A_D \big|_{\tau=0}$. Nevertheless, none of the numbers contained in Tables 6a, 7a, and 8a would exceed the one in their respective upper-left cell (i.e., the estimate of $A_D \big|_{\tau=0}$).

**Example 1:** The values of the elasticities of foreign demand for U.S. soybeans are presented in Table 6. The standard estimate ($A_D \big|_{\tau=0}$) is 0.60, attained under the hypothesis that transfer services are in perfectly elastic supply. The main observation is that relaxing this assumption produces little change on $A_D$, unless extremely inelastic supply schedules of transfer services are considered. Therefore, it seems quite unlikely that introducing explicitly the supply of transfer services into the trade model is of any help in increasing the accuracy of the empirical estimate of $A_D$.

As an aside, Table 6 (as well as Tables 7 and 8) shows three analytical results derived before:

(a) $A_D \big|_{\tau=0} > A_D \big|_{\tau_{aA} = \tau_{aA} = 0}$

(b) $A_D \big|_{\tau=0} \geq A_D \big|_{A_M = A_M, a_{Ma} = a_{Ma}}$

(c) $A_D \big|_{A_M = A_M, a_{Ma} = a_{Ma}} > A_D \big|_{\tau_{aA} = \tau_{aA} = 0}$
Table 6. Estimates of Foreign Demand Elasticity for U.S. Soybeans

a. Two Transfer Services Industries Completely Unrelated to Each Other

\[
\begin{pmatrix}
A_D \\
\tau_{aa} = \tau_{AA} = 0
\end{pmatrix}
\]

<table>
<thead>
<tr>
<th>(\tau_{aa})</th>
<th>0</th>
<th>1</th>
<th>10</th>
<th>100</th>
<th>(\infty)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.60</td>
<td>0.58</td>
<td>0.46</td>
<td>0.15</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0.60</td>
<td>0.58</td>
<td>0.46</td>
<td>0.15</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>0.59</td>
<td>0.57</td>
<td>0.45</td>
<td>0.15</td>
<td>0</td>
</tr>
<tr>
<td>100</td>
<td>0.55</td>
<td>0.53</td>
<td>0.43</td>
<td>0.15</td>
<td>0</td>
</tr>
<tr>
<td>(\infty)</td>
<td>0.52</td>
<td>0.51</td>
<td>0.41</td>
<td>0.15</td>
<td>0</td>
</tr>
</tbody>
</table>

b. One Transfer Services Industry

\[
\begin{pmatrix}
A_D \\
A_{M_A} = a_{M_A} = a_{M_a}
\end{pmatrix}
\]

<table>
<thead>
<tr>
<th>(\tau_{aa})</th>
<th>0</th>
<th>1</th>
<th>10</th>
<th>100</th>
<th>(\infty)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.60</td>
<td>0.58</td>
<td>0.48</td>
<td>0.17</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0.61</td>
<td>0.60</td>
<td>0.48</td>
<td>0.17</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>0.71</td>
<td>0.70</td>
<td>0.57</td>
<td>0.20</td>
<td>0</td>
</tr>
<tr>
<td>100</td>
<td>1.75</td>
<td>1.70</td>
<td>1.39</td>
<td>0.48</td>
<td>0</td>
</tr>
<tr>
<td>(\infty)</td>
<td>(\infty)</td>
<td>(\infty)</td>
<td>(\infty)</td>
<td>(\infty)</td>
<td>0.44</td>
</tr>
</tbody>
</table>
Example 2: Table 7 reports the foreign demand elasticity estimates for U.S. wheat. It is readily observable that the difference between the base case \( A_D \rvert_{\tau=0} = 0.79 \) and the values of \( A_D \) with less than perfectly elastic supply of transfer services is more sizable than in Example 1. It appears that there could be some motivation to further inquire about the nature of the transfer services industries and their behavioral parameters. In other words, depending on the need for precision in the demand elasticity estimate, it may be worthwhile to model explicitly the transfer services industries.

Example 3: The foreign demand elasticity estimates for Brazilian soybeans are contained in Table 8. When compared to the basis \( A_D \rvert_{\tau=0} = 5.92 \), the demand elasticities under less than perfectly elastic supplies of transfer services may exhibit important differences. This is so even within the range of elastic supplies \( (\tau_{AA} \leq 1, \tau_{aa} \leq 1) \); for instance, if there are two transfer services industries completely unrelated to each other and \( \tau_{AA} = \tau_{aa} = 1 \), \( A_D \) equals 3.58 (compared to \( A_D \rvert_{\tau=0} = 5.92 \)). It can also be seen that, if there is enough evidence suggesting that the transfer services industries are completely unrelated to each other, most efforts should be directed towards modeling explicitly the one dealing with Brazilian exports. This is because this one has much more potential to affect the estimated \( A_D \). The main result of this example is that, unless there are strong indications supporting the hypothesis that transfer services are in very elastic supply, it seems unwise to overlook the possible impact of the behavioral
Table 7. Estimates of Foreign Demand Elasticity for U.S. Wheat

a. Two Transfer Services Industries Completely Unrelated to Each Other

\[
\left( A_D \right| \tau_{AA} = \tau_{aA} = 0 )\]

<table>
<thead>
<tr>
<th>( \tau_{aa} )</th>
<th>0</th>
<th>1</th>
<th>2</th>
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<th>10</th>
<th>100</th>
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<tbody>
<tr>
<td>0</td>
<td>0.79</td>
<td>0.67</td>
<td>0.58</td>
<td>0.41</td>
<td>0.27</td>
<td>0.04</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0.78</td>
<td>0.66</td>
<td>0.57</td>
<td>0.40</td>
<td>0.27</td>
<td>0.04</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>0.73</td>
<td>0.62</td>
<td>0.54</td>
<td>0.39</td>
<td>0.27</td>
<td>0.04</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>0.69</td>
<td>0.59</td>
<td>0.52</td>
<td>0.38</td>
<td>0.26</td>
<td>0.04</td>
<td>0</td>
</tr>
<tr>
<td>100</td>
<td>0.51</td>
<td>0.46</td>
<td>0.41</td>
<td>0.32</td>
<td>0.23</td>
<td>0.04</td>
<td>0</td>
</tr>
<tr>
<td>( \infty )</td>
<td>0.45</td>
<td>0.40</td>
<td>0.37</td>
<td>0.29</td>
<td>0.22</td>
<td>0.04</td>
<td>0</td>
</tr>
</tbody>
</table>

b. One Transfer Services Industry

\[
\left( A_D \right| A_M = A_m, a_M = a_m)\]

<table>
<thead>
<tr>
<th>( \tau_{aa} )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>5</th>
<th>10</th>
<th>100</th>
<th>( \infty )</th>
</tr>
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<tbody>
<tr>
<td>0</td>
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<td>0.72</td>
<td>0.65</td>
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<td>0.07</td>
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<td>0.73</td>
<td>0.67</td>
<td>0.53</td>
<td>0.39</td>
<td>0.07</td>
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<tr>
<td>5</td>
<td>0.89</td>
<td>0.80</td>
<td>0.73</td>
<td>0.58</td>
<td>0.43</td>
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<tr>
<td>10</td>
<td>0.98</td>
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<tr>
<td>100</td>
<td>2.68</td>
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<td>0.23</td>
<td>0</td>
</tr>
<tr>
<td>( \infty )</td>
<td>( \infty )</td>
<td>( \infty )</td>
<td>( \infty )</td>
<td>( \infty )</td>
<td>( \infty )</td>
<td>( \infty )</td>
<td>0.18</td>
</tr>
</tbody>
</table>
Table 8. Estimates of Foreign Demand Elasticity for Brazilian Soybeans

a. Two Transfer Services Industries Completely Unrelated to Each Other

\[
\left( A_D \mid \tau_{AA} = \tau_{aA} = 0 \right):
\]

<table>
<thead>
<tr>
<th>( \tau_{aa} )</th>
<th>0</th>
<th>0.5</th>
<th>1</th>
<th>2</th>
<th>5</th>
<th>10</th>
<th>100</th>
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</thead>
<tbody>
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<td>4.46</td>
<td>3.58</td>
<td>2.57</td>
<td>1.39</td>
<td>0.79</td>
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<td>3.58</td>
<td>2.57</td>
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<td>0.09</td>
<td>0</td>
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<tr>
<td>10</td>
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<td>3.52</td>
<td>2.54</td>
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<td>0.78</td>
<td>0.09</td>
<td>0</td>
</tr>
<tr>
<td>100</td>
<td>4.96</td>
<td>3.90</td>
<td>3.21</td>
<td>2.37</td>
<td>1.33</td>
<td>0.77</td>
<td>0.09</td>
<td>0</td>
</tr>
<tr>
<td>( \infty )</td>
<td>4.00</td>
<td>3.28</td>
<td>2.78</td>
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<td>1.25</td>
<td>0.74</td>
<td>0.09</td>
<td>0</td>
</tr>
</tbody>
</table>

b. One Transfer Services Industry

\[
\left( A_D \mid A_M = A_{Ma}, aM = a_{Ma} \right):
\]

<table>
<thead>
<tr>
<th>( \tau_{aa} )</th>
<th>0</th>
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<th>1</th>
<th>2</th>
<th>5</th>
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<tr>
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<td>6.89</td>
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<td>( \infty )</td>
<td>( \infty )</td>
<td>( \infty )</td>
<td>( \infty )</td>
<td>( \infty )</td>
<td>( \infty )</td>
<td>( \infty )</td>
<td>0.09</td>
</tr>
</tbody>
</table>
characteristics of the transfer services supplies on the derived demand estimates.

The corollary of the present section is that the relevance of modeling specifically the transfer services in N-SPEM and SPEM in order to obtain better estimates of the derived commodity demand elasticities relies heavily on

(a) The nature of the transfer services industries and their elasticities of supply

(b) Direct commodity elasticities of supply and demand

(c) The values of market shares and relative prices

Depending on these characteristics, the bias introduced on the estimated $A_D$ by omitting an explicit formulation of transfer services supplies may be either positive or negative and sizable or not.\(^1\) Therefore, without further inspection of the trade system it is not safe to assume a priori that it is harmless to ignore completely the transfer services industries.

\(^1\)As it was pointed out previously, the most likely situation seems to be the one in which $A_D \bigg|_{\tau=0}$ is upwardly biased.
CHAPTER 5
SUMMARY AND IMPLICATIONS

Most spatial and non-spatial partial equilibrium models (SPEM and N-SPEM) assume either explicitly or implicitly that transfer services are in perfectly elastic supply. If this case is used as the basis or standard for comparison, the presence of a less than infinitely elastic supply of transfer services alters the empirical results obtained from those models.

It is seen that with a less than infinitely price-responsive supply of transfer services there is no longer a unique price transmission elasticity. One must define a "price transmission elasticity for supply," which indicates the relative change in the price ratio \( \frac{\text{Seller Price}}{\text{Buyer Price}} \) under a commodity demand shock, and a "price transmission for demand," which expresses the relative change in \( \frac{\text{Seller Price}}{\text{Buyer Price}} \) caused by a supply shock. The relative change in the price ratio \( \frac{\text{Seller Price}}{\text{Buyer Price}} \) is bigger when there is a supply shock than when there is a demand shock, unless transfer services are in perfectly elastic supply: in this situation, the relative changes under both shocks are the same.

When the trade model is composed of a single aggregate commodity supply facing a single aggregate commodity demand, the derived commodity supply and demand are less elastic than the standard, and less so the more inelastic the supply of transfer services. If commodity as well as transfer services elasticities increase in the long run, it is ambiguous
how the length of run affects the size of the difference between the standard derived elasticity and the derived elasticity estimated under a positively sloped supply schedule of transfer services. This is because a bigger commodity elasticity exacerbates that difference, while a bigger elasticity of supply of transfer services works in the opposite direction.

The results for the one-supply/one-demand models cannot be directly extrapolated to models that have more than one aggregate commodity supply and/or demand. This is so because now there are at least two commodity trade flows, and these can be attended by a single transfer services industry or by different transfer services industries related or not related to each other.

The conclusions of the one-supply/one-demand models apply also to more complex systems if there is a one-to-one relationship between trade flows and transfer services industries, and these transfer services industries are completely independent from each other. With these restrictions the standard derived commodity elasticities overestimate the "true" derived elasticities. The positive bias of the standard derived demand (supply) elasticity becomes bigger if the transfer services industries attending the trade flows originating in each seller (arriving at each buyer) are interdependent while the industries serving the trade flows arriving at each buyer (emanating from each seller) are completely unrelated. In the opposite situation the standard derived demand (supply) elasticity may be either less-positively or negatively biased. In all other cases, that is, a single transfer services industry
or different transfer services industries related to each other, it is not known if the standard derived commodity demand and supply elasticities are over- or underestimating the "true" derived commodity elasticities. However, it seems more plausible that the standard estimates have an upward bias in most situations.

Anything else equal, the derived commodity demand (supply) cannot be more inelastic than when the transfer services attending the trade flows arriving at each buyer (originating in each seller) are completely independent. On the other hand, the derived demand (supply) cannot be more elastic than when there is a single transfer services industry serving the trade flows reaching each buyer (emanating from each seller).

Three examples from world agricultural trade are given to illustrate the main conclusion of the paper: it cannot be said a priori that the characteristics of the supply of transfer services are irrelevant for the empirical estimation of derived commodity elasticities. Knowledge of other parameters of the system can be of help in assessing the need to explicitly model the supply of transfer services. To facilitate this task, a simple formula is presented.

The models used to derive the above results are simple and overlook some relevant real-world situations. Among these are the possibility of commodity substitution and/or complementarity, and the presence of storage. They also neglect the possibility of temporary disequilibrium and of substitution of transfer services for commodity. Moreover, the models are deterministic and there is no room for uncertainty. All of
these shortcomings of the present models offer a fertile field for further research.
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APPENDIX 1

\[ A_D \left|_{\tau = 0} - A_D \right|_{\tau_B = \tau_{BB} = 0} = \frac{[1 + \tau_{bb} d(1-r)]\tau_{BB} D^2 g (1-g)f}{\text{[denominator]}} + \frac{[1 + \tau_{BB} D(1-g)]\tau_{bb} d^2 r (1-r)(1-f)}{\text{[denominator]}} > 0 \quad (A1.1) \]

where: \([\text{denominator}] = 1 + \tau_{BB} D(1-g) + \tau_{bb} d(1-r) + \tau_{BB}\tau_{bb} Dd(1-g)(1-r)\]

\[ A_D \left|_{\tau_B = \tau_{bb} = 0} - A_D \right|_{\tau_B = \tau_{bb} = 0} = \frac{[1+\tau_{bb} d(1-r)](\tau_{BB}(1-f) - \tau_{bb} f)D(1-g)+(1-f)]\tau_{bb} Dd(1-r)}{\text{[denominator][divisor]}} + \frac{[1+\tau_{BB} D(1-g)](\tau_{bb} f-\tau_{bb} (1-f))d(1-r)+f]\tau_{BB} Dd(1-g)}{\text{[denominator][divisor]}} > 0 \quad (A1.2) \]

where: \([\text{denominator}] = \text{the same as in equation (A1.1)}\).

\[ [\text{divisor}] = 1 + \tau_{BB} D(1-g) + \tau_{bb} d(1-r) + (\tau_{BB}\tau_{bb} - \tau_{BB}\tau_{bb})Dd(1-g)(1-r)\]

\[ A_D \left|_{\tau_B = \tau_{bb} = 0} - A_D \right|_{\tau_B = \tau_{bb} = 0} = \frac{\tau_{BB} Dd(1-g)(1-f)[1+\tau_{BB} D(1-g)]}{\text{[denominator][divisor][denominator][divisor]}} + \frac{\tau_{bb} Dd(1-r)f[1+\tau_{bb} d(1-r)]}{\text{[denominator][divisor][denominator][divisor]}} > 0 \quad (A1.3) \]

where \([\text{denominator}] = \text{the same as in equation (A1.1)}\).

By transitivity, expressions (A1.1), (A1.2), and (A1.3) allow us to derive the following relationships:

\[ A_D \left|_{\tau = 0} - A_D \right|_{\tau = 0} > 0 \quad (A1.4) \]
\[ A_D \bigg|_{\tau=0} - A_D \bigg|_{B_M = B_M; \ b_M = b_M} > 0 \] (A1.5)
APPENDIX 2

Each of the derivatives intervening in expression (118) can be stated in elasticity form in the following way:

\[
\frac{\partial^2 Q_D}{\partial A_P} = - \frac{A_P}{A_Q} \left( - \frac{\partial^2 Q_D}{\partial A_P^2} \frac{A_Q}{A_P} \right) \\
= - A_D \frac{A_Q}{A_P} \tag{A2.1}
\]

\[
B_{QB} = - \frac{B_P}{B_Q} \left( - B_{QB} \right) \frac{B_Q}{B_P} \\
= - D \frac{B_Q}{B_P} \tag{A2.2}
\]

\[
a_{Q_a} = \frac{a_p}{a_Q} a_{Q_a} \frac{a_Q}{a_p} \\
= s \frac{a_Q}{a_p} \tag{A2.3}
\]

\[
A_{M_A} = \frac{A_Q}{A_M} A_{M_A} \frac{A_M}{A_Q} \\
= \tau_{AA} \frac{A_M}{A_Q} \tag{A2.4}
\]
Substituting expressions (A2.1) through (A2.7) into equation (118),

\[
-A_{DQ}^{A_P} = - D \frac{B_Q^{A_P}}{B_p^{A_P}} - D \frac{B_Q^{A_P}}{B_p^{A_P}} s \frac{a_Q^{A_P}}{a_p^{A_P}} \frac{a_M^{A_P}}{a_Q^{A_P}} - s \frac{a_Q^{A_P}}{a_p^{A_P}} \]

where

\[
\text{[denominator]} = 1 + \left( \tau_{AA} \frac{A^{A_Q}}{A_Q^{A_P}} + \tau_{aa} \frac{a^{A_Q}}{a_Q^{A_P}} - \tau_{Aa} \frac{a^{A_Q}}{a_Q^{A_P}} - \tau_{aA} \frac{a^{A_Q}}{a_Q^{A_P}} \right) s \frac{a_Q^{A_P}}{a_p^{A_P}} - \left( \tau_{AA} \frac{A^{A_Q}}{A_Q^{A_P}} \tau_{aa} \frac{a^{A_Q}}{a_Q^{A_P}} - \tau_{Aa} \frac{a^{A_Q}}{a_Q^{A_P}} \tau_{aA} \frac{a^{A_Q}}{a_Q^{A_P}} \right) \left( - D \frac{B_Q^{A_P}}{B_p^{A_P}} s \frac{a_Q^{A_P}}{a_p^{A_P}} - \tau_{AA} \frac{A^{A_Q}}{A_Q^{A_P}} \right) \left( - D \frac{B_Q^{A_P}}{B_p^{A_P}} \right)
\]
Setting \( \frac{A_Q}{B_Q} = p \) (i.e., \( p \) is the share of seller A in the overall trade), and rearranging (A2.8),

\[
A_D = \frac{D + Ds \tau_{aa} \frac{a_M}{a_p} + s(1-p) \frac{B_p}{a_p}}{[\text{denominator}]} \quad (A2.9)
\]

where \([\text{denominator}]=\)

\[
\frac{B_p}{a_p} + \left[ \tau_{AA} \frac{A_M}{A_p} (1-p) + \tau_{aa} \frac{a_M}{a_p} p - \tau_{aA} \frac{A_M}{A_p} p - \tau_{aA} (1-p) \frac{a_M}{a_p} \right] s \frac{B_p}{a_p} +
\]

\[
(\tau_{AA} \tau_{aa} - \tau_{aA} \tau_{aA}) Ds \frac{A_M}{a_p} \frac{a_M}{a_p} + \tau_{AA} D \frac{A_M}{A_p} +
\]

Finally:

\[
A_D = \frac{D + \tau_{aa} Ds(n-1) + sn(1-p)}{[\text{denominator}]} \quad (119)
\]

where \([\text{denominator}]=mp + [\tau_{AA} m(n-1)(1-p) + \tau_{aa} m(n-1)p - \tau_{aA} n(m-1)p -
\]

\[
\tau_{aA} m(n-1)(1-p) s + (\tau_{AA} \tau_{aa} - \tau_{aA} \tau_{aA} )Ds(m-1)(n-1) + \tau_{AA} D(m-1)
\]

\[
D = - \frac{B_Q}{B_Q} \frac{B_p}{B_Q}
\]

\[
s = \frac{a_Q}{a_Q}
\]

\[
\tau_{ij} = \frac{\tau_{ij}}{\tau_{ij}} \frac{\tau_{ij}}{\tau_{ij}}, \quad i=A,a; \quad j=A,a
\]
\[ p = \frac{A_Q}{B_Q} \leq 1 \]
\[ m = \frac{B_p}{A_p} \gg 1 \]
\[ n = \frac{B_p}{a_p} \gg 1 \]

and the restrictions
\[ \tau_{AA} \gg \tau_{Aa} \left( \frac{p}{1-p} \right) \gg 0 \]
\[ \tau_{aa} \gg \tau_{aA} \left( \frac{1-p}{p} \right) \gg 0 \]
The derived demand is more inelastic the smaller the direct elasticity of supply of transfer services to trade "A".

\[
\frac{\partial A_D}{\partial \tau_{aa}} = - \frac{[D + \tau_{aa} Ds(n-1) + sn(1-p)] [sn(m-1)(1-p) + \tau_{aa} Ds(n-1)(m-1) + D(m-1)]}{[\text{denominator}]^2}
\]

\[
= - (A_D)^2 (m-1) < 0
\]  \hspace{1cm} (A3.1)

where \([\text{denominator}] = mp + [\tau_{aa} n(m-1)(1-p) + \tau_{aa} m(n-1)p - \tau_{aa} m(n-1)(1-p)]s + (\tau_{aa} \tau_{aa} - \tau_{aa} \tau_{aa} Ds(m-1)(n-1) + \tau_{aa} D(m-1)\)

The derived demand is more inelastic the smaller the direct elasticity of supply of transfer services to trade "a".

\[
\frac{\partial A_D}{\partial \tau_{aa}} = \frac{[\text{denominator}][Ds(n-1)] - [D + \tau_{aa} Ds(n-1) + sn(1-p)][sn(m-1)p + \tau_{aa} Ds(m-1)(n-1)]}{[\text{denominator}]^2}
\]

which, upon simplification, yields equation (A3.2).  \hspace{1cm} (A3.2)

\[
\frac{\partial A_D}{\partial \tau_{aa}} = \frac{-[\tau_{aa} Dn(m-1)p + \tau_{aa} Dm(n-1)(1-p) + \tau_{aa} \tau_{aa} D(m-1)(n-1) + mp(1-p)]s^2(n-1)}{[\text{denominator}]^2} < 0
\]

where \([\text{denominator}]\) is the same as in equation (A3.1).
The derived demand is more inelastic the bigger the cross elasticity of supply of transfer services "A" with respect to trade "a".

\[
\frac{\partial A^D}{\partial \tau_{aa}} = \frac{[D + \tau_{aa} D_s(n-1) + sn(1-p)] [np + \tau_{AA} D(n-1)] s(m-1)}{[\text{denominator}]^2} > 0 \quad (A3.3)
\]

where [denominator] is the same as in equation (A3.1).

The derived demand is more inelastic the bigger the cross elasticity of supply of transfer services "a" with respect to trade "A".

\[
\frac{\partial A^D}{\partial \tau_{AA}} = \frac{[D + \tau_{aa} D_s(n-1) + sn(1-p)] [m(1-p) + \tau_{AA} D(m-1)] s(n-1)}{[\text{denominator}]^2} > 0 \quad (A3.4)
\]

where [denominator] is the same as in equation (A3.1).

The derived demand is more inelastic the smaller the supply elasticity of seller "a".

\[
\frac{\partial A^D}{\partial s} = \frac{[Z + (X+Y)s][\tau_{aa} D(n-1) + n(1-p)] - (X+Y)[D + \tau_{aa} D_s(n-1) + sn(1-p)]}{[\text{denominator}]^2}
\]

\[
= \frac{Z[\tau_{aa} D(n-1) + n(1-p)] - (X+Y)D}{[\text{denominator}]^2}
\]

\[
= \frac{pm[\tau_{aa} D(n-1)+n(1-p)]+\tau_{AA}Dn(m-1)(1-p)-XD+\tau_{AA}D^2(m-1)(n-1)}{[\text{denominator}]^2}
\]
\[
\frac{mn(p(1-p)+\tau_{AA}^nDn(m-1)p+\tau_{AA}^nDm(n-1)(1-p)+\tau_{AA}^mD^2(m-1)(n-1))}{[\text{denominator}]^2} = \frac{[np + \tau_{AA}^nD(n-1)][m(1-p) + \tau_{AA}^nD(m-1)]}{[\text{denominator}]^2} > 0 \quad (A3.5)
\]

where [denominator] is the same as in equation (A3.1).

\[
X = \tau_{AA}^n(m-1)(1-p)+\tau_{AA}^m(n-1)p-\tau_{AA}^m(n-1)p-\tau_{AA}^m(n-1)(1-p)
\]

\[
Y = (\tau_{AA}^n + \tau_{AA}^m)D(m-1)(n-1)
\]

\[
Z = p m + \tau_{AA}^n D (m-1)
\]

The derived demand is more inelastic the higher the margin costs of trade "A" with respect to the price paid by buyer "B" (i.e., the bigger m).

\[
\frac{\delta A_D}{\delta m} = - \frac{[D+\tau_{AA}^nDs(n-1)+sn(1-p)][\tau_{AA}^n(1-p)+\tau_{AA}^m(n-1)p-\tau_{AA}^m(n-1)(1-p)]s}{[\text{denominator}]^2} - \frac{[D+\tau_{AA}^nDs(n-1)+sn(1-p)][(\tau_{AA}^n+\tau_{AA}^m)Ds(n-1)+\tau_{AA}^Dp]}{[\text{denominator}]^2} < 0 \quad (A3.6)
\]

where [denominator] is the same as in equation (A3.1).

A more inelastic direct demand does not imply a more inelastic derived demand.
\[
\frac{\partial A_D}{\partial D} = \frac{[U + (W+V)D][1+\tau_{aa}s(n-1)] - [D + \tau_{aa}Ds(n-1) + sn(1-p)](W+V)}{[\text{denominator}]^2}
\]

\[
= \frac{U[1+\tau_{aa}s(n-1)] - sn(1-p)(W+V)}{[\text{denominator}]^2}
\]

\[
= \frac{U - \tau_{AA}sn(m-1)(1-p)[1+\tau_{aa}s(n-1)] - sn(1-p)W}{[\text{denominator}]^2}
\]

\[
= \frac{[p + \tau_{aa}s(n-1) - \tau_{AA}s(n-1)(1-p)][1+\tau_{aa}s(n-1)]}{[\text{denominator}]^2}
\]

\[
= \frac{\tau_{AA}sn(m-1)p[1+\tau_{aa}s(n-1)] + \tau_{AA}\tau_{aA}^2n(m-1)(n-1)(1-p)}{[\text{denominator}]^2}
\]

\[
= \frac{[p + \tau_{aa}s(n-1) - \tau_{AA}s(n-1)(1-p)][m + \tau_{aa}sn(m-1) - \tau_{AA}sn(m-1)]}{[\text{denominator}]^2}
\]

(A3.7)

where [denominator] is the same as in equation (A3.1).

\[U = pm + [\tau_{AA}n(m-1)(1-p) + \tau_{aa}m(n-1)p - \tau_{AA}n(m-1)p - \tau_{aA}m(n-1)(1-p)]s\]

\[V = \tau_{AA}(m-1)[1+\tau_{aa}s(n-1)]\]

\[W = -\tau_{AA}\tau_{aA}s(m-1)(n-1)\]
Therefore \( \frac{\partial A_A^D}{\partial D} \geq 0 \iff [m + \tau_{aa} sm(n-1) - \tau_{AA} sn(m-1)] \geq 0. \)

Also, \( \tau_{aa} m(n-1) > \tau_{AA} n(m-1) \Rightarrow \frac{\partial A_A^D}{\partial D} > 0. \)

A smaller share of seller "A" in total trade (\( p \)) does not imply that its derived demand (\( A_D^e \)) is more elastic.

\[
\begin{align*}
\frac{\partial A_D^e}{\partial p} &= \frac{-sn(G+Fp) - [D + \tau_{aa} Ds(n-1) + sn(1-p)]F}{[\text{denominator}]^2} \\
&= \frac{-snG - [D + \tau_{aa} Ds(n-1) + sn]F}{[\text{denominator}]^2} \\
&= \frac{\tau_{AA} s^2 n(n-1)[m + \tau_{aa} D(m-1)] - [\tau_{AA} s n(m-1) + F][D + \tau_{aa} Ds(n-1) + sn]}{[\text{denominator}]^2} \\
&= \frac{\tau_{AA} D^2 s(n-1)(n-1) - [m + \tau_{aa} s n(m-1) - \tau_{AA} s n(m-1)][D + \tau_{aa} Ds(n-1) + sn]}{[\text{denominator}]^2} \\
&= \frac{[m + \tau_{aa} s n(m-1)]\tau_{AA} Ds(n-1)}{[\text{denominator}]^2} \\
&= \frac{\tau_{aa} Ds(n-1)[\tau_{AA} s n(m-1) - \tau_{aa} s n(m-1) - m]}{[\text{denominator}]^2} \\
&= \frac{[m + \tau_{aa} s n(m-1) - \tau_{AA} s n(m-1)][D + \tau_{aa} Ds(n-1) + sn]}{[\text{denominator}]^2}
\end{align*}
\]
\[
- \frac{[D+(\tau_{aa} + \tau_{aA})D_s(n-1)+s^n][m+\tau_{aa} sm(n-1) - \tau_{AA} s n(m-1)]}{[\text{denominator}]^2}
\]

where [denominator] is the same as in equation (A3.1)

\[
F = m + [(\tau_{aa} + \tau_{aA})m(n-1) - (\tau_{AA} + \tau_{Aa})n(m-1)]s
\]

\[
G = [D+\tau_{aa} Ds(n-1)+ns]\tau_{AA} (m-1) - [m+\tau_{AA} D(m-1)]\tau_{AA} s(n-1)
\]

Therefore, \( \frac{\partial^2 A}{\partial p} \geq 0 \iff [m+\tau_{aa} sm(n-1) - \tau_{AA} s n(m-1)] \leq 0 \)

and \( \tau_{aa} m(n-1) > \tau_{AA} n(m-1) \Rightarrow \frac{\partial^2 A}{\partial p} < 0. \)

A higher relative margin cost of trade "a" (n) has an ambiguous effect on the derived demand of seller "A" (A_D).

\[
\frac{\partial^2 A_D}{\partial n} = \frac{[m+\tau_{aa} (1-p) m]sm+\tau_{AA} D(m-1)]\tau_{AA} D+(1-p)]s}{[\text{denominator}]^2} - \frac{[\tau_{AA} \tau_{aa} - \tau_{Aa} \tau_{aa}]\tau_{AA} D+(1-p)]D_s^2(m-1)+[D-\tau_{aa} Ds+\tau_{AA} D+(1-p)]s}{[\text{denominator}]^2}
\]

\[
\frac{[m+\tau_{aa} (1-p) m]sm-(\tau_{AA} \tau_{aa} - \tau_{Aa} \tau_{aa})D_s(m-1)+\tau_{AA} D(m-1)]\tau_{AA} D+(1-p)]s}{[\text{denominator}]^2} + \frac{(\tau_{aa} D_s - D) J}{[\text{denominator}]^2}
\]
\[ \tau_{aa} D_s[mp+\tau_{AA} D(m-1)-\tau_{Aa} s(m-1)p] \]
\[ \frac{s(l-p)(mp-\tau_{aa} p-\tau_{Aa} (1-p))s+\tau_{AA} \tau_{Aa} D_s(m-1)+\tau_{AA} D(m-1)-DJ}{[\text{denominator}]^2} \]

\[ \tau_{Aa} D_s(m-1)p+\tau_{aa} D_s(m-1)+\tau_{Aa} \tau_{aa} D_s(m-1) \]
\[ \frac{[\tau_{aa} D(m-1)+m(l-p)]{p+\tau_{aa} D-\tau_{aa} p-\tau_{Aa} (1-p)]s}s}{[\text{denominator}]^2} \]

(A3.9)

where [denominator] is the same as in equation (A3.1).

\[ J = \tau_{AA} s(m-1)(1-p)+\tau_{aa} samp-\tau_{Aa} s(m-1)p-\tau_{aa} sm(1-p)+\tau_{AA} \tau_{aa} \tau_{Aa} \tau_{aa} D_s(m-1) \]

Therefore, \( \frac{\partial^2 D}{\partial n^2} \geq 0 \iff \{p + \tau_{aa} D - [\tau_{aa} p - \tau_{Aa} (1-p)]s\} \geq 0. \)
From all possible values allowed for $\tau_{AA}$, the one that makes $\tau_{AA} = 0$ bigger is $\tau_{AA} = 0$. In this case, equation (133) becomes

$$\tau_{AA} = T_{AA} = 0$$

In this case, equation (133) becomes

$$\tau_{AA} = T_{AA} = 0$$

Subtracting expression (A4.1) from expression (126)

$$\tau_{AA} = T_{AA} = 0$$

Therefore,

$$\tau_{AA} = T_{AA} = 0$$

Therefore,

$$\tau_{AA} = T_{AA} = 0$$