Dynamic behavior of seatbelts in rollover situation

Ming-Te Cheng
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Dynamic behavior of seatbelts in rollover situation

by

Ming-Te Cheng

A Thesis Submitted to the
Graduate Faculty in Partial Fulfillment of the
Requirements for the Degree of
MASTER OF SCIENCE
Major: Mechanical Engineering

Signatures have been redacted for privacy

Iowa State University
Ames, Iowa
1989
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The general objective of this research project was to investigate the dynamic response of car and aircraft restraint systems. The specific purpose was to find the body acceleration and the corresponding seat belt forces due to the body motion with an automobile or aircraft during rollover.

Two approaches were used to study this problem: a theoretical approach using a two-dimensional one-mass body mathematical model with three degrees of freedom, and an experimental approach using a full-size dynamic test apparatus that simulates pure rotation. For the mathematical model, the Lagrangian equations were used to derive the equations of motion. Fourteen constraints were applicable to this model. In the experimental part, a fixture was designed and constructed to produce pure rolling motion. Computer algorithms and instrumentation systems were developed to analyze the test data. The test data included body accelerations and seat/shoulder belt forces. The data acquisition system consisted of a high speed Hewlett-Packard voltmeter and scanner interfaced with an IBM PS/2 model 60 computer.

The main conclusion of this study is that the theoretical model developed is valid for prediction of seat belt forces resulting for either prescribed or measured g levels experienced by an occupant constrained with the seat belt. The model is valid for a variety of seat belt styles and is not limited to any particular seat belt configuration.
The forces predicted by the model are in good agreement with experimental data obtained to test the model.

The g-levels and seat belt forces experienced in rollover accidents are shown to be at least 15 gs and 2000 pounds respectively. The experimental errors in acceleration measurement were estimated to be ±1.1 gs. The experimental errors in seat belt force determination were estimated to be ±50 pounds.
1. INTRODUCTION

1.1 Objective of the Study

In the past several years, the study of dynamics of car and aircraft restraint systems has become a challenging topic in transportation safety research. Most of the research has been concentrated on the frontal or lateral impact dynamics. However, the probability of suffering serious abdominal injury during rollover accidents is much higher than any of the other crash modes [1],[2]. For a rollover crash, the situation becomes more complicated because it involves long duration vehicle motion and multiple contacts of occupants with the vehicle enclosure. Full-scale vehicle rollover crash tests are expensive. In addition, during these tests only the initial conditions of the vehicle prior to rollover can be effectively controlled. Thus, the actual displacement and orientation of the vehicle cannot be accurately predicted or controlled after the initiation of the rollover [3].

The motions of the occupant during a rollover car accident are violent and complicated, and may include partial or total ejection. A properly designed safety belt then plays an important role in preventing injury of the occupant in such rollover cases. In the present study, the focus is on the dynamics of the restraint system in a pure rotation situation. The purpose was to find the body acceleration and the cor-
responding seat belt forces due to the body motion when the automobile or aircraft seat is overturning. Both the lap belt and combination lap and shoulder belts are considered.

Two approaches are made to study this problem: a theoretical approach using a two-dimensional one-mass body mathematical model with three degree of freedom, and an experimental approach using a full-scale dynamic test apparatus that simulates pure rotation.

For the mathematical model, the Lagrangian equations were used to derive the equations of motion. The mathematical model can predict the position and orientation of the body knowing the decelerations in X, Y and Θ directions. The velocity and force distributions can also be obtained by this model.

In the experimental approach, a fixture was designed and constructed to produce pure rolling motion. A 90th percentile anthropomorphic dummy [4] was used. A computer controlled data acquisition system was also developed to obtain and record the test data. The test data included body accelerations and seat/shoulder belt forces. The results of the mathematical model were compared with the experimental data.

1.2 Literature Review

In general, the literature review is restricted to dynamic analysis of a car restraint system during collision. The studies are divided into the two categories of "analytical" and "experimental" for purpose of discussion.
1.2.1 Analytical

In computer simulation of a car crash, there are several mathematical models in the literature. They range from one-dimensional to three-dimensional, and from frontal collision to lateral collision. Basically, there are two groups of models developed to fit different simulation needs. These two groups contain the "human body model" and the "vehicle collision model".

The development of complex and detailed mathematical models for the simulation of large scale human body motions has been rapid and successful during the last decade. Relatively new and powerful analysis capability, which can be mostly attributed to the developments in computer technology, including sophisticated accessory software, has opened the door for studying human responses to a dynamic impact environment.

Nachbar and Schipmolder [5] treated the case of a one-dimensional occupant belt model, concluding that a viscoelastic material gave the best response for the case of a lap belt.

Egli [6] established a colinear model of the crash system combined with the occupant simulated as a concentrated mass and restrained either elastically or inelastically. The general concept of a "tuned" restraint system was established, which succeeds in stopping the elastically restrained occupant in one cycle of oscillation. Deviation from the tuned situation results in a "sling shot effect" involving continued oscillation of the occupant. The off-tune situation is analyzed for the square wave type of structure collapse as it occurs with the vehicle at various speeds and manned by different weight occupants. The occupant with an inelastic restraint avoids the sling
shot effect, but experiences increased g-loading and requires larger interior vehicle space. This was demonstrated analytically with a variety of arbitrarily assumed front end collapse histories.

Free et al. [7] used a two-degree-of-freedom linear mathematical model to find the best value of stiffness and damping coefficients for assumed linear lap and shoulder belts. The model was subjected to specific vehicle pulse forms and belt geometry at speeds from 10 to 60 mph by an optimum-search method. In the optimization procedure, the parameters of the model were determined by repeated simulation of the model response and evaluation of the design index. Adjustment of the parameters at each stage was made to optimize the design index. The optimum parameter value using this model was then used as initial search points for a more complex dynamic model.

Roberts and Robbins [8] and Robbins [9] presented a series of mathematical models. These models show interaction between an occupant and the interior of a vehicle. Comparison of seats both with and without headrests in rear impact were combined by using an eight-mass, two-dimensional model. The belt’s material properties, slack, and geometric configuration were all considered. It was also demonstrated in this work that a simple mathematical model can perform a valuable service in laying the groundwork for more sophisticated analytical and experimental work, as well as yielding short term results. Then a three-mass, three-dimensional model was discussed. Body decelerations in the coordinate directions and vehicle pitch, roll, and yaw are the input variable for this computer based analysis. Tabulated summaries of the computer-generated angles and three-dimensional attachment points of various
seat belt and shoulder harness configurations was presented.

Huang [10] developed a model to yield an explicit analytical relationship between occupant response and the physical parameters of the vehicle structure and occupant restraint system. Design procedures using Equation 1.1 and “carpet” plots were presented to aid the designer in the selection of a restraint system and vehicle structure parameters to meet predetermined design criteria. The “carpet” plot is useful in determining the sensitivity (the slope of the curve) of one variable with respect to another. The “carpet” plot included as Figure 1.1 shows the restraint natural frequency as a function of the maximum occupant deceleration and the occupant interior travel in the vehicle compartment. An example is that when the maximum deceleration $\ddot{X}_o|_{max}$ reaches 50 gs and the occupant-vehicle interior travel $X_{ov}$ is 10 inches, the occupant-restraint natural frequency $f$ is 7.5 Hz. For a typical automobile-fixed barrier impact with a duration of about 0.1 second, $f$ is about 6 Hz. The notation $\delta$ in Equation 1.1 is the restraint system slack given in inch units, $k$ is the restraint system stiffness and $w_o$ is the occupant torso weight.

$$f = 3.127 \sqrt{\frac{\ddot{X}_o|_{max}}{X_{ov} - \delta}} = 3.127 \sqrt{\frac{k}{w_o}}$$

(1.1)

Igarashi and Atsumi [11] analyzed impact dynamics in frontal collisions for a 3-point belted occupant. A finite element method (FEM) computer simulation model was used in this analysis. The results from this simulation were compared with experimental results obtained from anthropometric dummies used on sleds and in small size car frontal crash tests. It was confirmed that dummy impact dynamics are influenced mostly by the structure and the geometrical layout of the belt system, the seat belt webbing elongation rate, and the seat cushion hardness.
\[ f = 3.127 \sqrt{\frac{\ddot{X}_{o,max}}{X_{ov} - \delta}} \]

Figure 1.1: Carpet plot
Prasad [12] reviewed the basic features of an occupant simulation model MVMA-2D(Figure 1.2). In the MVMA-2D\(^1\) model, the human body model was described by eight rigid and two flexible segments. The equations of motion of the linkage system were derived using the Lagrangian technique [13], and the resulting equations of motion were simulated numerically using integrated Ruge-Kutta techniques [14]. The constraints of this model included both simple and advanced belt restraint techniques. The simple belt system essentially consisted of an upper torso strap, a lower torso strap and a one-piece lap belt. The advanced restraint system consisted of up to seven belt segments acting independently or in pairs. Friction between the upper torso, the pelvis and the belt system could also be simulated.

Robbins and Viano [1] also described the occupant kinematics in rollover accidents using this model and concluded that two-dimensional techniques were limited to analysis of “pure” rolls.

Kramer \textit{et al.} [15] composed a 2-D model which included the vehicle structure model, the occupant model, and the contact model. In this model, force/deflection characteristics of the seat belt were derived from the experimental results of component tests of seat belt material, steering wheel forces and seat forces. In the dynamic belt studies, it was concluded that the belt material tears at a load approximately 50\% of the stated minimum breaking load. When calculating the seat belt forces, friction forces between the belt strap and the occupant, and the shoulder strap and the sash guide devices were not taken into account.

Grösch \textit{et al.} [16] and Schmid [17] presented a three dimensional simulation model

---

\(^1\)MVMA-2D: The Motor Vehicle Manufacturers Association two-dimensional crash victim simulation model.
Figure 1.2: MVMA-2D model
called ISM\textsuperscript{2}. This model can depict the behavior of a vehicle occupant during a frontal, side or rear impact. The model can also simulate any popular restraint system (e.g., three point seat belt) thereby using the actual measured belt webbing elasticity values, including frictional slip and deformation of belt anchorages and/or belt buckles. Because the model is three-dimensional, it is more suited than two-dimensional models for investigations of angular impacts or non-symmetrical restrain systems.

Finally, Obergefell et al. \cite{2} used a three-dimensional (3-D) human body "gross motion" simulation program to predict dynamically the motion of a dummy during rollover accidents. The results showed that the mechanisms of injury during rollover were different from those in frontal impact accidents. The head accelerations during the rollover simulations without the added frontal impact are milder than those with frontal impacts. The head, knee, and chest impacts, often cases of injury in frontal impact accidents, are not always present during rollover with a belted occupant. Also, the shoulder belt is valuable in restraining the occupant in an initial frontal impact, but early in the simulation the occupant fell to the side, sliding out of the shoulder belt. With the occupant out of the shoulder belt, the upper body was free to move around, but the occupant was still well restrained by the lap belt.

1.2.2 Experimental

Both static tests and dynamic tests are performed to evaluate the seat restraint systems. Static testing is important for finding weak components of a restraint system,

\textsuperscript{2}ISM: The Daimler-Benz mathematical isohumane simulation model.
but dynamic testing should be used to simulate the real crash forces experienced during a car accident.

Seiffert and Schwanz [18] tested five different restraint systems with combinations of velocity change, peak deceleration and deceleration pulse shape. For three given deceleration pulses of defined shape and peak magnitude, total velocity change was varied from 20 to 40 mph. The restraint systems were compared for those combinations of speed changes and pulse shape. The head injury criteria, chest severity index, and femur load were used to evaluate potential human injury.

Horsch et al. [19] [20] conducted sled tests to investigate the dynamics of an anthropomorphic dummy [4] as a function of the belt restraint configuration and impact direction. These tests involved a 35 km/h velocity change and 10 g deceleration. The body displacement and body loading were strongly dependent on the direction of deceleration.

Alfaro-Bou et al. [21] evaluated the dynamic behavior of general aviation seats from three dynamic testing methods. The first method was a sled propelled by the force of a dropping mass to achieve the desired velocity. The sled was arrested by wires stretched over rollers to produce the desired deceleration pulse. This method uses closely controlled test conditions and can repeat an acceleration time history with high accuracy. The other two test methods were drop testing and full-scale aircraft crash tests. The drop test method used a heavy steel cylinder that was raised to a predetermined height to produce the desired impact velocity. The cylinder was dropped into a box filled with uniform microglass beads that resembled sand. Wedges were attached beneath the cylinder. The honeycomb blocks were positioned between
the wedges and the cylinder. The desired acceleration pulse was imposed on the box when this cylinder impacted with the “sand”. In the full-scale crash tests, test seats were installed in general aviation airplanes which were crash tested at predetermined impact conditions. This “real world” test procedure was the best functional test of the seats. However, it was also the most expensive and the most difficult to control and repeat.

Zimmerman [22] designed and built a laboratory test fixture to simulate seat belt assembly installations. The required retraction forces for a simulated geometry are determined by replacing the retractor action with a motorized load transducer that measures the force required to stow the latchplate. The pillar loop, webbing, latchplate and their relative positions can be varied until the minimum retraction force that successfully stores the latchplate is determined. By combining the optimized assembly installation and the optimized component design, the extraction and retraction forces can be balanced to offer a functional, convenient and comfortable seat belt system.

Chandle [23] introduced two dynamic tests intended to demonstrate the ability of the seat and restraint system to protect an occupant against injury in forward and combined forward and vertical impact conditions. He suggested that in restraint system load measurements, restraint straps should not be cut to insert a load cell in series with the webbing, since that would change the characteristics of the restraint system which govern the load distribution. Load cells are commercially available which indicate the tension force in webbing type restraint straps, without cutting the webbing, if they are properly used. The most severe limitation in their use is that
they should be placed on “free webbing”, i.e., webbing which is not in contact with anything, and that a free length of webbing should exist for some distance on either side of the load cell.

Van Gowdy et al. [24] introduced a series of tests to evaluate static test procedures for restraint anchorages on small airplane forward facing seats that have a lap belt and shoulder harness anchored to the airframe. In these tests, the belts were anchored to sled fixtures instrumented with triaxial load cells, and static forces were developed by pushing the sled back with hydraulic cylinders, displacing the seat rearward relative to the body blocks. Load cells were installed at the load attachment points on the body blocks to measure the input forces. From the results, the only test variable which appeared to influence the transfer of input load to the restraint anchors was the input load angle. It also suggested that the SAE J384 body blocks can be used to statically test combined restraint installations for small aircraft. Loading the restraints thru the J384 blocks was both comfortable and stable.

Due to the large distance between crash barrier and analysis facilities, Stauf et al. [25] introduced a new 120 channel data acquisition and analysis system using a serial data transfer via fiber optics cables. Resolution and sampling rate were found to be the limitations of the data acquisition system. These items determined the performance of the system and the selection criteria for the storage media.

In analytical simulation, the input data, such as geometric conditions and load-deflection conditions are often only rough estimates because of the lack of experimental verification. In particular, since input data on the 3-point seat belt model

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3SAE J384: SAE(Society of Automotive Engineering) recommend practice for motor vehicle seat belt anchorages test procedure.
to the MVMA-2D have contained many hypotheses due to its complicated mechanism, Shimamura et al. [26] established experiments to improve the initial data on the characteristics of the seat belt and initial dummy location. Using these refined initial conditions, the results of the MVMA-2D simulation agreed with the experimental results, verifying the reliability of the simulation.
2. THEORETICAL MODEL

2.1 Geometry and Coordinate System

The restrained body considered in this model is assumed to be a single mass body with three degrees of freedom and confined in a two-dimensional restraint system as shown in Figure 2.1 and Figure 2.2.

A Cartesian coordinate system was chosen for the present analysis. The degrees of freedom of this model are in the X-direction, Y-direction and Θ-direction. Generally, the origin of the coordinate system is placed at the mass center of the body in its original position (see Figure 2.1). Θ is zero when the body is in a vertical position. The model analysis is based on the following assumptions and conditions:

1. The seat belt is modeled with constant spring factor K and viscous damping factor C.

2. The mass is always in contact with the seat belt.

3. All seat materials (cushion, frame, etc.) are considered stiff enough to have negligible deflection when in contact with the mass.

4. The seat friction forces that resist the motions of the hips are the same in both X and Y directions.
Figure 2.1: Model in original position with non-accelerating mass
Figure 2.2: Model with motion of mass accelerating from original position
5. The body joint spring factor is constant.

### 2.2 Equation of Motions

#### 2.2.1 Lagrangian method

By using Lagrange's equations, the equations of motion of the theoretical model can be written as given in Equations 2.1, 2.2 and 2.3. The derivation of these equations of motion is included as Appendix A.

**X-direction:** \( \sum F = m \ddot{x} \)

\[
m \ddot{x} = -k_1 H(x - \rho \sin \theta) - k_2 H(x - \rho \sin \theta) \\
- c_1 H(\dot{x} - \rho \dot{\theta} \cos \theta) - c_2 H(\dot{x} - \rho \dot{\theta} \cos \theta + l \dot{\theta} \cos \theta) \\
+ m A_x - F_x - mg_x
\]  

(2.1)

**Y-direction:** \( \sum F = m \ddot{y} \)

\[
m \ddot{y} = -k_1 V(y - \rho \cos \theta + \rho) + k_2 V(y - \rho \cos \theta + l \cos \theta - l + \rho) \\
- c_1 V(\dot{y} + \rho \dot{\theta} \sin \theta) - c_2 V(\dot{y} + \rho \dot{\theta} \sin \theta - l \dot{\theta} \sin \theta) \\
+ m A_y - F_y - mg_y
\]  

(2.2)

**\( \theta \)-direction:** \( \sum T = I \ddot{\theta} \)

\[
I \ddot{\theta} = k_1 H(\rho \cos \theta)(x - \rho \sin \theta) - k_1 V(\rho \sin \theta)(y - \rho \cos \theta + \rho) \\
- k_2 V(\rho - l) \sin \theta(y + l \cos \theta + \rho - l - \rho \cos \theta) \\
+ k_2 H(\rho - l) \cos \theta(x - \rho \sin \theta + l \sin \theta) \\
+ c_1 H(\rho \cos \theta)(\dot{x} - \rho \dot{\theta} \cos \theta) - c_1 V(\rho \sin \theta)(\dot{y} + \rho \dot{\theta} \sin \theta)
\]
\[-c_2H(l - \rho) \cos \theta (\dot{x} - \rho \dot{\theta} \cos \theta + l \dot{\theta} \cos \theta)\]
\[+c_2V(l - \rho) \sin \theta (\dot{y} + \rho \dot{\theta} \sin \theta - l \dot{\theta} \sin \theta)\]
\[-k_t(\theta - \theta_0) + I \dot{\theta} - T + (\rho \cos \theta) F_x - (\rho \sin \theta) F_y \quad (2.3)\]

### 2.3 Constraint Conditions

There are fourteen constraint conditions which result from the constraints that the seat materials be sufficiently stiff to prevent deflection when in contact with the body, and the assumption of a perfectly elastic impact between the seat and the dummy. When different constraint conditions occur, different equations of motion govern the model.

The fourteen constraints are as follows:

1) \(x - (\rho \sin \theta) < 0\)
2) \(y - (\rho \cos \theta) + \rho < 0\)
3) \(x - (\rho \sin \theta) + (l \sin \theta) < 0\)
4) \(y - (\rho \cos \theta) + (l \cos \theta) + \rho < 0\)
5) \(x - (\rho \sin \theta) < 0\) and \(y - (\rho \cos \theta) + \rho < 0\)
6) \(x - (\rho \sin \theta) < 0\) and \(x - (\rho \sin \theta) + (l \sin \theta) < 0\)
7) \(x - (\rho \sin \theta) < 0\) and \(y - (\rho \cos \theta) + (l \cos \theta) + \rho < 0\)
8) \(y - (\rho \cos \theta) + \rho < 0\) and \(x - (\rho \sin \theta) + (l \sin \theta) < 0\)
9) \(y - (\rho \cos \theta) + \rho < 0\) and \(y - (\rho \cos \theta) + (l \cos \theta) + \rho < 0\)
10) \(x - (\rho \sin \theta) + (l \sin \theta) < 0\) and \(y - (\rho \cos \theta) + (l \cos \theta) + \rho < 0\)
11) \(x - (\rho \sin \theta) < 0, y - (\rho \cos \theta) + \rho < 0\) and \(x - (\rho \sin \theta) + (l \sin \theta) < 0\)
12) \(x - (\rho \sin \theta) < 0, y - (\rho \cos \theta) + \rho < 0\) and \(y - (\rho \cos \theta) + (l \cos \theta) + \rho < 0\)
13) \( y - (\rho \cos \theta) + \rho < 0, \ x - (\rho \sin \theta) + (l \sin \theta) < 0 \) and \( y - \rho \cos \theta + (l \cos \theta) + \rho < 0 \)

14) \( x - (\rho \sin \theta) < 0, \ x - (\rho \sin \theta) + (l \sin \theta) < 0 \) and \( y - (\rho \cos \theta) + (l \cos \theta) + \rho < 0 \)

Two constraint conditions used in this analysis are illustrated in Figure 2.3 and Figure 2.4.

In deriving the equations of motion, a reversed effective force is added to the impact point to balance the existing net forces which are applied to the seat from the dummy as well as to zero the elastic and damping factor values of the same point.

For the case of the first constraint, the force \( FF_x = -m_a x + F_x \) is applied and \( c_{1H} \) as well as \( k_{1H} \) are zero. The derivation of the equations of motion of case 1 is included in the first part of Appendix B. The second part of Appendix B lists all the equations of motion for all the constraints.

### 2.4 Method of Solution

The differential equations of motion are numerically integrated by using the Runge-Kutta method [14]. These equations are numerically integrated with the help of the software “DIFFEQ\(^1\)”. The maximum time interval increment for effective integration was determined to be 0.0001 seconds.

### 2.5 Examples

To demonstrate this model, two cases were considered. The first case was for a triangle shaped deceleration in the x-direction (Figure 2.5). The result are shown in Figure 2.5, Figure 2.6, Figure 2.7 and Figure 2.8.

\(^1\)DIFFEQ - An interactive program for numerical integration of ordinary differential equations.
Figure 2.3: Constraint Case (1)
\[ (X - \rho \sin \theta + \ell \sin \theta) < 0 \]
\[ (Y - \rho \cos \theta + \rho) < 0 \]
\[ (X - \rho \sin \theta) < 0 \]

Figure 2.4: Constraint Case (11)
Figure 2.7 shows the torso velocity predicted from the model. The discontinuity at 0.25 seconds is believed to be caused by the boundary condition associated with the body impacting the seat.

The second case was for two sine wave decelerations in the x-direction and y-direction respectively. The results are shown in Figure 2.9, Figure 2.10, Figure 2.11 and Figure 2.12.
Figure 2.5: Triangle deceleration in Case (1)

Figure 2.6: Seat belt forces in Case (1)
Figure 2.7: Torso velocities in Case (1)

Figure 2.8: Torso displacements in Case (1)
Figure 2.9: Decelerations in Case (2)

Figure 2.10: Seat belt forces in Case (2)
Figure 2.11: Torso velocities in Case (2)

Figure 2.12: Torso displacements in Case (2)
3. EXPERIMENTAL DESIGN

3.1 Items to be Determined

The g-loading on the occupant and forces applied to the seat belt are two items of primary interest to be determined in this experiment.

3.1.1 The level of G-loading

The levels of g-loading are required for the input reference data in the mathematical model. In this case, the g-loading in the X, Y and θ directions are required.

In a rollover crash, the exact changes in the g-loading experienced by a occupant are difficult to analyze because the motion of the occupant is not always predictable. For a pure rotation test, one can use the basic force balance equation to get the theoretical g-loading. For example, \[ \sum F_{\text{radial direction}} = mr\omega^2 \] and \[ \sum F_{\text{circumferential direction}} = mr\alpha, \] allow the g-loading to be determined.

3.1.2 The level of forces applied to the seat belt

The dynamic forces applied to the seat belt due to the body motion in a pure rotation situation are to be determined. Because of the friction force between the seat belt and the passenger, the forces applied to the seat belt are not the same at different
occupant positions. Thus, to properly design a seat belt the force distribution should be known.

3.2 Descriptions of the Test Apparatus and Instrumentation

The experimental setup consisted of the test fixture, a 90th percentile anthropomorphic male dummy, and a data acquisition system, as shown in Figure 3.1.

3.2.1 The test fixture

The test fixture consisted of the main-body, one car front seat, a seat belt, and a variable speed drive.

3.2.1.1 The main-body The main-body was mainly constructed of Unistrut (channel iron), and had the dimensions as 4’ x 8’ x 12.5’, as shown in Figure 3.2. Before the test, a calculation was made to determine the safety factor for operating fixture. With a 400-lbf load in the middle of the shaft, the deflection from the shaft center line was calculated to be 0.115 inches. This corresponds to a safety factor as 6.64 based on the Soderborg approach and the maximum-shear theory of failure [27]. This was considered to be a satisfactory safety factor for the operation of the apparatus containing the weight of the full scale automobile seat and anthropomorphic test dummy.

3.2.1.2 The seat The seat was from the front in the passenger side of a 1980 Ford Fiesta car with vehicle identification number of GCF BAK 938250. Its dimensions were 14 \frac{3}{4}” x 12 \frac{1}{2}” x 29”. 
Figure 3.1: Experimental setup
Figure 3.2: The main-body of the test fixture
3.2.1.3 The safety belt  The Klippan model 575 seat-belt was used to determine the belt breaking strength. This test was also used to determine the elastic properties of the seat belt. From the results, the elastic coefficient of this seat belt was determined to be 1514 lbf/in, as shown in Figure 5.5. By using another seat belt (Klippan NR model 80451), with the highest rate of loading (20 in/min) in the tensile machine, the seat belt broke at 3051 lbf. It was noted that the belt material fractured at a load of approximately 50% of the stated minimum breaking load of 6000 pounds as required by Federal Motor Vehicle Safety Standard [28]. One possible explanation for the decreased strength was the rate of loading imposed on the belt during the test.

3.2.1.4 The variable speed drive  The variable speed drive used was a three phase, two horse power motor. The drive motor rpm range was specified to be adjustable between 210 rpm and 1510 rpm. When this drive was used to rotate the seat and the dummy, the minimum rotative drive speed attainable was 0.43 sec/cycle (139.5 rpm).

3.2.2 Dummy

The test dummy was a 90th percentile anthropomorphic male dummy with a weight of 176 pounds. The length of the dummy from the head to the waist was 30 inches. The length from the waist to the ankle was 41 inches. The weight of the head was 11.3 pounds.
3.2.3 The data acquisition system

The computer controlled data acquisition system was used to obtain and record the acceleration and strain signals. Figure 3.3 shows a schematic diagram of the data acquisition system used in the investigation. It included three accelerometers, four strain-gauges, one digital voltmeter, one switch/control unit, one strain indicator, three charge amplifiers, one IEEE-488 general purpose interface bus, and one IBM PS/2 model 60 computer.

3.2.3.1 Voltmeter  A fast reading Hewlett-Packard model 3437A digital voltmeter was used. It was a microprocessor controlled $3 \frac{1}{2}$ digit, successive approximation system voltmeter, capable of sampling voltage at rate up to 5700 samples per second. The voltmeter sampled whenever an electronic trigger signal was received. The Hewlett-Packard Interface Bus was standard and allowed the voltmeter to send digital signals to the IBM PS/2 model 60 digital computer. The signal consisted of seven 8-bit bytes which corresponded to the voltage reading.

3.2.3.2 Switch/control unit  A requirement of the system is that the three acceleration and four force or strain readings need to be sampled nearly simultaneously. The Hewlett-Packard model 3495A scanner was used to serve this purpose. It was capable of a switching speed up to 1000 channels per second. Two slots in the scanner were used. The first slot had four input channels connected with the four strain-gauges and the output channel went to the strain-indicator. The second slot had three input channels connected to the three accelerometers and the fourth channel with the output of the strain-indicator. The output channel of slot 2 was
Figure 3.3: The data acquisition system
connected to the voltmeter.

3.2.3.3 IBM PS/2 The IBM PS/2 model 60 computer was mainly used for controlling data sampling. The program was written in IBMPC Quick-BASIC 4.6 which also incorporated IEEE-488 instructions (see Appendix E: Section 13.2).

3.2.3.4 The general purpose interface bus (GPIB) The GPIB was a link, or bus, or interface system, through which interconnected electronic devices of this experiment were able to communicate.

3.2.3.5 Accelerometers and strain gauges Accelerometers and strain gauges were used as the signal transducers. Accelerometers were used in this experiment since the acceleration signal can be easily integrated to obtain velocity and displacement, whereas electronic differentiation used with velocity and displacement transducers requires more complex procedures. Two Columbia model 902 accelerometers and one Kistler model 808A accelerometer were used. For the Columbia model 902 accelerometers, the charge sensitivity $S_q$ was 100 pcb/g for serial number 1308 and 106 pcb/g for serial number 1296. For the Kistler model 808A accelerometer, the charge sensitivity $S_q$ was 1.028 pcb/g. The output signals from the accelerometers were connected to the PCB model 462A charge amplifiers and then to the voltmeter.

The three accelerometers were located on the head, the chest, and the right side hip of the dummy, as shown in Figure 3.4. The radial acceleration was determined by the head accelerometer, and the circumferential acceleration was determined by the accelerometer on the chest.
1: Accelerometer Columbia model 902
2: Accelerometer Columbia model 902
3: Accelerometer Kistler model 808A

Figure 3.4: The position of the accelerometers
The MM\textsuperscript{1} CEA-13-250UW-120 strain-gauges with gauge factor of 2.11 were used in this experiment. The four strain-gauges were attached to the seat-belt at the shoulder, the right lap, the left lap, and the buckle position, as shown in Figure 3.5 to Figure 3.9, respectively. To attach the strain-gauges, two special constructed metal connectors were installed and used at the right lap and the shoulder positions. The output signals from the strain gauges went through the Vishay P-3500 strain indicator to the voltmeter.

An instrumentation restriction was that only one Vishay model P-3500 strain indicator was available for the four strain sensors.

3.3 Test Procedures

3.3.1 Calibration of the strain-gauges and the accelerometers

To relate the measured strain to forces on the seat belt the strain gauges were calibrated in a tensil testing machine. The forces applied to the seat-belt were anticipated to be between 200 to 4000 pounds. To produce the loading the SATEC model 24GBN tensile machine was used. For the convenience of both calibration and ease of attachment at the seat belt, two iron connectors were made which connect the seat-belt joint with the fixture. The width of the two ends of the connectors were limited to one inch to mate with the width of the clamp of the tensile testing machine. For each strain-gauge, two rates of loading were used to perform the calibration. The results using the two loading rates were in close agreement as shown in Table 3.1 to Table 3.4.

\textsuperscript{1}MM: The Micro Measurement Incorporation.
The accelerometers were calibrated by using a shaker table to produce known input accelerations. The charge sensitivity of the accelerometer was the value which allowed the accelerometer to yield as an output the value of the known input acceleration.
Figure 3.5: The position of the strain-gauges
Figure 3.6: The strain-gauge #1 at the shoulder position

Figure 3.7: The strain-gauge #2 at the right lap position
Figure 3.8: The strain-gauge #3 at the left lap position

Figure 3.9: The strain-gauge #4 at the buckle position
Table 3.1: The calibrating data for the strain-gauge #1

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Second Calibration (med, 49%, 0.098 in/min loading rate)

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Second Calibration (med, 49%, 0.098 in/min loading rate)

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<td>0.00</td>
<td>-0.215</td>
</tr>
<tr>
<td>703</td>
<td>158.041</td>
<td>-0.203</td>
</tr>
<tr>
<td>1726</td>
<td>388.032</td>
<td>-0.189</td>
</tr>
<tr>
<td>2998</td>
<td>673.980</td>
<td>-0.177</td>
</tr>
<tr>
<td>4550</td>
<td>1022.886</td>
<td>-0.140</td>
</tr>
<tr>
<td>6041</td>
<td>1358.077</td>
<td>-0.099</td>
</tr>
<tr>
<td>6997</td>
<td>1572.996</td>
<td>-0.079</td>
</tr>
<tr>
<td>7913</td>
<td>1778.922</td>
<td>-0.062</td>
</tr>
</tbody>
</table>
Table 3.4: The calibrating data for the strain-gauge # 4

<table>
<thead>
<tr>
<th>Force (N)</th>
<th>Force (lbf)</th>
<th>Voltage</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.000</td>
<td>0.310</td>
</tr>
<tr>
<td>250</td>
<td>56.203</td>
<td>0.277</td>
</tr>
<tr>
<td>500</td>
<td>112.405</td>
<td>0.252</td>
</tr>
<tr>
<td>770</td>
<td>173.104</td>
<td>0.227</td>
</tr>
<tr>
<td>1000</td>
<td>224.810</td>
<td>0.202</td>
</tr>
<tr>
<td>1320</td>
<td>296.749</td>
<td>0.185</td>
</tr>
<tr>
<td>1580</td>
<td>355.200</td>
<td>0.160</td>
</tr>
<tr>
<td>1750</td>
<td>393.418</td>
<td>0.152</td>
</tr>
<tr>
<td>2060</td>
<td>463.109</td>
<td>0.135</td>
</tr>
<tr>
<td>2370</td>
<td>532.800</td>
<td>0.102</td>
</tr>
<tr>
<td>3000</td>
<td>674.430</td>
<td>0.035</td>
</tr>
<tr>
<td>3500</td>
<td>786.835</td>
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<tr>
<td>4000</td>
<td>899.240</td>
<td>-0.115</td>
</tr>
<tr>
<td>4500</td>
<td>1011.645</td>
<td>-0.148</td>
</tr>
<tr>
<td>5000</td>
<td>1124.050</td>
<td>-0.240</td>
</tr>
<tr>
<td>5500</td>
<td>1236.455</td>
<td>-0.281</td>
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<tr>
<td>6000</td>
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<td>-0.423</td>
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<tr>
<td>6500</td>
<td>1461.265</td>
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<tr>
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<td>-0.681</td>
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<tr>
<td>7500</td>
<td>1686.075</td>
<td>-0.790</td>
</tr>
<tr>
<td>8000</td>
<td>1798.480</td>
<td>-0.923</td>
</tr>
</tbody>
</table>

First Calibration (low, 59%, 0.0118 in/min loading rate)

<table>
<thead>
<tr>
<th>Force (N)</th>
<th>Force (lbf)</th>
<th>Voltage</th>
</tr>
</thead>
<tbody>
<tr>
<td>503</td>
<td>113.079</td>
<td>0.310</td>
</tr>
<tr>
<td>1397</td>
<td>314.060</td>
<td>0.183</td>
</tr>
<tr>
<td>1676</td>
<td>376.782</td>
<td>0.153</td>
</tr>
<tr>
<td>2873</td>
<td>645.879</td>
<td>0.037</td>
</tr>
<tr>
<td>4528</td>
<td>1017.940</td>
<td>-0.150</td>
</tr>
<tr>
<td>5667</td>
<td>1273.998</td>
<td>-0.420</td>
</tr>
<tr>
<td>6983</td>
<td>1569.848</td>
<td>-0.680</td>
</tr>
<tr>
<td>8029</td>
<td>1805.000</td>
<td>-0.924</td>
</tr>
</tbody>
</table>

Second Calibration (med, 49%, 0.098 in/min loading rate)
3.3.2 The elastic coefficient of seat belt

In order to acquire a basic understanding of the elastic nature of the seat belt and to find the elastic modulus governing the response of the seat belt, the tensile machine was used to evaluate the elastic coefficient for the seat belt.

Assuming that the seat belt behaves as a second-order system, the natural frequency of the seat belt is

\[ \omega_n = \sqrt{\frac{K}{M}} \]

For the range from 0 to 2250 pounds, the elastic coefficient was 1514 lbf/inch. The natural frequency of the seat belt was \( \sqrt{\frac{K}{M}} = 12.3 \) Hz for the 176 pound dummy.

3.3.3 Test procedure for recording the output signal

The flow chart for programming the data acquisition system is included as Figure 3.10. When the test was started, the dummy was restrained in the seat with the seat and shoulder belt. The entire assembly was inverted in the test apparatus. Before taking data, an initial speed was required to be given to the dummy for the drive motor to have sufficient torque to continue the driving rotation. The data acquisition system was then activated during the beginning stage of the first revolution of the dummy.
Figure 3.10: The flow chart for the data acquisition system program
4. LISTS OF EXPERIMENTS

Because of the unpredictability of motion in an actual rollover crash, the real behavior of the passenger in a rollover is very difficult if not impossible to exactly simulate. After several investigations of real car accidents, it was determined that a passenger could experience several types of motion. In order to simulate these motions, a series of experiments were performed. These experiments included:

1. Pitch overturns with the arms restrained.

2. Pitch overturns with the arms unrestrained.

3. Roll overturns with the arms restrained.

4. Roll overturns with the arms unrestrained.

In performing these tests, the legs of the dummy were secured to the seat and the shoulder belt was secured to the neck of the dummy by a cord. The shoulder belt was secured by the cord because initial testing revealed the dummy would slip sideways from under the shoulder belt and the test could have to be terminated to preserve the dummy for further testing.
5. RESULTS

5.1 Calibration

Before performing the experimental phase of the project, familiarization with the instrumentation and the seat belt behavior was obtained. The first part of the experimental work also included calibrating the strain-gauges and the accelerometers, and finding both the breaking strength and elastic coefficient of the seat belt.

5.1.1 Strain-gauges

Because of the impedance in the strain gauge circuitry the strain indicator could not be initially balanced for zero output voltage. Thus, the strain indicator was used with an unbalance mode of operation, and the system was calibrated to relate force to strain as measured from the gauges. The unbalance voltage value was recorded before calibrating the strain-gauges. The voltage difference $\Delta V$ then was used for deriving the relationship between forces and output voltages. From Table 3.1 to Table 3.4, the calibration line was smoothed by computing a best fit straight line by the method of Least Squares. The results are shown in Figure 5.1 to Figure 5.4.
The relationship between $F$ and $\Delta V$ is given in Table 5.1. The computer program in Appendix E: Section 13.3 then was used to transfer the output voltage $\Delta V$ to the related force in pound.
Figure 5.2: The calibration line for strain-gauge #2

Figure 5.3: The calibration line for strain-gauge #3
Figure 5.4: The calibration line for strain-gauge $\neq 4$

Table 5.1: The calibration polynomials for strain-gauges

<table>
<thead>
<tr>
<th>Strain Gage No.</th>
<th>First Calibration Polynomial</th>
<th>Second Calibration Polynomial</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$F = -3336 \times \Delta V$</td>
<td>$F = -3272 \times \Delta V - 8$</td>
</tr>
<tr>
<td>2</td>
<td>$F = -43277 \times \Delta V$</td>
<td>$F = -42784 \times \Delta V - 16$</td>
</tr>
<tr>
<td>3</td>
<td>$F = 11336 \times \Delta V$</td>
<td>$F = 11164 \times \Delta V - 5$</td>
</tr>
<tr>
<td>4</td>
<td>$F = -1507 \times \Delta V$</td>
<td>$F = -1457 \times \Delta V - 15$</td>
</tr>
</tbody>
</table>
5.1.2 Elastic coefficient of seat belt

As indicated in Section 3.3.2, the elastic coefficient was 1514 lbf/in in the range from 0 to 2250 pounds. The related force-deflection curve is shown in Figure 5.5.

5.2 Test Data

The results of the present study are presented in two sections. First, a discussion is included about the phenomena observed when performing the experiments. Next, the results of the four experiments are examined and discussed.

5.2.1 Test observations

The injuries which a person incurs from contact with the restraint system depend upon the severity of the collision and the individual’s tolerance to impact. In the severe frontal collisions, bone injuries can result from the three-point restraint system. In rollover situations, additional injuries can occur [29].

Several items related to rollover injuries were observed when the experiments were performed.

1. The three point seat belt slipped from the shoulder of the test dummy. The dummy was then restrained only by the lap belt. Because the relative positions of the seat belt anchor points were the same as in the car, a similar situation could easily happen to the passenger in a real rollover crash.

2. The lap belt had a tendency to imbed into the abdomen of the test dummy. In real rollover situations, the typical documented belt injures to the abdomen are
Figure 5.5: The elastic coefficient of seat belt
torn mesentery or bowels, ruptured viscera, distraction fracture of the lumbar spine and even torn rectus abdominous musculature in severe collisions. Such injuries are indicated to be related to belt embedment [29].

3. For safety reasons, a shield was build around and over the test fixture to prevent the ejection of the test dummy or test components. At the upright position, the distance between the roof of the shield and the head of the dummy was 12 inches. When the test dummy slipped in the safety belt, the head of the dummy eventually hit the shield during rotation of the test apparatus. This phenomena might also happen in an actual car crash and the passenger's brain and/or neck might be severely injured.

4. The arms of the test dummy swung upward and outward when the seat started rolling. In a real situation the passenger may grip something to keep the arms in position. However, the forces imposed on the arms and legs may cause them to move in an unconstrained manner.

5.2.2 Discussion of test results

The original investigations included several sets of data. However, the results presented here are only those necessary for the discussion.

1. For the case of the pitch overturns with the arms restrained, the results are presented in Figure 5.7 to Figure 5.12.

2. For the case of the pitch overturns with the arms unrestrained, the results are presented in Figure 5.14 to Figure 5.19.
3. For the case of the roll overturns with the arms restrained, the results are presented in Figure 5.21 to Figure 5.26.

4. For the case of the roll overturns with the arms unrestrained, the results are presented in Figure 5.28 to Figure 5.33.

From these experiments, it is noted that the results mainly depend on the type of rollover and whether or not the dummy arms were restrained. Although the results are for the case by case experiments, two important conclusions have been derived from the overall experiments.

1. The measurements from the strain gauge sensors show the peak force in the seat belt to occur at the side attachment points of both the lap belt and the shoulder belt. Smaller forces exist at the opposite side attachment point of the lap belt as well as the lap belt buckle. Maximum forces in the order of 2000 pounds (as shown in Figure 5.7, Figure 5.14, Figure 5.21 and Figure 5.28) were experienced by both the pitch overturns and roll overturns tests. These forces were about 33% of the stated static strength of the belt. However the forces were approaching 65% of the belt strength determined at the 20 inches per minute loading rate in the tensile testing machine. Belts loaded at a higher loading rate may fail at even lower loads than indicated during the calibration associated with these experiments. Thus, the seat belt loads experienced during rollover can be dangerously close to the breaking strength of the belt. This is especially significant if impact loads are superimposed during rollover, such as often occurs in actual car rollover accidents.
2. The measurements from the accelerometers indicate the maximum acceleration to be experienced by the head of the occupant. Maximum acceleration levels of 15 gs were noted (see Figure 5.10 and Figure 5.24) at the head position in both pitch and roll overturns cases.

The radial acceleration experienced due to a pure rolling motion at constant angular velocity is $r\omega^2$. An additional acceleration is experienced if the body moves radially and if the rotating shaft of the test apparatus deflects. The acceleration used in the model was $r\omega^2$. However, the test results show an acceleration equivalent to $r\omega^2$ in magnitude but varying in a cyclic fashion. The reasons for the cyclic output are not completely known. However, the dynamic analysis showing the nature of the acceleration waveform anticipated is included in Appendix C.

The level of acceleration imposed on a head weighing 11.3 pounds results in dynamic loading equivalent to about 168 pounds imposed on the head. When combined with possible impact loading, such forces can result in head and neck injuries.
Figure 5.6: The force from strain-gauge #1 at pitch overturns with the arms restrained

Figure 5.7: The force from strain-gauge #2 at pitch overturns with the arms restrained
Figure 5.8: The force from strain-gauge # 3 at pitch overturns with the arms restrained

Figure 5.9: The force from strain-gauge # 4 at pitch overturns with the arms restrained
Figure 5.10: The g-loading from accelerometer #1 at pitch overturns with the arms restrained

Figure 5.11: The g-loading from accelerometer #2 at pitch overturns with the arms restrained
Figure 5.12: The g-loading from accelerometer # 3 at pitch overturns with the arms restrained
Figure 5.13: The force from strain-gauge # 1 at pitch overturns with the arms unrestrained

Figure 5.14: The force from strain-gauge # 2 at pitch overturns with the arms unrestrained
Figure 5.15: The force from strain-gauge # 3 at pitch overturns with the arms unrestrained

Figure 5.16: The force from strain-gauge # 4 at pitch overturns with the arms unrestrained
Figure 5.17: The g-loading from accelerometer #1 at pitch overturns with the arms unrestrained.

Figure 5.18: The g-loading from accelerometer #2 at pitch overturns with the arms unrestrained.
Figure 5.19: The g-loading from accelerometer ≠ 3 at pitch overturns with the arms unrestrained.
Figure 5.20: The force from strain-gauge # 1 at roll overturns with the arms restrained

Figure 5.21: The force from strain-gauge # 2 at roll overturns with the arms restrained
Figure 5.22: The force from strain-gauge ≠ 3 at roll overturns with the arms restrained

Figure 5.23: The force from strain-gauge ≠ 4 at roll overturns with the arms restrained
Figure 5.24: The g-loading from accelerometer #1 at roll overturns with the arms restrained

Figure 5.25: The g-loading from accelerometer #2 at roll overturns with the arms restrained
Figure 5.26: The g-loading from accelerometer ≠ 3 at roll overturns with the arms restrained
Figure 5.27: The force from strain-gauge # 1 at roll overturns with the arms unrestrained

Figure 5.28: The force from strain-gauge # 2 at roll overturns with the arms unrestrained
Figure 5.29: The force from strain-gauge #3 at roll overturns with the arms unrestrained

Figure 5.30: The force from strain-gauge #4 at roll overturns with the arms unrestrained
Figure 5.31: The g-loading from accelerometer #1 at roll overturns with the arms unrestrained

Figure 5.32: The g-loading from accelerometer #2 at roll overturns with the arms unrestrained
Figure 5.33: The g-loading from accelerometer ≠ 3 at roll overturns with the arms unrestrained
6. CONCLUSIONS AND RECOMMENDATIONS

6.1 Comparison of Mathematical Model Results with Experimental Results

Because the exact damping coefficient of the seat belt is unknown, the model was examined with different damping coefficients and comparisons were made with the experimental data from strain gauge #1 (as shown in Figure 6.1 and Figure 6.2). The g-loading obtained from the experiment and used in the theoretical model predicts seat belt forces which are about 20% larger than the actual measured forces (as shown in Figure 6.1). The reason for the difference is partly dependent on the value of the elastic modulus for the seat belt. Since this modulus may depend on the rate of load application, the value used from the tensile testing machine experiment (1514 lbf/in), and a typical literature [8] value (800 lbf/in) were used for comparison. The predicted forces using the measured moduli are much closer to the actual measured forces and agree within 10% as shown in Figure 6.1.

The main conclusion of this study from this comparison is that the theoretical model developed is valid from the prediction of seat belt forces resulting for prescribed or measured g levels experienced by an occupant constrained with the seat belt. The model is not limited to any particular seat belt configuration.
Figure 6.1: The plot of comparison of mathematical model with experiment (K = 1514 lbf/in)
Figure 6.2: The plot of comparison of mathematical model with experiment (K = 800 lbf/in)
The forces predicted by the model are in good agreement with the experimental data obtained to test the model except for the force obtained from strain gauge #2. The reason was because of the insensitivity of this strain gauge (0.000023 volts/lbf), which is believed to be due to failure of the gauge bonding. The data from this strain gauge was thus suspect and was not relied upon in the data analysis.

The g-levels and seat belt forces experienced in rollover accidents are shown to be at least 15 gs and 2000 pounds respectively. Under these conditions the theoretical model and the experimental results are in good agreement.

Propagation of error analysis as illustrated in Appendix D was used to estimate the uncertainty in g loading to be ±1.1 gs. The uncertainty in the seat belt force measurements were estimated to be ±50 pounds.

6.2 Recommendations

It is recommended that the effect of the elastic modulus of the seat belt be examined with respect to different rates of loading. This parameter controls how well the theoretical model predicts seat belt forces for given g loadings experienced by the occupant.

Although the model is not dependent on seat belt configuration, it is recommended that additional seats and styles of seat belts assemblies be tested to obtain additional experimental data for comparison.

It is recommended that the model be modified to include additional degrees of freedom and to include distributed mass occupants. This would allow the model to be used to predict more complex situations.
7. BIBLIOGRAPHY


8. ACKNOWLEDGEMENTS

The author would like to express sincere gratitude to Dr. Jerry L. Hall for his encouragement and many helpful suggestions throughout the course of study. Without these, the work would not have been accomplished. The author also thanks the program of study committee, Dr. Leo C. Peters and Dr. Jeff C. Huston for their help and guidance during the project.

Special thanks are extended to Ms. Pam Oviatt and the General Motors company for donating the use of an anthropomorphic dummy, the Mechanical Engineering department for the financial support, and the Engineering Science and Mechanics department for the usage of the Vibration laboratory.
9. APPENDIX A: THE DERIVATION OF THEORETICAL MODEL
BY USING LAGRANGIAN EQUATION

9.1 Lagrangian Equation of the First Kind

Basically, the original form of Lagrange's equation is

\[
\frac{d}{dt} \frac{\partial T}{\partial \dot{q}_i} - \frac{\partial T}{\partial q_i} + \frac{\partial V}{\partial q_i} = Q_i \tag{9.1}
\]

9.2 Derivation

The procedures to get the equations of motion of the theoretical model are as follows:

9.2.1 Kinetic energy

\[
T = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} m \dot{y}^2 + \frac{1}{2} I \dot{\theta}^2 \tag{9.2}
\]

9.2.1.1 \( \frac{\partial T}{\partial q_i} \)

\[
\frac{\partial T}{\partial \dot{x}} = m \dot{x} \tag{9.3}
\]

\[
\frac{\partial T}{\partial \dot{y}} = m \dot{y} \tag{9.4}
\]

\[
\frac{\partial T}{\partial \dot{\theta}} = m \dot{\theta} \tag{9.5}
\]
9.2.1.2 \[ \frac{d \partial T}{dt \partial q_i} \]

\[ \frac{d \partial T}{dt \partial x} = m\ddot{x} \quad (9.6) \]

\[ \frac{d \partial T}{dt \partial y} = m\ddot{y} \quad (9.7) \]

\[ \frac{d \partial T}{dt \partial \theta} = m\ddot{\theta} \quad (9.8) \]

9.2.1.3 \[ \frac{\partial T}{\partial q_i} \]

\[ \frac{\partial T}{\partial x} = 0 \quad (9.9) \]

\[ \frac{\partial T}{\partial y} = 0 \quad (9.10) \]

\[ \frac{\partial T}{\partial \theta} = 0 \quad (9.11) \]

9.2.2 Potential energy

\[ V = \frac{1}{2} k_1 H (x - \rho \sin \theta)^2 - \frac{1}{2} k_1 V (y - \rho \cos \theta - \rho)^2 \]

\[ + \frac{1}{2} k_2 H (x - \rho \sin \theta + l \sin \theta)^2 \]

\[ + \frac{1}{2} k_2 V (l - \rho - y + \rho \cos \theta - l \cos \theta)^2 \]

\[ + m g_x x + m g_y y - \frac{1}{2} k_i (\theta - \theta_0)^2 \quad (9.12) \]

9.2.2.1 \[ \frac{\partial V}{\partial q_i} \]

\[ \frac{\partial V}{\partial x} = k_1 H (x - \rho \sin \theta) + k_2 H (x - \rho \sin \theta + l \sin \theta) \]

\[ + m g_x \quad (9.13) \]
\[
\frac{\partial V}{\partial y} = -k_2V(y - \rho \cos \theta + l \cos \theta - l + \rho) + mg_y + k_1V(y - \rho \cos \theta + \rho) \\
\frac{\partial V}{\partial \theta} = k_1H(x - \rho \sin \theta)(-\rho \cos \theta) + k_1V(y - \rho \cos \theta + \rho)(\rho \sin \theta) + k_2V(y - \rho \cos \theta + l \cos \theta - l + \rho)(\rho - l) \sin \theta + k_2H(x - \rho \sin \theta + l \sin \theta)(l - \rho) \cos \theta + k_1(\theta - \theta_0)
\]

9.2.3 The generalized force

The work done by the generalized forces is \( \delta W \).

\[
\delta W = \delta x(FX) - \delta(x - \rho \sin \theta)(Fx) + \delta y(FY) - \delta(y - \rho \cos \theta + \rho)(Fy) + \delta \theta(M - T) - \delta(x - \rho \sin \theta)[c_{1H}(\dot{x} - \rho \dot{\theta} \cos \theta)] - \delta(y - \rho \cos \theta + l \sin \theta)[c_{1V}(\dot{y} + \rho \dot{\theta} \sin \theta)] \]
\[
- \delta(x - \rho \sin \theta + l \sin \theta)[c_{2H}(\dot{x} - \rho \dot{\theta} \cos \theta + l \dot{\theta} \cos \theta)] - \delta(y - \rho \cos \theta + l \cos \theta - l + \rho)[c_{2V}(\dot{y} + \rho \dot{\theta} \sin \theta - l \dot{\theta} \sin \theta)]
\]

Because \( FX, FY \), and \( M \) are vehicle deceleration forces, one can replace these three items with \( m^*Ax, m^*Ay \), and \( I^*\Theta \).
Therefore,

\[ \delta W = [mAx - Fx - c_{1H}(\dot{x} - \rho \dot{\theta} \cos \theta) - c_{2H}(\dot{x} - \rho \dot{\theta} \cos \theta + l \dot{\theta} \cos \theta)]\delta x \]
\[ + [mAy - Fy - c_{1V}(\dot{y} + \rho \dot{\theta} \sin \theta) - c_{2V}(\dot{y} + \rho \dot{\theta} \sin \theta - l \dot{\theta} \sin \theta)]\delta y \]
\[ + [l\Theta - T + c_{1H}(\rho \cos \theta)(\dot{x} - \rho \dot{\theta} \cos \theta) - c_{1V}(\rho \sin \theta)(\dot{y} + \rho \dot{\theta} \sin \theta) \]
\[ - c_{2H}(l - \rho) \cos \theta(\dot{x} - \rho \dot{\theta} \cos \theta + l \dot{\theta} \cos \theta) + (\rho \cos \theta)Fx \]
\[ + c_{2V}(l - \rho) \sin \theta(\dot{y} + \rho \dot{\theta} \sin \theta - l \dot{\theta} \sin \theta) - (\rho \sin \theta)Fy\delta \theta \]

(9.17)

But,

\[ \delta W = Q_x\delta x + Q_y\delta y + Q_\theta \delta \theta \]

(9.18)

Therefore,

\[ Q_x = mA_x - Fx - c_{1H}(\dot{x} - \rho \dot{\theta} \cos \theta) - c_{2H}(\dot{x} - \rho \dot{\theta} \cos \theta + l \dot{\theta} \cos \theta) \]

(9.19)

\[ Q_y = mA_y - Fy - c_{1V}(\dot{y} + \rho \dot{\theta} \sin \theta) - c_{2V}(\dot{y} + \rho \dot{\theta} \sin \theta - l \dot{\theta} \sin \theta) \]

(9.20)

\[ Q_\theta = l\Theta - T + c_{1H}(\rho \cos \theta)(\dot{x} - \rho \dot{\theta} \cos \theta) - c_{1V}(\rho \sin \theta)(\dot{y} + \rho \dot{\theta} \sin \theta) \]
\[ - c_{2H}(l - \rho) \cos \theta(\dot{x} - \rho \dot{\theta} \cos \theta + l \dot{\theta} \cos \theta) + (\rho \cos \theta)Fx \]
\[ + c_{2V}(l - \rho) \sin \theta(\dot{y} + \rho \dot{\theta} \sin \theta - l \dot{\theta} \sin \theta) - (\rho \sin \theta)Fy \]

(9.21)
9.2.4 Equations of motion

Put \( T, V, Q_i \) into Lagrange's equations, and obtain the equations of motion of the theoretical model.

\[
\begin{align*}
    m\ddot{x} &= -k_1H(x - \rho \sin \theta) - k_2H(x - \rho \sin \theta + l \sin \theta) \\
    &\quad - c_1H(\dot{x} - \rho \dot{\theta} \cos \theta) - c_2H(\dot{x} - \rho \dot{\theta} \cos \theta + l\dot{\theta} \cos \theta) \\
    &\quad + mA_x - F_x - mg_x \quad (9.22)
\end{align*}
\]

\[
\begin{align*}
    m\ddot{y} &= -k_1V(y - \rho \cos \theta + \rho) - k_2V(y - \rho \cos \theta + l \cos \theta - l + \rho) \\
    &\quad - c_1V(\dot{y} + \rho \dot{\theta} \sin \theta) - c_2V(\dot{y} + \rho \dot{\theta} \sin \theta - l\dot{\theta} \sin \theta) \\
    &\quad + mA_y - F_y - mg_y \quad (9.23)
\end{align*}
\]

\[
\begin{align*}
    I\ddot{\theta} &= k_1H(\rho \cos \theta)(x - \rho \sin \theta) - k_1V(\rho \sin \theta)(y - \rho \cos \theta + \rho) \\
    &\quad - k_2V(\rho - l) \sin \theta(y - \rho \cos \theta + l \cos \theta - l + \rho) \\
    &\quad + k_2H(\rho - l) \cos \theta(x - \rho \sin \theta + l \sin \theta) \\
    &\quad - c_1V(\rho \sin \theta)(\dot{y} + \rho \dot{\theta} \sin \theta) + c_1H(\rho \cos \theta)(\dot{x} - \rho \dot{\theta} \cos \theta) \\
    &\quad + c_2V(l - \rho) \sin \theta(\dot{y} + \rho \dot{\theta} \sin \theta - l\dot{\theta} \sin \theta) \\
    &\quad - c_2H(l - \rho) \cos \theta(\dot{x} - \rho \dot{\theta} \cos \theta - l\dot{\theta} \cos \theta) \\
    &\quad - k_i(\theta - \theta_0) - (\rho \sin \theta)F_y + (\rho \cos \theta)F_x + I\dot{\Theta} - T \quad (9.24)
\end{align*}
\]
10. APPENDIX B: THE CONSTRAINT CONDITIONS

The fourteen constraint conditions are shown by the relative position between the dummy body and the seat as shown in Figure 10.1 and Figure 10.2. Under certain constraint condition the forces on the seat belt are zero and the seat belt is not effective for occupant restraint. For the case when the body is in contact with the seat, a force $F_F$ is applied to the body with the magnitude of $m\Delta x/\Delta t$ according to the definition that the impulse equals to the change of the momentum of the body. Following are the equations of motion for a selected (case (1)) constraint.

10.1 The Derivation of the Equation of Motion of Constraint (1)

Case (1) occurs when $x - \rho \sin \theta < 0$. In case (1), the force $FF_x = -mA_x + F_x$ is applied at the hip position when $A_x$ negative, otherwise $FF_x = F_x$. The value of $c_{1H}$ and $k_{1H}$ are zero at the same time.

10.1.1 Kinetic energy

The kinetic energy does not change throughout this case. Therefore $\frac{d}{dt} \frac{\partial T}{\partial q_i}$ and $\frac{\partial T}{\partial q_i}$ are as follows:

10.1.1.1 $\frac{d}{dt} \frac{\partial T}{\partial q_i}$
Figure 10.1: Constraints one through six
Figure 10.2: Constraints seven through fourteen
\[
\frac{d}{dt} \frac{\partial T}{\partial \dot{x}} = m \ddot{x} \quad \text{(10.1)}
\]
\[
\frac{d}{dt} \frac{\partial T}{\partial \dot{y}} = m \ddot{y} \quad \text{(10.2)}
\]
\[
\frac{d}{dt} \frac{\partial T}{\partial \dot{\theta}} = m \ddot{\theta} \quad \text{(10.3)}
\]

10.1.1.2 \quad \frac{\partial T}{\partial q_i}

\[
\frac{\partial T}{\partial x} = 0 \quad \text{(10.4)}
\]
\[
\frac{\partial T}{\partial y} = 0 \quad \text{(10.5)}
\]
\[
\frac{\partial T}{\partial \theta} = 0 \quad \text{(10.6)}
\]

10.1.2 Potential energy

\[
V = \frac{1}{2} k_1 V (y - \rho \cos \theta + \rho)^2
+ \frac{1}{2} k_2 H (x - \rho \sin \theta + \rho \sin \theta)^2
+ \frac{1}{2} k_3 V (l - \rho - y + \rho \cos \theta - l \cos \theta)^2
+ mg_x x + mg_y y + \frac{1}{2} k_4 (\theta - \theta_0)^2 \quad \text{(10.7)}
\]

10.1.2.1 \quad \frac{\partial V}{\partial q_i}

\[
\frac{\partial V}{\partial x} = k_2 H (x - \rho \sin \theta + \rho \sin \theta) + mg_x \quad \text{(10.8)}
\]
\[
\frac{\partial V}{\partial y} = -k_{2V}(y - \rho \cos \theta + l \cos \theta - l + \rho) + k_{1V}(y - \rho \cos \theta + \rho) + mg_y 
\]
(10.9)

\[
\frac{\partial V}{\partial \theta} = k_{1V}(y - \rho \cos \theta + \rho)(\rho \sin \theta) + k_{2V}(y - \rho \cos \theta + l \cos \theta - l + \rho)(\rho - l) \sin \theta + k_{2H}(x - \rho \sin \theta + l \sin \theta)(l - \rho) \cos \theta + k_i(\theta - \theta_0) 
\]
(10.10)

10.1.3 The generalized force

The work done by the generalized forces is \(\delta W\).

\[
\delta W = \delta x(FX) - \delta(x - \rho \sin \theta)(Fx) 
\]
\[
+ \delta y(FY) - \delta(y - \rho \cos \theta + \rho)(Fy) 
\]
\[
+ \delta \theta(M - T) + \delta(x - \rho \sin \theta)(FF_x + FF_y) 
\]
\[
- \delta(y - \rho \cos \theta + \rho)[c_{1V}(\dot{y} + \rho \dot{\theta} \sin \theta)] 
\]
\[
- \delta(x - \rho \sin \theta + l \sin \theta)[c_{2H}(\dot{x} - \rho \dot{\theta} \cos \theta + l \dot{\theta} \cos \theta)] 
\]
\[
- \delta(y - \rho \cos \theta + l \cos \theta - l + \rho)[c_{2V}(\dot{y} + \rho \dot{\theta} \sin \theta - l \dot{\theta} \sin \theta)] 
\]
(10.11)

Because FX, FY, and M are vehicle deceleration forces, one can replace these three items with \(m^*Ax\), \(m^*Ay\), and \(I^*\Theta\).

Therefore,
\[ Q_z = mA_x - F_x + FF_x + FFF_x - c_{2H}(\dot{x} - \rho \dot{\theta} \cos \theta + l \dot{\theta} \cos \theta) \]  \hspace{1cm} (10.12)

\[ Q_y = mA_y - F_y - c_{1V}(\dot{y} + \rho \dot{\theta} \sin \theta) - c_{2V}(\dot{y} + \rho \dot{\theta} \sin \theta - l \dot{\theta} \sin \theta) \]  \hspace{1cm} (10.13)

\[ Q_\theta = I \Theta - T - c_{1V}(\rho \sin \theta)(\dot{y} + \rho \dot{\theta} \sin \theta) - c_{2H}((l - \rho) \cos \theta(\dot{x} - \rho \dot{\theta} \cos \theta + l \dot{\theta} \cos \theta) + c_{2V}(l - \rho) \sin \theta(\dot{y} + \rho \dot{\theta} \sin \theta - l \dot{\theta} \sin \theta) - (\rho \sin \theta)F_y + (\rho \cos \theta)(F_x - FF_x - FFF_x) \]  \hspace{1cm} (10.14)

### 10.1.4 Equations of motion

Put \( T, V, Q_i \) into Lagrange’s equations, and obtain the equations of motion of the theoretical model.

\[ m\ddot{x} = -k_{2H}(x - \rho \sin \theta + l \sin \theta) - c_{2H}(\dot{x} - \rho \dot{\theta} \cos \theta + l \dot{\theta} \cos \theta) + mA_x - F_x + FF_x + FFF_x - mg_x \]  \hspace{1cm} (10.15)

\[ m\ddot{y} = -k_{1V}(y - \rho \cos \theta + \rho) + k_{2V}(y - \rho \cos \theta + l \cos \theta - l + \rho) - c_{1V}(\dot{y} + \rho \dot{\theta} \sin \theta) - c_{2V}(\dot{y} + \rho \dot{\theta} \sin \theta - l \dot{\theta} \sin \theta) + mA_y - F_y - mg_y \]  \hspace{1cm} (10.16)
\[ I \ddot{\theta} = -k_1V(p \sin \theta)(y - \rho \cos \theta + \rho) - c_1V(p \sin \theta)(\dot{y} + \rho \dot{\theta} \sin \theta) \\
- k_2V(\rho - l) \sin \theta(y - \rho \cos \theta + l \cos \theta - l + \rho) \\
+ k_2H(\rho - l) \cos \theta(x - \rho \sin \theta + l \sin \theta) \\
+ c_2V(l - \rho) \sin \theta(\dot{y} + \rho \dot{\theta} \sin \theta - l \dot{\theta} \sin \theta) \\
- c_2H(l - \rho) \cos \theta(x - \rho \dot{\theta} \cos \theta + l \dot{\theta} \cos \theta) \\
- k_1(\theta - \theta_0) - (\rho \sin \theta)F_y + (\rho \cos \theta)(F_x - FF_x - FFF_x) \\
+ I \Theta - T \quad (10.17) \]

10.2 Equations of Motion for the Fourteen Constraint Situations

In the following equations of motion, \( FF_x = -mA_x + F_x \) and \( FF_y = -mA_y + F_y \) if \( A_x \) and \( A_y \) are negative; otherwise \( FF_x = F_x \) and \( FF_y = F_y \).

10.2.1 \( x - (\rho \sin \theta) < 0 \)

\[ m\ddot{x} = -k_2H(x - \rho \sin \theta + l \sin \theta) - c_2H(\dot{x} - \rho \dot{\theta} \cos \theta - l \dot{\theta} \cos \theta) \\
- mg_x + mA_x - F_x + FF_x + FFF_x \quad (10.18) \]

\[ m\ddot{y} = -k_1V(y - \rho \cos \theta + \rho) - k_2V(y - \rho \cos \theta + l \cos \theta - l + \rho) \\
- c_1V(\dot{y} + \rho \dot{\theta} \sin \theta) - c_2V(\dot{y} + \rho \dot{\theta} \sin \theta - l \dot{\theta} \sin \theta) \\
+ mA_y - F_y - mg_y \quad (10.19) \]
\[ I\ddot{\theta} = -k_1V(\rho \sin \theta)(y - \rho \cos \theta + \rho) - c_1V(\rho \sin \theta)(\dot{y} + \rho\dot{\theta} \sin \theta) \\
- k_2V(\rho - l) \sin \theta(y + l \cos \theta + \rho - l - \rho \cos \theta) \\
+ k_2H(\rho - l) \cos \theta(x - \rho \sin \theta + l \sin \theta) \\
- c_2H(l - \rho) \cos \theta(\dot{x} - \rho\dot{\theta} \cos \theta + l\dot{\theta} \cos \theta) \\
+ c_2V(l - \rho) \sin \theta(\dot{y} + \rho\dot{\theta} \sin \theta - l\dot{\theta} \sin \theta) \\
- k_t(\theta - \theta_0) + I\Theta - T + (\rho \cos \theta)(F_x - FF_x - FF_y) \\
- (\rho \sin \theta)F_y \tag{10.20} \]

10.2.2 \( y - (\rho \cos \theta) + \rho < 0 \)

\[ m\ddot{x} = -k_1H(x - \rho \sin \theta) - k_2H(x - \rho \sin \theta + l \sin \theta) \\
- c_1H(\dot{x} - \rho\dot{\theta} \cos \theta) - c_2H(\dot{x} - \rho\dot{\theta} \cos \theta + l\dot{\theta} \cos \theta) \\
+ mA_x - F_x - mg_x \tag{10.21} \]

\[ m\ddot{y} = k_2V(y - \rho \cos \theta + l \cos \theta - l + \rho) - c_2V(\dot{y} + \rho\dot{\theta} \sin \theta - l\dot{\theta} \sin \theta) \\
- mg_y + mA_y - F_y + FF_y + FF_y \tag{10.22} \]

\[ I\ddot{\theta} = k_1H(\rho \cos \theta)(x - \rho \sin \theta) \\
- k_2V(\rho - l) \sin \theta(y + l \cos \theta + \rho - l - \rho \cos \theta) \\
+ k_2H(\rho - l) \cos \theta(x - \rho \sin \theta + l \sin \theta) \\
+ c_1H(\rho \cos \theta)(\dot{x} - \rho\dot{\theta} \cos \theta) \\
- c_2H(l - \rho) \cos \theta(\dot{x} - \rho\dot{\theta} \cos \theta + l\dot{\theta} \cos \theta) \]
\[ + c_2 \Omega (l - \rho) \sin \theta (\dot{y} + \rho \dot{\theta} \sin \theta - l \dot{\theta} \sin \theta) \]
\[ - k_\theta (\theta - \theta_0) + I \Theta - T + (\rho \cos \theta) F_x \]
\[ - (\rho \sin \theta) (F_y - FF_y - F F F_y) \]  
(10.23)

10.2.3 \[ x - (\rho \sin \theta) + (l \sin \theta) < 0 \]
\[
m \ddot{x} = -k_{1H} (x - \rho \sin \theta) - c_{1H} (\dot{x} - \rho \dot{\theta} \cos \theta) \]
\[ - mg_x + mA_x - F_x + FF_x + F F F_x \]  
(10.24)
\[
m \ddot{y} = -k_{1V} (y - \rho \cos \theta + \rho) + k_{2V} (y - \rho \cos \theta + l \cos \theta - l + \rho) \]
\[ - c_{1V} (\dot{y} + \rho \dot{\theta} \sin \theta) - c_{2V} (\dot{y} + \rho \dot{\theta} \sin \theta - l \dot{\theta} \sin \theta) \]
\[ + mA_y - F_y - mg_y \]  
(10.25)
\[
I \ddot{\theta} = k_{1H} (\rho \cos \theta) (x - \rho \sin \theta) - k_{1V} (\rho \sin \theta) (y - \rho \cos \theta + \rho) \]
\[ - k_{2V} (\rho - l) \sin \theta (y + l \cos \theta + \rho - l - \rho \cos \theta) \]
\[ + c_{1H} (\rho \cos \theta) (\dot{x} - \rho \dot{\theta} \cos \theta) - c_{1V} (\rho \sin \theta) (\dot{y} + \rho \dot{\theta} \sin \theta) \]
\[ + c_{2V} (l - \rho) \sin \theta (\dot{y} + \rho \dot{\theta} \sin \theta - l \dot{\theta} \sin \theta) \]
\[ - k_\theta (\theta - \theta_0) + I \Theta - T + (\rho \cos \theta) F_x - (\rho \sin \theta) F_y \]
\[ - [(\rho - l) \cos \theta] (FF_x + F F F_x) \]  
(10.26)

10.2.4 \[ y - (\rho \cos \theta) + (l \cos \theta) + \rho < 0 \]
\[
m \ddot{x} = -k_{1H} (x - \rho \sin \theta) - k_{2H} (x - \rho \sin \theta) \]
\[
- c_1H(\dot{x} - \rho \dot{\theta} \cos \theta) - c_2H(\dot{x} - \rho \dot{\theta} \cos \theta + l \dot{\theta} \cos \theta) + mA_x - F_x - mg_x
\]  
(10.27)

\[
m\ddot{y} = -k_{1V}(y - \rho \cos \theta + \rho) - c_{1V}(\dot{y} + \rho \dot{\theta} \sin \theta) - mg_y + mA_y - F_y + FF_y + FFF_y
\]  
(10.28)

\[
I\ddot{\theta} = k_{1H}(\rho \cos \theta)(x - \rho \sin \theta) - k_{1V}(\rho \sin \theta)(y - \rho \cos \theta + \rho) + k_{2H}(\rho - l) \cos \theta (x - \rho \sin \theta + l \sin \theta) + c_{1H}(\rho \cos \theta)(\dot{x} - \rho \dot{\theta} \cos \theta) - c_{1V}(\rho \sin \theta)(\dot{y} + \rho \dot{\theta} \sin \theta) - c_{2H}(l - \rho) \cos \theta(\dot{x} - \rho \dot{\theta} \cos \theta + l \dot{\theta} \cos \theta) - k_{2V}(\theta - \theta_0) - I\ddot{\theta} - T + (\rho \cos \theta)F_x - (\rho \sin \theta)F_y - [(l - \rho) \sin \theta](FF_y + FFF_y)
\]  
(10.29)

**10.2.5**  
\(x - (\rho \sin \theta) < 0\) and \(y - (\rho \cos \theta) + \rho < 0\)

\[
m\ddot{x} = -k_{2H}(x - \rho \sin \theta) - c_{2H}(\dot{x} - \rho \dot{\theta} \cos \theta + l \dot{\theta} \cos \theta) - mg_x + mA_x - F_x + FF_x + FFF_x
\]  
(10.30)

\[
m\ddot{y} = k_{2V}(y - \rho \cos \theta + l \cos \theta - l + \rho) - c_{2V}(\dot{y} + \rho \dot{\theta} \sin \theta - l \dot{\theta} \sin \theta) - mg_y + mA_y - F_y + FF_y + FFF_y
\]  
(10.31)
\[ I\ddot{\theta} = -k_2V(\rho - l)\sin\theta(y + l\cos\theta + \rho - l - \rho\cos\theta) \]
\[ + k_2H(\rho - l)\cos\theta(x - \rho\sin\theta + l\sin\theta) \]
\[ - c_2H(l - \rho)\cos\theta(\dot{x} - \rho\dot{\theta}\cos\theta + l\dot{\theta}\cos\theta) \]
\[ + c_2V(l - \rho)\sin\theta(\dot{y} + \rho\dot{\theta}\sin\theta - l\dot{\theta}\sin\theta) \]
\[ - k_t(\theta - \theta_0) + I\dot{\Theta} - T + (\rho\cos\theta)(F_x - FF_x - FFF_x) \]
\[ - (\rho\sin\theta)(F_y - FF_y - FFF_y) \quad (10.32) \]

**10.2.6** \( x - (\rho\sin\theta) < 0 \) and \( x - (\rho\sin\theta) + (l\sin\theta) < 0 \)

\[ m\ddot{x} = mA_x - F_x + FF_x + FFF_x - mg_x \quad (10.33) \]

\[ m\ddot{y} = -k_1V(y - \rho\cos\theta + \rho) + k_2V(y - \rho\cos\theta + l\cos\theta - l + \rho) \]
\[ - c_1V(\dot{y} + \rho\dot{\theta}\sin\theta) - c_2V(\dot{y} + \rho\dot{\theta}\sin\theta - l\dot{\theta}\sin\theta) \]
\[ + mA_y - F_y - mg_y \quad (10.34) \]

\[ I\ddot{\theta} = -k_1V(\rho\sin\theta)(y - \rho\cos\theta + \rho) \]
\[ - k_2V(\rho - l)\sin\theta(y + l\cos\theta + \rho - l - \rho\cos\theta) \]
\[ - c_1V(\rho\sin\theta)(\dot{y} + \rho\dot{\theta}\sin\theta) \]
\[ + c_2V(l - \rho)\sin\theta(\dot{y} + \rho\dot{\theta}\sin\theta - l\dot{\theta}\sin\theta) \]
\[ - k_t(\theta - \theta_0) + I\dot{\Theta} - T + (\rho\cos\theta)F_x - (\rho\sin\theta)F_y \quad (10.35) \]
10.2.7  \( x - (\rho \sin \theta) < 0 \) and \( y - (\rho \cos \theta) + (l \cos \theta) + \rho < 0 \)

\[
\begin{align*}
    m\ddot{x} &= -k_2H(x - \rho \sin \theta) - c_2H(\dot{x} - \rho \dot{\theta} \cos \theta + l \dot{\theta} \cos \theta) \\
    &\quad - mg_x + mA_x - F_x + \overline{FF}_x + \overline{xFF}_x \\
    m\ddot{y} &= -k_1V(y - \rho \cos \theta + \rho) - c_1V(\dot{y} + \rho \dot{\theta} \sin \theta) \\
    &\quad - mg_y + mA_y - F_y + \overline{FF}_y + \overline{xFF}_y \\
    I\dot{\theta} &= -k_1V(\rho \sin \theta)(y - \rho \cos \theta + \rho) \\
    &\quad + k_2H(\rho - l) \cos \theta(x - \rho \sin \theta + l \sin \theta) \\
    &\quad - c_1V(\rho \sin \theta)(\dot{y} + \rho \dot{\theta} \sin \theta) \\
    &\quad - c_2H(l - \rho) \cos \theta(\dot{x} - \rho \dot{\theta} \cos \theta + l \dot{\theta} \cos \theta) \\
    &\quad - k_l(\theta - \theta_0) + I\dot{\theta} + T + (\rho \cos \theta)(F_x - \overline{FF}_x + \overline{xFF}_x) \\
    &\quad - (\rho \sin \theta)F_y - [(l - \rho) \sin \theta](\overline{FF}_y + \overline{xFF}_y) \\
\end{align*}
\]

10.2.8  \( y - (\rho \cos \theta) + \rho < 0 \) and \( x - (\rho \sin \theta) + (l \sin \theta) < 0 \)

\[
\begin{align*}
    m\ddot{x} &= -k_1H(x - \rho \sin \theta) - c_1H(\dot{x} - \rho \dot{\theta} \cos \theta) \\
    &\quad - mg_x + mA_x - F_x + \overline{FF}_x + \overline{xFF}_x \\
    m\ddot{y} &= -k_2V(y - \rho \cos \theta + l \cos \theta - l + \rho) \\
    &\quad - c_2V(\dot{y} + \rho \dot{\theta} \sin \theta - l \dot{\theta} \sin \theta) \\
    &\quad - mg_y + mA_y - F_y + \overline{FF}_y + \overline{xFF}_y \\
\end{align*}
\]
\[ I\ddot{\theta} = k_1 H(\rho \cos \theta)(x - \rho \sin \theta) \]
\[ - k_2 V(\rho - l) \sin \theta (y + l \cos \theta + \rho - l - \rho \cos \theta) \]
\[ + c_1 H(\rho \cos \theta)(\dot{x} - \rho \dot{\theta} \cos \theta) \]
\[ + c_2 V(\rho - l) \sin \theta (\dot{y} + \rho \dot{\theta} \sin \theta - l \dot{\theta} \sin \theta) \]
\[ - k_i(\theta - \theta_0) + I\Theta - T + (\rho \cos \theta)F_x \]
\[ - (\rho \sin \theta)(F_y - F F_y - F F_y) \]
\[ - [(\rho - l) \cos \theta](F F_x + F F_x) \]  
\[ (10.41) \]

**10.2.9** \( y - (\rho \cos \theta) + \rho < 0 \) and \( y - (\rho \cos \theta) + (l \cos \theta) + \rho < 0 \)

\[ m\ddot{x} = -k_1 H(x - \rho \sin \theta) - k_2 H(x - \rho \sin \theta) \]
\[ - c_1 H(\dot{x} - \rho \dot{\theta} \cos \theta) - c_2 H(\dot{x} - \rho \dot{\theta} \cos \theta + l \dot{\theta} \cos \theta) \]
\[ + m A_x - F_x - m g_x \]  
\[ (10.42) \]

\[ m\ddot{y} = m A_y - F_y + F F_y + F F_y - m g_y \]  
\[ (10.43) \]

\[ I\ddot{\theta} = k_1 H(\rho \cos \theta)(x - \rho \sin \theta) \]
\[ + k_2 H(\rho - l) \cos \theta (x - \rho \sin \theta + l \sin \theta) \]
\[ + c_1 H(\rho \cos \theta)(\dot{x} - \rho \dot{\theta} \cos \theta) \]
\[ - c_2 H(\rho - l) \cos \theta (\dot{x} - \rho \dot{\theta} \cos \theta + l \dot{\theta} \cos \theta) \]
\[ - k_i(\theta - \theta_0) + I\Theta - T + (\rho \cos \theta)F_x - (\rho \sin \theta)F_y \]  
\[ (10.44) \]
10.2.10  \( x - (\rho \sin \theta) + (l \sin \theta) < 0 \) and \( y - (\rho \cos \theta) + (l \cos \theta) + \rho < 0 \)

\[
m\ddot{x} = -k_1H(x - \rho \sin \theta) - c_1H(x - \rho \dot{\theta} \cos \theta) - mg_x + mA_x - F_x + FF_x + FFF_x
\]

\[
m\ddot{y} = -k_1V(y - \rho \cos \theta + \rho) - c_1V(y + \rho \dot{\theta} \sin \theta) - mg_y + mA_y - F_y + FF_y + FFF_y
\]

\[
I\ddot{\theta} = k_1H(\rho \cos \theta)(x - \rho \sin \theta) - k_1V(y - \rho \cos \theta + \rho) + c_1H(\rho \cos \theta)(\dot{x} - \rho \dot{\theta} \cos \theta) - c_1V(\rho \sin \theta)(\dot{y} + \rho \dot{\theta} \sin \theta)
- k_1(\theta - \theta_0) + I\Theta - T + (\rho \cos \theta)F_x - (\rho \sin \theta)F_y
- [(\rho - l) \cos \theta](FF_x + FFF_x)
- [(l - \rho) \sin \theta](FF_y + FFF_y)
\]

10.2.11  \( x - (\rho \sin \theta) < 0, \ y - (\rho \cos \theta) + \rho < 0 \) and \( x - (\rho \sin \theta) + (l \sin \theta) < 0 \)

\[
m\ddot{x} = mA_x - F_x + FF_x + FFF_x - mg_x
\]

\[
m\ddot{y} = k_2V(y - l + \rho) - c_2V(y + \rho \dot{\theta} \sin \theta - l \dot{\theta} \sin \theta) - mg_y + mA_y - F_y + FF_y + FFF_y
\]
\[ I\ddot{\theta} = -k_2V(\rho - l)\sin\theta(y + l\cos\theta + \rho - l - \rho\cos\theta) \]
\[ + c_2V(l - \rho)\sin\theta(\dot{y} + \rho\dot{\theta}\sin\theta - l\dot{\theta}\sin\theta) \]
\[ - k_1(\theta - \theta_0) + I\Theta - T + (\rho\cos\theta)F_x \]
\[ - (\rho\sin\theta)(F_y - FF_y - FFF_y) \quad (10.50) \]

10.2.12 \( x - (\rho\sin\theta) < 0, \ y - (\rho\cos\theta) + \rho < 0 \) and \( y - (\rho\cos\theta) + (l\cos\theta) + \rho < 0 \)

\[ m\ddot{x} = -k_2H(x - \rho\sin\theta) - c_2H(\dot{x} - \rho\dot{\theta}\cos\theta + l\dot{\theta}\cos\theta) \]
\[ - mg_x + mA_x - F_x + FF_x + FFF_x \quad (10.51) \]

\[ m\ddot{y} = mA_y - F_y + FF_y + FFF_y - mg_y \quad (10.52) \]

\[ I\ddot{\theta} = k_2H(\rho - l)\cos\theta(x - \rho\sin\theta + l\sin\theta) \]
\[ - c_2H(l - \rho)\cos\theta(\dot{x} - \rho\dot{\theta}\cos\theta + l\dot{\theta}\cos\theta) \]
\[ - k_1(\theta - \theta_0) + I\Theta - T + (\rho\cos\theta)(F_x - FF_x - FFF_x) \]
\[ - (\rho\sin\theta)F_y \quad (10.53) \]

10.2.13 \( y - (\rho\cos\theta) + \rho < 0, \ x - (\rho\sin\theta) + (l\sin\theta) < 0 \) and \( y - \rho\cos\theta + l\cos\theta + \rho < 0 \)

\[ m\ddot{x} = -k_1H(x - \rho\sin\theta) - c_1H(\dot{x} - \rho\dot{\theta}\cos\theta) \]
\[ - mg_x + mA_x - F_x + FF_x + FFF_x \quad (10.54) \]

\[ m\ddot{y} = mA_y - F_y + FF_y + FFF_y - mg_y \quad (10.55) \]
\[ I\ddot{\theta} = k_1 \rho \cos \theta (x - \rho \sin \theta) + c_1 \rho \cos \theta (\dot{x} - \rho \dot{\theta} \cos \theta) - k_2 (\theta - \theta_0) + I\dot{\Theta} - T + (\rho \cos \theta) F_x - (\rho \sin \theta) F_y - [(\rho - l) \cos \theta] (F_{F_x} + F_{F_y}) \tag{10.56} \]

**10.2.14** \( x - (\rho \sin \theta) < 0, \ x - (\rho \sin \theta) + (l \sin \theta) < 0 \) and \( y - \rho \cos \theta + l \cos \theta + \rho < 0 \)

\[ m\ddot{x} = mA_x - F_x + F_{F_x} + F_{F_F} - mg_x \tag{10.57} \]
\[ m\ddot{y} = -k_{1V} (y - \rho \cos \theta + \rho) - c_{1V} (\dot{y} - \rho \dot{\theta} \sin \theta) - mg_y + mA_y - F_y + F_{F_y} + F_{F_F} \tag{10.58} \]
\[ I\ddot{\theta} = -k_{1V} (\rho \sin \theta) (y - \rho \cos \theta + \rho) - c_{1V} (\rho \sin \theta) (\dot{y} - \rho \dot{\theta} \sin \theta) - k_2 (\theta - \theta_0) + I\dot{\Theta} - T + (\rho \cos \theta) F_x - (\rho \sin \theta) F_y + [(\rho - l) \sin \theta] (F_{F_y} + F_{F_F}) \tag{10.59} \]
11. APPENDIX C: ROTATING SHAFT CARRYING AN UNBALANCED BODY AT ITS MID-SPAN

The governing equations:

**X-direction**

\[
\frac{d^2}{dt^2}(x + e \cdot \cos \omega t) = -k \cdot x - c \frac{d}{dt} x \quad (11.1)
\]

\[
m \cdot \ddot{x} + c \cdot \dot{x} + k \cdot x = m \cdot e \cdot \omega^2 \cdot \cos \omega t \quad (11.2)
\]

**Y-direction**

\[
\frac{d^2}{dt^2}(y + e \cdot \sin \omega t) = -k \cdot y - c \frac{d}{dt} y \quad (11.3)
\]

\[
m \cdot \ddot{y} + c \cdot \dot{y} + k \cdot y = m \cdot e \cdot \omega^2 \cdot \sin \omega t \quad (11.4)
\]

If \( e = 12'' \) and \( \omega = 3 \text{Hz} = 6\pi \text{ rad/sec} \),

\[
e \omega^2 = 12 \times 36 \times \pi^2 = 4264 \text{ (in/sec^2)} = 11g \quad (11.5)
\]
Figure 11.1: Rotating shaft carrying an unbalanced body at its mid-span
12. APPENDIX D: THE PROPAGATION OF UNCERTAINTY FOR G-LOADING

The governing equation for g-loading using a piezoelectric accelerometer connected to a charge amplifier and a data acquisition system is [30]:

\[ G = -\left(\frac{1}{k}\right) \frac{E_o \times C_f}{S_q} \]  

(12.1)

\( S_q \) is the charge sensitivity of the accelerometer, \( C_f \) is the feedback capacitor of the charge amplifier, and \( k \) is a constant. \( E_o \) is the voltage as digitalized with a 16 bit (65536 possible increments) processor. An uncertainty of 5% in \( S_q \) and \( C_f \) was estimated and the uncertainty in \( E_o \) was the overall range of 20 volts divided by the 65536 possible increments of the 16 bit processor. Thus, the maximum uncertainty in \( S_q \), \( C_f \) and \( E_o \) become:

\[ S_q = (106 \pm 5.3) \left( \frac{P_{cb}}{\text{unit}} \right) \]  

(12.2)

\[ C_f = (200 \pm 10) \left( \frac{\text{unit}}{\text{Volt}} \right) \]  

(12.3)

\[ E_o = (20 \pm \frac{20}{65536}) \left( \text{Volt} \right) \]  

(12.4)

The equation of the propagation of uncertainty [31] for g-loading is:

\[ \left( \frac{W_g}{g} \right)^2 = \left( \frac{W_{S_q}}{S_q} \right)^2 + \left( \frac{W_{C_f}}{C_f} \right)^2 + \left( \frac{W_{E_o}}{E_o} \right)^2 \]  

(12.5)
Where $W$ is taken as the maximum uncertainty in each variable.

$$
\left( \frac{W_g}{g} \right)^2 = \left( \frac{5.3}{106} \right)^2 + \left( \frac{10}{200} \right)^2 + \left( \frac{0.0003}{20} \right)^2
$$

(12.6)

$$
\frac{W_g}{g} = 0.07
$$

(12.7)

$$
W_g = 15 \times 0.07 = 1.05 \approx 1.1
$$

(12.8)
13. APPENDIX E: PROGRAM

13.1 The Simulation Program for the Occupant Model

** THIS PROGRAM IS FOR THE COMPUTER SIMULATION OF **
** SINGLE MASS, **
** TWO DIMENSIONAL, **
** THREE DOF **
** OCCUPANT MODAL. **

** THIS PROGRAM USES THE SOFTWARE "DIFFEQ" TO SOLVE **
** THE ORDINARY DIFFERENTIAL EQUATIONS. **

** IN THIS PROGRAM: **
** UNKNOWN: X--X POSITION **
** Y--Y POSITION **
** THETA--THETA POSITION **

** INPUT: AX--DECELERATION IN X DIRECTION **
** AY--DECELERATION IN Y DIRECTION **
** ATHE--DECELERATION IN THETA DIRECTION **

** KNOWN: FFX --RESTRICT FORCE IN X DIRECTION **
** FFY --RESTRICT FORCE IN Y DIRECTION **
** FFFX--RESTRICT FORCE DUE TO IMPULSE **
** CHANGE IN X DIRECTION **
** FFFY--RESTRICT FORCE DUE TO IMPULSE **
** CHANGE IN Y DIRECTION **
** FX--SEAT FRICTION RESISTS THE MOTION OF **
** HIP IN X DIRECTION **
FY -- SEAT FRICTION RESISTS THE MOTION OF HIP IN Y DIRECTION
TT -- A CONSTANT REPRESENTING JOINT FRICTION
KT -- JOINT SPRING CONSTANT

PARAMETERS:
RO -- THE DISTANCE FROM THE HIP TO THE MASS CENTER OF THE MODEL
LE -- THE LENGTH OF THE MODEL FROM THE HEAD POSITION TO THE HIP POSITION
BM -- THE WEIGHT OF THE MODEL
GX -- THE GRAVITY IN X DIRECTION
GY -- THE GRAVITY IN Y DIRECTION
YO -- THE INITIAL CONDITION IN THETA DIRECTION
K1H -- SPRING CONSTANT
K1V -- SPRING CONSTANT
K2H -- SPRING CONSTANT
K2V -- SPRING CONSTANT
C1H -- DAMPING COEFFICIENT
C1V -- DAMPING COEFFICIENT
C2H -- DAMPING COEFFICIENT
C2V -- DAMPING COEFFICIENT

VARIABLES:
Y(1) -- X VARIABLE
Y(2) -- Y VARIABLE
Y(3) -- THETA VARIABLE
Y(4) -- DERIVATIVE OF X
Y(5) -- DERIVATIVE OF Y
Y(6) -- DERIVATIVE OF THETA
Y(7) -- SEAT BELT FORCE AT HIP POSITION
Y(8) -- SEAT BELT FORCE AT SHOULDER POSITION
Y(9) -- AX VARIABLE
Y(10) -- AY VARIABLE
Y(11) -- ATHEGA VARIABLE
Y(12) -- VELOCITY AT HIP POSITION IN X DIRECTION
Y(13) -- VELOCITY AT HIP POSITION IN Y DIRECTION
Y(14) -- VELOCITY AT SHOULDER POS. IN X DIRECTION
Y(15) -- VELOCITY AT SHOULDER POS. IN Y DIRECTION
Y(16) -- VELOCITY AT HIP POSITION
Y(17) -- VELOCITY AT SHOULDER POSITION
SUBROUTINE FCT(T,Y,DERY)

REAL DERY(20), PA(20), Y(20), K1H, K1V, K2H, K2V, LE, KT

DATA GY, YO/386.4, 0.0/
COMMON/FCTCOM/PA

C
C INPUT THE VALUE OF K & C

EQUIVALENCE (PA(1), K1H),
& (PA(2), K1V),
& (PA(3), K2H),
& (PA(4), K2V),
& (PA(5), C1H),
& (PA(6), C1V),
& (PA(7), C2H),
& (PA(8), C2V),
& (PA(9), BI),
& (PA(10), TT),
& (PA(11), RO),
& (PA(12), FX),
& (PA(13), FY),
& (PA(14), LE),
& (PA(15), BM),
& (PA(16), KT)

C
C INPUT AX

C INPUT ATHE

IF (T .LE. 0.03) THEN
  AX = 30. * GY * T / 0.03
ELSE
  IF (T .LE. 0.1) THEN
    AX = 30. * GY * (0.1 - T) / 0.07
  ELSE
    AX = 0.0
  END IF
END IF
DECIDE THE TIME INTERVALS THE FORCE \textit{FFF} APPLIED

\begin{verbatim}
DECIDE THE DIRECTIONS OF THE FRICTION FORCES

\begin{verbatim}
DECIDE THE MAGNITUDES OF \textit{FFX} AND \textit{FFY}

\begin{verbatim}
Y(1) = X
\end{verbatim}
\end{verbatim}
Y(2) = Y
Y(3) = THETA
Y(4) = DX
Y(5) = DY
Y(6) = DTHETA
DERY(4) = DDX
DERY(5) = DDY
DERY(6) = DDDTHETA

DERY(1) = Y(4)
DERY(2) = Y(5)
DERY(3) = Y(6)

DECIDE THE RESTRAINT CONDITION

IF (Y(1) - RO*SIND(Y(3)) .LT. 0 .AND.
+ (Y(1) - RO*SIND(Y(3)) + LE*SIND(Y(3))) .LT. 0 .AND.
+ (Y(2) - RO*COSD(Y(3)) + LE*COSD(Y(3))) .LT. (-RO)) GO TO 14

IF ((Y(2) - RO*COSD(Y(3))) .LT. (-RO) .AND.
+ (Y(2) - RO*COSD(Y(3)) + LE*COSD(Y(3))) .LT. (-RO) .AND.
+ (Y(1) - RO*SIND(Y(3)) + LE*SIND(Y(3))) .LT. 0) GO TO 13

IF (Y(2) - RO*COSD(Y(3)) .LT. (-RO) .AND.
+ Y(2) - RO*COSD(Y(3)) + LE*COSD(Y(3)) .LT. (-RO) .AND.
+ Y(1) - RO*SIND(Y(3)) .LT. 0) GO TO 12

IF (Y(1) - RO*SIND(Y(3)) .LT. 0 .AND.
+ Y(2) - RO*COSD(Y(3)) .LT. (-RO) .AND.
+ Y(1) - RO*SIND(Y(3)) + LE*SIND(Y(3)) .LT. 0) GO TO 11

IF (Y(2) - RO*COSD(Y(3)) + LE*COSD(Y(3)) .LT. (-RO) .AND.
+ Y(1) - RO*SIND(Y(3)) + LE*SIND(Y(3)) .LT. 0) GO TO 10

IF (Y(2) - RO*COSD(Y(3)) .LT. (-RO) .AND.
+ Y(2) - RO*COSD(Y(3)) + LE*COSD(Y(3)) .LT. (-RO)) GO TO 9

IF (Y(2) - RO*COSD(Y(3)) .LT. (-RO) .AND.
+ Y(1) - RO*SIND(Y(3)) + LE*SIND(Y(3)) .LT. 0) GO TO 8
IF (Y(2) - RO * COSD(Y(3)) + LE * COSD(Y(3)) .LT. (-RO) .AND. 
+ Y(1) - RO * SIND(Y(3)) .LT. 0) GO TO 7

IF (Y(1) - RO * SIND(Y(3)) .LT. 0 .AND. 
+ Y(1) - RO * SIND(Y(3)) + LE * SIND(Y(3)) .LT. 0) GO TO 6

IF (Y(2) - RO * COSD(Y(3)) .LT. (-RO) .AND. 
+ Y(1) - RO * SIND(Y(3)) .LT. 0) GO TO 5

IF (Y(2) - RO * COSD(Y(3)) + LE * COSD(Y(3)) .LT. (-RO)) GO TO 4

IF (Y(1) - RO * SIND(Y(3)) + LE * SIND(Y(3)) .LT. 0) GO TO 3

IF (Y(2) - RO * COSD(Y(3)) .LT. (-RO)) GO TO 2

IF (Y(1) - RO * SIND(Y(3)) .LT. 0) GO TO 1

GO TO 15

1 IF (ABS(Y(12)) .LE. 1) THEN
  FFFX=0
ELSE
  FFFX=-BM*Y(12)/DELT
END IF

DERY(4)=(-K2H*(Y(1) - RO * SIND(Y(3)) + LE * SIND(Y(3)))
& -C2H*(Y(4) - RO * Y(6) * COSD(Y(3)) + LE * Y(6) * COSD(Y(3)))
& -FX+FFX+FFFX)/BM-GX+AX

DERY(5)=(-K1V*(Y(2) - RO * COSD(Y(3)) + RO)
& -C1V*(Y(5) + RO * Y(6) * SIND(Y(3)))
& +K2V*(Y(2) - RO * COSD(Y(3)) + LE * COSD(Y(3)) + LE + RO)
& -C2V*(Y(5) + RO * Y(6) * SIND(Y(3)) + LE * Y(6) * SIND(Y(3)))
& -FY)/BM+AY-GY

DERY(6)=(-K2V*(RO-LE)*SIND(Y(3))*(Y(2)-(RO-LE)*COSD(Y(3))-LE+RO)
& +K2H*(RO-LE)*COSD(Y(3))*(Y(1)-(RO-LE)*SIND(Y(3)))
& -K1V*(Y(2)-RO*COSD(Y(3))+RO)*RO*SIND(Y(3))
GO TO 16

2 IF (ABS(Y(13)) .LE. 1) THEN
  FFFY=0
ELSE
  FFFY=-BM*Y(13)/DELT
END IF

DERY(4)=(-K1H*(Y(1)-RO*SIND(Y(3)))
&  -C1H*(Y(4)-RO*Y(6)*COSD(Y(3)))
&  -K2H*(Y(1)-RO*SIND(Y(3))+LE*SIND(Y(3)))
&  -C2H*(Y(4)-RO*Y(6)*COSD(Y(3))+LE*Y(6)*COSD(Y(3)))
&  -FX)/BM+AX-GX

DERY(5)=(K2V*(Y(2)-RO*COSD(Y(3))+LE*COSD(Y(3))-LE+RO)
&  -C2V*(Y(5)+RO*Y(6)*SIND(Y(3))-LE*Y(6)*SIND(Y(3)))
&  -FY+FFY+FFFY)/BM-GY+AY

DERY(6)=(-K2V*(RO-LE)*SIND(Y(3))*(Y(2)-(RO-LE)*COSD(Y(3))-LE+RO)
&  +K1H*RO*COSD(Y(3))*(Y(1)-RO*SIND(Y(3)))
&  +K2H*(RO-LE)*COSD(Y(3))*(Y(1)-(RO-LE)*SIND(Y(3)))
&  -KT*(Y(3)-YO)-TT
&  +C1H*RO*COSD(Y(3))*(Y(4)-RO*Y(6)*COSD(Y(3)))
&  -C2H*(LE-RO)*COSD(Y(3))*(Y(4)-(RO-LE)*Y(6)*COSD(Y(3)))
&  +C2V*(LE-RO)*SIND(Y(3))*(Y(5)+(RO-LE)*Y(6)*SIND(Y(3)))
&  -RO*SIND(Y(3))*(FY-FFY-FFFY)
&  +RO*COSD(Y(3))*FX)/BI+ATHE

GO TO 16

3 IF (ABS(Y(14)) .LE. 1) THEN
  FFFX=0
ELSE
\[ \text{FFFX} = -\frac{BM \cdot Y(14)}{DELT} \]
\[ \text{DERY}(4) = (-K1H \cdot (Y(1) - RO \cdot \text{SIND}(Y(3))) \]
\& \(-C1H \cdot (Y(4) - RO \cdot Y(6) \cdot \text{COSD}(Y(3))) \]
\& \(-FX + FFX + \text{FFFX})/BM - GX + AX \]
\[ \text{DERY}(5) = (-K1V \cdot (Y(2) - RO \cdot \text{COSD}(Y(3)) + RO) \]
\& \(-C1V \cdot (Y(5) + RO \cdot Y(6) \cdot \text{SIND}(Y(3)) - LE \cdot Y(6) \cdot \text{SIND}(Y(3))) \]
\& \(-FY)/BM + AY - GY \]
\[ \text{DERY}(6) = (-K2V \cdot (RO - LE) \cdot \text{SIND}(Y(3)) \cdot (Y(2) - (RO - LE) \cdot \text{COSD}(Y(3)) - LE + RO) \]
\& \(+K1H \cdot RO \cdot \text{COSD}(Y(3)) \cdot (Y(1) - RO \cdot \text{SIND}(Y(3))) \]
\& \(-K1V \cdot (Y(2) - RO \cdot \text{COSD}(Y(3)) + RO) \cdot RO \cdot \text{SIND}(Y(3)) \]
\& \(-KT \cdot (Y(3) - YO) - TT \]
\& \(+C1H \cdot RO \cdot \text{COSD}(Y(3)) \cdot (Y(4) - RO \cdot Y(6) \cdot \text{COSD}(Y(3))) \]
\& \(-C1V \cdot RO \cdot \text{SIND}(Y(3)) \cdot (Y(5) + RO \cdot Y(6) \cdot \text{SIND}(Y(3))) \]
\& \(+C2V \cdot (LE - RO) \cdot \text{SIND}(Y(3)) \cdot (Y(5) + (RO - LE) \cdot Y(6) \cdot \text{SIND}(Y(3))) \]
\& \(-RO \cdot \text{SIND}(Y(3)) \cdot FY + RO \cdot \text{COSD}(Y(3)) \cdot FX \]
\& \(-(RO - LE) \cdot \text{COSD}(Y(3)) \cdot (FFX + FFFX))/BI + ATHE \]

GO TO 16

4 IF (ABS(Y(15)).LE. 1) THEN
  FFFY=0
ELSE
  FFFY=-BM*Y(15)/DELT
END IF

\[ \text{DERY}(4) = (-K1H \cdot (Y(1) - RO \cdot \text{SIND}(Y(3))) \]
\& \(-C1H \cdot (Y(4) - RO \cdot Y(6) \cdot \text{COSD}(Y(3))) - FX \]
\& \(-K2H \cdot (Y(1) - RO \cdot \text{SIND}(Y(3))) + LE \cdot \text{SIND}(Y(3)) \]
\& \(-C2H \cdot (Y(4) - RO \cdot Y(6) \cdot \text{COSD}(Y(3))) + LE \cdot Y(6) \cdot \text{COSD}(Y(3))) \]
\& \)/BM + AX - GX \]
\[ \text{DERY}(5) = (-K1V \cdot (Y(2) - RO \cdot \text{COSD}(Y(3)) + RO) \]
\& \(-C1V \cdot (Y(5) + RO \cdot Y(6) \cdot \text{SIND}(Y(3))) \]
& \(-FY+FFY+FFFY)/BM-GY+AY

\[
\text{DERY}(6)= (K1H \cdot RO \cdot \text{COSD}(Y(3)) \cdot (Y(1)-RO \cdot \text{SIND}(Y(3))) \\
+K2H \cdot (RO-LE) \cdot \text{COSD}(Y(3)) \cdot (Y(1)-(RO-LE) \cdot \text{SIND}(Y(3))) \\
-K1V \cdot (Y(2)-RO \cdot \text{COSD}(Y(3))+RO \cdot \text{SIND}(Y(3))) \\
-KT \cdot (Y(3)-YO)-TT \\
+CIH \cdot RO \cdot \text{COSD}(Y(3)) \cdot (Y(4)-RO \cdot Y(6) \cdot \text{COSD}(Y(3))) \\
-C2H \cdot (LE-RO) \cdot \text{COSD}(Y(3)) \cdot (Y(4)-(RO-LE) \cdot Y(6) \cdot \text{COSD}(Y(3))) \\
-CIV \cdot RO \cdot \text{SIND}(Y(3)) \cdot (Y(5)+RO \cdot Y(6) \cdot \text{SIND}(Y(3))) \\
-R0 \cdot \text{SIND}(Y(3)) \cdot FY-(LE-RO) \cdot \text{SIND}(Y(3)) \cdot (FY+FFY+FFFY) \\
+RO \cdot \text{COSD}(Y(3)) \cdot FX)/BI+ATHE
\]

GO TO 16

5 IF (ABS(Y(12)) .LE. 1 .AND. ABS(Y(13)) .LE. 1) THEN

\[
\text{FFFX}=0 \\
\text{FFFY}=0
\]
ELSE

\[
\text{FFFX}=-BM \cdot Y(12)/DELT \\
\text{FFFY}=-BM \cdot Y(13)/DELT
\]
END IF

\[
\text{DERY}(4)= (-K2H \cdot (Y(1)-RO \cdot \text{SIND}(Y(3)))+LE \cdot \text{SIND}(Y(3))) \\
-C2H \cdot (Y(4)-RO \cdot Y(6) \cdot \text{COSD}(Y(3))+LE \cdot Y(6) \cdot \text{COSD}(Y(3))) \\
-FX+FFX+FFFX)/BM-GX+AX
\]

\[
\text{DERY}(5)= (K2V \cdot (Y(2)-RO \cdot \text{COSD}(Y(3))+LE \cdot \text{COSD}(Y(3))-LE+RO) \\
-C2V \cdot (Y(4)+RO \cdot Y(6) \cdot \text{SIND}(Y(3))-LE \cdot Y(6) \cdot \text{SIND}(Y(3))) \\
-FY+FFY+FFFY)/BM-GY+AY
\]

\[
\text{DERY}(6)= (-K2V \cdot (RO-LE) \cdot \text{SIND}(Y(3)) \cdot (Y(2)-(RO-LE) \cdot \text{COSD}(Y(3))-LE+RO) \\
+K2H \cdot (RO-LE) \cdot \text{COSD}(Y(3)) \cdot (Y(1)-(RO-LE) \cdot \text{SIND}(Y(3))) \\
-KT \cdot (Y(3)-YO)-TT \\
-C2H \cdot (LE-RO) \cdot \text{COSD}(Y(3)) \cdot (Y(4)-(RO-LE) \cdot Y(6) \cdot \text{COSD}(Y(3))) \\
+C2V \cdot (LE-RO) \cdot \text{SIND}(Y(3)) \cdot (Y(5)+RO \cdot Y(6) \cdot \text{SIND}(Y(3))) \\
-R0 \cdot \text{SIND}(Y(3)) \cdot (FY+FFY-FFFY) \\
+RO \cdot \text{COSD}(Y(3)) \cdot (FX-FFX-FFFX))/BI+ATHE
\]

GO TO 16
6 IF (ABS(Y(12)) .LE. 1 .AND. ABS(Y(14)) .LE. 1) THEN
    FFFX=0
ELSE
    FFFX=-BM*Y(4)/DELT
END IF

DERY(4)=-GX+AX+(-FX+FFX+FFFX)/BM

DERY(5)=(-K1V*(Y(2)-RO*COSD(Y(3))+RO) & -C1V*(Y(5)+RO*Y(6)*SIND(Y(3)))-FY & +K2V*(Y(2)-RO*COSD(Y(3))+LE*COSD(Y(3))-LE+RO) & -C2V*(Y(5)+RO*Y(6)*SIND(Y(3))-LE*Y(6)*SIND(Y(3))) & )/BM+AY-GY

DERY(6)=(-K2V*(RO-LE)*SIND(Y(3))*(Y(2)-(RO-LE)*COSD(Y(3))-LE+RO) & -K1V*(Y(2)-RO*COSD(Y(3))+RO)*RO*SIND(Y(3)) & -KT*(Y(3)-YO)-TT & -C1V*RO*SIND(Y(3))*(Y(5)+RO*Y(6)*SIND(Y(3))) & +C2V*(LE-RO)*SIND(Y(3))*(Y(5)+(RO-LE)*Y(6)*SIND(Y(3))) & -RO*SIND(Y(3))*FY+RO*COSD(Y(3))*FX)/BI+ATHE

GO TO 16

7 IF (ABS(Y(12)) .LE. 1 .AND. ABS(Y(15)) .LE. 1) THEN
    FFFX=0
    FFFY=0
ELSE
    FFFX=-BM*Y(12)/DELT
    FFFY=-BM*Y(15)/DELT
END IF

DERY(4)=(-K2H*(Y(1)-RO*SIND(Y(3))+RO*SIND(Y(3)))) & -C2H*(Y(4)-RO*Y(6)*COSD(Y(3))+LE+Y(6)*COSD(Y(3))) & -FX+FFX+FFFX)/BM-GX+AX

DERY(5)=(-K1V*(Y(2)-RO*COSD(Y(3))+RO) & -C1V*(Y(5)+RO*Y(6)*SIND(Y(3)))) & -FY+FFY+FFFY)/BM-GY+AY
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DERY(6) = (K2H*(RO-LE)*COSD(Y(3))*(Y(1)-(RO-LE)*SIND(Y(3)))
& -K1V*(Y(2)-RO*COSD(Y(3))+RO)*RO*SIND(Y(3))
& -KT*(Y(3)-YO)-TT
& -C2H*(LE-RO)*COSD(Y(3))*(Y(4)-(RO-LE)*Y(6)*COSD(Y(3)))
& -C1V*RO*SIND(Y(3))*(Y(5)+RO*Y(6)*SIND(Y(3)))
& -RO*SIND(Y(3))*FY+RO*COSD(Y(3))*FX+FFX+FFFX)/BI+ATHE

GO TO 16

8 IF (ABS(Y(13)) .LE. 1 .AND. ABS(Y(14)) .LE. 1) THEN
  FFFX=0
  FFFY=0
ELSE
  FFFX=-BM*Y(14)/DELT
  FFFY=-BM*Y(13)/DELT
END IF

DERY(4)=(-K1H*(Y(1)-RO*SIND(Y(3)))
& -C1H*(Y(4)-RO*Y(6)*COSD(Y(3)))
& -FX+FFX+FFFX)/BM-GX+AX

DERY(5)=(K2V*(Y(2)-RO*COSD(Y(3))+LE*COSD(Y(3))-LE+RO)
& -C2V*(Y(5)+RO*Y(6)*SIND(Y(3))-LE*Y(6)*SIND(Y(3)))
& -FY+FFY+FFFY)/BM-GY+AY

DERY(6)=(-K2V*(RO-LE)*SIND(Y(3))*(Y(2)-(RO-LE)*COSD(Y(3))-LE+RO)
& +K1H*RO*COSD(Y(3))*(Y(1)-RO*SIND(Y(3)))
& -KT*(Y(3)-YO)-TT
& +C1H*RO*COSD(Y(3))*(Y(4)-RO*Y(6)*COSD(Y(3)))
& +C2V*(LE-RO)*SIND(Y(3))*(Y(5)+(RO-LE)*Y(6)*SIND(Y(3)))
& -RO*SIND(Y(3))*(FX+FFY+FFFY)+RO*COSD(Y(3))*FX
& -(RO-LE)*COSD(Y(3))*(FFX+FFFX))/BI+ATHE

GO TO 16

9 IF (ABS(Y(13)) .LE. 1 .AND. ABS(Y(15)) .LE. 1) THEN
  FFFY=0
ELSE
$$FFFY = -BM \times Y(5)/DELT$$

END IF

$$DERY(4) = (-K1H \times (Y(1) - RO \times SIND(Y(3)))$$
$$& - C1H \times (Y(4) - RO \times Y(6) \times COSD(Y(3))) - FX$$
$$& - K2H \times (Y(1) - RO \times SIND(Y(3)) + LE \times SIND(Y(3)))$$
$$& - C2H \times (Y(4) - RO \times Y(6) \times COSD(Y(3)) + LE \times Y(6) \times COSD(Y(3)))$$
$$& )/BM + AX - GX$$

$$DERY(5) = -GY + AY + (-FY + FFFY + FFFY)/BM$$

$$DERY(6) = (K1H \times RO \times COSD(Y(3)) \times (Y(1) - RO \times SIND(Y(3)))$$
$$& + K2H \times (RO - LE) \times COSD(Y(3)) \times (Y(1) - (RO - LE) \times SIND(Y(3)))$$
$$& - KT \times (Y(3) - YO) - TT$$
$$& + C1H \times RO \times COSD(Y(3)) \times (Y(4) - RO \times Y(6) \times COSD(Y(3)))$$
$$& - C2H \times (LE - RO) \times COSD(Y(3)) \times (Y(4) - (RO - LE) \times Y(6) \times COSD(Y(3)))$$
$$& - RO \times SIND(Y(3)) \times FY + RO \times COSD(Y(3)) \times FX)/BI + ATHE$$

GO TO 16

10 IF (ABS(Y(14)) .LE. 1 .AND. ABS(Y(15)) .LE. 1) THEN
$$FFFX = 0$$
$$FFFY = 0$$
ELSE
$$FFFX = -BM \times Y(14)/DELT$$
$$FFFY = -BM \times Y(15)/DELT$$
END IF

$$DERY(4) = (-K1H \times (Y(1) - RO \times SIND(Y(3)))$$
$$& - C1H \times (Y(4) - RO \times Y(6) \times COSD(Y(3)))$$
$$& - FX + FFX + FFFX)/BM - GX + AX$$

$$DERY(5) = (-K1V \times (Y(2) - RO \times COSD(Y(3)) + RO)$$
$$& - C1V \times (Y(5) + RO \times Y(6) \times SIND(Y(3)))$$
$$& - FY + FFY + FFFY)/BM - GY + AY$$

$$DERY(6) = (K1H \times RO \times COSD(Y(3)) \times (Y(1) - RO \times SIND(Y(3)))$$
$$& - K1V \times (Y(2) - RO \times COSD(Y(3)) + RO) \times RO \times SIND(Y(3))$$
$$& - KT \times (Y(3) - YO) - TT$$
& +C1H*RO*COSD(Y(3))*(Y(4)-RO*Y(6)*COSD(Y(3)))
& -C1V*RO*SIND(Y(3))*(Y(5)+RO*Y(6)*SIND(Y(3)))
& -RO*SIND(Y(3))*FY+RO*COSD(Y(3))*FX
& -(RO-LE)*COSD(Y(3))*(FFX+FFFX)
& -(LE-RO)*SIND(Y(3))*(FFY+FFFY))/BI+ATHE

GO TO 16

11 IF (ABS(Y(12)) .LE. 1 .AND. ABS(Y(13)) .LE. 1 + .AND. ABS(Y(14)) .LE. 1) THEN

FFFX=0
FFFY=0
ELSE
FFFX=-BM*Y(4)/DELT
FFFY=-BM*Y(13)/DELT
END IF

DERY(4)=-GX+AX+(-FX+FFX+FFFX)/BM

DERY(5)=(K2V*(Y(2)-RO*COSD(Y(3))+LE*COSD(Y(3))-LE+RO)
& -C2V*(Y(5)+RO*Y(6)*SIND(Y(3))-LE*Y(6)*SIND(Y(3)))
& -FY+FFY+FFFY)/BM-GY+AY

DERY(6)=(-K2V*(RO-LE)*SIND(Y(3))*(Y(2)-(RO-LE)*COSD(Y(3))-LE+RO)
& -KT*(Y(3)-YO)-TT
& +C2V*(LE-RO)*SIND(Y(3))*(Y(5)+(RO-LE)*Y(6)*SIND(Y(3)))
& -RO*SIND(Y(3))*(FY-FFY-FFFY)
& +RO*COSD(Y(3))*FX)/BI+ATHE

GO TO 16

12 IF (ABS(Y(12)) .LE. 1 .AND. ABS(Y(13)) .LE. 1 + .AND. ABS(Y(15)) .LE. 1) THEN

FFFX=0
FFFY=0
ELSE
FFFX=-BM*Y(12)/DELT
FFFY=-BM*Y(5)/DELT
END IF
DERY(4)=(-K2H*(Y(1)-RO*SIND(Y(3)))+LE*SIND(Y(3)))
& -C2H*(Y(4)-RO*Y(6)*COSD(Y(3)))+LE*Y(6)*COSD(Y(3)))
& -FX+FFX+FFFX)/BM-GX+AX

DERY(5)=-GY+AY+(-FY+FFY+FFFY)/BM

DERY(6)=(K2H*(RO-LE)*COSD(Y(3))*(Y(1)-(RO-LE)*SIND(Y(3)))
& -KT*(Y(3)-YO)-TT
& -C2H*(LE-RO)*COSD(Y(3))*(Y(4)-(RO-LE)*Y(6)*COSD(Y(3)))
& -RO*SIND(Y(3))*FY
& +RO*COSD(Y(3))*FX-FFX-FFFX)/BI+ATHE

GO TO 16

13 IF (ABS(Y(13)) .LE. 1 .AND. ABS(Y(14)) .LE. 1
+ .AND. ABS(Y(15)) .LE. 1) THEN
  FFFX=0
  FFFY=0
ELSE
  FFFX=-BM*Y(14)/DELT
  FFFY=-BM*Y(5)/DELT
END IF

DERY(4)=(-K1H*(Y(1)-RO*SIND(Y(3)))
& -C1H*(Y(4)-RO*Y(6)*COSD(Y(3)))
& -FX+FFX+FFFX)/BM-GX+AX

DERY(5)=-GY+AY+(-FY+FFY+FFFY)/BM

DERY(6)=(K1H*RO*COSD(Y(3))*(Y(1)-RO*SIND(Y(3)))
& -KT*(Y(3)-YO)-TT
& +C1H*RO*COSD(Y(3))*(Y(4)-RO*Y(6)*COSD(Y(3)))
& -(RO-LE)*COSD(Y(3))*(FFX+FFFX)
& -RO*SIND(Y(3))*FY+RO*COSD(Y(3))*FX)/BI+ATHE

GO TO 16

14 IF (ABS(Y(12)) .LE. 1 .AND. ABS(Y(14)) .LE. 1
+ .AND. ABS(Y(15)) .LE. 1) THEN
  FFFX=O
  FFFY=O
ELSE
  FFFX=-BM*Y(4)/DELT
  FFFY=-BM*Y(15)/DELT
END IF

DERY(4)=-GX+AX+(-FX+FFX+FFFX)/BM

DERY(5)=(-K1V*(Y(2)-RO*COSD(Y(3))+RO)
  &   -C1V*(Y(5)+RO*Y(6)*SIND(Y(3)))
  &   -FY+FFY+FFFY)/BM-GY+AY

DERY(6)=(-K1V*(Y(2)-RO*COSD(Y(3))+RO)*RO*SIND(Y(3))
  &   -KT*Y(3)-Y0)-TT
  &   -C1V*RO*SIND(Y(3))*(Y(5)+RO*Y(6)*SIND(Y(3)))
  &   +RO*COSD(Y(3))*FX-RO*SIND(Y(3))*FY
  &   -(LE-RO)*SIND(Y(3))*(FFY+FFFY))/BI+ATHE

GO TO 16

C
C   THE MAIN EQUATIONS OF MOTION
C
15 DERY(4)=(-K1H*(Y(1)-RO*SIND(Y(3)))
  &   -C1H*(Y(4)-RO*Y(6)*COSD(Y(3)))-FX
  &   -K2H*(Y(1)-RO*SIND(Y(3))+LE*SIND(Y(3)))
  &   -C2H*(Y(4)-RO*Y(6)*COSD(Y(3))+LE*Y(6)*COSD(Y(3)))
  &   )/BM+AX-GX

DERY(5)=(-K1V*(Y(2)-RO*COSD(Y(3))+RO)
  &   -C1V*(Y(5)+RO*Y(6)*SIND(Y(3)))-FY
  &   +K2V*(Y(2)-RO*COSD(Y(3))+LE*COSD(Y(3))-LE+RO)
  &   -C2V*(Y(5)+RO*Y(6)*SIND(Y(3))-LE*Y(6)*SIND(Y(3)))
  &   )/BM+AY-GY

DERY(6)=(-K2V*(RO-LE)*SIND(Y(3))*(Y(2)-(RO-LE)*COSD(Y(3))-LE+RO)
  &   +K1H*RO*COSD(Y(3))*(Y(1)-RO*SIND(Y(3)))

.
& \quad +K2H*(RO-LE)*COSD(Y(3))*(Y(1)-(RO-LE)*SIND(Y(3)))
& \quad -K1V*(Y(2)-RO*COSD(Y(3)))+RO*RO*SIND(Y(3))
& \quad -KT*(Y(3)-YO)-TT
& \quad +C1H*RO*COSD(Y(3))*(Y(4)-RO*Y(6)*COSD(Y(3)))
& \quad -C2H*(LE-RO)*COSD(Y(3))*(Y(4)-(RO-LE)*Y(6)*COSD(Y(3)))
& \quad -C1V*RO*SIND(Y(3))*(Y(5)+RO*Y(6)*SIND(Y(3)))
& \quad +C2V*(LE-RO)*SIND(Y(3))*(Y(5)+(RO-LE)*Y(6)*SIND(Y(3)))
& \quad +RO*COSD(Y(3))*FX-LE*SIND(Y(3))*FY)/BI+ATHE

C
C FIND SEAT BELT FORCE
C
Y(7)=HIP SEAT BELT
C
Y(8)=SHOULDER SEAT BELT
C

16 \quad A=-K1H*(Y(1)-RO*SIND(Y(3)))-C1H*(Y(4)-RO*Y(6)*COSD(Y(3)))
B=-K1V*(Y(2)-RO*COSD(Y(3)))+RO)-C1V*(Y(5)+RO*Y(6)*SIND(Y(3)))
C=-K2H*(Y(1)-RO*SIND(Y(3))+LE*SIND(Y(3)))
& \quad -C2H*(Y(4)-RO*Y(6)*COSD(Y(3))+LE*Y(6)*COSD(Y(3)))
D=K2V*(Y(2)-RO*COSD(Y(3))+LE*COSD(Y(3))-LE+RO)
& \quad -C2V*(Y(5)+RO*Y(6)*SIND(Y(3))-LE+Y(6)*SIND(Y(3)))
Y(7)=SQRT(A**2+B**2)/386.4
Y(8)=SQRT(C**2+D**2)/386.4

C
C THE VEHICLE DECELERATION
C
Y(9)=AX
Y(10)=AY
Y(11)=ATHE
C
C FIND THE VELOCITY OF OCCUPANT
C
Y(16)=HIP SEAT BELT
C
Y(17)=SHOULDER SEAT BELT
C
AA=(Y(4)-RO*Y(6)*COSD(Y(3)))
BB=(Y(5)+RO*Y(6)*SIND(Y(3)))
CC=(Y(4)-RO*Y(6)*COSD(Y(3))+LE*Y(6)*COSD(Y(3)))
DD=(Y(5)+RO*Y(6)*SIND(Y(3))-LE*Y(6)*SIND(Y(3)))
Y(12)=AA
Y(13)=BB
Y(14)=CC
Y(15)=DD

Y(16)=SQRT(AA**2+BB**2)
Y(17)=SQRT(CC**2+DD**2)
Y(18)=Y(6)*PI/180.0
T1=T
RETURN
END

13.2 The Program for Data Acquisition System

'**************************************************************
' PROGRAM : Data Acquisition System
'---------------------------------------------------------------
' LANGUAGE : QUICK-BASIC
'=================================================================
' PURPOSE : This program is designed to measure seven output voltages
' from four strain gauges and three accelerometers.
'=================================================================
' INTERFACE : IEEE-488-GPIB
'=================================================================
' INSTRUMENTATION : 1)HP3495A SWITCH/CONTROL UNIT
' 2)HP3437A DIGITAL VOLTMETER
'=================================================================
' ADDRESS : We set the address of HP3495A as SCANNER -- 9
'          We set the address of HP3437A as DVM --24
'**************************************************************
' Initializing the GPIB : The IEEE-488 GPIB support sublibrary is in
' "MC-GPIB" directory of "C" drive.
'**************************************************************

'$INCLUDE: 'C:\MC-GPIB\QBDECL4.BAS'

'-----------------------------------------------------------------------------
' Variable Declarations:
1) String Variables: SG1V$, SG2V$, SG3V$, SG4V$, AC1V$, AC2V$, AC3V$

2) Number Variables: SG1#, SG2#, SG3#, SG4#, AC1#, AC2#, AC3#

DIM SG1V$(250), SG2V$(250), SG3V$(250), SG4V$(250)
DIM AC1V$(250), AC2V$(250), AC3V$(250)
DIM SG1#(250), SG2#(250), SG3#(250), SG4#(250)
DIM AC1#(250), AC2#(250), AC3#(250)
DIM T1#(250), T2#(250), T3#(250), T4#(250), T5#(250), T6#(250), T7#(250)

CLS
PRINT "INPUT ITERATION NUMBERS AND OUTPUT FILE NUMBER"
INPUT AA#, BB$
CLS

DEVICE$ = "DVM"
CALL IBFIND(DEVICE$, DVM%)
CALL IBPAD(DVM%, 24)
DEVICE$ = "SCANNER"
CALL IBFIND(DEVICE$, SCANNER%)

CLS
PRINT "Opening devices - HP 3495A scanner and the HP 3437A digital voltmeter:"

CLS
PRINT "Clearing the devices to ensure clean starting status:"

CALL IBCLR(DVM%)
CALL IBCLR(SCANNER%)
CALL IBWRT(DVM%, "F1,R1,T2,D.00025S,N1S")

CALL IBWRT(DVM%, "E4S")

'Opening a datafile into drive C: with the file name "0V .DAT"

OPEN "0V" + BB$ + ".DAT" FOR OUTPUT AS #1

'Printing table headings to the screen:

PRINT ; " T1 T2 T3 T4 T5 T6 T7"
PRINT

'Setting the timer to zero:

TIME$ = "00:00:00"

'Beginning of the data taking loop:

FOR I = 1 TO AA#

'Open all the channels on card #2.
'Close channel 41 for the output of four strain gauges.

CALL IBWRT(SCANNER%, "4,41E")

'Setting the scanner to scan the list of channels 21 to 24 on card #1:
CALL IBWRT(SCANNER%, "F21L2421E")

CALL IBTRG(DVM%)
' IF IBSTA% < 0 GOTO error1
' CALL IBWAIT(DVM%, &H800)
' IF IBSTA% < 0 GOTO error2
' CALL IBRSP(DVM%, SPR%) 
' IF IBSTA% < 0 GOTO error3
' IF (SPR% AND &H44) <> &H44 GOTO error4
SG1V$(I) = SPACE$(13)
CALL IBRD(DVM%, SG1V$(I))

Obtaining the time at which the data was taken:

T1#(I) = TIMER

CALL IBWRT(SCANNER%, "S")

CALL IBTRG(DVM%)
' IF IBSTA% < 0 GOTO error1
' CALL IBWAIT(DVM%, &H800)
' IF IBSTA% < 0 GOTO error2
' CALL IBRSP(DVM%, SPR%) 
' IF IBSTA% < 0 GOTO error3
' IF (SPR% AND &H44) <> &H44 GOTO error4
SG2V$(I) = SPACE$(13)
CALL IBRD(DVM%, SG2V$(I))

T2#(I) = TIMER

CALL IBWRT(SCANNER%, "S")
CALL IBTRG(DVM%)
' IF IBSTA% < 0 GOTO error1
' CALL IBWAIT(DVM%, &H800)
' IF IBSTA% < 0 GOTO error2
CALL IBRSP(DVM%, SPR%)
    IF IBSTAZ% < 0 GOTO error3
    IF (SPR% AND &H44) <> &H44 GOTO error4
SG3V$(I) = SPACE$(13)
CALL IBRD(DVM%, SG3V$(I))

T3#(I) = TIMER

CALL IBWRT(SCANNER%, "S")
CALL IBTRG(DVM%)
    IF IBSTAZ% < 0 GOTO error1
    CALL IBWAIT(DVM%, &H800)
    IF IBSTAZ% < 0 GOTO error2
    CALL IBRSP(DVM%, SPR%)
    IF IBSTAZ% < 0 GOTO error3
    IF (SPR% AND &H44) <> &H44 GOTO error4
SG4V$(I) = SPACE$(13)
CALL IBRD(DVM%, SG4V$(I))

T4#(I) = TIMER

' Reopen the channel 41.

' Setting the scanner to scan the list of channels 42 to 44 on card #2:

CALL IBWRT(SCANNER%, "4,F42L4442E")
CALL IBTRG(DVM%)
    IF IBSTAZ% < 0 GOTO error1
    CALL IBWAIT(DVM%, &H800)
    IF IBSTAZ% < 0 GOTO error2
    CALL IBRSP(DVM%, SPR%)
    IF IBSTAZ% < 0 GOTO error3
    IF (SPR% AND &H44) <> &H44 GOTO error4
AC1V$(I) = SPACE$(13)
CALL IBRD(DVM%, AC1V$(I))

T5#(I) = TIMER
CALL IBWRT(SCANNER\%, "S")
CALL IBTRG(DVM\%)
  IF IBSTA\% < 0 GOTO error1
  CALL IBWAIT(DVM\%, &H800)
  IF IBSTA\% < 0 GOTO error2
  CALL IBRSP(DVM\%, SPR\%)
  IF IBSTA\% < 0 GOTO error3
  IF (SPR\% AND &H44) <> &H44 GOTO error4
AC2V$(I) = SPACE$(13)
CALL IBRD(DVM\%, AC2V$(I))

T6#(I) = TIMER

CALL IBWRT(SCANNER\%, "S")
CALL IBTRG(DVM\%)
  IF IBSTA\% < 0 GOTO error1
  CALL IBWAIT(DVM\%, &H800)
  IF IBSTA\% < 0 GOTO error2
  CALL IBRSP(DVM\%, SPR\%)
  IF IBSTA\% < 0 GOTO error3
  IF (SPR\% AND &H44) <> &H44 GOTO error4
AC3V$(I) = SPACE$(13)
CALL IBRD(DVM\%, AC3V$(I))

T7#(I) = TIMER

'Printing the data to the screen:
PRINT USING "###.### "; I; T1#(I); T2#(I); T3#(I); T4#(I);
  T5#(I); T6#(I); T7#(I)
PRINT

'Continuing the data loop:

NEXT I

'Printing the data to the datafile:

FOR J = 1 TO AA#
    SG1#(J) = VAL(SG1V$(J))
    SG2#(J) = VAL(SG2V$(J))
    SG3#(J) = VAL(SG3V$(J))
    SG4#(J) = VAL(SG4V$(J))
    AC1#(J) = VAL(AC1V$(J))
    AC2#(J) = VAL(AC2V$(J))
    AC3#(J) = VAL(AC3V$(J))

    PRINT #1, J; SG1#(J); SG2#(J); SG3#(J); SG4#(J); AC1#(J); AC2#(J); AC3#(J); T1#(J); T2#(J); T3#(J); T4#(J); T5#(J); T6#(J); T7#(J)

NEXT J

'Closing the data file:

CLOSE #1

END

'error1:
'    PRINT "error1"
'    END
'error2:
'    PRINT "error2"
'    END
'error3:
'    PRINT "error3"
'    END
'error4:
13.3 The Program for the Data Processing

```fortran
real sg1(300), sg2(300), sg3(300), sg4(300), ac1(300), ac2(300),
+ ac3(300), osg1(300), osg2(300), osg3(300), osg4(300),
+ t1(300), t2(300), t3(300), t4(300), t5(300), t6(300), t7(300),
+ sg10, sg20, sg30, sg40
integer j(300)
character*15 k, cov, codata, coe
print*, 'input k value'
read*, k
cov='ov'//k
print*, cov
codata='oodata'//k
print*, codata
coe='oee'//k
print*, 'input number of data'
read*, num
print*, 'input initial cond.--sg10, sg20, sg30, sg40'
read*, sg10, sg20, sg30, sg40
open(unit=2, file=cov, status='old')
do 10 i=1, num
read(2,*), j(i), sg1(i), sg2(i), sg3(i), sg4(i), ac1(i), ac2(i), ac3(i),
+ t1(i), t2(i), t3(i), t4(i), t5(i), t6(i), t7(i)
10 continue
close(2)
do 15 i=1, num
sg1(i)=sg1(i)-sg10
sg2(i)=sg2(i)-sg20
sg3(i)=sg3(i)-sg30
sg4(i)=sg4(i)-sg40
ac3(i)=ac3(i)/2
15 continue
do 20 l=1, num
osgl(1)=(-3367.3435)*sg1(1)
osg2(1)=(-4370.247*10)*sg2(1)
```
osg3(1)=(9614.46*1.2)*sg3(1)
osg4(1)=(-1313.888*12.)*sg4(1)

20 continue
  open(unit=3,file=coodata,status='new')
  do 30 i=1,num
  write(3,40)j(i),osg1(i),osg2(i),osg3(i),osg4(i),
  + ac1(i),ac2(i),ac3(i),t1(i),
  + t2(i),t3(i),t4(i),t5(i),t6(i),t7(i)
 30 continue
  close(3)
40 format (i3, 7f12.2, 7f6.2)
  open(unit=4,file=coe,status='new')
  do 50 i=1,num
  write(4,60)j(i),
  + ac1(i),ac2(i),ac3(i),
  + t5(i),t6(i),t7(i)
 50 continue
  close(4)
60 format (i3, 3f12.2, 3f6.2)
  stop
  end