ULTRASONIC NDE OF ADHESIVE BONDS : THE INVERSE PROBLEM

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INTRODUCTION

Over the past quarter century, a wide variety of ultrasonic techniques have been developed to determine the phase velocity and thickness of elastic plates. Techniques to measure the phase velocity include toneburst [1-4], separable pulse methods [5-7], and spectroscopy [8-11]. These classical methods require that the specimen be thick enough such that two successive echoes from the front and the back faces of the specimen, respectively, be separable in the time domain. Kinra and Dayal [12], developed a through transmission technique which removes this particular limitation of the classical methods. This technique works satisfactorily for the measurement of the phase velocity for specimens whose thickness is greater than one-half of the wavelength; for thinner specimens, however, their numerical algorithm runs into convergence problems. Moreover, their numerical algorithm cannot be used to determine thickness at any wavelength. The reasons for their convergence problems are discussed in detail by Iyer, Hanneman and Kinra [13]. They demonstrated that a detailed sensitivity analysis is a necessary pre-requisite for the development of a robust inversion algorithm. Accordingly, a new inversion scheme based on the method of least squares was developed by Iyer and Kinra to determine thickness from the measurements of phase, magnitude and complex spectrum, respectively, [14-17]. In all of the above ultrasonic methods only one parameter can be determined i.e., an accurate knowledge of thickness is required to determine the wavespeed and vice versa. This defines the central objective of the present work: In this paper we present a technique for determining, simultaneously, the thickness and wavespeed of a thin layer. This is useful in characterizing the cohesive properties of thin adhesive layers in an adhesively-bonded joints where the properties of the cured bond may be significantly different from that of the bulk adhesive. Results are presented for thin aluminum plates, by way of calibration, and for adhesively bonded joints.

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THEORY

An elastic plate immersed in an elastic medium is shown in Figure 1. Consider a one-dimensional, time harmonic, longitudinal displacement wave traveling in the positive x-direction given by $u_i(x,t) = \exp[i(\omega t - k_0 x)]$, incident upon the plate. Under steady state conditions, the reflected and the transmitted displacement waves from the plate may be expressed as $u_r(x,t) = A_r \exp[i(\omega t + k_0 x)]$ and $u_t(x,t) = A_t \exp[i(\omega t - k_0 x)]$, respectively. Here A_r and A_t are the complex amplitudes of the reflected and transmitted waves, respectively; $k_j = \omega s_j$, is the wavenumber in medium j; s_j = slowness in medium j = inverse of wavespeed, c_j. In addition, there are steady state displacement waves within the plate which can be expressed, by the superposition of the right going and left going waves, as

$$u_s(x,t) = A\exp[i(\omega t - k_1 x)] + B\exp[i(\omega t + k_1 x)] \tag{1}$$

In order to solve the unknown complex amplitudes, A_t, A_r, A and B, the following boundary conditions, obtained by enforcing the continuity of displacements and stresses at the boundaries of the plate, are applied:

$$@\ x = 0 \qquad u_i + u_r = u_s \ : \qquad \sigma_i + \sigma_r = \sigma_s \tag{2a}$$

$$@\ x = h \qquad u_s = u_t \ : \qquad \sigma_s = \sigma_t \tag{2b}$$

where the associated stress fields are obtained from Hooke's Law:

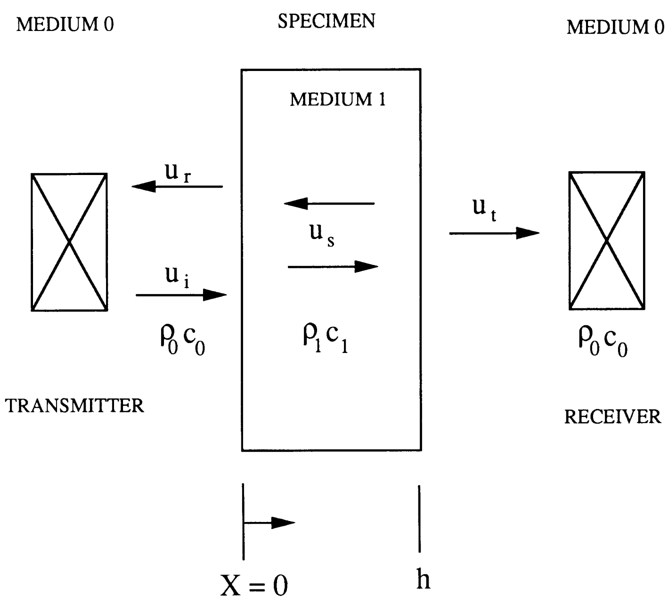

Figure 1. Schematic of steady state waves in an elastic plate immersed in an elastic medium.

$$\sigma_j = (\lambda_k + 2\mu_k)\frac{\partial u_j}{\partial x} \qquad j=i,r,t,s; \quad k=0 \; for \; j=i,r,t \qquad k=1 \; for \; j=s \tag{3}$$

λ, μ are the Lame constants. Solving for A_t from the above set of Equations (2) yield

$$A_t = \frac{(1 - R_{01}^2)\exp[i\omega h(s + s_0)]}{\exp[i2\omega hs] - R_{01}^2} \tag{4}$$

$$R_{01} = \{1 - \rho_0 s/\rho_1 s_0\} \,/\, \{1 + \rho_0 s/\rho_1 s_0\} \tag{5}$$

For brevity, the subscript "1" has been removed from s. The main objective of this paper is to determine h and s, simultaneously, from measured spectrum, A_t. The reader is referred to Ref. [16], for details of experimental procedures and the necessary apparatus to measure the complex amplitude ratio, A_t. However, for sake of continuity, a brief summary is presented here. We perform two independent measurements: the specimen is removed from the path of the transmitter to the receiver, and the reference signal f(t) is recorded on the oscilloscope; the specimen is inserted in the path of the transmitter to the receiver, and the specimen signal g(t) is recorded. Let $F^*(\omega)$ and $G^*(\omega)$ be, respectively, the Fourier Transforms of f(t) and g(t); the "transfer function" of the plate, $H^*(\omega)$, is defined as $H^*(\omega) = G^*(\omega)/F^*(\omega)$. It has been shown by Hanneman and Kinra [18,19], $H^*(\omega) \equiv A_t$.

INVERSE PROBLEM

The "inverse problem" is defined to be a problem of determining s and h, simultaneously, from the measurements of $H^*(\omega)$. In this section, we develop a successive approximation procedure called the Gaussian least squares differential correction method [20]; this method is an generalization of Newton's root solving method for finding the x-values satisfying the non-linear equation of the form F(x) = 0. Since $H^*(\omega)$ is measured at discrete frequencies, $\omega_j = 1,2,\ldots.N$, over the useful bandwidth of the transducer, Eq.(4) can be written in a discretized form as

$$H_j^* = \frac{(1 - R_{01}^2)\exp[i\omega_j hs_0(1 + s/s_0)]}{\exp[i2\omega_j hs_0 s/s_0] - R_{01}^2} \tag{6}$$

where R_{01}, given by Eq. (5), is a function of (s/s_0) only; and we have judiciously separated our two unknown parameters, h and s, into $p_1=hs_0$ and $p_2=s/s_0$. We now introduce $\tilde{H}^*_j(p_1,p_2)$ as the theoretical transfer function (given by the right hand side of Eq.(4)), in order to distinguish it from the experimentally measured transfer function H^*_j. It is now desired to determine that particular values of p_1 and p_2, viz. p_{1r} and p_{2r}, that will minimize the sum of squares of the residuals, namely,

$$E(p_1,p_2) = \frac{1}{N}\sum_{j=1}^{N} \Delta e_j^2 \equiv \frac{1}{N}[\Delta H^*]^T[\Delta H^*] \tag{7}$$

where

$$\Delta e_j = \| H_j^* - \tilde{H}_j^*(p_1,p_2) \| \tag{8}$$

and

$$[\Delta H]^T = \{ \Delta e_1 \quad \Delta e_2 \quad \ldots\ldots \quad \Delta e_N \} \tag{9}$$

Assume that initial guesses (*current* estimates), $\{P_c\}^T = \{p_{1c}\ ,\ p_{2c}\}$ of the unknown p-values are available and that they are not too far from the "true" values, $\{P_r\}^T = \{p_{1r}\ ,\ p_{2r}\}$, such that

$$\{P_r\} = \{P_c\} + \{\Delta P\} \tag{10}$$

The current residuals corresponding to the current estimates are calculated from Eq.(7) as

$$\Delta e_{jc} = H_j^* - \tilde{H}_j^*(p_{1c},p_{2c}) \tag{11}$$

The residuals corresponding to the "true" values $\{P_r\}$ then can be estimated from the Taylor series expansion about $\{P_c\}$ as:

$$\Delta e_{jr} = \Delta e_{jc} - \sum_{i=1}^{2} \left(\frac{\partial \tilde{H}_j^*}{\partial p_i} \bigg|_c \right) \Delta p_i + \textit{higher order terms} \qquad j = 1,2,\ldots..N \tag{12}$$

If we now truncate the Taylor series of Eq.(12) at the linear terms, then the left hand side is only an improved estimate of the residuals instead of the the exact values. We then have the linearly predicted residuals, Δe_{jp}, as

$$\Delta e_{jp} = \Delta e_{jc} - \sum_{i=1}^{2} \left(\frac{\partial \tilde{H}_j^*}{\partial p_i} \bigg|_c \right) \Delta p_i \qquad j = 1,2,\ldots..N \tag{13}$$

where, for brevity, we have introduced $\tilde{H}_j^* = \| \tilde{H}_j^* \|$, subscript "p" denotes predicted residuals. In a matrix form, these are conveniently represented as

$$\{\Delta H_p\} = \{\Delta H_c\} - [A]\{\Delta P\} \tag{14}$$

where

$$[A] = \begin{bmatrix} \frac{\partial \tilde{H}_1^*}{\partial p_1} & \frac{\partial \tilde{H}_2^*}{\partial p_2} \\ \frac{\partial \tilde{H}_2^*}{\partial p_1} & \frac{\partial \tilde{H}_2^*}{\partial p_2} \\ \cdots & \cdots \\ \frac{\partial \tilde{H}_N^*}{\partial p_1} & \frac{\partial \tilde{H}_N^*}{\partial p_2} \end{bmatrix}_c \quad ; \quad \{\Delta P\} = \begin{Bmatrix} \Delta p_1 \\ \Delta p_2 \end{Bmatrix} \tag{15}$$

Keeping in mind that our objective is to minimize $E(p_1,p_2)$, we now need to determine the approximate differential corrections, $\{\Delta P\}^T = \{\Delta p_1,\Delta p_2\}$, that will minimize $E_p(p_1,p_2)$ given by

$$E_p(p_1,p_2) = \frac{1}{N}\{\Delta H_p^*\}^T\{\Delta H_p^*\} \tag{16}$$

or

$$E_p(p_1,p_2) = \frac{1}{N}[\{\Delta H_c^*\} - [A]\{\Delta P\}]^T\ [\{\Delta H_c^*\} - [A]\{\Delta P\}] \tag{17}$$

The necessary conditions for minimization of E_p, see Ref. [20], is given by

$$\nabla_{\Delta P} E = 0 \Rightarrow \{\Delta P\} = ([A]^T[A])^{-1}[A][\Delta H_c] \tag{18}$$

The Inversion Algorithm

The Gaussian Algorithm may be summarized in the following steps: (1). Input parameter starting estimates for $\{P_c\}^T = \{p_{1c}, p_{2c}\}$; (2). Compute $H_j^*(p_1,p_2)$ at the current estimates; (3). Compute the matrix of partial derivatives, [A], at the current estimates; (4). Form the error E_c. (5). Determine the differential corrections, $\{\Delta P\}$ from Eq.(16); (6). Form the Error E_p. (7). Upon convergence terminate the process. Otherwise obtain new current estimate by adding the differential corrections, $\{\Delta P\}$, to the current estimates, $\{P_c\}$ and go to step (2). The convergence criteria used to terminate the process was $\Sigma \Delta p_i^2 < 10^{-8}$.

CALIBRATION RESULTS

By means of calibration, we first tested thin aluminum plates immersed in water. Experiments were conducted on specimens having thicknesses of 0.254 mm, 0.514 mm 0.634 mm, and 1.62 mm, at 10 MHz so that h/λ varied from $0.4 \le h/\lambda \le 2.6$. Each measurement was replicated ten times (i.e. specimen removed and replaced each time).

Assuming that the thickness is known, the "true" wavespeed, $c_{\text{"true"}}$, was determined by techniques described in Ref. [16]. It was found to be 6.35 mm/μs $\pm$ 0.06. The experimental data generated to determine the "true" values was then used to determine h and c simultaneously, using the Gauss_Newton algorithm described in the previous section. The results are presented in Table 1. It can be seen that excellent agreement between the "true" values and the measured values was obtained for both thickness and wavespeed.

The results for adhesively-bonded joints are presented in Table 2. Once again the reader is referred to Ref. [16] for details of the sample preparation and the testing procedures. To obtain a "true" estimate of the wavespeed in the adhesive, a 6 mm thick epoxy specimen was prepared from bulk material and its wavespeed at 2.25 MHz. was found to be 2.20 mm/μs. In the investigation of adhesively bonded joints, only magnitude information was used to deduce h and c. Setting our objective to determine h and c to within $\pm$5% of the "true" values, we find that the measured values for wavespeed are consistently higher than the "true" values. It is conjectured that in-situ properties of the adhesive layer may be significantly different its bulk properties. We also observe that for specimen #8, the measured values of wavespeed at 20 MHz, c = 2.76 mm/μs, is higher than that measured at 5 MHz, c = 2.48 mm/μs. The reason could be that the adhesive material exhibits dispersive behaviour.

Table 1. Calibration Results on Aluminum to Determine c and h simultaneosly

Frequency : 10 MHz. Mode : Through Transmission
Density : 2.78 gm/cc c("true") : 6.35 mm/μs $\pm$ 0.06

h/λ	**h (micrometer) (mm)$\pm$0.025**	**h (NDT) (mm)**	**c (NDT) (mm/μs)**
0.4	0.254	0.254 $\pm$ 0.002	6.30 $\pm$ 0.02
0.8	0.520	0.514 $\pm$ 0.003	6.30 $\pm$ 0.03
1.0	0.635	0.634 $\pm$ 0.006	6.38 $\pm$ 0.03
2.5	1.613	1.616 $\pm$ 0.016	6.36 $\pm$ 0.02

CONCLUSION

Hitherto, the problem of UNDE has been posed in one of two ways: Given h find c; Given c find h. In this paper we have developed a technique whereby one can determine both c and h, simultaneously. This technique is particularly useful in the characterization of thin adhesive layers in adhesively bonded joints where the thickness of the cured bond is not known and the wavespeed of the cured bond may be

Table 2. Results on Adhesively Bonded Joints to determine c and h simultaneously from the Magnitude Spectrum.

Mode : Through Transmission (Direct Contact)
c"true": 2.20 mm/μs @ 2.25 MHz.
Density: 1.27 gm/cc.

Sp. ID #	Freq. MHz	h/λ	h "true" mm ± 0.02	h "NDT" mm	Error mm	c "NDT" mm/μs	Error mm/μs
2	20.0	0.6	0.06	0.08	0.02	2.44	0.24
8	5.0	0.1	0.05	0.05	0.00	2.48	0.28
8	20.0	0.4	0.05	0.06	0.01	2.76	0.56
10	10.0	0.3	0.06	0.07	0.01	2.55	0.35
11	7.5	0.1	0.04	0.05	0.01	2.34	0.14
11	10.0	0.1	0.04	0.06	0.02	2.36	0.16

significantly different from that of the bulk adhesive. Excellent results were obtained for the case of thin metal plates whereas values of h and c to within ±5% of the "true" values were not obtained for the case of adhesively-bonded joints. The reasons for this error is being inverstigated. The numerical algorithm developed to determine c and h, may be readily extended to include density as a third unknown parameter; this will be the subject our future communication.

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