EFFECT OF INTERFACE ROUGHNESS ON THE REFLECTION COEFFICIENT

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INTRODUCTION

All surfaces and interfaces between dissimilar materials are rough. The extent to which this roughness affects the reflection of elastic waves has been the subject of study for many decades. A recent book by Ogilvy [1] provides a comprehensive review and summary of this subject. The problem is of particular interest in ultrasonic nondestructive evaluation. For example, it has been shown that for typical inspection geometries, rough surfaces degrade the signal-to-noise ratio for scattering defects [2] - [3].

Most of the theories developed so far for rough surfaces are based on phenomenological approaches. Rough surfaces are treated as structureless surface with certain statistical characteristics. Approximate solutions are derived based on asymptotic analysis and physical intuition. In this paper, an attempt is made to develop a micromechanics approach to study the interaction of waves with rough interfaces. In this approach, a rough surface is viewed as a distribution of asperities on a (smooth) reference surface. From this point view, the rough surface becomes a collection of infinite number of scatterers whose shapes and locations are known statistically. To remedy the multiple scattering problem, a differential self-consistent scheme (DSCS) is used in conjunction with the backscattering signal strength formula [4]. First, we consider the backscattering from only one (typical) asperity on the surface. Once the single scatterer problem is solved, the DSCS allows us to derive an initial value problem whose solution will approximate the multiple scattering result.

To develop such a micromechanics approach, deterministic rough surfaces are considered in this paper. Specifically, we assume (i) the rough surface is a distribution of *identical* asperities; (ii) the distribution of asperities is *uniform* along the surface. In other words, the averages of asperity shape and distribution of a random surface are used in this paper. It should be pointed out that these assumptions are not essential to the theory. As discussed in the next section, similar procedures can be developed when the shape and distribution of the asperities are random variables. The results for random rough surfaces will be published in a separate paper.

Review of Progress in Quantitative Nondestructive Evaluation, Vol. 12
Edited by D.O. Thompson and D.E. Chimenti, Plenum Press, New York, 1993

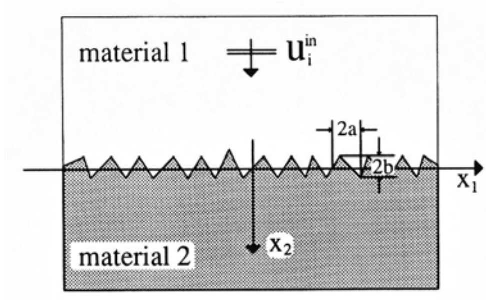

Fig. 1 A rough interface between dissimilar materials

PROBLEM STATEMENT

Consider a rough interface between two dissimilar, isotropic, linearly elastic materials of infinite extent. A Cartesian coordinate system (x_1, x_2, x_3), as shown in Fig. 1, is chosen such that the deviation of the material interface from the smooth reference surface $x_2 = 0$ is given by the function $x_2 = h(x_1)$. Usually, interface roughness is random, *i.e.*, h is part of a continuous random process. The randomness of the interface can be represented by a statistical height distribution $p(h)$, where $p(h)dh$ is the probability of any interface point being at a distance between h and h + dh away from the mean plane $x_2 = 0$. Without loss of generality, we assume

$$\langle h \rangle = \int_{-\infty}^{\infty} hp(h)dh = 0 \quad . \tag{1}$$

From the micromechanics of point view, the rough interface can be viewed as a distribution of asperities along the smooth reference surface as shown in Fig. 1. Three parameters are involved in describing the interface. They are the asperity width $2a$, height $2b$, and density

$$c = aN/L \quad , \tag{2}$$

where N is the number of asperities within distance $2L$. The randomness of the rough interface can be characterized by treating a, b, and c as random variables. However, in this paper, we take them to be deterministic parameters, or equivalently, take them to be their spatial averages. It is shown in next section, c is related to the mean square roughness of the interface. Therefore, c can be viewed as a measure of the degree of roughness.

For convenience, we call the material in the upper half-space material 1, the one in the lower half space material 2. Let λ_i, μ_i be the Lame constants and ρ_i be the mass density, where the subscript $i = 1, 2$ corresponds to the material with which these constants are associated.

Next, let a plane, longitudinal, time-harmonic wave travel in the direction of the positive x_2 axis from $x_2 = -\infty$. Assume the wave has an amplitude factor u_0 and frequency ω. If the steady state term $\exp(-i\omega t)$, which is common to all field variables, is omitted, the displacement field generated by this plane wave in material 1 can be written as

$$u_i^{in} = u_0 \delta_{i2} \exp(ik_L^{(1)} x_2) \quad , \tag{3}$$

where δ_{ij} is the Kronecker delta and $k_L^{(1)}$ is the longitudinal wavenumber in material 1.

Because of the interaction between the incident wave and the interface irregularities, the wave field near the interface is very complicated. In fact, the random portion of the wave field must be described statistically. However, far from the interface, the coherent (or average) portion of the wave field will be dominated by longitudinal plane waves [3]. Therefore, the total displacement for the coherent wave field may be written as

$$u_i = \begin{cases} u_i^{in} + R(c)u_0 \exp(-ik_L^{(1)}x_2) \ , \ x_2 < 0 \\ T(c)u_0 \exp(ik_L^{(2)}) \qquad\qquad , \ x_2 > 0 \end{cases} , \tag{4}$$

where R and T are termed the reflection and transmission coefficients, respectively. Their dependence on the interface roughness is explicitly indicated in (4) by denoting them as functions of c, although R and T also depend on frequencies.

The total displacement field given by (4) implies that, as far as the coherent field is concerned, the rough interface may be treated effectively as a smooth interface with an effective reflection coefficient that depends on the roughness. The objective of this paper is to find R and T in terms of c and the incident frequency.

BACKSCATTERING AMPLITUDE

Backscattering from the rough interface contains two components. One is from the asperities, the other is from the smooth interface. In this section, the signal strength formula derived by Auld [4] will be used to obtain the backscattering from a smooth interface and from a single asperity on the interface, respectively.

Backscattering Signal Strength Formula

For a two transducer system, Auld [1] has derived a steady-state reciprocal relation which can be applied to flaw detection and characterization. Transducer I with power P produces the incident field. Transducer II is the receiver. The ratio of received electrical signal strength over incident signal strength is denoted by Γ. Auld's formula gives the change of Γ due to scattering by an imperfection:

$$\delta\Gamma = [(E_{\mathrm{II}})_{\mathrm{flaw}} - (E_{\mathrm{II}})_{\mathrm{no\,flaw}}]/(E_{\mathrm{I}})_{\mathrm{flaw}} , \tag{5}$$

where E_{I} and E_{II} are the strengths of the electrical signals in transducer I and II, respectively. For backscattering, (5) is simplified to

$$\delta\Gamma = -\frac{i\omega}{4P}\int_S (\sigma_{kj}^{(2)}u_k^{(1)} - \sigma_{kj}^{(1)}u_k^{(2)})n_j dS \ , \tag{6}$$

where S is an arbitrary surface which surrounds the scatterer and n_j is the normal of the surface defined positive inward. The quantities $\sigma_{kj}^{(1)}$ and $u_k^{(1)}$ are the stress and displacement fields induced by the exciting transducer I with power P in the absence of the scatterer, while $\sigma_{kj}^{(2)}$ and $u_k^{(2)}$ are the stress and displacement fields in the presence of the scatterer. One may also call $\sigma_{kj}^{(1)}$ and $u_k^{(1)}$ the incident fields and call $\sigma_{kj}^{(2)}$ and $u_k^{(2)}$ the total fields.

Backscattering from the Interface

In this section, we consider a smooth interface with reflection coefficient R. For the incident wave given in (3), the relevant displacement and stresses on the interface are:

Incident fields: $u_2^{(1)} = u_2^{in} = u_0$, $\sigma_{22}^{(1)} = \sigma_{22}^{in} = ik_L^{(1)} u_0 (\lambda_1 + 2\mu_1)$ (7)

Total fields: $u_2^{(2)} = u_2 = u_0(1+R)$, $\sigma_{22}^{(2)} = \sigma_{22} = ik_L^{(1)} u_0 (\lambda_1 + 2\mu_1)(1-R)$. (8)

If the entire lower half-space is considered as a scatterer, the backscattering from the lower half-space can be calculated through (6). For practical purposes, let us assume that the incident beam has a bounded cross-section. Let (-L,L) denote the insonified region on the interface by the incident beam. Then, (6) becomes

$$\delta\Gamma_1 = -\frac{i\omega}{4P}\int_{-L}^{L}(\sigma_{22}u_2^{in} - \sigma_{22}^{in}u_2)dx_1 \quad . \tag{9}$$

Making use of (7) - (8) in (9) yields

$$\delta\Gamma_1 = u_0^2\left(\frac{-i\omega}{4P}\right)\left[4i(\lambda_1 + 2\mu_1)k_L^{(1)}RL\right] \ , \tag{10}$$

where, again, L is the half-length of the insonified region by the incident beam on the interface.

<u>Backscattering from an Asperity</u>

Consider a single asperity on the interface. When the incident wave u_i^{in} is given by (3), the total wave field can be decomposed into three components. For example, the total wave fields in material 1 ($x_2 < 0$) can be written as

$$u_i^{(2)} = u_i^{in} + u_i^r + u_i^s \ , \ \sigma_{ij}^{(2)} = \sigma_{ij}^{in} + \sigma_{ij}^r + \sigma_{ij}^s \quad , \tag{11a,b}$$

where $u_i^{in}, \sigma_{ij}^{in}$ are the incident waves, u_i^r, σ_{ij}^r are the reflected waves from the interface in the absence of the asperity, and u_i^s, σ_{ij}^s are the scattered waves from the asperity in the absence of the interface.

To obtain the backscattering from the asperity only, we choose the incident field in (6) to be

$$u_i^{(1)} = u_i^{in} + u_i^r \ , \ \sigma_{ij}^{(1)} = \sigma_{ij}^{in} + \sigma_{ij}^r \tag{12a,b}$$

Substitution of (11a,b) and (12a,b) into (6) yields

$$\delta\Gamma_2 = u_0^2\left(\frac{-i\omega}{4P}\right)\left[i(\lambda_1 + 2\mu_1)k_L^{(1)}af(R)\right] \ , \tag{13}$$

where, in many cases of practical interest,

$$f(R) = pR^2 + 2qR + r \quad , \tag{14}$$

and p, q and r are constants dependent of the frequency and asperity geometry.

DIFFERENTIAL SELF-CONSISTENT SCHEME

Differential self-consistent scheme (DSCS) has been used extensively in the area of micromechanics of composite materials. In this section, the DSCS is used to derive a differential equation for the effective reflection coefficient $R(c)$ defined in (4).

The DSCS is based on the notion of incremental construction of the backscattering amplitude by adding one asperity at a time to the interface. Suppose that at a given asperity density (measure of roughness) c, the interface is treated as a smooth one with effective reflection coefficient $R(c)$. The backscattering amplitude from this effectively smooth interface can be obtained from (10). The fundamental assumption of DSCS is that when an additional asperity is added to the interface, the change in backscattering due to this addition is the backscattering from a single interface asperity. This procedure results in an initial value problem for the effective reflection coefficient R. To accomplish the DSCS procedure, let us consider the following three problems:

Problem 1: Assume the interface has an asperity density $c = aN/(L - a)$.

For this problem, if the rough interface is treated as a smooth interface with effective reflection coefficient $R(c)$, the backscattering from the effectively smooth interface can be calculated from (10)

$$\delta\Gamma_1(c) = u_0^2\left(\frac{-i\omega}{4P}\right)\left[4i(\lambda_1 + 2\mu_1)k_L^{(1)}R(c)L\right] \quad . \tag{15}$$

Problem 2: Assume the interface has an asperity density $c_1 = a(N+1)/L$.

Again, if the rough interface is treated as a smooth interface with effective reflection coefficient $R(c_1)$, the backscattering from the effectively perfect interface also can be calculated from (10)

$$\delta\Gamma_1(c_1) = u_0^2\left(\frac{-i\omega}{4P}\right)\left[4i(\lambda_1 + 2\mu_1)k_L^{(1)}R(c_1)L\right] \quad . \tag{16}$$

Problem 3: Assume an asperity of width $2a$ is located on the interface having effective reflection coefficient $R(c)$, where $c = aN/(L - a)$.

It is conceivable that the total backscattering in this problem contains two components. One is from the asperity, the other is from the effective interface. The component from the asperity is given by (13)

$$\delta\Gamma_2 = u_0^2\left(\frac{-i\omega}{4P}\right)\left[i(\lambda_1 + 2\mu_1)k_L^{(1)}af[R(c)]\right] \quad . \tag{17}$$

The component from the effective smooth interface with effective reflection coefficient $R(c)$ is given by (15). Therefore, the total backscattering in Problem 3 is the sum of (15) and (17).

On the other hand, let us consider an interface having asperity density aN/L, *i.e.*, there are N asperities in the region $[-L, L]$. If we want to add an additional asperity of width $2a$ to this region, we must rearrange the existing N asperities so that a region of length $2a$ becomes available to accommodate the new one. Therefore, after the addition, the actual region occupied by the previous N asperities is reduced to $(2L - 2a)$. This means that the asperity density outside the newly added asperity is $aN/(L - a)$ instead of the original aN/L. However, the actual asperity density in $[-L, L]$ should become $a(N+1)/L$ after the addition. This observation indicates that Problem 3 can be equivalently stated as the consequence of adding one more asperity to an interface with asperity density aN/L. Since both Problem 3 and Problem 2 have the same asperity density $a(N+1)/L$, they should have the backscattering.

Based on the reasoning above, the DSCS states that the backscattering amplitude from Problem 2 is the sum of those from Problem 1 and Problem 3, namely,

$$\delta\Gamma_1(c_1) = \delta\Gamma_1(c) + \delta\Gamma_0(c) \quad . \tag{18}$$

Substituting (15) - (17) into (18) yields

$$4R(c_1)L = 4R(c)L + af[R(c)] \quad , \tag{19}$$

or

$$\frac{R(c_1) - R(c)}{c_1 - c} = \frac{af[R(c)]}{4L(c_1 - c)} \quad . \tag{20}$$

Since

$$\frac{a}{L(c_1 - c)} = \frac{1}{1-c} \quad ,$$

it follows from (21) that

$$\frac{R(c_1) - R(c)}{c_1 - c} = \frac{f[R(c)]}{4(1-c)} \quad . \tag{21}$$

In the limit $c_2 \to c$, (20) becomes

$$\frac{dR}{dc} = \frac{f(R)}{4(1-c)} \quad , \tag{22}$$

which is a first order differential equation for the reflection coefficient R as a function of asperity density c.

An initial condition is required to uniquely determine $R(c)$ from (22). Since c is the asperity density, or a measure of the roughness, it is obvious that $c = 0$ means no asperity, *i.e.*, smooth interface. In this case, the reflection coefficient R from a smooth interface is well know [5]

$$R(0) = R_0 = \frac{\rho_1/k_L^{(1)} - \rho_2/k_L^{(2)}}{\rho_1/k_L^{(1)} + \rho_2/k_L^{(2)}} \quad . \tag{23}$$

With (23), the differential equation (22) can be solved to yield a unique solution. To illustrate this general approach, an example is given in the next section.

EXAMPLE AND DISCUSSION

Consider an interface between Al and air. The surface asperities of the aluminum are assumed rectangular as shown in Fig. 2. It also assumed that both a and b are much smaller than the incident wavelength. Therefore, the scattered field due to one asperity can be approximated by the quasi-static solution. In doing so, the three constants in (14) are obtained as,

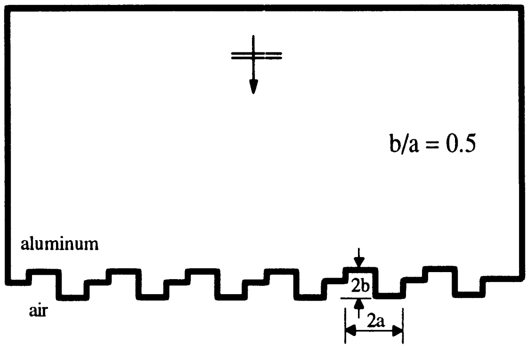

Fig. 2 Aluminum plate with square asperities

$$p=\beta-\alpha-\gamma\exp(-ik_L b) \;,\; q=2[i\gamma\sin(k_L b)-\beta] \;,\; r=\alpha+\beta+\gamma\exp(ik_L b) \quad (24\text{-}26)$$

where

$$\alpha=-2\cos(2k_L b) \;,\; \beta=(2iak_L/\pi)(c_L/c_T)^2(1-\upsilon)\cos^2(k_L b) \;, \quad (27\text{-}28)$$

$$\gamma=(ak_L \ln 2/\pi)(c_L/c_T)^2(1-\upsilon)\sin^2(k_L b) \;. \quad (29)$$

In (24) - (29), c_L and c_T are, respectively, the longitudinal and shear phase velocities in Al, and υ is the Poisson's ratio of aluminum.

Make use of these constants in (22) and (23), we arrive at

$$R(c)=\frac{w(q+d)(1-c)^{d/2}+d-q}{p[1-w(1-c)^{d/2}]}=|R(c)|e^{i\theta} \;, \quad (30)$$

where

$$d=(q^2+pr)^{1/2} \;,\; w=(p+q-d)/(p+q+d) \;. \quad (31)$$

This gives the reflection coefficient as a function of asperity density c. The frequency dependence of R comes from the constants p, q, and r.

To relate c to the surface roughness, we use the root mean square height, σ, to measure the degree of roughness. For this example, it is found that

$$\sigma=\sqrt{\frac{1}{L}\int_0^L h(x)dx}=b\sqrt{c} \;. \quad (32)$$

Substitution of (32) into (30) yields

$$R(\sigma)=\frac{w(q+d)(b^2-\sigma^2)^{d/2}+(d-q)b^2}{p[b^2-w(b^2-\sigma^2)^{d/2}]}=|R(\sigma)|e^{i\theta} \;. \quad (33)$$

Note that (33) is valid for low frequencies because of the use of quasi-static solution in the scattering problem. It is found that for $k_T a \le 1$ the amplitude of the reflection coefficient is unit, i.e., $|R(\sigma)|=1$. This is in agreement with the result of [6]. However, the phase of the reflection coefficient, θ, is affected by the roughness. Fig. 3 shows θ as a function of $k_T a$ for several values of the root mean square roughness, σ.

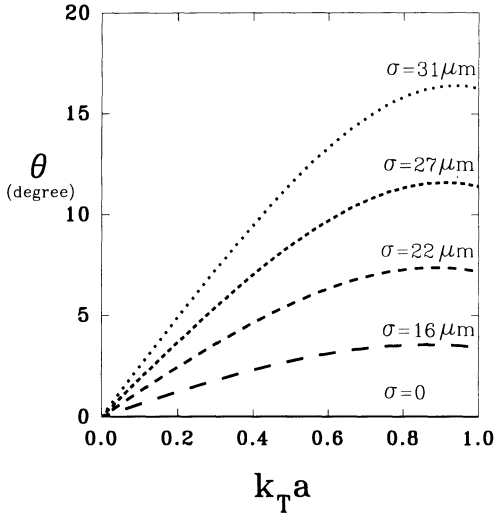

Fig. 3 Phase of the reflection coefficient vs. frequency for various roughness

As one can see from the derivation that the solution given by (33) is an approximation. It is valid for small $k_T b \ll 1$, which can be either interpreted as small roughness or low frequency. Therefore, (33) can be viewed as an asymptotic solution for small $k_T b$. The fact that roughness changes the phase of R, but not the amplitude of R seems to indicate the effect of roughness on the amplitude is secondary to the effect of roughness on the phase. This observation confirms the validity of the phase-screen approximation [3], which assumes that only the phase of the wave field is affected by surface roughness.

ACKNOWLEDGMENTS

This work is supported in part by NSF through Grant MSS-9119291.

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