Wavelet transform based techniques for ultrasonic signal processing

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Wavelet transform based techniques for ultrasonic signal processing

by

Sanand Prasad

A Thesis Submitted to the
Graduate Faculty in Partial Fulfillment of the
Requirements for the Degree of
MASTER OF SCIENCE
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Major: Electrical Engineering

Iowa State University
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CHAPTER 1. INTRODUCTION

1.1 Overview of Ultrasonic Nondestructive Testing

The science of nondestructive testing deals with obtaining the material or structural properties of a component by studying the interaction of the component with some form of energy. Nondestructive testing may be employed during manufacture or service of a component to assess its quality and determine the suitability of retaining it in service. These methods are used extensively to inspect aircraft structures and components, railroad wheels, concrete structures, nuclear power plants, gas pipelines and other structures where it is important to ascertain the ability of the components to withstand stresses and prevent failure after prolonged use. The popular nondestructive testing methods are X-ray, ultrasonic, eddy current and infrared techniques. Active measurement techniques like radiography, ultrasonics and eddy current methods interrogate the specimen with a burst of energy and analyze the return signal to deduce the properties of the specimen under test. Passive measurement techniques like infrared imaging analyze the thermal energy given off by a sample. Ultrasonic testing methods are popular because it is relatively easy to generate and detect ultrasonic energy. Ultrasonic test equipment is fairly inexpensive and the testing does not require special enclosures like X-ray testing methods. Ultrasonic methods can also be applied to obtain the characteristics of surfaces at any depth within a component.
and is not limited to just the top surface. These reasons make ultrasonic testing a very popular nondestructive testing technique [1].

Ultrasonic testing of materials is used either to detect flaws in a component or to measure the properties of a material. This involves injecting a burst of high energy at ultrasonic frequencies into the sample by means of a transducer. The length of the input pulse varies from a short pulse for high-resolution flaw detection to a long tone burst for precise attenuation and velocity measurements. The injected wave travels through the sample and interacts with the material. The return echo from the sample is picked up using another transducer and this represents the signature of the material at various depths along the path of travel of the pulse. The echo is then analyzed to determine the properties of the sample under test. Inhomogeneities that may be present in the sample represent discontinuities in the acoustic impedance of the medium. The velocity of the injected ultrasonic pulse is altered at these discontinuities and the return echo shows spikes at these points. The location of these spikes with respect to a reference point gives the location of the discontinuity [2].

Ultrasonic instrumentation mainly consists of a pulse generator to generate the ultrasonic pulses, a receiver to gather the echoes from the sample under study, a display and analysis system and one or two transducers depending on the mode of operation. The basic system model is shown in Figure 1.1 [3], [4].

The transmitter or pulser generates electrical pulses of high energy. The output of this system is ideally a broadband spike of sufficient amplitude containing uniform spectral density (a delta function). Such a function is not realizable physically but can be approximated by a pulse of large amplitude and very fast fall times. Commercial systems operate in the frequency range of 1-10 MHz. However, special applications
Figure 1.1: A general ultrasonic system model

employ systems operating at 10 KHz to test highly attenuative materials and systems operating at upto 1 GHz for high resolution measurements of low-loss materials are available. The pulser unit has controls for adjusting the energy, damping and repetition rate of the input pulses.

As mentioned before, an ultrasonic test system may use one or two transducers depending on the mode of operation. In the pitch-catch mode two separate transducers are used for transmission and reception. In the pulse-echo mode, however, the same transducer is used both as a transmitter and receiver. The transducer utilizes a piezoelectric material crystal to convert the input electrical pulse to ultrasonic waves while transmitting and vice versa. The ultrasonic energy is coupled to the test specimen by means of an acoustical coupling medium, usually water or a gel.

The receiver is a high gain amplifier which picks up the return echo from the sample, amplifies it and performs other functions such as pulse shaping. The receiver
has controls for adjusting the gain and bandwidth.

The analysis and display system implements any post-processing that might be necessary to extract the relevant information. In modern systems, the echo signal is digitized and powerful digital signal processing techniques are used. The display may be generated in either A, B or C formats, depending on the application. An A-scan display plots the returned echo as a function of time. Calibrated in terms of the material constants, this may also be interpreted as a plot of the received echo versus the depth of the sample. A B-scan is a collection of A-scans arranged in a two-dimensional format. A C-scan is a collection of the peaks of A-scans over a region of the sample and represents the acoustical properties of the sample surface at a certain depth.

The above testing method, however, is limited by two factors. First, the amount of input pulse energy required is quite high if the sample thickness is large. Second, the resolution that can be obtained is limited by reflections from the microstructure of the specimen under test. This is called background clutter or grain noise.

Testing at very high frequencies tends to amplify the grain noise or the backscatter from the grain structure of the material under test. This means that flaws which are of a size comparable to the grains of the material, are hard to detect. Increasing the resolution of testing only worsens the problem whereas decreasing the resolution reduces the probability of detection to very low values.

Techniques that have been used to reduce the effects of backscatter include time-domain averaging of the return echoes and correlation methods [5]-[7]. Since uncorrelated noise usually has a zero mean, averaging tends to improve the signal to noise ratio (SNR). Averaging $2^m$ signals improves the SNR by $3m$ dB. A minimum
of 32 averages is recommended for relatively noise-free signals and 128-512 averages may be required for moderately noisy signals. Correlation receivers employ white-noise as investigating inputs. White-noise is approximated by pseudorandom m-ary sequences. These sequences are easy to generate and duplicate. By correlating the return echo with a delayed version of the input, SNR improvement is achieved. The improvement in SNR depends on the duration of correlation and the length of the code used [6].

Current research and development activities in the field of ultrasonic nondestructive testing are focused on designing better test equipment, using signal processing methods to improve the signal to noise ratio, modeling and developing new imaging techniques. Digital circuitry is being employed extensively in modern ultrasonic instrumentation. Computers are being used to control the operation of test systems and the process of ultrasonic testing is being completely automated. The availability of fast analog-to-digital converters and advances in VLSI have facilitated the use of digital signal processing techniques. Complex signal processing methods are being employed to improve the SNR and obtain better images. Modeling the ultrasonic phenomenon plays an important role in detection problems where the SNR is too low. Using fast computers, numerical techniques that mimic the ultrasonic measurement process are being developed. Advanced imaging techniques using phased arrays of transducers, holography and synthetic aperture focusing methods are being investigated.
1.2 Problem Statement

The problem of interest in this research is the detection of 'hard-alpha' regions in titanium samples. Hard-alpha regions in titanium alloys are regions of high oxygen and nitrogen concentration. These regions are brittle inclusions inside the material. There are two known types of hard-alpha inclusions. They are termed Type I and Type II defects. A Type I defect is one which is often accompanied by voids and cracks, making it possible to detect it using ultrasonic NDE techniques. Type II defects are 'aluminum-rich alpha-stabilized segregation region in a titanium alloy, with a hardness only slightly higher than the adjacent matrix' [8]. During the process of manufacture or use of a titanium component, hard-alpha regions act as stress centers. This leads to initiation of cracks in the material and possible failure of the component. Detection of these inclusions, therefore, becomes crucial during manufacture or routine inspection. The devastating effects of hard-alpha inclusions in aircraft engine components have been well documented. The air crash in Sioux City, IA in 1989 is attributed to a hard-alpha inclusion initiated failure of an engine turbine blade.

The detection of hard-alpha inclusions by ultrasonic methods is complicated by the presence of backscatter from the host material. Backscatter places a fundamental restriction on the size of the inhomogeneity that can be detected by ultrasonic testing methods. Further, ultrasonic grain noise signals from titanium samples is highly correlated with signals caused by the hard-alpha inclusion. Presence of this correlated grain noise along with the fact that the acoustic impedance difference between any hard-alpha inclusion and the host material is of the order of 10%, makes the detection of hard-alpha inclusion a challenging task.
Figure 1.2 shows a photomicrograph of a Type I hard-alpha region in a titanium sample. This defect is accompanied by a large void. This is typical of a Type I defect, which is frequently, but not always, associated with voids or cracks. The white region surrounding the void is a layer of stabilized alpha. Surrounding this is a region of enlarged alpha grains of titanium.

Given the complex nature of the task at hand, conventional ultrasonic processing methods do not provide a way of detecting hard-alpha inclusions. The probability of detecting these inclusions using simple thresholding techniques is extremely low. Advanced signal processing concepts are required to improve the chances of detection.
In looking for an analysis technique, two things are of primary importance:

- The signature response of hard-alpha inclusion is time-localized.
- The spectral properties of this time-localized region is needed for detection purposes.

Figure 1.3 illustrates the above concepts. The titanium sample under test can be thought of as being made up of a hypothetical grid. The investigating ultrasonic wavefront enters the sample at one end and the signatures are gathered at the opposite face. If one of these blocks contains a hard-alpha inclusion, this would manifest itself as a time-localized signal in the received signature. This time-localized signal is part of the response from the other blocks in the grid along the line of travel of the investigating wavefront. To detect this inclusion, an analysis technique which can bring out the spectral differences of this region relative to the host material is needed. This translates into a need for obtaining the joint time-frequency distribution of the signal.

To summarize, a technique capable of maintaining the time-localization of the hard-alpha inclusion in a titanium sample in the frequency domain is required. Sophisticated analysis techniques like these are necessary because of the presence of highly correlated grain noise in titanium samples.

1.3 Scope of Thesis

Chapter 1 examines the general principles of ultrasonic testing of materials and introduces the limitations of the technique. An introduction to the problem of hard-alpha detection using ultrasonic methods is given and the justification for using a relatively sophisticated time-frequency technique for analysis is provided.
The utility, or the lack of it, of the Fourier transform in analyzing non-stationary signals is examined in chapter 2. Three of the more popular time-frequency distributions, the Wigner-Ville, the Short-Time Fourier Transform and the Wavelet Transform, are explained and compared. It will also be shown that the Wavelet transform represents the most effective way of carrying out the analysis. A survey of the methods which have been used in the past to detect non-stationary signals using time-frequency methods is provided at the end of the chapter.

An overview of the simulation study used in this research and the proposed technique for hard-alpha detection is provided in chapter 3. A hypothesis test used to evaluate the performance of the proposed algorithm is also explained. A few detection examples are also provided.

Chapter 4 includes some discussions on the proposed technique, a comparison
with other known techniques for solving the hard-alpha problem and suggests some future research directions.
CHAPTER 2. TIME-FREQUENCY DISTRIBUTIONS

This chapter examines the concept of joint time-frequency distributions of signals. These distributions are primarily used in the analysis of signals whose properties evolve with time, i.e., non-stationary signals. The initial section explores the utility of the Fourier transform, an important tool in spectral analysis, in such an analysis procedure. The need for using a joint time-frequency technique is brought out. The latter sections will examine three popular time-frequency analysis methods and a comparison will be made among these. An example which brings out the features of all the three distributions is also provided. The final section contains a review of techniques for detecting non-stationary signals using these time-frequency mappings.

2.1 Non-stationary Signal Analysis with the Fourier Transform

The primary tool for performing spectral analysis has been the Fourier transform. The Fourier transform maps a signal in the time domain into the frequency domain using complex exponential basis functions. The analysis equation of the Fourier transform is given by,

\[ X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt \] (2.1)

The power of the Fourier transform lies in the fact that it can decompose a signal
into its constituent-frequencies and provide the relative magnitude and phase of each frequency component [21]. The disadvantage is the loss of time information in the magnitude spectra. The exponential basis functions used in the Fourier transform are infinite in duration. This leads to spreading of any time localization or abrupt changes in the signal over the entire frequency axis. Although the time localization information is embedded in the phase spectrum, difficulties associated with estimating the true phase spectrum has prevented its use. Any time localization of the input signal is therefore lost when the magnitude spectrum is used.

A classic case of the above is the Fourier transform of an impulse signal. In the time-domain, the dirac-delta is localized but the magnitude of the delta function is spread over the entire frequency spectrum. This is illustrated in Figure 2.1. The magnitude of the Fourier transform does not have any information regarding the time of occurrence of the delta function.

Nonstationary signal analysis using the Fourier magnitude spectrum would, therefore, be inappropriate. The time information carried by the input signal is, however, embedded in the Fourier-phase spectrum. This follows from the following

\[
x(t) \quad \text{abs}(X(f))
\]

\[
t \quad f
\]

Figure 2.1: The dirac-delta and its Fourier magnitude spectrum
property of the Fourier transform shown in equation 2.2.

\[ \text{Given: } x(t) \leftrightarrow X(f), \text{then: } x(t - t_0) \leftrightarrow e^{-j2\pi f t_0} X(f) \tag{2.2} \]

The phase spectrum can be processed to obtain information relating to time. Unfortunately, processing the Fourier phase spectrum involves the problem of unwrapping. The Fourier phase spectrum is a discontinuous function and integer multiples of \(2\pi\) should be added to make it continuous. This is a complicated process and usually not resorted to.

A good way of circumventing this problem is to introduce a window in the time domain. By sectioning parts of the signal and computing the Fourier transforms of these windowed signals, a measure of time can be introduced into Fourier analysis. The width of this window is altered depending on the degree of nonstationarity associated with the signal. This is the concept of the Short-Time Fourier transform (STFT), one of the popular methods of time-frequency analysis.

The concept of time-frequency analysis stems from the necessity to study time-varying spectra. In the next few sections the various methods available for analyzing time-varying spectra, namely, the STFT, the Wigner-Ville distribution and the Wavelet transform, will be examined. In order to facilitate comparison between the three techniques, decompositions of a simple non-stationary signal using each of the methods are shown.

### 2.2 The Short-Time Fourier Transform

The Short-Time Fourier Transform (STFT) uses a single window to compute the time-frequency spectrum of a signal. The input signal is first windowed in the
time domain. The Fourier transform of the windowed sections of a signal constitutes it’s STFT. Many windows like the rectangular window and the exponential window have been proposed depending on the modeling problem at hand. The STFT can be defined by the relation,

$$S(t, \Omega) = \int_{-\infty}^{\infty} x(\tau) g^*(\tau - t) e^{-j\Omega \tau} d\tau$$  \hspace{1cm} (2.3)$$

For a discrete time signal, the STFT is defined as [13],

$$S(n, \omega) = \sum_{m=-\infty}^{\infty} x[m] g[n - m] e^{-j\omega m}$$  \hspace{1cm} (2.4)$$

The analysis window, $g[n]$, is normalized such that $g[0] = 1$. The STFT represents the local behavior of the signal $x[n]$ through the the sliding window $g[n - m]$. An implicit assumption made in the STFT is that the windowed sections of the signal are stationary.

The STFT analysis can be thought of as a filtering operation on the signal using a modulated filter bank. The analysis window, $g[n]$, represents the filter and the exponential basis functions modulate this filter to obtain a modulated filter bank. On a time-frequency plane the STFT amounts to sampling the signal uniformly on both the time and frequency axes. The time-bandwidth product of the window used corresponds to the areas as shown in Figure 2.2. The time-bandwidth product is lower bound by the “Heisenberg uncertainty principle”. This means that

$$\Delta t \Delta f \geq \frac{1}{4\pi}$$

The major disadvantage of the STFT is the trade-off in time-frequency resolution. Due to the “Heisenberg” lower bound and the use of the same window throughout, a signal can be studied with either high time or frequency resolution but
The analysis window can be chosen to be narrow in time or frequency such that it satisfies the "Heisenberg" lower bound. If the time-resolution is desired, then the window chosen is narrow. This results in a very poor frequency resolution and vice-versa.

By choosing a suitable sampling value for the frequency axis, the discrete time STFT can be converted to the discrete STFT. This means that efficient FFT techniques can be used to compute the STFT. The set of basis functions generated by such a sampled modulated filter bank is orthonormal only if the window function, $g[n]$, is poorly localized either in time or frequency. Hence, to obtain good time or frequency resolution, most discrete STFT analysis procedures resort to redundancy or oversampling.
2.2.1 Example of the short-time Fourier transform

Figure 2.3 shows the STFT of three transient signals separated both in time and frequency. A rectangular window of length 32 was used for the analysis. This represents a compromise in the time-frequency resolution obtained. The third transient is resolved but the first two are smeared out. To obtain a better time resolution a shorter window would have to be used but this will have to be at the expense of a poorer frequency resolution. This illustrates the time-frequency resolution trade-off that is inherent in the STFT.

Figure 2.3: Short-time Fourier transform of transient signals
2.3 The Wigner-Ville Distribution

The Wigner-Ville distribution (WV) of a continuous signal is defined as \[10]\n
\[ W_f(t, \Omega) = \int_{-\infty}^{\infty} e^{-j\Omega \tau} f(t + \frac{\tau}{2}) f^*(t - \frac{\tau}{2}) d\tau \]  \hspace{1cm} (2.5) \]

For a discrete-time signal the WV distribution is given by \[11]\n
\[ W_f(n, \omega) = 2 \sum_{k=-\infty}^{\infty} e^{-j2\omega k} f(n + k) f^*(n - k) \]  \hspace{1cm} (2.6) \]

The Wigner-Ville distribution of a signal is a bilinear transform. This means that the WV of the sum of two signals is not the sum of the WVs of the signals. Specifically,

\[ W_{f+g}(n, \omega) = W_f(n, \omega) + W_g(n, \omega) + 2Re\{W_{f+g}(n, \omega)\} \]  \hspace{1cm} (2.7) \]

The presence of the cross terms \(2Re\{W_{f+g}(n, \omega)\}\) in equation 2.7 introduces artifacts in the time-frequency distribution obtained using the WV. Conventional linear system techniques can no longer be used to analyze signals decomposed with the WV.

Some of the other properties of the WV, however, make it a good tool for the analysis of non-stationary signals. As an example, the WV of a time-limited signal is restricted to the same time-interval in the time-frequency plane. Similarly a bandlimited signal has the same frequency support in the WV domain. All these imply that the WV of a nonstationary signal is guaranteed to be localized on the time-frequency plane.

Another difficulty in applying the WV distribution to discrete-time signals is the fact that the WV distribution is periodic in \(\pi\). The spectrum of a discrete signal is
periodic in $2\pi$. So when the WV distribution of a discrete-time signal is computed, there may be aliasing in the time-frequency domain. This aliasing is avoided if the signal is analytic or a bandlimited signal is sampled at a rate which is at least twice the Nyquist frequency. An analytic signal is defined as one which has no negative frequencies. This means that the spectrum of an analytic signal is limited to the range $0 - \pi$ and is zero in the range $\pi - 2\pi$. The WV of such a signal would, therefore, have no aliasing problems. When a bandlimited signal is sampled at twice its Nyquist frequency, the spectrum of the sampled signal is bandlimited to the interval $0 - \pi$. Again, the WV of such a signal does not suffer from aliasing problems. The disadvantage, however, is in having to deal with the redundancy which arises due to oversampling.

Figure 2.4 illustrates the spectrum of a bandlimited signal and the frequency support of its WV decomposition. The spectrum of this is bandlimited to the range $0 - \omega_a$ and $\omega_b - 2\pi$. It is easy to see how aliasing could occur in the WV distribution if the difference between $\omega_b$ and $\omega_a$ is less than $\pi$.

The redundancy in the discrete time Wigner-Ville distribution can be reduced by sampling along the frequency axis. By doing so, efficient FFT methods can be utilized. The computation of the Wigner-Ville distribution of a signal reduces to computing the sequence $f(n + k)f^*(n - k)$ as a function of $k$ for every sample, $n$, in the input signal and then computing the DFT of this new sequence.

2.3.1 Example of the Wigner-Ville distribution

Figure 2.5 shows the WV distribution of three transient signals separated both in time and frequency. The input signal used here is not analytic and hence there
is a lot of aliasing which tends to clutter the time-frequency plane. The bilinear nature of the Wigner-Ville distribution also adds to the clutter in the time-frequency representation of the signal. An analytic signal can be obtained by using the Hilbert transform. By doing so the input signal of Figure 2.5 can be bandlimited to the interval 0 to $\pi$. This eliminates the aliasing and hence most of the clutter in the time-frequency representation. This is illustrated in Figure 2.6. The contour plot shown indicates a finite support region on the time-frequency plane for the three transients in the signal.
Figure 2.5: Wigner-Ville distribution of transient signals
Figure 2.6: Wigner-Ville distribution of analytic signals
2.4 The Wavelet Transform

The Wavelet transform (WT) is the latest technique to emerge for processing signals with time-varying spectra. The WT is defined in terms of basis functions obtained by compression/dilation and shifting of a 'mother wavelet'. Mathematically, the wavelet coefficients are given by,

\[ WT_f (t, a) = \int f(\tau) h_{a,t}^*(\tau) d\tau, \]  

(2.8)

where,

\[ h_{a,\tau} = \frac{1}{\sqrt{a}} h \left( \frac{t - \tau}{a} \right) \]  

(2.9)

Equation 2.9 is the shifted and compressed version of the mother wavelet h(t). The time-shift is \( \tau \) and the frequency scale is \( a \).

The wavelet synthesis equation consists of summing up all the projections of the signal onto the wavelets. This is represented by equation 2.10.

\[ f(t) = \int \int WT_f(\tau, a) h_{a,\tau}(t) \frac{d\tau da}{a^2} \]  

(2.10)

Discretization of the time-scale parameters leads to the equation,

\[ h_{j,k}(t) = a_0^{-j} h \left( a_0^{-j} t - kT \right) \]  

(2.11)

With reference to equation 2.9, the scale \( a = a_0^j \) and the time-shift \( \tau = k a_0^j T \). \( T \) is the sampling frequency of the input signal. A signal sampled at the scale \( a1 \), i.e., with \( j = 1 \) in the discretized parameters, roughly corresponds to the frequency \( f1 \) and a signal at the scale \( a2 \) corresponds to the frequency \( f2 \). The discretized scale and time-shift parameters, \( a \) and \( \tau \), are both dependent on the value of \( a_0 \). This
correlation between the scale and shift parameters also implies that a signal at the scale \(a_2\) is subsampled at a rate \(\frac{f_1}{f_2}\) of the scale \(a_1\), assuming \(a_1 < a_2\). In other words, as we move up in scale, we move lower in frequency. The sampling frequency can thus be reduced from scale to scale in accordance with Nyquist's rule to avoid redundancy.

Figure 2.7 illustrates this multiresolution property of the wavelet transform. The frequency sampling shown here is on a 'dyadic' scale, i.e., \(a_0 = 2\) in equation 2.11. A value of \(a_0 = 2\) represents a frequency stepping in an octave-by-octave fashion. This is particularly useful since the subsampling is by a factor of two, which implies dropping every other sample.

![Dyadic sampling of the wavelet time-frequency plane](image)

Figure 2.7: Dyadic sampling of the wavelet time-frequency plane
The value of \( a_0 \) determines the orthonormality of the basis functions generated by the wavelet function. \( a_0 \) close to 1 constitutes a redundant case. Any function, \( h(t) \), of finite energy and time support can then be used as a mother wavelet and perfect signal reconstruction is possible without any restrictive conditions on \( h(t) \). Sparse sampling on the frequency axis with \( a_0 = 2 \) yields orthonormal basis functions only for special choices of \( h(t) \). The theory of wavelet frames provides a framework which encompasses the two extreme cases discussed above. It provides for a way to balance redundancy by choosing \( a_0 \) between 1 and 2 and placing restrictions on \( h(t) \) to achieve signal reconstruction, depending on the application [20]. Signal analysis and recognition are some applications where orthogonality is not critical and redundancy is usually resorted to but applications such as signal coding and compression that employ transform techniques require a strictly orthogonal basis function set.

Figure 2.8 shows the Mexican Hat function, which is used as the mother wavelet in this study. The Mexican Hat function is given by equation 2.12.

\[
h(t) = (1 - t^2)e^{-t^2/2}
\]

(2.12)

The dyadic sampling scheme with \( a_0 = 2 \) was used leading to an octave-by-octave basis filtering of the input signal. Figure 2.9 shows time-shifted and compressed Mexican hat function at two scales.

Figure 2.10 shows the raised cosine function given by equation 2.13, where \( \omega_0 \) is the bandwidth and \( \omega_1 \) is the center frequency of a system.

\[
f(t) = [1 - \cos(\omega_1 t)]\cos(\omega_0 t)
\]

(2.13)

The response of an inclusion to an ultrasonic pulse can be modeled by the raised cosine function. The wavelet transform can be seen as the correlation of the signal.
Figure 2.8: A basic wavelet

Figure 2.9: Compressed and shifted wavelets
The raised cosine function resembles the Mexican Hat function of Figure 2.8 to a great extent. This makes the Mexican Hat a good choice for ultrasonic signal processing applications.

The advantage of the wavelet representation is the ability to use different windows at different frequencies. The wavelet transform looks at the signal with poor time-resolution and high frequency resolution at lower frequencies and vice-versa. The areas of the individual windows are still lower bound by the "Heisenberg" principle. Also, the width of the frequency window increases in a logarithmic fashion but maintaining the $\frac{\text{bandwidth}}{\text{center freq.}}$ ratio a constant. This is referred to as constant-Q analysis in literature [15]. Figure 2.11 shows the coverage of the time-frequency plane using the wavelet representation.
Figure 2.11: Time-frequency coverage of the WT
2.4.1 Examples of the wavelet transform

Figure 2.12 shows the phase-plane or the time-frequency representation of a delta function. The Mexican hat was used as the basis function. A delta function is time-limited but has components at all scales or frequencies.

Figure 2.12: Dirac-delta and it’s wavelet transform
Figure 2.13 shows the phase-plane of three transient signals separated both in time and frequency. Three distinct clusters are seen on the phase plane. These clusters are separated both in time and frequency. This demonstrates the time-frequency resolving ability of the wavelet transform.
2.4.2 Interpretations of the wavelet transform

There have been many interpretations of the wavelet transform. The most obvious is the one which views it as an inner product, and hence treats it as the correlation between the wavelet and signal vectors. In certain applications the wavelet transform has been seen as a projection of the signal onto a subspace spanned by the basis functions generated by shifted and scaled versions of the mother wavelet.

An interesting analogy to the wavelet transform is one which compares wavelet analysis to a microscope [15]. The magnification is determined by the value of $a^j_0$. Then the microscope is moved to the location of interest in the time-frequency plane by shifting it. After a coarse examination, the field of vision can be narrowed by zooming in, equivalent to changing the value of $j$. Minute details can also be observed by choosing smaller steps, i.e., $a^j_0 T$.

In digital signal processing, the discrete wavelet transform with a dyadic sampling scheme is seen as a multi-resolution decomposition of a signal using Quadrature Mirror Filter (QMF) pairs [22], [23]. The wavelet is a bandpass filter and the wavelet transform is performed by filtering the signal with a set of octave-band filters and subsampling the output of each filter to sample the multiresolution signals at their respective Nyquist frequencies. Such a scheme is shown in Figure 2.14.

The filter $g[n]$ is a halfband lowpass FIR filter and the filter coefficients of this filter is the solution to the scaling function given by equation 2.14.

\[ \phi(x) = \sum_{n=-\infty}^{\infty} c_n \phi(2x - n) \quad (2.14) \]

The filter $h[n]$ is a halfband highpass FIR filter and the coefficients of this filter
Figure 2.14: QMF filter pair implementation of the wavelet transform

The coefficients of $h[n]$ and $g[n]$ are also related by the expression

$$h[L - 1 - n] = (-1)^n g[n]$$  \hspace{1cm} (2.16)

where $L$ is the length of the FIR filter.

The wavelet transform is defined by the scale and shift equations. Recursive filtering using this filter bank produces the wavelet coefficients at the output of the highpass filters and the detail signal at the output of the final lowpass filter. It can be shown that the basis functions represented by the recursive highpass filtering is orthonormal and spans the space of band-limited functions in the interval $(-2\pi, -\pi) \cup (\pi, 2\pi)$. The halfband lowpass filter at the end of the tree represents orthonormal basis functions which span the space of band-limited functions in the interval $(-\pi, \pi)$. Together, the filter bank produces an orthonormal basis function set that spans the space $(-2\pi, 2\pi)$, i.e., the entire space of bandlimited discrete signals. This scheme
therefore represents a decomposition of the input signal onto a set of orthonormal basis functions.

Given a set of orthonormal basis functions which satisfy equation 2.14, the coefficients of the lowpass filter can be computed from equation 2.14. The coefficients of the halfband highpass filter can then be computed as a solution of the shifting function or from the equation 2.16. Different wavelets have been synthesized starting with different sets of orthonormal functions satisfying the scaling function. An extensive study of these can be found in reference [17].

2.5 Some Comparisons of the Time-Frequency Methods

In this section, the three time-frequency techniques described earlier will be compared. The justification for choosing the wavelet transform as the method for analysis will also be provided.

The Wigner-Ville decomposition, as stated earlier, is a bilinear transform. This means that the sum of the Wigner distributions of two signals is not the same as the distribution of the sum of the signals. Conventional linear system methods cannot be applied with the Wigner distribution.

The Short-time Fourier transform requires the user to set the size of the window and the step size. This means that there is an inherent trade-off in terms of time-frequency resolution. Increased time-resolution means reduced frequency resolution and vice-versa due to the “Heisenberg” lower bound. Also, the same window is used throughout the frequency spectrum of interest. The STFT can thus be viewed as recursive bandpass filtering of the signal with a window of constant bandwidth. This is illustrated in Figure 2.15.
The Wavelet transform deals with the “Heisenberg” bound in a unique way. At higher frequencies, when the signal is varying faster, a window of high time resolution, or equivalently, low frequency resolution is used. At lower frequencies, poor time resolution and high frequency resolution is used. This window size is built into the transform and does not require any user intervention. It has also been hypothesized that most naturally occurring signals do follow this pattern. The response of the human ear has also been modeled in this fashion. The wavelet decomposition amounts to recursively filtering the signal with a filter whose bandwidth increases in a logarithmic manner. This is illustrated in Figure 2.16.

The STFT and Wavelet transforms are both linear transforms as opposed to the Wigner-Ville distribution. The cross terms that appear as a result of the bilinear nature of the Wigner-Ville distribution makes it unattractive to use.

Between the STFT and wavelet transforms, the wavelet transform does not suffer from the drawbacks of time-frequency trade-offs. Further, the user does not have to specify any window dimensions in the wavelet transform. The wavelet transform is also flexible in the sense that the user can specify any frequency sampling rate, i.e., any value of $a_0$ between 1 and 2 can be chosen. The dyadic sampling scheme is popular because it involves subsampling by a factor of 2 at each step, which is a simple decimation operation with discrete signals. Further, for some choices of the mother wavelet, $h(t)$, an orthonormal basis function set can be generated, thereby eliminating redundancy. Perfect reconstruction filters can also be constructed for this scheme by a simple interchange of the analysis filter coefficients, leading to an efficient analysis and synthesis method. This sums up the reasons for having chosen the wavelet transform as the method for analysis.
Figure 2.15: Frequency domain coverage of the STFT

Figure 2.16: Frequency domain coverage of the WT
2.6 Techniques for Non-stationary Signal Analysis

Time-frequency mapping techniques are used to study non-stationary signals, i.e., signals whose frequency changes with time. These techniques overcome the stationarity requirement imposed by the Fourier Transform. By mapping the non-stationary signal onto a joint time-frequency plane, a time dependence is introduced into the frequency domain analysis techniques. Interestingly, a time-frequency mapping can be compared to a musical score. A musical score is a collection of different scales played at different instants. Thus a score is a joint function of time and frequency. As non-stationary signals are localized in time, it is natural to introduce time dependence into the analysis of non-stationary signals.

This section provides a review of detection methods that have been used to detect non-stationary signals using time-frequency mapping techniques. The popular time-frequency methods used have been the Wigner-Ville (WV) distribution, the Short-Time Fourier Transform (STFT) and more recently, the Wavelet Transform (WT). The techniques to be discussed here will therefore concentrate on these distributions.

Non-stationary analysis techniques can be divided into two classes based on the approach used. One, methods which rely upon existence of signal models or a priori information and two, methods which do use signal models.

1. If the signal to be detected is known a priori, then this signal model is used in the detection process. A maximum likelihood estimate using a known signal model can be computed. This estimate can be a least square sense fit to the signal model. This situation is common in communication systems, where a signal model can be constructed easily.

2. If no a priori information about the signal to be detected is known, statistical
properties of the signal are used. The signal is modeled as a random signal with a certain probability distribution. With some knowledge about the degradation process or the-noise-statistics, a detection scheme can be devised. This situation often arises in signal detection problems in NDE and radar.

2.6.1 Wigner-Ville distribution based techniques

The WV distribution is a bilinear transform. This introduces artifacts into the time-frequency distributions obtained using this technique. All detection schemes based on the WV distribution have to overcome this handicap.

One way of characterizing signals is via their local and global moments. Local moments are determined either by considering the WV distribution as a function of frequency for a fixed time or vice versa. These moments carry information regarding the time or frequency variations of the signal. Global moments are moments over the entire time-frequency plane. If a signal can be localized either in time or frequency, then local moments can be compared with the global moments to detect a non-stationary signal [11]. These techniques have been used to detect seismic, speech and sonar signals degraded by noise [26], [27].

The problem of signal detection has also been addressed as one of finding the best rank one approximation to a model dependent matrix. This model dependent matrix is obtained from a two-dimensional model of the signal or from the WV distribution of the signal model. A least-squares sense fit can be obtained to the signal model using this approach. This technique is based on the existence of a signal model and has found applications in communication systems [24].

Time-varying filtering methods have been proposed to detect signals using the
WV distribution. These are referred to as signal estimation and synthesis techniques. Both signal amplitude and phase have been synthesized using this method. The algorithm is based on synthesizing a signal whose WV distribution best approximates a given time-frequency function. It uses the maximum eigenvalues and eigenvector of the model signal and synthesizes an approximation to fit the model [25].

2.6.2 Short-time Fourier transform based techniques

The main limitation of the STFT is the trade off in time and frequency resolution. A signal can be studied with good time resolution or frequency resolution but not both. This is because the time-bandwidth product is lower bound by the ‘Heisenberg principle’. This constrains the time-bandwidth product to be greater than or equal to $\frac{1}{4\pi}$. The STFT also implicitly assumes that the signal inside the window is stationary and uses complex exponential basis functions to map the windowed portion to the frequency domain. Techniques using the STFT to detect non-stationary signals have to accommodate the time-resolution trade off. The selection of the window to be used is dependent upon whether a better time or frequency resolution is required. The length of the window should be sufficiently narrow to ensure that the signal is stationary within the window span.

An interesting approach to detecting nonstationary signals using the STFT is by designing windows to match the signal to be detected. This approach has been outlined in reference [14]. A one-sided exponential window function whose rate of decay can be varied is used. It is shown that under the hypothesis of noise only the STFT decomposition corresponds to a chi-square distribution and under noise plus transient to a non-central chi-square distribution.
The split-spectrum techniques used in ultrasonic NDE have also been likened to a time-frequency analysis using the STFT [31]. These techniques are based on the premise that the signal of interest is located in a particular frequency region. The analysis is performed using a rectangular window and the Fourier basis functions. The inverse Fourier transform of the windowed segments are computed to obtain a time-frequency distribution. These methods are empirical and a rigorous mathematical proof of these has not been provided.

2.6.3 Wavelet transform based techniques

The wavelet transform is a linear transform combining the advantages of both the WV distribution and the STFT. This does suggest that the techniques that benefit from WV and STFT analysis can serve as candidates for analysis using the wavelet transform [29].

One way of detecting transients using the WT is described in reference [28]. A transient represents a particular pattern on a wavelet time-frequency plane. This means that a transient can be represented by a few coefficients on the time-frequency plane. This pattern can be used to detect a signal buried in noise.

Wavelet de-noising techniques are methods which strive to improve the SNR of the input signal. These techniques describe schemes which attempt to reject noise by damping or thresholding in the wavelet domain. The thresholds or damping factors are chosen by the user depending on the data being analyzed [32], [33].

The WT being a relatively new technique, detection schemes using the WT are still in their infancy. Issues such as designing wavelets to suit particular applications are being addressed. The next few years should see a lot of developments in this
area. Efficient computation methods of the WT have also been developed. The fast wavelet transform which uses a pair of Quadrature Mirror Filters recursively to give a multiresolution decomposition is one such technique [17].
CHAPTER 3. PROPOSED TECHNIQUE AND HYPOTHESIS TESTING

This chapter explains the proposed wavelet transform based detection scheme to identify hard-alpha inclusions in titanium samples. The first section describes the data set used in the experiment. The definition of the Signal-to-noise ratio (SNR) measure used is also provided. The wavelet transform based detection scheme is explained in the second section. The apriori information used in the detection algorithm and the three-step detection process are explained. The performance of the proposed algorithm is evaluated using a hypothesis test. This hypothesis test and the results of the evaluation are provided in the fourth section. The last section shows the wavelet decomposition of the flaw, noise and flaw+noise signals. The ability of the wavelet transform to limit the flaw signal to a few frequency scales is central to the development of the algorithm. This is evident from the wavelet decompositions provided. Finally, a few detection examples are shown to prove the efficacy of the proposed technique.

3.1 Experimental Setup

The data set used in the research consists of background clutter signals or noise signals obtained experimentally from titanium samples using conventional A-scan
techniques. A simulated spherical hard-alpha inclusion was then added and this additive model formed the data set used in the analysis.

The noise data set consisted of 51 A-scans obtained from a Ti-6-4 block. The transducer used was a 0.5" focused transducer with a focal length of 2". The total length of the acquired data was 10μsec. A sampling frequency of 100 MHz was used, giving 1000 data points. The total scan area was 1.125" by 1.125" with scan spacing of 0.025". A 0.4 mm diameter spherical inclusion with an acoustical impedance difference of 10% from the host material was added.

The Signal-to-noise ratio (SNR) of the signal was defined as the ratio of the peak of the flaw signal and the peak of the noise signal in a trace. Table 3.1 shows the characteristics of the data set that was used. As is typical with hard-alpha signals, the SNR of the input signal is very low. The average SNR of the signals used in this study was 0.8.

<table>
<thead>
<tr>
<th>Total number of samples analyzed</th>
<th>51</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max. signal SNR</td>
<td>1.51</td>
</tr>
<tr>
<td>Min. signal SNR</td>
<td>0.78</td>
</tr>
<tr>
<td>Average SNR</td>
<td>0.80</td>
</tr>
</tbody>
</table>

Figure 3.1 shows a typical A-scan noise signal while Figure 3.2 shows a simulated hard-alpha inclusion signal. Figure 3.3 shows the flaw signal added to an A-scan background clutter signal. The SNR of the signal in this example is 1.5.
Figure 3.1: Background clutter signal

Figure 3.2: A simulated hard-alpha inclusion signal
Figure 3.3: Noise with added flaw signal
3.2 Proposed Detection Technique

The proposed technique for hard-alpha detection using the wavelet transform consists of three steps. The \textit{a priori} information used in the process and the proposed algorithm are explained in this section.

\textit{A priori} Information:

The first step in the technique is to obtain the region of support of the flaw signal on the wavelet time-frequency plane. This is done by examining the flaw energy distribution across frequency scales. The wavelet decomposition is carried out over eight scales on an octave-by-octave basis and this tends to localize the flaw signal to certain frequency scales. The scales for decomposition was obtained empirically. This represents the \textit{a priori} information used in the process. Figure 3.4 illustrates this concept.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure3_4.png}
\caption{\textit{A priori} information used}
\end{figure}
For each test datum,

Step 1) The wavelet transform of the input signal is obtained. The wavelet transform is computed over the same scales as in the previous step. The noise signal tends to be scattered over the entire phase-plane. Due to the poor SNR of the input signal the flaw signal is hidden and is not readily visible (WT).

Step 2) After the wavelet decomposition, the frequency of support of the flaw signal or the scales of the wavelet transform in which most of the energy of the flaw signal is concentrated, as determined from the \textit{a priori} information, is chosen (SEL).

Step 3) The geometric mean of the subset of the multiresolution signals obtained from the wavelet decomposition is then computed (GM). The geometric mean is defined as

\[ y[n] = \left( \prod_{i=1}^{N} x_i[n] \right)^{1/N} \quad (3.1) \]

The geometric mean process has been proved to be a relatively robust operation in processing ultrasonic signals with high clutter levels [31]. The geometric mean process has been compared with other techniques like the arithmetic mean, which have been used in the past to process ultrasonic signals. These form a class of techniques which have been collectively referred to as frequency diverse filtering.

The output of the geometric mean process is a signal of higher SNR relative to the input. Thresholding techniques can be now used at the output of this filter to detect any hard-alpha inclusion. The increase in SNR is brought about by the combination of selecting only the flaw support region from the wavelet time-frequency plane and computing the geometric mean. Figure 3.5 illustrates the proposed system concept.
Figure 3.5: Proposed system for hard-alpha detection
3.3 System Performance Evaluation

To evaluate the performance of the proposed system, hypothesis testing techniques are used. Hypothesis testing involves computing the output under the two hypotheses, noise only - \( X_0 \) and flaw+noise - \( X_1 \), with the inputs modeled as random signals. The main goals of hypothesis testing is to study the system performance in terms of

1. SNR improvement

The measures of performance used in hypothesis testing are the Probability of Detection (POD) and Probability of False alarm (POF). These are defined as,

\[
\text{NoFlaw} - X_0 : POF = \int_T^\infty p(y|x_0)dy \\
\text{Flaw} - X_1 : POD = \int_T^\infty p(y|x_1)dy
\]

where \( T \) is the threshold used at the output of the system. The POD and POF measures define the performance of the system as a function of the threshold level used at the output. The Neyman-Pearson criteria is used to select the threshold levels. The advantages of this technique are that it does not require any knowledge of prior probabilities and for a given POF measure, it maximizes the POD.

Implicit in the development of the algorithm is the assumption that the input is Gaussian. The process of hypothesis testing will be explained by stepping through the system shown in Figure 3.5, starting at the input and moving towards the output. The aim is to compute the probability density function of the output under the two hypotheses, given that the input is Gaussian.
1: Signal Model
The peak value of the input signal $x$ without flaw is modeled as $N(0, \sigma^2)$ whereas the signal with flaw is modeled as $N(m, \sigma^2)$.

\begin{align}
\text{NoFlaw: } X_0 &\sim N(0, \sigma^2) \\
\text{Flaw: } X_1 &\sim N(m, \sigma^2)
\end{align} \tag{3.4, 3.5}

2: Wavelet Transform
Since the Wavelet Transform is a linear transform, the output signals are Gaussian.

\begin{align}
\text{NoFlaw: } X_{0,i} &\sim N(0, \sigma_i^2) \\
\text{Flaw: } X_{1,i} &\sim N(m_i, \sigma_i^2)
\end{align} \tag{3.6, 3.7}

where $i$ denotes the $i$-th slice in the wavelet time-frequency plane. From linear system theory,

\[ m_i = E[W(a,b)_{a=a_i,0 \leq b \leq B}] \]

\[ m_i = E[x] \int h \left( \frac{t-b}{a} \right)_{a=a_i,0 \leq b \leq B} dt \] \tag{3.8}

3: For theoretical simplicity, only two slices of choice are considered.

4: Geometric Mean Filter
The two slices selected in step 3 are subjected to a geometric mean filter. This is defined by the process,

\[ Y = (X_{j,1} X_{j,2})^{1/2} \] \tag{3.9}

With the above process, it is required to obtain the distribution of $Y$ under both hypothesis, $X_0$ and $X_1$. 

The method adopted to obtain the pdf of Y is:

i. Fix $X_{j,2}$ at $x_{j,2}$.

ii. Then,

$$f(y) = \int_{-\infty}^{\infty} \frac{2|y|f(x_{j,1}) f(x_{j,2})}{|x_{j,2}|} dx_{j,2}$$

(3.10)

Using this technique, the pdf of Y is:

1. Under no-flaw hypothesis:

$$f(y) = \int_{-\infty}^{\infty} \frac{|y| e^{\frac{-y^4}{2x_{0,2}^2\sigma_1^2}} e^{\frac{-x_{0,2}^2}{2\sigma_2^2}}}{\pi \sigma_1 \sigma_2 |x_{0,2}|} dx_{0,2}$$

Substituting $u = x_{0,2}^2$ in the above equation, the integral can be obtained from standard tables or using symbolic computation software packages [35]. Under no-flaw hypothesis the distribution of the output, $y$, is shown in equation 3.11

$$f(y) = \frac{2 |y|}{\pi \sigma_1 \sigma_2} K(0, \frac{y^2}{\sigma_1 \sigma_2})$$

(3.11)

where $K$ is the modified Bessel function of the second kind. 2. Under flaw hypothesis:

$$f(y) = \frac{2 |y|}{\pi \sigma_1 \sigma_2} \int_{-\infty}^{\infty} \frac{\exp\left[-\left(\frac{(x_{j,2}-m_1)^2}{2\sigma_1^2}\right)\right] \exp\left[-\left(\frac{(x_{j,2}-m_2)^2}{2\sigma_2^2}\right)\right]}{x_{j,2}} dx_{j,2}$$

(3.12)

The Neyman-Pearson criteria is now used to compute the POD/POF or the ROC curves for the system. The method of computing the performance curves is to start by assigning a value for the POF. This involves estimating a value of $T$ that satisfies

$$POF = \int_T^{\infty} p(y|x_0)dy$$

(3.13)
This value of T is then used with

\[ POD = \int_T^\infty p(y|x_1) dy \]  \hspace{1cm} (3.14)

to determine the value of the POD. This process is repeated for various values of the POF.

The ROC curves were computed numerically using equations 3.11, 3.12 and 3.3. Figure 3.6 shows the POD vs POF curves plotted for an input mean value equal to 0.8. A mean value of 0.8 was chosen because this represented the average SNR of the data set used. The values of \( m_1, m_2 \) were computed from equation 3.8 and these were used in equation 3.12.

Figure 3.6 indicates the results of the simulations carried out. These figures were obtained from the 51 data samples analyzed. A threshold value was set and the number of traces which had at least one value greater than the threshold were counted. This number divided by 51 gave the POF measure in the case of noise only signals and POD in the case of flaw+noise signals. This represents one point on the curve. To obtain all the points, the threshold value was changed and the process was repeated.

Figure 3.6 also shows the POD vs POF curve for an optimal matched filter for comparison. The matched filter uses maximum \textit{a priori} information and is the best solution to a detection problem. This therefore represents the upper bound to a detection problem [38].
Figure 3.6: ROC curves for proposed detection algorithm - Theory and simulation

KEY (SNR = 0.8)

- Simulation
- --- NP Theory
- - - - Matched Filter
3.4 Detection Examples

This section contains some plots obtained from the simulations. All simulations were implemented using MATLAB Version 3.5f running on a DEC/ULTRIX platform with X/Motif graphical interface [34].

The wavelet decomposition was carried out as the inner product of the shifted and compressed wavelets and the input signal. The Mexican hat function shown in Figure 2.8 was used as the basis function. The Mexican Hat was chosen because it is easily implemented and satisfies the frame conditions for \( a_0 = 2 \). The sampling in frequency was thus carried out on an octave-by-octave basis.

The multiresolution signals obtained were combined together to obtain the 2-dimensional time-frequency plots shown. Time and frequency are shown along the X and Y axes and the magnitude of the signal at each point on the time-frequency plane is displayed along the Z axis.

Figure 3.7 shows the wavelet decomposition of the simulated hard-alpha signal. The flaw signal shows finite support on the frequency axis. This is the basis for the process of selecting a part of the phase-plane as input to the geometric mean filter.

Figure 3.8 shows the decomposition of the background noise signal obtained from titanium samples. The noise signal or the A-scan trace is modeled as white noise colored by the transducer spectrum. This colored noise was found to have components at all frequency scales on the time-frequency plane. This is evident from the wavelet decomposition of the noise signal.
Figure 3.7: Flaw wavelet decomposition

Figure 3.8: Noise wavelet decomposition
Figure 3.9 shows the decomposition of the flaw and noise signals added together. This is the simulated titanium sample with a hard-alpha inclusion. Since the wavelet transform is a linear transform, this decomposition is a sum of the flaw and noise decompositions shown in Figure 3.7 and Figure 3.8, respectively. Since the SNR of the signal is too low, the flaw signal is masked off completely by the noise.
Figure 3.10 shows the distribution of the energy of the flaw signal and Figure 3.11, the noise signal along the frequency scales. The energy of the flaw signal reaches a peak at the frequency scale 4. The frequency scales 4 and 5 are therefore chosen from the time-frequency plane as the input to the geometric-mean filter. By adopting this strategy, most of the noise energy is rejected leading to an improvement in the SNR.
Figure 3.11: Noise energy distribution across frequency scales

Three detection examples are shown on the following pages. These illustrate the input signals used and the resulting output of the geometric mean filter. These plots show signals of various SNR under the two cases- flaw+noise and noise only. A significant peak shows up at the flaw location in the first case, which allows the output to be thresholded to detect the presence of an inclusion. Table 3.2 lists the particulars of the input and output signals in these examples. Table 3.3 shows the input SNR and output SNR of the signals. The input and output SNRs shown are
peak SNR values defined as the peak of flaw signal divided by the peak of the noise signal in an A-scan trace. The SNR enhancement achieved by the process is also computed for each case. The SNR enhancement in decibels (dB) is calculated using equation 3.15. A multiplication factor of 20 is used in the enhancement calculations because of the voltage ratios used to compute the input and output SNR values.

\[
SNR \text{ Enhancement} = 20 \log \left( \frac{\text{Output} \text{SNR}}{\text{Input} \text{SNR}} \right) \tag{3.15}
\]

**Table 3.2: Example input and output signals**

<table>
<thead>
<tr>
<th>Example</th>
<th>Input signals</th>
<th>Output signals</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Flaw+noise</td>
<td>Flaw+noise</td>
</tr>
<tr>
<td></td>
<td>Noise only</td>
<td>Noise only</td>
</tr>
<tr>
<td>1</td>
<td>Figure 3.12</td>
<td>Figure 3.14</td>
</tr>
<tr>
<td>2</td>
<td>Figure 3.16</td>
<td>Figure 3.18</td>
</tr>
<tr>
<td>3</td>
<td>Figure 3.20</td>
<td>Figure 3.22</td>
</tr>
</tbody>
</table>

**Table 3.3: SNR figures for the detection examples**

<table>
<thead>
<tr>
<th>Example</th>
<th>Input SNR</th>
<th>Output SNR</th>
<th>SNR enhancement(dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.50</td>
<td>3.20</td>
<td>6.0 dB</td>
</tr>
<tr>
<td>2</td>
<td>1.03</td>
<td>1.60</td>
<td>3.8 dB</td>
</tr>
<tr>
<td>3</td>
<td>0.90</td>
<td>1.61</td>
<td>5.1 dB</td>
</tr>
</tbody>
</table>
Figure 3.12: Flaw+noise input: example 1

Figure 3.13: Noise only input: example 1
Figure 3.14: Filter output - flaw+noise: example 1

Figure 3.15: Filter output - noise only: example 1
Figure 3.16: Flaw+noise input: example 2

Figure 3.17: Noise only input: example 2
Figure 3.18: Filter output - flaw+noise: example 2

Figure 3.19: Filter output - noise only: example 2
Figure 3.20: Flaw+noise input: example 3

Figure 3.21: Noise only input: example 3
Figure 3.22: Filter output - flaw+noise: example 3

Figure 3.23: Filter output - noise only: example 3
CHAPTER 4. DISCUSSIONS

This thesis describes the use of the wavelet transform to address the problem of hard-alpha detection. This new technique with the ability to use different windows at different frequencies, tends to model the signal in an efficient way. The concept of using lower frequency resolution at higher frequency scales and higher resolution at lower frequency scales is an effective way to model naturally occurring signals. The same is true with regard to the issue of the the time resolution too. Using this technique allows the flaw signal or the simulated hard-alpha inclusion to be localized to a finite region in the time-frequency domain. The ability to localize information in both time and frequency domains plays a key role in enhancing the ability to discriminate the hard-alpha inclusion signal from the background clutter signal.

Other approaches to hard-alpha detection have relied on model based techniques which assumes the existence of a priori signal models. Three such methods have been outlined in references [38] and [39]. The most successful of these techniques is the matched-filter method. Given a known signal, the matched filter is the most efficient way of detecting this signal buried in noise [38]. The approximated frequency response of the optimal matched filter is given by, \( H(f) \approx \frac{F^*(f)}{S(f)} \) where \( F^*(f) \) is the conjugate of the flaw frequency response and \( S(f) \) is the noise power spectrum, used to whiten the input spectrum. This technique was used with both white and colored noise.
inputs and SNR improvement been reported in both cases. Excellent performance measures were also obtained as evinced by the POD vs POF plots in Figure 3.6. The drawback of this method is the need for a priori knowledge of the flaw frequency response, which is rarely possible.

The second technique which has been studied is the use of split-spectrum processing methods [38]. These techniques have previously been shown to improve SNR and detectability of flaws in ultrasonic test signals. These experiments, however, were carried out on signals obtained from stainless steel samples. These have considerably less background clutter and the input SNR is quite high. The same technique used with titanium samples did produce results in certain cases and depended on proper selection of various parameters. This method uses the Fourier Transform to model the signals and for subsequent analysis. The problems in using the Fourier Transform for analyzing non-stationary signals has been explained before. The main disadvantage of this method lies in the sensitivity of the technique to the choice of parameters.

Neural network classifiers have also been applied to the problem of hard-alpha detection [39]. Neural networks need to be trained using test signals and this again assumes existence of flaw models. Neural network models and statistical analysis techniques were applied to achieve improved probability of detection.

The proposed technique for hard-alpha detection uses a two-fold approach. First, the wavelet transform places the flaw signal in a finite frequency region. In choosing this region most of the noise power is eliminated, thereby, improving the SNR. Second, the geometric mean filter processes this section of the time-frequency plane to obtain a time domain output. This again improves the probability of detection.
The advantages in using the wavelet transform to model the flaw signal are again two-fold. The problem of determining the width of the window used in analysis is eliminated and the wavelet transform provides a logarithmic coverage of the frequency axis. This means that the frequency range of the transducer can be covered efficiently.

The geometric mean filter has been shown to be fairly robust and insensitive to changes in the parameters like the frequency support used at the input [30]. This again provides for improved reliability of the proposed technique.

An average SNR gain of 4~5 dB was achieved using this technique. The performance measure or the POD/POF curve shown in Figure 3.6 shows fairly good agreement between theory and simulation. The slope of this curve, which determines the efficacy of the technique, does indicate some improvement in the detectability of the hard-alpha inclusion. The biggest advantage of the proposed technique is the use of relatively little a priori information. Given this fact, the performance curves do agree with other techniques which tend to use less a priori information [28].

The theoretical curves were generated using two “slices” from the wavelet time-frequency plane. More complicated mathematical principles have to be used to analyze a geometric mean filter with more input “slices”. Additional improvement in performance of the filter is anticipated if more “slices” are used.

4.1 Future Research Directions

The problem of hard-alpha detection is complicated by the fact that the SNR of the input signal is very low. Model-based techniques have to be used when dealing with such low SNR signals. Development of more models of hard-alpha inclusions is an ongoing effort [9]. Availability of such models may result in more reliable
model-based techniques. Since the wavelet transform is an effective way to model non-stationary signals, model-based techniques can be combined with the wavelet transform to obtain better performance.

Alternate choices of wavelets can be explored to model the signal more efficiently. An interesting branch of wavelet transform theory is the design of wavelets to suit a particular application. Such modifications with the window function has been reported with detection schemes using the Short-Time Fourier Transform [14].

Wavelet de-noising is another promising approach. Using the statistical properties of the signal and noise, wavelet de-noising tends to improve the SNR of the input signal.

The wavelet transform has aroused a lot of interest in the signal processing community in the past few years. Detection schemes using the wavelet transform are being developed continually. A start has been made here to address a complicated problem using a new technique. This has yielded fairly good results. As better detection schemes are developed and better models become available for the hard-alpha inclusion signals, wavelet transform based methods can be expected to become more commonplace in attempts to solve the problem.
BIBLIOGRAPHY


