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The application of aquifer testing evaluation techniques to oxidized till confining units in Central Iowa

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The application of aquifer testing evaluation techniques to oxidized till confining units in Central Iowa

by

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NOMENCLATURE

A = coefficient used by Hazen method, dimensionless

b = saturated thickness of the aquifer, ft.

d_{xx} = particle size for which xx% of particles are smaller, m

h = head, ft.

h_i = head in the ith well, ft.

H = saturated thickness at the beginning of the test, ft.

H_o = initial saturated thickness of the aquifer, ft.

K = hydraulic conductivity, cm/s

Q = discharge rate of pumping well, gpm

r = radial distance from the pumping well, ft.

r_i = radial distance of the ith well, ft.

r_w = radius of the pumping well, ft.

s = drawdown calculated by the Theis equation, ft.

s_d = dimensionless drawdown, dimensionless

s_i = observed drawdown in the ith well, ft.

s_{jc} = correction drawdown using the Jacob's correction, ft.

s_{pi} = calculated drawdown in the ith well, ft.

$s_w(t)$ = drawdown in the pumping well at time t, ft.

S = storativity, dimensionless

S_c = storage coefficient, dimensionless

S_s = specific storage, ft^{-1}

S_y = specific yield, dimensionless

S_{ya} = apparent specific yield, dimensionless

t = time since the start of pumping, min.

T = transmissivity, cm^2/s

T' = initial transmissivity, cm^2/s

u = well function coefficient, dimensionless

$V_{DC}(t)$ = volume of drawdown cone at time t, ft^3

V_p = volume of water removed from the aquifer, ft^3

$W(u)$ = confined aquifer well function, dimensionless

x = linear coordinate in the x direction, ft.

y = linear coordinate in the y direction, ft.

z = linear coordinate in the z direction, ft.

σ_1 = tracking function for the Masch and Denny Method, dimensionless

Φ = force potential

ϕ_i = negative logarithm to the base 2 of the particle size d_i, dimensionless

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INTRODUCTION

The ability to predict the flow of groundwater has always been a concern when attempting to remediate a groundwater contamination problem. In the past, a majority of the attention was given to contamination problems that occurred in typical aquifer materials (i.e., sand and gravel or limestone). Recently, government regulations have forced the focus to be shifted toward materials that have typically been considered confining units, such as till and other low hydraulic conductivity material.

A large number of the remedial activities focus on the removal of a contaminant such as petroleum products or pesticides from these low hydraulic conductivity materials. Although these materials will yield appreciable quantities of water, pumping in these areas is generally typified by a small radius of influence and low discharge rates. This makes groundwater recovery operations in these units costly and time consuming.

The lack of proper design plagues many remediation sites throughout Iowa. Many of the problems that currently exist are due to lack of data and available modeling techniques. Many researchers have approached this problem using numerical methods and other techniques. Solutions of this type are useful in a research setting, but lose their appeal when confined by the monetary constraints imposed by the consulting industry. The literature review performed prior to the preparation of this paper revealed few articles that discuss the effect of pumping in these formations. In settings of this type, little or no data is available dealing with the use of commonly accepted analytical solutions, such as the Theis method. The use of aquifer testing in these types of formations could be immense.

It is usually assumed that these till materials have low hydraulic conductivities (10^{-4} to 10^{-10} cm/s) (Freeze and Cherry, 1979). Due to this large range of conductivities, tracking of a contaminant plume in this material is almost impossible. This indicates the need for a quick method of determining the hydraulic conductivity of a site. Slug testing has become popular with a large number of consultants because of it's ease of performance and data analysis. Unfortunately, this method only provides information about a small area surrounding the well. Conversely, aquifer testing provides an average value for the aquifer parameters throughout the area of influence. The parameters determined by the aquifer testing should provide a better idea of the hydraulic characteristics of the aquifer. Because of this, it is necessary to test the applicability of existing analytical solutions when applied to testing in these non-typical formations.

To test the applicability of existing solutions, an aquifer test was performed in a till unit near Ames, Iowa. The test consisted of the pumping of a fully penetrating well for twenty-four hours. Water levels were measured in twenty-one wells surrounding the pumping well. Following the completion of the testing, several variations of the Theis solution were used to model the drawdown data. The results of the testing and a comparison of the different methods is discussed in the following report.

The goals of the aquifer test at this site were as follows:

- Examine the response of an unconfined till unit to pumping.
- Determine the applicability of current confined aquifer analysis techniques.
- Determine the effect of various correction schemes on the parameter determinations.
- Compare the hydraulic conductivity obtained from mechanical analysis, slug and aquifer testing.
- Compare the hydraulic conductivity and specific yield obtained from the various particle size relationships available.

These and other relationships will be examined and discussed in the text that follows.

SITE INFORMATION

The site is located in Field 5 of the Iowa State University (ISU) Agronomy/Agricultural Engineering Farm west of the city of Ames, Iowa. The location of the aquifer test site is shown in Figure 1. Figure 2 shows the location of the test site within Field 5. The aquifer test site is currently seeded in grass and is not expected to be cropped in the near future.

The well field is approximately centered between the surrounding field drainage tiles. These tiles are spaced approximately 120 feet on center. The presence of these tiles causes the water table in the area to experience a natural decline. This decline is highest immediately following large precipitation events, although it quickly declines. The tiles impact the overall hydrology of the site, however the effect on the water table during the aquifer testing is minimal.

Appendix A (Lemar, 1991) describes the methods and materials used for drilling, sampling and well completion. This Appendix also describes the installation of the wells and well nests at the site.

Following completion, the wells were surveyed and spatial locations determined. The placement and numbering of the wells is shown in Figure 3. These top of casing (TOC) elevations, along with coordinates and radial distances from the pumping well are shown in Table 1.

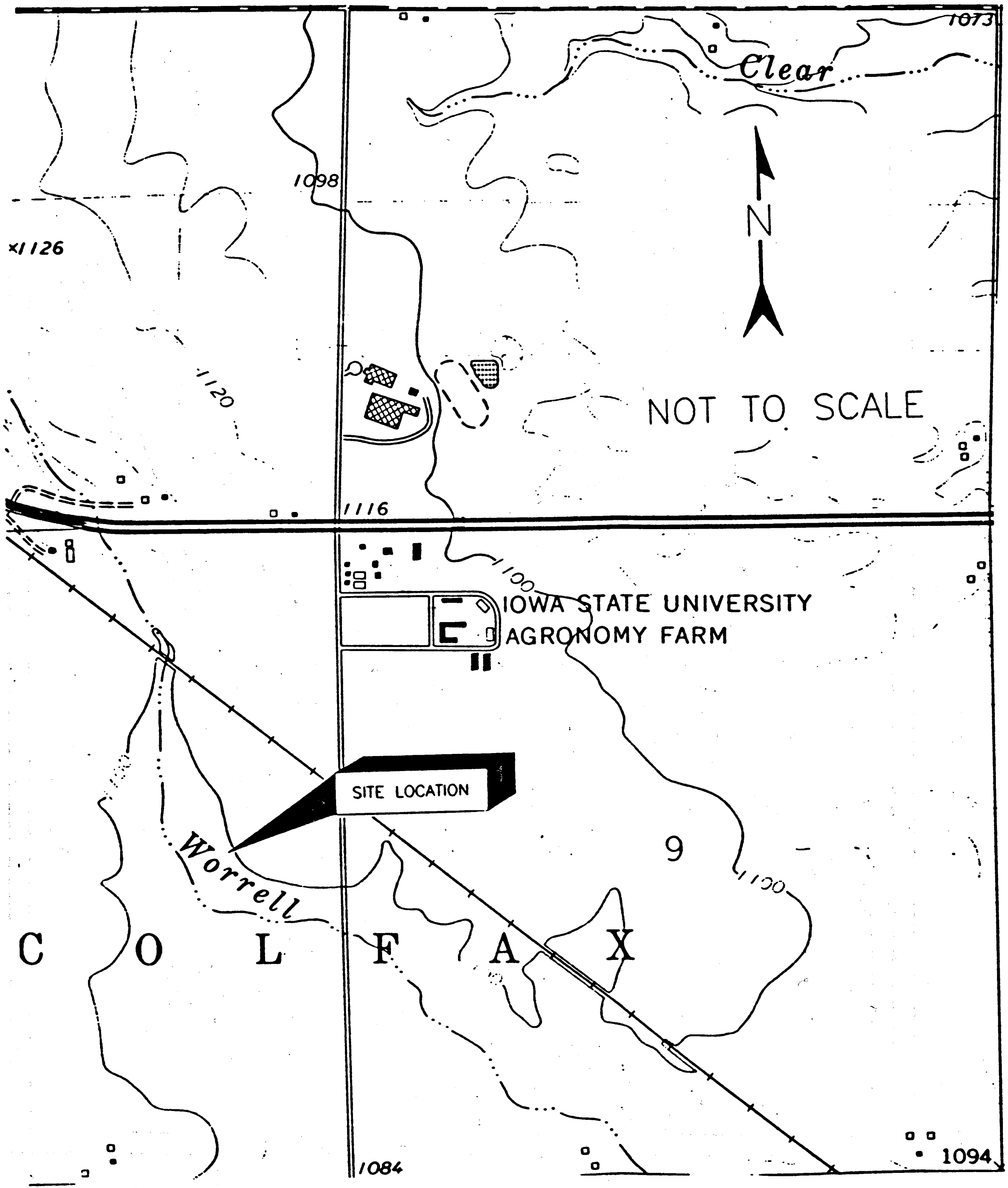

Figure 1. Location of aquifer test site

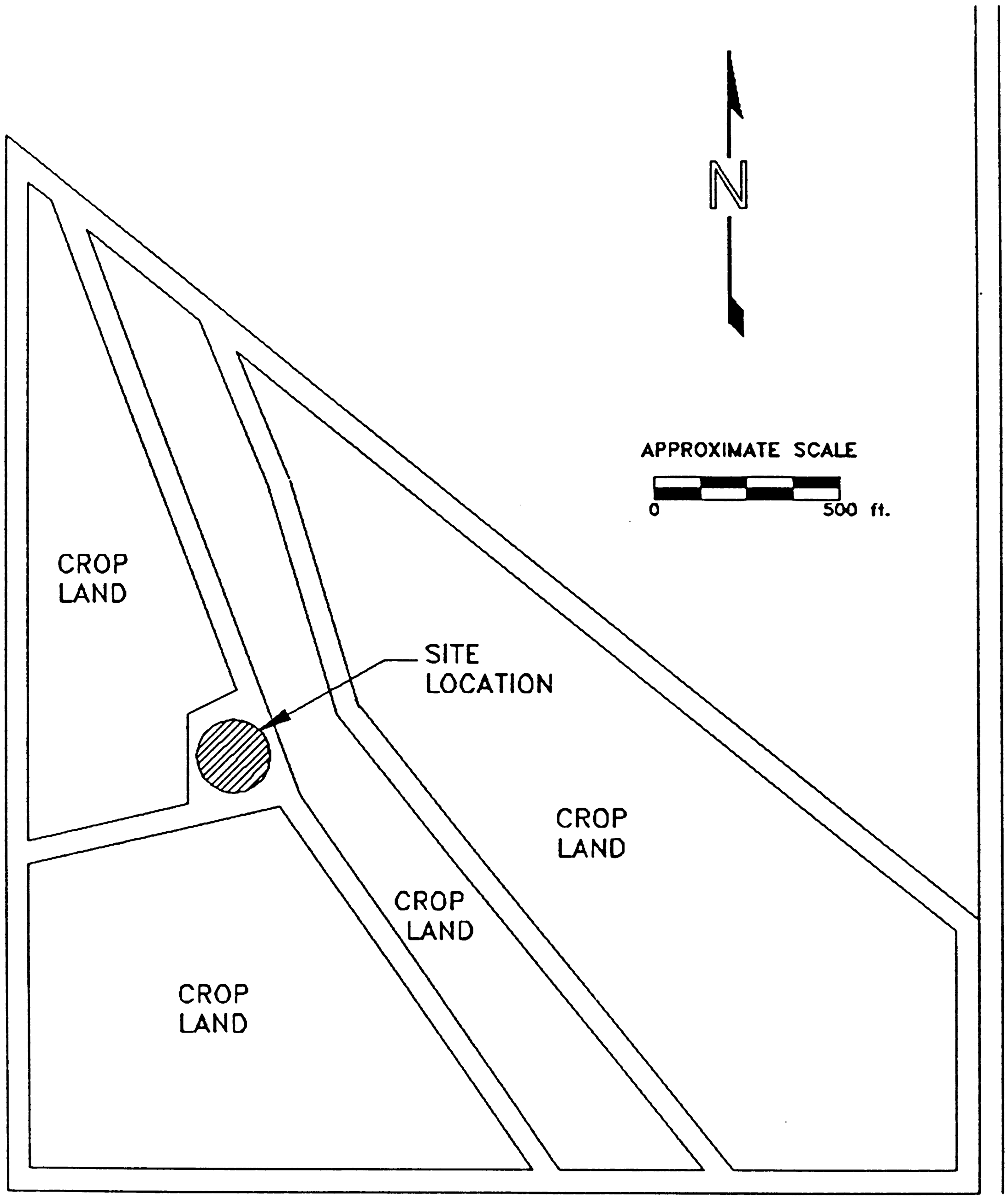

Figure 2. Location of well field in Field 5

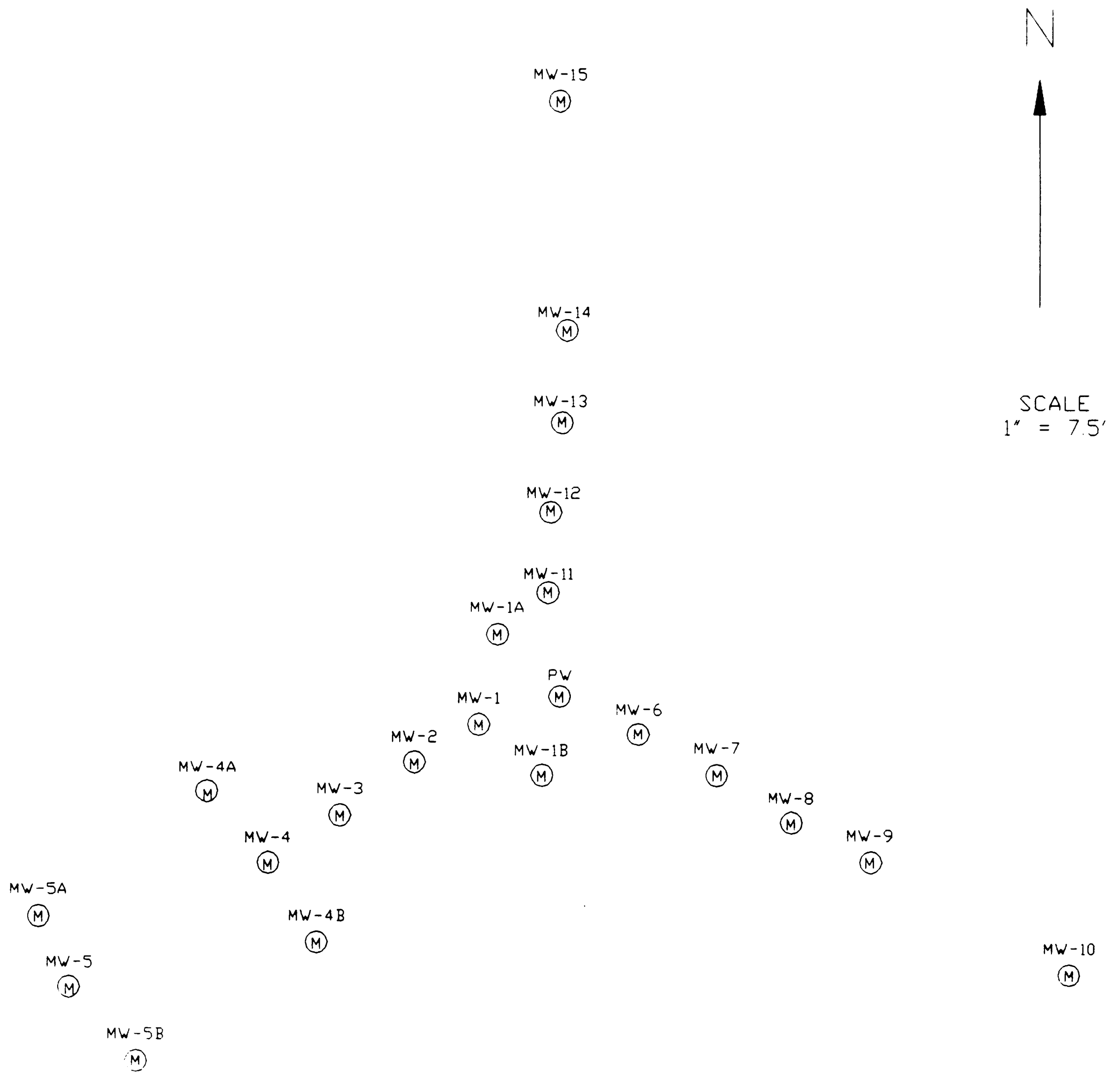

Figure 3. Spatial locations of wells

Table 1. Spatial Location of Monitoring Wells

Well No.	X[a] Coordinate ft.	Y[a] Coordinate ft.	Radial[b] Distance ft.	Top of Casing Elevation ft.
1	0.72	2.83	2.92	16.63
1A	2.98	0.85	3.10	16.58
1B	-1.88	2.05	2.78	16.69
2	0.63	5.51	5.55	16.48
3	0.40	8.59	8.60	16.48
4	0.30	11.57	11.57	16.46
4A	3.44	12.12	12.60	16.34
4B	-2.88	11.39	11.75	16.60
5	0.00	19.60	19.60	16.17
5A	2.64	19.22	19.40	16.31
5B	-3.22	18.88	19.15	16.18
6	-2.29	-1.70	2.85	16.65
7	-4.91	-3.27	5.90	16.87
8	-7.49	-4.80	8.90	16.77
9	-10.02	-6.38	11.88	16.92
10	-16.76	-10.41	19.73	16.95
11	3.29	-1.28	3.53	16.64
12	5.77	-2.88	6.45	16.64
13	8.20	-4.60	9.40	16.34
14	10.91	-6.30	12.60	16.64
15	17.81	-10.01	20.43	16.53
PW	0.00	0.00	0.00	16.69

[a] X and Y coordinates were determined using PW as the (0,0) point of the coordinate system. The direction of the coordinate axes is shown in Figure 3.

[b] Radial distance is measured with respect to PW.

HYDROGEOLOGY

For the purpose of this investigation, the scope was limited to the uppermost 15 to 20 feet. Figure 4 shows the general stratigraphy of the area.

Groundwater was encountered at a depth of approximately five feet while drilling. Upon completion of drilling and well development, the water table rose to approximately 0.5 to 1.5 feet below the TOC. Based on the apparent hydraulic conductivity difference between the oxidized and unoxidized till, it appears that a majority of groundwater flow is occurs in the oxidized till. Based on this assumption, the saturated thickness was measured as the height of saturated soil above the unoxidized till layer. The saturated thickness of the area ranges from 7.84 to 10.55 feet with an average thickness of 9.13 feet. The approximate depth of the unoxidized till was determined from samples extruded from shelby tubes obtained during drilling at the site. There appears to be no significant trend in the slope of the underlying confining layer at this site. Groundwater elevations and saturated thicknesses for each well measured on July 13, 1990 are given in Table 2.

Geology

The geology consists of approximately 63 feet of late Wisconsin till overlying older Pre-Illinoian till units and coarse-grained sediments. The upper 12.5 feet of the till is oxidized (Simpkins, 1990). Bedrock is encountered at a depth of approximately 300 feet. This bedrock provides water for several residential wells in the area.

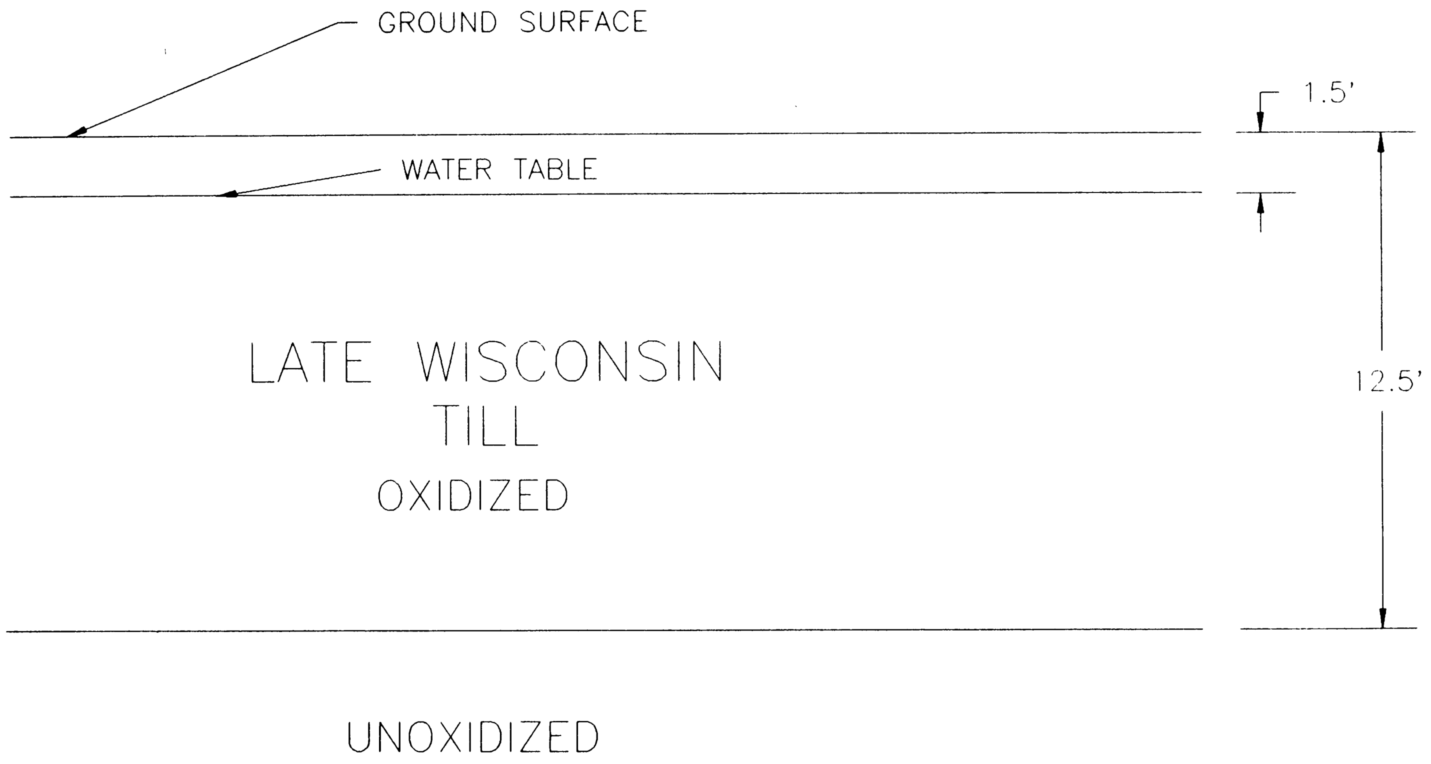

Figure 4. General Stratigraphy of the area

Table 2. Hydrogeological data

Well No.	Top of Casing Elevation ft.	Depth[a] to Water ft.	Ground Water Elevation ft.	Saturated Thickness ft.
1	16.63	2.42	14.21	10.08
1A	16.58	2.29	14.29	(1)[b]
1B	16.69	2.36	14.33	(1)
2	16.48	2.31	14.17	10.04
3	16.48	2.43	14.05	9.43
4	16.46	2.50	13.96	8.86
4A	16.34	2.51	13.83	(1)
4B	16.60	2.53	14.07	(1)
5	16.17	2.52	13.65	8.35
5A	16.31	2.73	13.58	(1)
5B	16.18	2.42	13.76	(1)
6	16.65	2.18	14.47	7.84
7	16.87	2.33	14.54	7.97
8	16.77	2.15	14.62	8.14
9	16.92	2.22	14.70	8.12
10	16.95	1.91	15.04	8.32
11	16.64	2.28	14.36	8.92
12	16.64	2.24	14.40	10.25
13	16.34	1.94	14.40	10.55
14	16.64	2.24	14.40	9.45
15	16.53	2.20	14.33	10.30
PW	16.69	2.33	14.36	8.56

[a] Depth to water on August 2, 1990
[b] (1) = Cannot be determined exactly

Structural Properties

The structural properties of the soils taken during the drilling of the site are shown in Table 3. The porosity of the oxidized till material ranged from 29% to 33% with an average of 32%. The mass water content ranged from 15% to 18%, with an average of 18%. The bulk density of the material averaged 1.84 g/cm with a range of 1.79 to 1.89 g/cm.

From the particle size analysis, the USDA soil classifications for the oxidized and unoxidized layers were found to be loam, clay loam or sandy clay loam. All soils taken from the site exhibited similar particle size distributions. The particle size distribution and USDA classification of the oxidized and unoxidized tills are surprisingly similar.

Hydrogeology

Water levels obtained just prior to the start of the pumping test are shown in Table 4. Using methods described by Pinder et al. (1981), the horizontal and vertical gradients were determined. This method assumes the phreatic surface is linear across the site. This assumption should be reasonable, even though the drainage to the tiles will generate a slightly parabolic surface. Using the x and y coordinates and elevations of the center of the screen, the gradients shown in Table 5 were calculated.

Table 3. Structural properties of area soils

Well No.	Moisture Content %	Bulk Density g/cm	Porosity %
1	17.17	1.83	31.97
1A	(1)[a]	(1)	(1)
1B	(1)	(1)	(1)
2	17.65	1.82	32.34
3	(1)	(1)	(1)
4	16.28	1.85	31.23
4A	(1)	(1)	(1)
4B	(1)	(1)	(1)
5	15.40	1.89	29.74
5A	(1)	(1)	(1)
5B	(1)	(1)	(1)
6	18.34	1.79	33.46
7	18.03	1.85	31.23
8	17.65	1.83	31.97
9	17.70	1.83	31.97
10	18.27	1.83	31.97
11	17.51	1.85	31.23
12	18.00	1.82	32.34
13	17.39	1.86	30.86
14	18.16	1.81	32.71
15	17.62	1.84	31.60
PW	18.52	1.84	31.60
Average[b]	17.58	1.84	31.75
Maximum	18.52	1.89	33.46
Minimum	15.40	1.79	29.74

[a] (1) = Sample not obtained while drilling

[b] Average = Arithmetic average

Table 4. Groundwater elevations prior to aquifer testing

Well No.	Ground Water Elevation ft.
1	14.21
1A	14.29
1B	14.33
2	14.17
3	14.05
4	13.96
4A	13.83
4B	14.07
5	13.65
5A	13.58
5B	13.76
6	14.47
7	14.54
8	14.62
9	14.70
10	15.04
11	14.36
12	14.40
13	14.40
14	14.40
15	14.33
PW	14.36

Table 5. Water table gradients at the site

Direction	Gradient Magnitude (ft/ft)
X	0.019
Y	0.034
Z	0.006

The radial gradient had a magnitude of 0.039 ft/ft and was generally in a northwesterly direction. The vertical gradient was determined to be in an upward (positive) direction. This may be due to the influence of the tiles in the area. Well nest 4 and 5 indicate a higher vertical gradient exists at the outer edge of the well field. Prior to testing, the three dimensional gradient had a magnitude of 0.044 ft/ft.

The groundwater flow contours are shown in Figure 5. The direction of groundwater flow is also shown in this figure.

Due to the tiles discussed previously, the water table at the site experiences a natural decline. During the aquifer test, the natural decline was approximately 0.01 ft/day. It was felt this decline did not influence the results, due to the relatively short time of the aquifer testing.

Hydraulic Conductivity

The unoxidized till appears to form a lower confining layer for the phreatic (unconfined) aquifer. Although no hydraulic testing was performed in the confining layer, research and a review of existing literature indicates the hydraulic conductivity of this layer to be less than 10^{-6} cm/s.

Hazen Method

In order to approximate the hydraulic conductivity from the particle size analysis, the Hazen approximation was used. The Hazen formula is as given below:

$$K\text{-}Ad_{10} \tag{1}$$

For d_{10} in mm and K in cm/s, the coefficient A is equal to 1.0 (Freeze and Cherry, 1979).

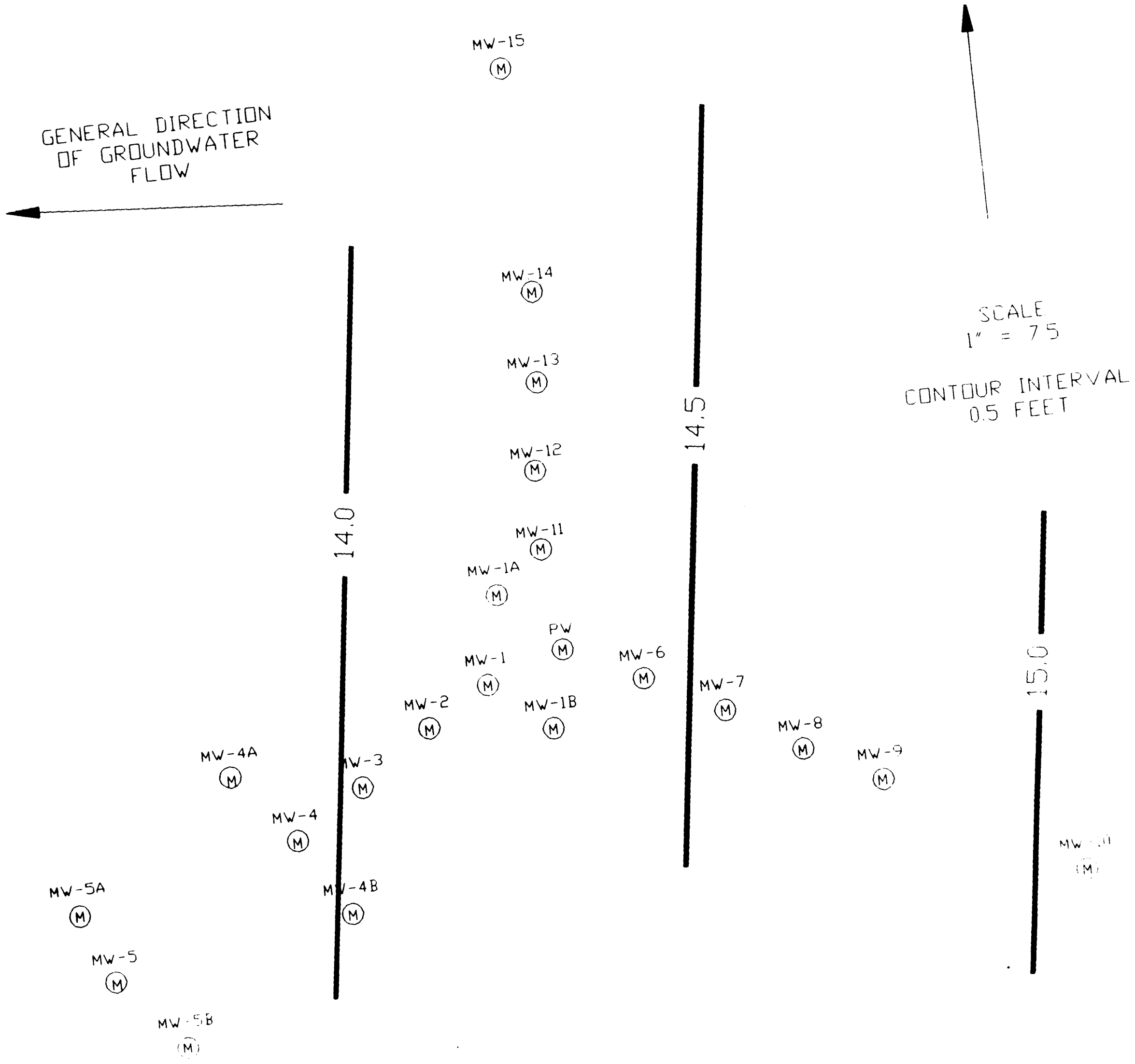

Figure 5. Groundwater flow contours

The Hazen approximation was used to approximate the hydraulic conductivity. The average d_{10} for the samples tested was 0.002 mm. The Hazen approximation yielded a hydraulic conductivity of 4×10^{-6} cm/s.

Masch and Denny Method

In an attempt to produce a better estimate of the hydraulic conductivity, the Masch and Denny method was used to account for the spread of the gradation curve (Freeze and Cherry, 1979) (Masch and Denny, 1966). The average grain sizes are shown in Table 6.

Table 6. Pertinent Soil Classifications for use with the Masch and Denny Method

Rating	Size (mm)	ϕ[a]
d_5	0.001	9.966
d_{16}	0.005	7.644
d_{50}	0.070	3.387
d_{84}	0.400	1.322
d_{95}	1.500	-0.585

[a] ϕ is equal to the negative logarithm to the base 2 of the particle diameter.

Using the equation shown below, the estimating parameter was determined.

$$\sigma_1 = \frac{(\phi_{16}+\phi_{84})}{4} + \frac{(\phi_5-\phi_{95})}{6.6} \tag{2}$$

From the above equations and the data presented in Table 6 above, a σ_1 of 3.17 was determined. Although the work presented by Masch and Denny does not allow an estimate of the hydraulic conductivity, an estimate of the maximum hydraulic conductivity was made. Tracking the value of σ_1 and the above referenced d_{50} on the graph presented in Masch and Denny and using the predictive techniques mentioned in their text, a maximum hydraulic conductivity of 4×10^{-3} cm/s was predicted.

Review of the work by Masch and Denny, reveals the above referenced equations were developed using sand and gravel only. No reference is made as to whether or not this method can be extended to the soils encountered at the test site.

A comparison of the results of the Hazen and Masch and Denny approximations show the two methods vary by three orders of magnitude. It is important to point out that both methods were developed using much coarser material then is present at the test site. Again, this points out the fact that little research has been performed on non-typical aquifer materials. It is very difficult to analyze the "correctness" of either of the estimates without determining the permeability of the soil through laboratory experiments.

WELL DEVELOPMENT PROCEDURES

Prior to aquifer testing, each well was thoroughly developed. Development was performed to stabilize and remove any excess sediments from the gravel pack. The well development was accomplished by surging fresh water into and out of the well bore. This procedure is similar to that recommended by the United States Environmental Protection Agency (USEPA) for low yield formations (USEPA, 1985). Literature indicates this procedure may increase the production of the well by as much as 10% (Driscoll, 1988).

The well development was accomplished using a vacuum pump and tap water. During this procedure, care was taken to assure that a greater volume of water was removed from the formation than was introduced. Several of the wells required multiple developments to reduce the turbidity of the water.

After the completion of well development activities, the wells were allowed to equilibrate for approximately one week prior to any further testing at the site.

SLUG TESTING

Following development, each well was slug tested to evaluate the hydraulic conductivity of the formation surrounding the well. The testing was performed by pumping the wells down to a point below the screened interval and then measuring the time/recovery data.

Several tests were terminated prior to full recovery, due to the slow recovery of the wells. A majority of the wells were retested to evaluate the accuracy of the initial test. Wells which exhibited long recovery times were not retested.

The slug test data was evaluated using Hvorslev's method (Hvorslev, 1951). In order to perform the analysis efficiently, a computer program was developed. A copy of this computer program is available from the author upon request.

Table 7 lists the hydraulic conductivity obtained from this testing. As the table shows, the hydraulic conductivity of the wells ranges from 1.31×10^{-4} (MW-4) to 8.71×10^{-6} cm/s (MW-5b). The variance, arithmetic mean and geometric mean of the hydraulic conductivity data are given in the table. The geometric mean is recommended for use by most groundwater texts (Fetter, 1980).

Using the hydraulic conductivities and the saturated aquifer thicknesses, the geometric mean of the transmissivity was determined to be 0.039 cm^2/s. The calculated transmissivity ranges from 0.001 to 0.087 cm^2/s.

A comparison of the conductivities determined from particle size analyses and the slug testing reveals the prediction made

Table 7. Results of slug testing using Hvorslev's method

WELL NO	Hydraulic Conductivity cm/s	Saturated Thickness ft	Transmissivity T cm^2/s	Basic Time Lag minutes
MW-1	1.87E-04	10.08	0.0575	3.88
MW-2	1.64E-04	10.19	0.0509	4.01
MW-3	8.71E-05	9.43	0.0250	8.02
MW-4	1.31E-04	8.86	0.0354	5.17
MW-5	4.73E-06	8.50	0.0012	142.41
MW-6	2.73E-04	7.84	0.0652	2.32
MW-7	3.05E-04	7.97	0.0741	2.07
MW-8	2.10E-04	8.14	0.0521	3.20
MW-9	2.56E-04	8.12	0.0634	2.65
MW-10	2.81E-04	8.32	0.0713	2.25
MW-11	1.83E-04	8.92	0.0498	3.69
MW-12	2.59E-04	10.25	0.0809	2.60
MW-13	2.71E-04	10.55	0.0871	2.37
MW-14	2.55E-04	9.45	0.0734	2.54
MW-15	2.22E-05	10.30	0.0070	30.22
PW	1.88E-04	8.95	0.0513	3.76
MEANS				
ARITHMETIC	1.93E-04	9.13	0.0530	14.49
GEOMETRIC	1.40E-04	NA[a]	0.0387	NA
VARIANCE	8.39E-09	0.87	0.0006	1215.31
MAXIMUM	3.05E-04	10.55	0.0871	142.41
MINIMUM	4.73E-06	7.84	0.0012	2.07

[a] NA indicates not applicable

by Masch and Denny to apparently provide a more correct result. This is hard to evaluate however, because the Masch and Denny method could only provide a maximum value of hydraulic conductivity. It does, however, point out that these materials are more permeable than the Hazen method indicates. This is extremely important in that the Hazen method is used more frequently than the Masch and Denny method.

AQUIFER TESTING

The aquifer test was started at 1027 hours on August 2, 1990 and completed at 1026 hours on August 3, 1990. Thus, the total duration of the aquifer test was 23 hours and 56 minutes. The well designated as PW was used as the pumping well.

The flow rate was checked and adjusted every three minutes throughout the test in an attempt to maintain a constant flow rate of 600 ml/min. The actual average flow rate during the test was 599 ml/min. The flow rate ranged from a high of 610 ml/min to a low of 593 ml/min. The variation was less than 1.8%, which is well within the experimental error of the aquifer test. Walton (1988) indicates the flow rate must be held to tolerance of ±10% for accurate results. A total of 227 gallons was extracted from the aquifer during the test.

Data Collection

The data were collected using an electronic tape. The method used in this and previous aquifer tests at this site indicate that this technique is accurate to approximately ±0.01 ft (0.3 cm) (Jones, 1990).

Prior to the start of the test, the depth to water was measured in each well. This was used as the static level for the aquifer test. In the early stages of the test, water levels were taken only in the first tier of wells (i.e., three feet radially). The next tier of wells were measured only after drawdown was observed in the inner tier of wells. Approximately two and half hours after the start of testing, measurements were made in every well on this and each subsequent measurement pass. Table 8 shows the drawdown results after 1/2, 5, 15 and 24 hours.

Table 8. Drawdown values in wells at specified times

	1/2 HR	12 HR	24 HR
WELL NO	DRAWDOWN, ft.		
MW-1	0.18	0.78	0.99
MW-1A	0.15	0.73	0.92
MW-1B	0.30	1.24	1.66
MW-2	0.13	0.67	0.86
MW-3	0.05	0.42	0.58
MW-4	0.02	0.30	0.45
MW-4A	0.01	0.22	0.34
MW-4B	0.00	0.31	0.46
MW-5	NA[a]	0.12	0.19
MW-5A	NA	0.13	0.20
MW-5B	NA	0.13	0.21
MW-6	0.34	0.96	1.21
MW-7	0.17	0.67	0.87
MW-8	0.08	0.51	0.67
MW-9	0.03	0.35	0.47
MW-10	NA	0.17	0.22
MW-11	0.20	0.79	1.01
MW-12	0.06	0.44	0.59
MW-13	0.05	0.35	0.47
MW-14	0.02	0.25	0.36
MW-15	NA	0.12	0.17
PW	0.90	2.44	3.38

[a] NA indicates not measured

During the test all of the wells experienced noticeable drawdown. The twenty-four hour drawdown ranged from a high of 1.66 feet in MW-1B to a low of 0.19 feet in MW-5. The inner tier of wells appear to be approaching a quasi-steady state. Drawdown of less than 0.02 feet per hour was measured in all wells at the termination of the test.

AQUIFER TEST ANALYSIS

The data obtained during the aquifer test was analyzed to determine the aquifer parameters of transmissivity (T) and specific yield (S_y). Because of the lack of published data on aquifer tests in this material, little is known about the best analysis approach to use. The methods chosen for this paper are as follows:

1. Theis method
    a. No corrections
    b. Jacob's Correction
    c. Omission of early time data
    d. Combination of b and c

2. Thiem method (Steady State method)

3. Water balance method
    a. McWhorter's Method
    b. Sen's Method

The methods and governing equations used for analysis will be discussed in detail in the sections that follow.

Due to the inability of the above solutions to account for partially penetrating wells, the data from the well nests was not analyzed. Neuman (1975) indicates that fully penetrating wells will minimize the impact of delayed yield in an aquifer.

Computer Analysis

In order to efficiently analyze the aquifer test data, a numerical type curve fitting program was developed. This program uses the concept of least squares minimization to obtain a solution.

The use of this method has several advantages. A partial list is given below:

- Speed. The solution method is more efficient than conventional curve matching.
- Consistent results. The least squares program will repeatedly find the same solution when presented the same data. This is very unlikely with standard curve matching.
- Evaluation of deviation from the theoretical solution. Estimates of the error (root mean error and maximum deviation) are directly obtainable using this method.

The author was first exposed to this method in an article by McElwee, 1980.

Theis Method

The cartesian coordinate governing equation for flow in a confined aquifer is as follows:

$$\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} = \frac{S}{T} \frac{\partial \Phi}{\partial t} \tag{3}$$

The governing equation for radial flow in a homogeneous and isotropic confined aquifer is given below:

$$\frac{\partial^2 \Phi}{\partial r^2} + \frac{1}{r} \frac{\partial \Phi}{\partial r} = \frac{S_c}{T} \frac{\partial \Phi}{\partial t} \tag{4}$$

In 1935, C.V. Theis developed an analytical solution to the above equation. The solution was derived from existing solutions to the analogous heat flow equations. The Theis solution is shown below.

$$s = \frac{Q}{4\pi T} W(u) \qquad (5)$$

$$W(u) = \int_{u}^{\infty} \frac{e^{-u}}{u} du \qquad (6)$$

$$u = \frac{r^2 S_c}{4Tt} \qquad (7)$$

The assumptions used in the development of the Theis solutions are as follows:

- Flow is entirely horizontal and radial
- Pumping well fully penetrates aquifer and pumps at a constant rate
- No water is stored within the wells
- Aquifer is homogeneous and isotropic and has infinite extent and constant thickness
- Aquifer base is impermeable and horizontal

The reader should recall the relationship between transmissivity (T) and hydraulic conductivity (K) is as shown below.

$$T = Kb \qquad (8)$$

Strictly speaking, the use of the Theis solution for unconfined aquifers is incorrect due to the constantly changing saturated thickness. The variance in the saturated thickness results in a varying transmissivity throughout the test.

Although the Theis equation was developed strictly for horizontal flow, many have used this solution to determine aquifer parameters of unconfined aquifers. Bear (1979) indicates that for small drawdowns, an unconfined aquifer can be treated as a confined aquifer. Streltsova (1972) indicates that the Theis solution can be used with fair accuracy when applied to unconfined aquifer testing. Jacob (1950) indicates the use of the Theis solution in analysis of unconfined aquifers yields nearly correct solutions so long as the drawdown is small in comparison to the saturated thickness.

The assumption made by Theis of constant storativity also provides problems when using the Theis solution for unconfined aquifers. Most literature indicates the storativity is not constant throughout an aquifer test in an unconfined aquifer. Youngs and Smiles (1963) indicate the specific yield of an unconfined aquifer is a function of time and radial distance from the pumping well. The authors state this arises from the hysteresis effect of the wetting and drying of soil. However, Ferris (1965) states "there is little justification for the premise that the storage coefficient of a water-table aquifer varies with the time of pumping inasmuch as such anomalous data are merely the results of trying to apply a two-dimensional flow formula to a three-dimensional problem."

The use of the Theis solution cannot account for any delayed yield (drainage) that occurs in an the aquifer. A review of the literature discussing delayed yield indicates that the early and late time data generally fit the Theis solution very well. Figure 6 depicts the delayed yield curve developed by

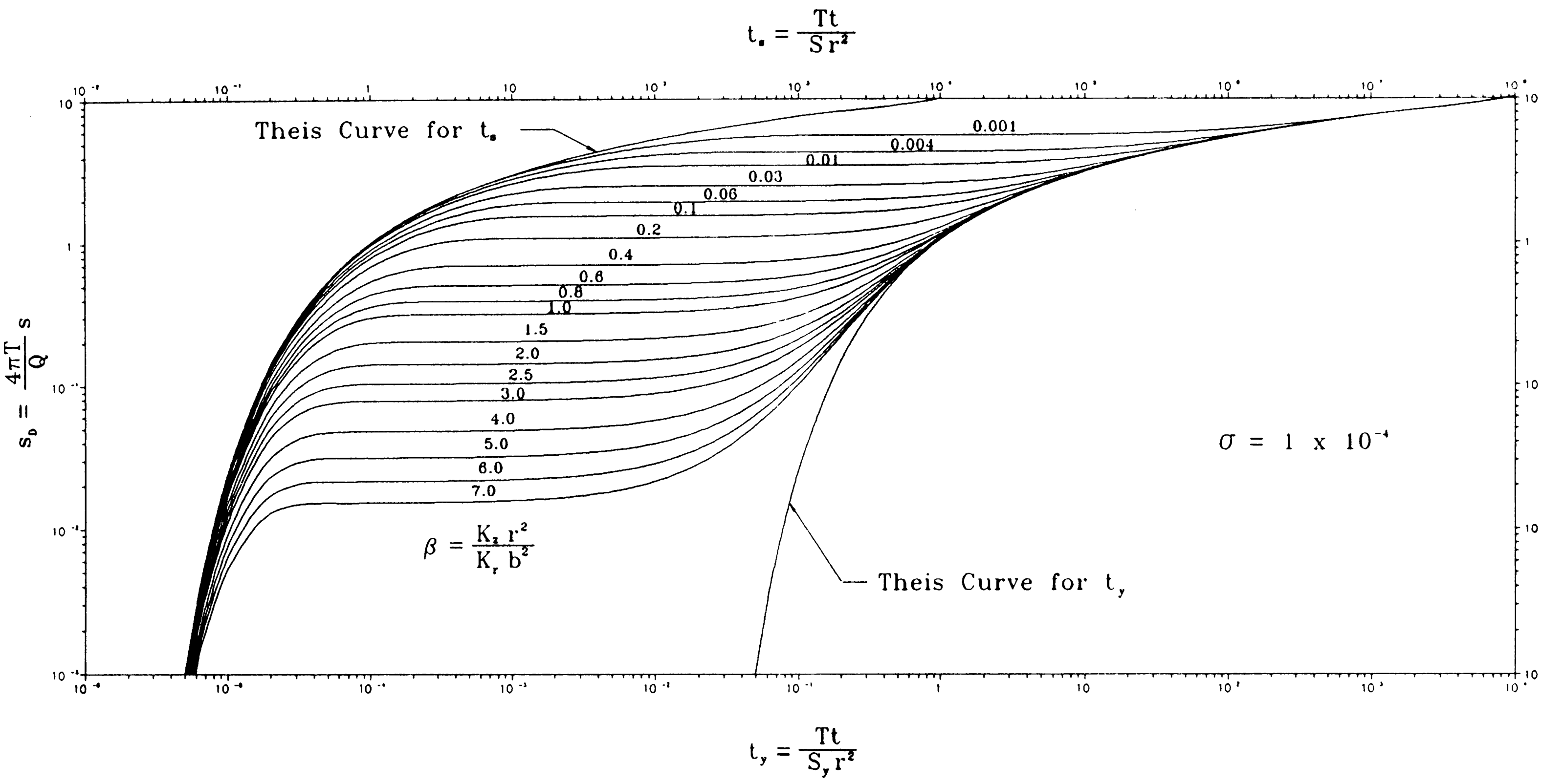

Figure 6. Delayed yield type curve

the author from data taken from Neuman (1975). Neuman (1972) indicates that the effects of delayed yield will dissipate at early and late times and at large radial distances from the well. This seems to correspond well to results obtained from the aquifer test analysis. In general, the fit of the Theis solution improves as the distance from the pumping well increases. This is a result of the dissipation of vertical flow as the distance from the pumping well increases.

Data Analysis

Using the program mentioned previously, the data from the monitoring wells were analyzed. Table 9 summarizes the results of this analysis. The results obtained from the computer program, including a graph of the solution, may be found in Appendix B (Lemar, 1991).

It is possible that the low hydraulic conductivity in the vicinity of the well may "fool" the analyst into determining that the aquifer has a high hydraulic conductivity due to the lack of response of the well. Should the well response (drawdown) lag behind the remainder of the aquifer, the Theis solution will "see" this part of the aquifer as being very productive (i.e., very little drawdown for a given pumping rate). Upon review of the aquifer and slug test results, some anomalous results were found to be present. A closer look at the slug test data in Table 7, reveals that MW-5 and MW-15 deviated significantly from the hydraulic conductivity of the other wells in the area. These results would seem to indicate that the areas surrounding these wells is very non-conductive. Warnings expressed by Walton (1988) indicate wells with extensively different time lags should be closely scrutinized prior to the use of the data. A closer look at the hydraulic conductivity of MW-15 as calculated from the

Table 9. Results of aquifer analysis using non-corrected Theis method

WELL NO	T cm^2/s	K cm/s	S_y
MW-1	0.1170	3.81E-04	0.0483
MW-2	0.1232	4.02E-04	0.0185
MW-3	0.1293	4.50E-04	0.0240
MW-4	0.1367	5.06E-04	0.0238
MW-5	0.1757	6.91E-04	0.0308
MW-6	0.1279	5.35E-04	0.0151
MW-7	0.1381	5.68E-04	0.0120
MW-8	0.1427	5.75E-04	0.0120
MW-9	0.1632	6.60E-04	0.0152
MW-10	0.2745	1.08E-03	0.0168
MW-11	0.1175	4.32E-04	0.0305
MW-12	0.1487	4.76E-04	0.0333
MW-13	0.1684	5.24E-04	0.0234
MW-14	0.1819	6.32E-04	0.0162
MW-15	0.3689	1.17E-03	0.0206
MEANS			
ARITHMETIC	0.1676	6.06E-04	0.0227
GEOMETRIC	0.1585	5.74E-04	0.0210
VARIANCE	0.0043	4.99E-08	0.0001
MAXIMUM	0.3689	1.17E-03	0.0483
MINIMUM	0.1170	3.81E-04	0.0120

aquifer testing, indicates this well to have the highest transmissivity of all those calculated during the test. The high transmissivity calculated for MW-15 would seem to indicate that the information provided by this well may be in error. However, analyzing the data from MW-10 (well with the second lowest time lag during slug testing) reveals it has a high calculated transmissivity. This would seem to indicate that the effect of large lag times is minimal. The explanation to this may lie in the fact that aquifer testing averages the permeability over time and space, whereas slug testing indicates the hydraulic conductivity of a small area.

The data obtained from the pumping well could not be analyzed using the least squares analysis program due to it's nature. The drawdown vs. time curves for the pumping well and the MW-1 are shown in Figure 7. Comparison of Figures 6 and 7 seems to indicate the possibility of delayed yield occurring in this aquifer setting. Due to time constraints, this possibility was not pursued. It is possible that the drawdown curve seen in Figure 7 may be explained by the large storage of the well bore when compared to the pumping rate. Again, this possibility was not explored.

Transmissivity The transmissivity of the aquifer as determined by aquifer testing, ranges from 0.13 cm^2/s to 0.37 cm^2/s with a geometric mean of 0.17 cm^2/s. Using the saturated thicknesses shown in Table 2, the hydraulic conductivity of the area surrounding the wells was determined. The hydraulic conductivity ranged from 1.20×10^{-3} cm/s to 3.81×10^{-4} cm/s. Fetter (1980) shows the hydraulic conductivity of silts and tills generally ranges from 10^{-4} cm/s to 10^{-6} cm/s.

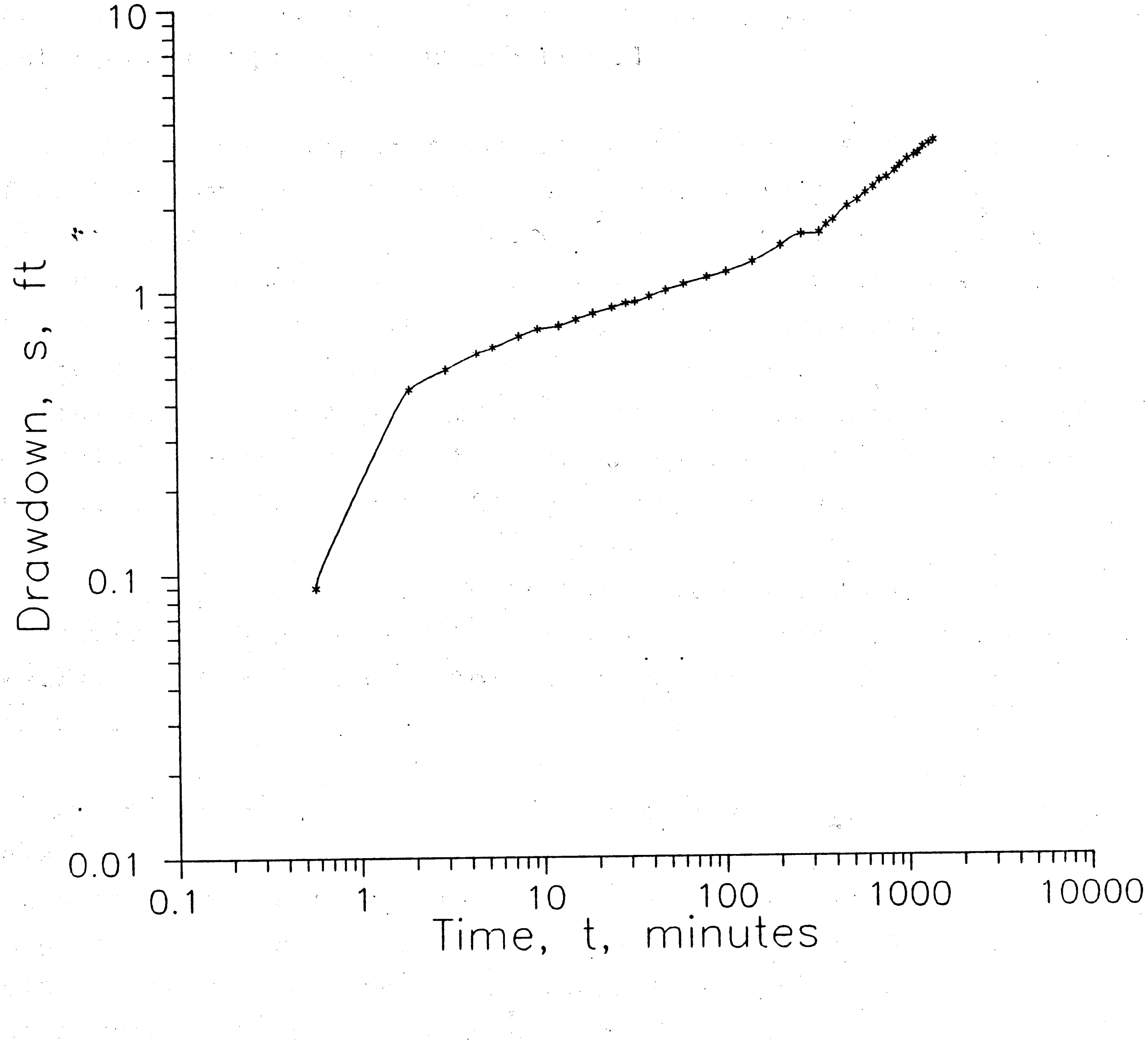

Figure 7. Drawdown curve for the pumping well

The relatively high hydraulic conductivity may be due to the high level of oxidation channels which existed in the oxidized portion of the till. Williams and Farvolden (1967) indicate oxidation channels are paths of relatively high permeability. They suggest the permeability of these channels is considerably greater than the permeability of the intergranular pore spaces in the till.

As Table 9 shows, transmissivity increases as the distance from the pumping well increases. This trend is shown graphically in Figure 8. This should be expected due to the decrease in saturated thickness close to the pumping well. As the saturated thickness decreases, the transmissivity decreases also. This leads to reduction in the transmissivity as the aquifer test proceeds. Therefore, the transmissivity calculated during the test is actually a weighted average of the transmissivity that existed during the test.

Storativity Results of the analysis indicates the storativity is generally the highest in the inner tier of wells. A graph of the storativity verses radial distance is shown in Figure 9.

Generally in an unconfined aquifer, the specific storage is assumed to be much smaller than the specific yield and thus ignored. Fetter (1980) indicates this assumption may not be true for fine grained materials. Neuman (1974) indicates the elastic storage of unconfined aquifers may be several orders of magnitude higher than most literature indicates.

The relationship shown below defines the storativity.

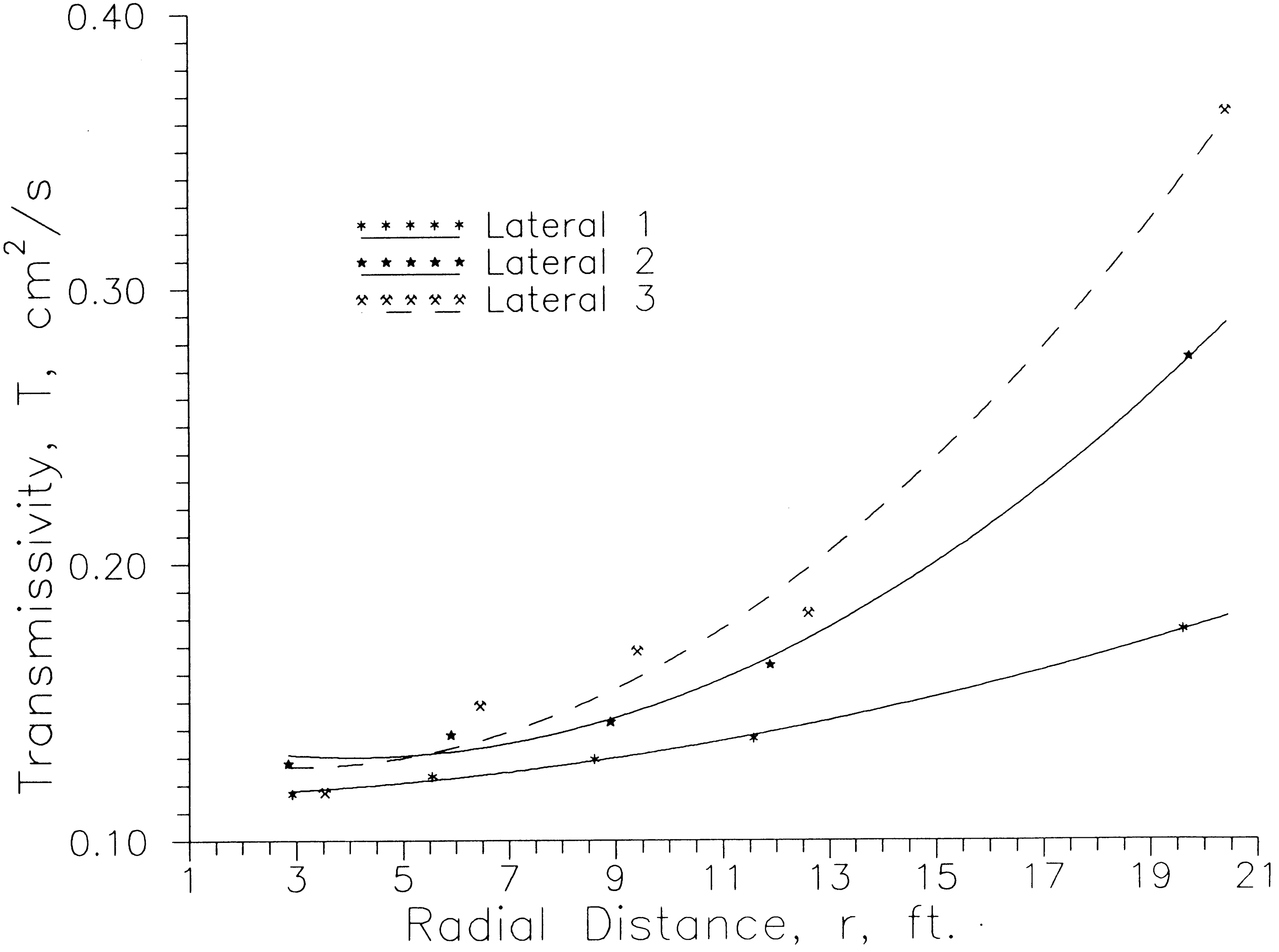

Figure 8. Transmissivity vs. distance graph

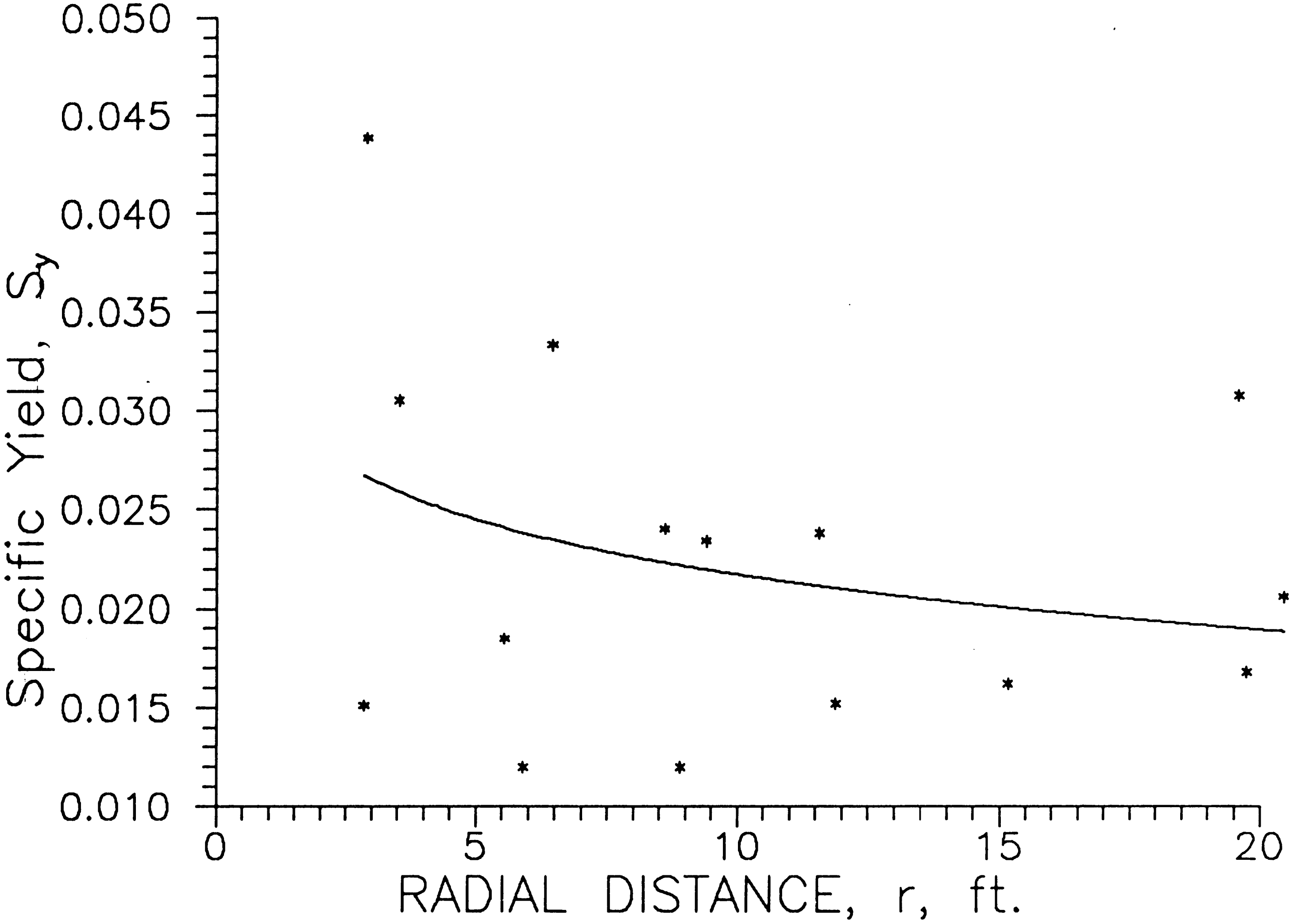

Figure 9. Specific yield vs. radial distance graph

$$S=S_y+S_sb \tag{9}$$

To investigate the validity of this assumption, a plot of storativity verses saturated thickness was developed. The investigation of this relationship produced no additional information. The relationship shown above may account for trends in the data. If the specific storage is within two orders of magnitude of the specific yield, the decrease in saturated thickness that occurs during pumping may have a major impact. This would explain the storativity generally increasing as the distance from the well increases. The variation exhibited in the inner tier of wells has not yet been explained.

McWhorter and Sunada (1977) define specific yield as the difference between the porosity and the specific retention. Many authors indicate the specific yield of an unconfined aquifer is approximately equal to the porosity of the aquifer material. As indicated previously, the average porosity was determined to be 17.87% which is much higher than the average specific yield of 2.22% determined during aquifer testing. This is related to the fact that the percent of the porosity in which water will flow (i.e., effective porosity) is generally very low. Estimates indicate the effective porosity of the material at the test site may be as low as 5% to 7%.

Fetter (1980) indicates the specific yield for sandy clay ranges from a minimum of 3% to a maximum of 12% with an average value of 7%. Work by Johnson (1967) shows the relationship between particle size and specific yield. Using the graphs provided in Johnson's paper, a specific yield of

10% was estimated. Other results compiled by Johnson, generally indicates that specific yields for materials similar to that occurring at the site ranges between 3% to 10%. The specific yield determined by aquifer testing is near the low end of these ranges. As noted earlier, the unoxidized and oxidized tills have a similar particle size distribution. This leads the methods discussed above to determine a specific yield that is very similar for both materials. Experience indicates the unoxidized till to have a much lower effective porosity (i.e., specific yield) than the oxidized till. It is the author's opinion this is an inconsistency in the grain size method for determining specific yield. This inconsistency again shows the lack of information about low hydraulic conductivity materials.

Johnson also indicates the porosity generally decreases with depth. Because the specific yield is directly related to the porosity, a decrease in porosity results in a reduction of specific yield. Data obtained from the well nests may help to show this, although the analyses have not been performed at this time.

Youngs and Smiles (1963) indicate the specific yield as determined by pumping tests in unconfined aquifers is highly dependent on the condition (drainage or recharging) of the water table at the start of the testing. They indicated the specific yield could vary from test to test by as much as three times depending on the drainage conditions prior to the test. According to the classifications presented by Youngs and Smiles, the pre-test water table was determine to be at condition 1. Condition 1 is typified by a predrained profile, indicating the water table has been lowered from a previously higher level. Condition 1 was shown to produce the highest specific yield of the three conditions.

Jacob's Correction

Due to its deficiencies when dealing with unconfined flow, modifications to the Theis equation to compensate for the decreasing saturated thickness are necessary. According to Jacob (1944), "the observed drawdown values must be 'corrected' to compensate for the decrease in the saturated thickness of the aquifer".

The governing equation for homogenous, isotropic flow in an unconfined aquifer is shown below.

$$\frac{\partial^2 h}{\partial x^2}+\frac{\partial^2 h}{\partial y^2}+\frac{\partial^2 h}{\partial z^2}=\frac{S_y}{K}\frac{\partial h}{\partial t} \tag{10}$$

If the Dupuit-Forchheimer (Dupuit) assumption is used, the unconfined radial flow governing becomes as shown below.

$$\frac{\partial^2 h^2}{\partial r^2}+\frac{1}{r}\frac{\partial h}{\partial r}=\frac{2S_y}{K}\frac{\partial h}{\partial t} \tag{11}$$

The above equation assumes the following:

- Specific yield is constant.
- water is released instantaneously from storage upon any decline of the phreatic surface.
- the release of water is linearly proportional to changes in storage.

The Dupuit assumption assumes a majority of the flow in the aquifer is horizontal. Thus, the solution obtained by using Dupuit neglects the seepage face. Proofs by Bear (1972) indicate the Dupuit approximation predicts the discharge exactly. However, the water table elevation computed is generally in error. McWhorter and Sunada (1977) state the Dupuit solution will only give correct water table elevations when:

$$\left(\frac{\partial h}{\partial x}\right)^2 \ll 1 \qquad (12)$$

Bear (1979), indicates the Dupuit is the most powerful assumption available for analyzing unconfined aquifers. As stated before, the Dupuit assumption considers the flow in the aquifer to be essentially horizontal. This may not be a correct assumption at early times during the test or at close distances to the pumping well, due to the large vertical flow components which occur in these situations.

Substituting Equations 13 and 14 into Equation 11 yields Equation 15.

$$h = H_o - s \qquad (13)$$

$$s_{jc} = s - \frac{s^2}{2H} \qquad (14)$$

$$\frac{\partial^2 s_{jc}}{\partial r^2} + \frac{1}{r}\frac{\partial s_{jc}}{\partial r} = \frac{H_o}{H_o - s}\,\frac{S_y}{KH_o}\,\frac{\partial s_{jc}}{\partial t} \qquad (15)$$

The governing equation (Equation 15) shown above, can be shown to produce an equation similar to the confined radial flow equation by substituting the following relationships.

$$S_{ya}=S_y\frac{H_o}{H_o-s} \qquad (16)$$

$$T'=KH_o \qquad (17)$$

With these modifications, the drawdown was reanalyzed. The results are shown and discussed below. The computer output resulting from this analysis is shown in Appendix C (Lemar, 1991).

Data Analysis

The use of Jacob's correction produced slightly higher values of transmissivity than those determined using the Theis method directly. This is expected since the method was developed to account for the decreasing saturated thickness. Table 10 shows the results obtained using Jacob's correction.

Although Jacob's correction does increase the value obtained for transmissivity, it does not increase it significantly. The geometric mean of data increased by only five percent.

The method had a varying impact on the specific yields calculated. The correction raised the maximum and lowered the minimum. Trends in the impact of the correction on the specific yield were almost impossible to detect. The overall impact was to increase the geometric mean of the specific yield by about 12%.

Table 10. Results of the aquifer analysis using Jacob's correction

WELL NO	T cm^2/s	K cm/s	S_{ya}	S_y
MW-1	0.1252	4.07E-04	0.0426	0.0413
MW-2	0.1308	4.27E-04	0.0181	0.0176
MW-3	0.1362	4.74E-04	0.0238	0.0230
MW-4	0.1427	5.28E-04	0.0238	0.0230
MW-5	0.1792	7.04E-04	0.0310	0.0300
MW-6	0.1420	5.94E-04	0.0139	0.0135
MW-7	0.1489	6.13E-04	0.0116	0.0113
MW-8	0.1520	6.13E-04	0.0118	0.0114
MW-9	0.1707	6.90E-04	0.0151	0.0147
MW-10	0.2802	1.10E-03	0.0168	0.0163
MW-11	0.1277	4.70E-04	0.0292	0.0283
MW-12	0.1556	4.98E-04	0.0329	0.0319
MW-13	0.1747	5.43E-04	0.0232	0.0225
MW-14	0.1875	6.51E-04	0.0162	0.0157
MW-15	0.3742	1.19E-03	0.0205	0.0199
MEANS				
ARITHMETIC	0.1752	6.34E-04	0.0220	0.0214
GEOMETRIC	0.1667	6.04E-04	0.0205	0.0199
VARIANCE	0.0042	4.86E-08	0.0001	0.0001
MAXIMUM	0.3742	1.19E-03	0.0426	0.0413
MINIMUM	0.1252	4.07E-04	0.0116	0.0113

Early Time Deletion Analysis

Neuman (1975) and Prickett (1965) indicated the drawdown data as the unconfined aquifer system approaches equilibrium (steady state), will follow the non-equilibrium (Theis) solution almost exactly.

The vertical flow components are greatest during the early part of the aquifer test. In order to reduce the confounding this may produce, the first hour of data was ignored. This cut-off point was determined by analysis of the curves developed during the Theis analysis. The data which did not correspond to the Theis curve was deleted.

Data Analysis

The deletion of the early time data produced lower values of hydraulic conductivity than did the two methods previously discussed. The results of this analysis are shown in Table 11. The computer output showing the analyzed data is shown in Appendix D (Lemar, 1991).

The decrease in hydraulic conductivity is noticed most in the inner wells. This results because the saturated thickness is decreasing rapidly in these wells. The early time data would have a higher transmissivity due the greater saturated thickness during this part of the test. Again however, the transmissivity only varied by approximately 8%.

The specific yield reacted similar to the transmissivity data in that it changed in relation to the radial distance. However, instead of decreasing closer to the well, it increased. The specific yield calculated for MW-1 increased by more than 30% following the deletion of the early time data. This indicates the specific yield is more sensitive to

Table 11. Results of the aquifer analysis using the time correction

WELL NO	T cm^2/s	K cm/s	S_y
MW-1	0.1022	3.33E-04	0.0617
MW-2	0.1089	3.56E-04	0.0239
MW-3	0.1221	4.25E-04	0.0260
MW-4	0.1357	5.02E-04	0.0242
MW-5	0.1717	6.75E-04	0.0308
MW-6	0.1013	4.24E-04	0.0314
MW-7	0.1203	4.95E-04	0.0168
MW-8	0.1266	5.10E-04	0.0149
MW-9	0.1585	6.40E-04	0.0160
MW-10	0.2716	1.07E-03	0.0171
MW-11	0.0958	3.53E-04	0.0484
MW-12	0.1351	4.32E-04	0.0390
MW-13	0.1543	4.80E-04	0.0265
MW-14	0.1809	6.28E-04	0.0165
MW-15	0.3690	1.18E-03	0.0208
MEANS			
ARITHMETIC	0.1569	5.67E-04	0.0276
GEOMETRIC	0.1456	5.27E-04	0.0252
VARIANCE	0.0050	5.80E-08	0.0002
MAXIMUM	0.3690	1.18E-03	0.0617
MINIMUM	0.0958	3.33E-04	0.0149

the early time deletion than is the transmissivity. Thus care must be exercised when determining how much of the early time data to delete.

Jacob's and Early Time Deletion

In an attempt to develop a better solution, the Jacob's and time corrections were combined. It was hoped these corrections would, when combined, help to dampen the effects of decreasing saturated thickness and vertical flow.

Data Analysis

The hydraulic conductivity values obtained reacted very much like the specific yield values in Trial 2. The mean value of hydraulic conductivity decreased, while both minimum and maximum value increased. Appendix E (Lemar, 1991) shows the computer output obtained from this round of testing. The data obtained from this testing is shown in Table 12.

The use of the Jacob's correction tended to dampen the impact of deletion of the early time data. The effect of the Jacob's correction on the time corrected data is very similar to curve smoothing in that it reduced the peaks and valleys in the data. The resulting mean hydraulic conductivity varied by only 6% from the unmodified data.

The combined effect of Jacob's and time deletion on the specific yield was very similar. It lowered the maximum and raised the minimum. However, the difference of the geometric means between the Jacob's/Time deletion and the unmodified data was still in excess of 30%.

Table 12. Results of the aquifer analysis using both Jacob's and time correction

WELL NO	T cm^2/s	K cm/s	S_{ya}	S_y
MW-1	0.1101	3.58E-04	0.0583	0.0566
MW-2	0.1161	3.79E-04	0.0234	0.0227
MW-3	0.1282	4.46E-04	0.0259	0.0251
MW-4	0.1413	5.23E-04	0.0242	0.0235
MW-5	0.1752	6.88E-04	0.0310	0.0300
MW-6	0.1140	4.77E-04	0.0289	0.0280
MW-7	0.1305	5.37E-04	0.0162	0.0158
MW-8	0.1351	5.44E-04	0.0147	0.0143
MW-9	0.1655	6.69E-04	0.0159	0.0154
MW-10	0.2650	1.04E-03	0.0174	0.0169
MW-11	0.1043	3.84E-04	0.0469	0.0455
MW-12	0.1410	4.51E-04	0.0388	0.0376
MW-13	0.1595	4.96E-04	0.0265	0.0257
MW-14	0.1854	6.44E-04	0.0165	0.0160
MW-15	0.3739	1.19E-03	0.0208	0.0202
MEAN				
ARITHMETIC	0.1630	5.89E-04	0.0270	0.0262
GEOMETRIC	0.1528	5.53E-04	0.0248	0.0241
VARIANCE	0.0047	5.32E-08	0.0001	0.0001
MAXIMUM	0.3739	1.19E-03	0.0583	0.0566
MINIMUM	0.1043	3.58E-04	0.0147	0.0143

Thiem Method

Before the development of the Theis solution, few methods were available for calculating the hydraulic conductivity of an aquifer. One of the more popular methods was the solution given by Thiem. The Thiem equation for phreatic aquifers is shown below.

$$K=\frac{Q}{\pi(h_2^2-h_1^2)}\ln\left(\frac{r_2}{r_1}\right) \tag{18}$$

The Thiem equation strictly applies only to a flat water table. This assumptions should not cause appreciable error, due to the small horizontal gradients that exist at this site.

This equation is only valid once the well field reaches quasi-steady state. Fetter (1980) states the Thiem method is likely to produce more accurate results than transient methods. Sen (1987) describes the Thiem method as one of the most reliable field methods for determining transmissivity. As previously stated, the aquifer appears to be approaching quasi-steady state near the end of the pump test. A longer test may help to indicate if steady state has been reached.

Neuman (1988) indicates the Thiem solution is invalid at large distances from the pumping well. Neuman continues, stating the Thiem solution is valid only in the range where the Jacob-Cooper semi-log approximation is valid.

The main drawback of the steady state analysis is the inability to determine the specific yield from the available information.

Data Analysis

The hydraulic conductivity values obtained using the Thiem solution are shown in Table 13. A plot of the elevation difference vs. the natural logarithm of the ratio of the radial distance is shown in Figure 10.

The conductivities range from 1.56×10^{-4} cm/s to 2.29×10^{-4} cm/s. The geometric and arithmetic means were equal to 1.84×10^{-4} cm/s. The transmissivities obtained using the Thiem formula have a much smaller range than the transmissivities obtained by the use of the Theis method.

As the Table shows, the hydraulic conductivity as determined by the Thiem equation is very consistent as compared to the other analysis methods. The mean of the Thiem results is very similar to those obtained from the slug test results.

The use of the drawdown values obtained from near the end of the test seems to indicate the aquifer is approaching steady state.

Water Balance Methods

Two water balance methods were discovered during the literature review. The first method was presented by McWhorter and Sunada (1977). The second method was presented in 1987 by Zekai Sen. The methods vary little in their theory, application or result. Therefore they will be discussed together.

The following assumptions were used to develop the water balance formulas:

Table 13. Hydraulic conductivity values obtained using the Thiem equation

Well No.	Radial Distance ft.	Top of Casing Elevation ft.	Water Elevation ft.	Hydraulic conductivity	
				ft/min	cm/s
1	2.92	16.63	13.22	3.19E-04	1.62E-04
2	5.55	16.48	13.31	3.65E-04	1.85E-04
3	8.60	16.48	13.47	3.82E-04	1.94E-04
4	11.57	16.46	13.51	4.01E-04	2.04E-04
5	19.60	16.17	13.46	4.51E-04	2.29E-04
6	2.85	16.65	13.26	3.12E-04	1.59E-04
7	5.90	16.87	13.67	3.29E-04	1.67E-04
8	8.90	16.77	13.95	3.31E-04	1.68E-04
9	11.88	16.92	14.23	3.25E-04	1.65E-04
10	19.73	16.95	14.82	3.07E-04	1.56E-04
11	3.53	16.64	13.35	3.21E-04	1.63E-04
12	6.45	16.64	13.81	3.22E-04	1.63E-04
13	9.40	16.34	13.93	3.37E-04	1.71E-04
14	12.60	16.64	14.04	3.47E-04	1.76E-04
15	20.43	16.53	14.16	3.68E-04	1.87E-04
PW	0.08	16.69	9.98	NA[a]	NA
				Arithmetic Mean	1.84E-04
				Geometric Mean	1.83E-04

[a] NA = Not calculable using this method

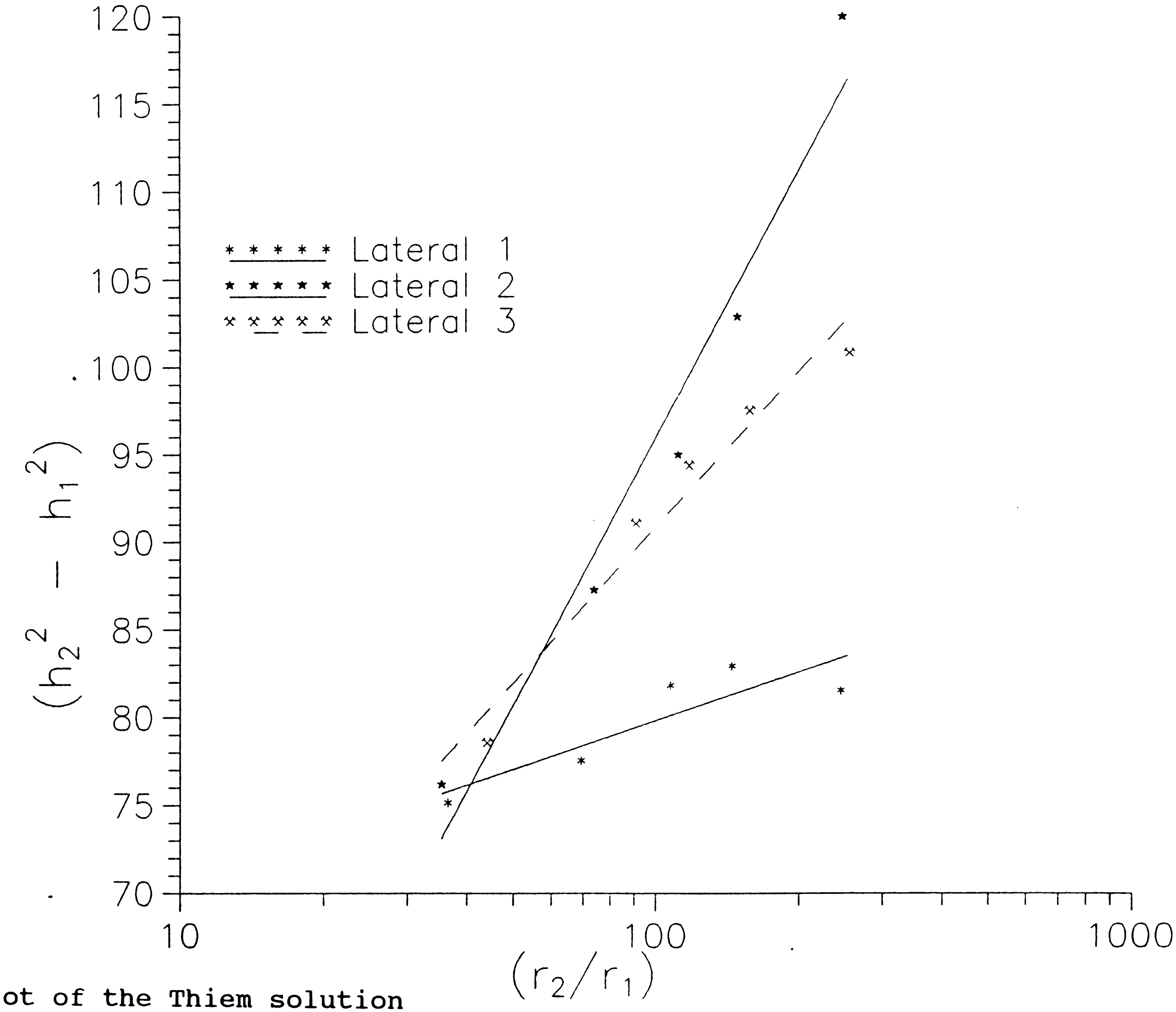

Figure 10. Plot of the Thiem solution

- Aquifer is homogenous, isotropic and infinite in radial extent
- Aquifer is confined from beneath
- Discharge well fully penetrates the aquifer
- Head loss through the screen and gravel pack is negligible
- The water table is horizontal prior to pumping
- No recharge occurs during the test
- All pumped water is removed from storage

The last assumption is perhaps the hardest to accept. Remson and Lang (1955) state "the coefficient of storage for water table conditions is essentially the same as the specific yield, which is the quantity of water yielded by gravity from saturated water bearing material expressed as a percentage or decimal fraction of the total volume of the material drained. The coefficient of storage is slightly but negligibly higher than the specific yield because it includes a small amount of storage derived from the compaction of the aquifer and expansion of water in the zone below the water table as the water table declines." Neuman (1975) indicates water is expelled from the aquifer due to not only draining of the pores, but also from expansion of the water and compression of the aquifer. This occurs at early and intermediate times during the pumping of an unconfined aquifer. The stages discussed by Neuman represent the transition in which flow changes from predominately vertical to horizontal.

Using the radial flow governing equation, the equation shown below was derived by Sen (1987).

$$S_y = \frac{Qt - \pi r_w^2 s_w(t)}{V_{DC}(t)} \tag{19}$$

The equation presented by McWhorter is shown below.

$$S_y = \frac{Qt}{V_{DC}(t)} \tag{20}$$

The main difference between the two equations is the subtraction of the volume of the well bore in the derivation by Sen. Although this may be important in large diameter wells, it is of little significance in the analysis of this site.

The volume calculations were made using the software package SURFER® by Golden Software. This software contains a utility for the determination of the volume contained between two three dimensional surfaces or a surface and plane. The use of this software was necessitated by the non-symmetrical drawdown cone produced by the pumping. The lack of symmetry is thought to be caused by the inhomogeneities in the aquifer. The use of SURFER® is also useful when trying to determine the location of the zero drawdown isopeth. The kriging was used to develop the contours needed to determine the volume of the drawdown cone.

Data Analysis

Table 14 depicts the values calculated using the methods discussed above.

Table 14. Determination of specific yield using the water balance method

Time hrs	Drawdown Cone Volume ft^3	Volume Pumped ft^3	Pumping Well Drawdown ft	Specific Yield [a]	Specific Yield [b]
0.66	163.59	0.833	0.95	0.0051	0.0050
5.62	391.34	7.144	1.60	0.0183	0.0182
15.57	651.47	19.782	2.76	0.0304	0.0303
23.85	814.55	30.309	3.38	0.0372	0.0371

[a] Specific Yield = Specific Yield determined by McWhorter and Sunada method

[b] Specific Yield = Specific Yield determined by Sen's method

As the table shows, the two methods produce very similar results. This is due to the small diameter of the well and the small drawdown in the well.

The results show the specific yield determined by water balance method to be higher than the specific yield predicted by the pumping tests. A graph of the specific yield vs. time is shown by Figure 11. Nwankwor et al. (1984) indicates the time required to obtain the ultimate specific yield increases considerably with decreasing mean grain size. Stallman (1971) indicates that in clean sand with typical water-table drawdown, less than 70% of ultimate specific yield will be attained after two days. This would seem to indicate that the specific yield determined by this method may not have yet reached the ultimate value.

Nwankwor et al. (1984), concludes that it is probable the water balance method will underestimate the specific yield during the initial part of the aquifer test. This is related

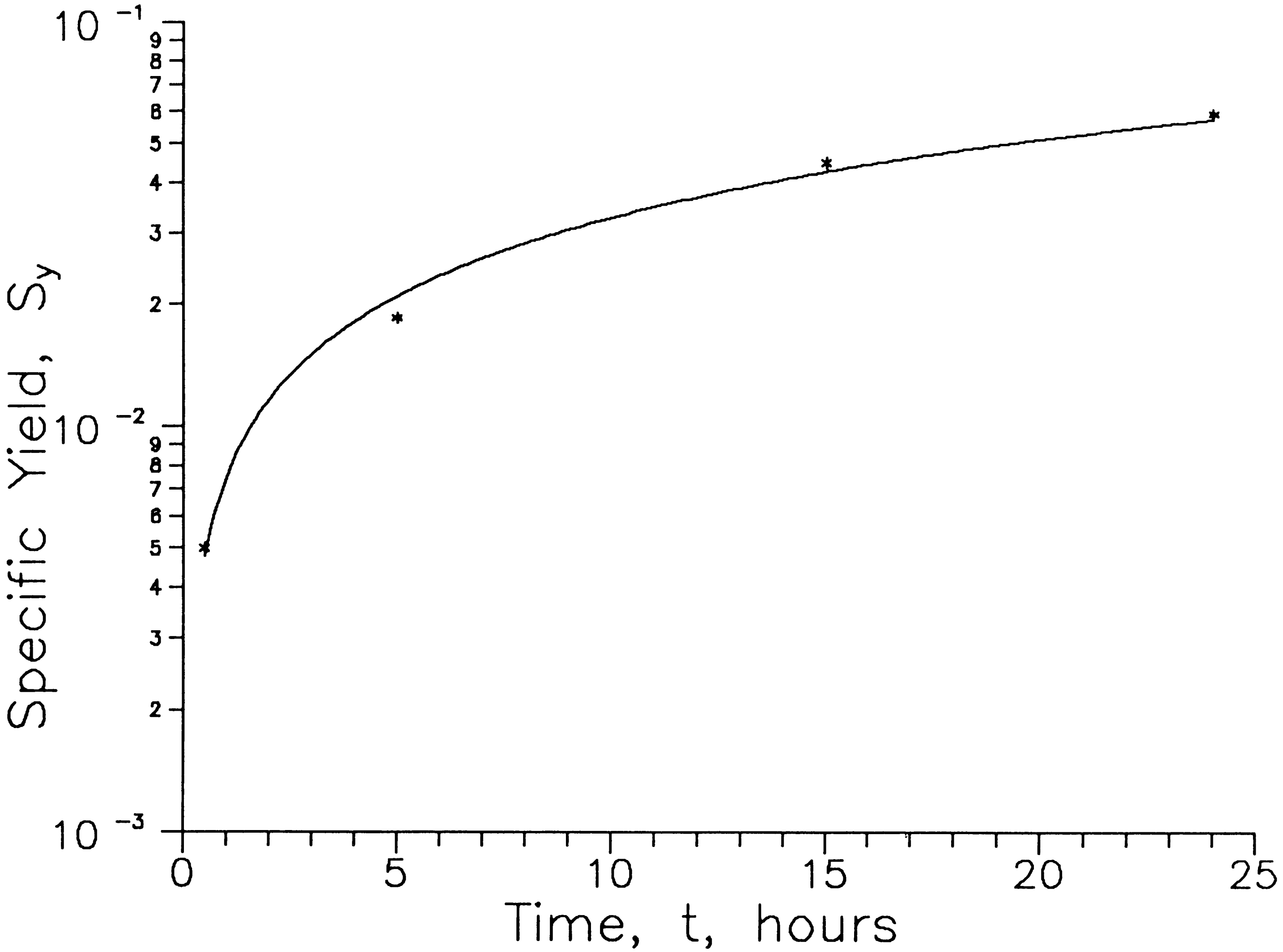

Figure 11. Water balance specific yield vs. time

to the rapid drainage of the aquifer and the effect of aquifer compactions and water expansion which can not be accounted for using the water balance method.

Many authors have debated the validity of the water balance methods. Neuman (1988) indicates the water balance method suggested by Sen, is invalid due to its failure to conserve mass. Neuman also criticizes the water balance method, due to its reliance on the Thiem method to determine the hydraulic conductivity. Sen and McWhorter both rely on the transmissivity to calculate the volume of the drawdown cone. This problem is avoided using the software mentioned previously.

One problem encountered using this method is the water level indicated by the fully penetrating well is actually an average water level at the radial distance of the well. This may tend to cause a distortion of the estimated shape of the water table. As the well nests indicate, this is much less of a problem further away from the well. This is due to the small vertical flow component in this area. Well nest 1 shows large differences between the twenty four hour drawdowns in the three wells. The twenty-four hour drawdown for the well nests are shown in Table 15.

Table 15. Twenty-four hour drawdown for the well nests

WELL	Drawdown ft.
MW-1	0.99
MW-1A	0.92
MW-1B	1.66
MW-4	0.45
MW-4A	0.34
MW-4B	0.46
MW-5	0.19
MW-5A	0.20
MW-5B	0.21

As the Table shows, the difference of the water table drawdown decrease at larger radial distances. The drawdown obtained from the shallow partially penetrating well provides the best estimate of the water table in the area. The installation of more shallow partially penetrating wells would be useful to more accurately depict the water table during pumping.

CONCLUSIONS

Table 16 shows the hydraulic conductivity values obtained from the various analysis methods. As this table shows, each analysis method produces very similar conductivities. The geometric means are the same order of magnitude. This would indicate that each method will produce useful information when used correctly. A hydraulic conductivity of 10^{-4} cm/s falls within the range of conductivities presented in the literature for materials of this type. It does, however, fall into the high end of the range.

Table 17 summaries the storativity values obtained during the analysis. Values of S_y obtained during the analysis were much lower than the literature would have predicted. However, all methods produced results of similar magnitude. Some variation was seen between the wells however, this was not significant. Of the four methods used, the water balance method predict the highest S_y. However, this may be due to the use of the fully penetrating wells to monitor the water table surface.

Based on the results included in this text, it would appear any of the three aquifer analysis solutions will produce similar results. However, several of the slug testing results varied an order of magnitude or more from the results of the aquifer testing. This may indicate that slug testing is not suitable for use as a remediation design tool in formations discussed in the paper.

Without further testing at this site, it is not clear which method will produce the "best" answer. Further aquifer testing will need to be performed at this site before conclusions can be reached. Tests should be performed at a range of flow rates

Table 16. Hydraulic conductivity of all analysis trials

WELL NO	TRIAL #1 cm/s	TRIAL #2 cm/s	TRIAL #3 cm/s	TRIAL #4 cm/s	SLUG TEST cm/s	Thiem cm/s
MW-1	3.81E-04	4.08E-04	3.33E-04	3.58E-04	1.87E-04	1.62E-04
MW-2	4.01E-04	4.26E-04	3.54E-04	3.78E-04	1.64E-04	1.85E-04
MW-3	4.21E-04	4.43E-04	3.97E-04	4.17E-04	8.15E-05	1.94E-04
MW-4	4.45E-04	4.64E-04	4.42E-04	4.60E-04	1.15E-04	2.04E-04
MW-5	5.72E-04	5.83E-04	5.59E-04	5.70E-04	4.73E-06	2.29E-04
MW-6	4.16E-04	4.62E-04	3.30E-04	3.71E-04	2.12E-04	1.59E-04
MW-7	4.49E-04	4.85E-04	3.92E-04	4.25E-04	2.41E-04	1.67E-04
MW-8	4.64E-04	4.95E-04	4.12E-04	4.40E-04	1.70E-04	1.68E-04
MW-9	5.31E-04	5.57E-04	5.16E-04	5.39E-04	2.06E-04	1.65E-04
MW-10	8.93E-04	9.12E-04	8.84E-04	5.85E-04	2.32E-04	1.56E-04
MW-11	3.82E-04	4.16E-04	3.12E-04	3.39E-04	1.62E-04	1.63E-04
MW-12	4.84E-04	5.06E-04	4.40E-04	4.59E-04	2.63E-04	1.63E-04
MW-13	5.48E-04	5.69E-04	5.02E-04	5.19E-04	2.84E-04	1.71E-04
MW-14	5.92E-04	6.10E-04	5.89E-04	6.03E-04	2.39E-04	1.76E-04
MW-15	1.20E-03	1.22E-03	1.20E-03	1.22E-03	2.27E-05	1.87E-04
PW	(1)	(1)	(1)	(1)	1.88E-04	(1)
MEAN						
ARITHMETIC	5.45E-04	5.70E-04	5.11E-04	5.12E-04	1.72E-04	1.84E-04
GEOMETRIC	5.16E-04	5.43E-04	4.74E-04	4.85E-04	1.27E-04	1.83E-04
VARIANCE	4.60E-08	4.44E-08	5.33E-08	4.22E-08	6.57E-09	3.75E-10
MAXIMUM	1.20E-03	1.22E-03	1.20E-03	1.22E-03	2.84E-04	2.29E-04
MINIMUM	3.81E-04	4.08E-04	3.12E-04	3.39E-04	4.73E-06	1.56E-04

Trial #1 - Non-corrected data
Trial #2 - Jacob's correction
Trial #3 - Time deletion
Trial #4 - Both Jacob's and time deletion

(1) - Not able to determine using this method

Table 17. Specific Yield of all analysis trials

WELL NO	TRIAL #1	TRIAL #2	TRIAL #3	TRIAL #4	WATER BALANCE	AVERAGE
MW-1	0.0483	0.0413	0.0617	0.0566	(2)	0.0520
MW-2	0.0185	0.0176	0.0239	0.0227	(2)	0.0207
MW-3	0.0240	0.0230	0.0260	0.0251	(2)	0.0245
MW-4	0.0238	0.0230	0.0242	0.0235	(2)	0.0236
MW-5	0.0308	0.0300	0.0308	0.0300	(2)	0.0304
MW-6	0.0151	0.0135	0.0314	0.0280	(2)	0.0220
MW-7	0.0120	0.0113	0.0168	0.0158	(2)	0.0140
MW-8	0.0120	0.0114	0.0149	0.0143	(2)	0.0132
MW-9	0.0152	0.0147	0.0160	0.0154	(2)	0.0153
MW-10	0.0168	0.0163	0.0171	0.0169	(2)	0.0168
MW-11	0.0305	0.0283	0.0484	0.0455	(2)	0.0382
MW-12	0.0333	0.0319	0.0390	0.0376	(2)	0.0355
MW-13	0.0234	0.0225	0.0265	0.0257	(2)	0.0245
MW-14	0.0162	0.0157	0.0165	0.0160	(2)	0.0161
MW-15	0.0206	0.0199	0.0208	0.0202	(2)	0.0204
PW	(1)	(1)	(1)	(1)	(2)	(3)
MEAN	0.0227	0.0214	0.0276	0.0262	0.0371	0.0245
VARIANCE	0.0001	0.0001	0.0050	0.0001	(2)	0.0001
MAXIMUM	0.0483	0.0413	0.3690	0.0566	(2)	0.0520
MINIMUM	0.0120	0.0113	0.0958	0.0143	(2)	0.0132

Trial #1 - Non-corrected data
Trial #2 - Jacob's correction
Trial #3 - Time deletion
Trial #4 - Both Jacob's and time deletion

(1) - Not able to determine using this method
(2) - Method determines specific yield for entire well field only
(3) - Not calculable

and durations. The values obtained using the methods in this report can then be used to evaluate which method will best reproduce the drawdowns observed in the aquifer.

DISCUSSION OF FUTURE RESEARCH

The potential of this site is seemingly endless. The following is a partial list of research that may wish to be attempted at the site.

In order to fully investigate the aquifer characteristics in this type of setting, an aquifer test of much longer duration is needed. This will help to evaluate the presences of delay gravity drainage occurring in this setting. It is felt a minimum of 72 hours is necessary to reveal the presence of delayed yield in this setting. A test of one week or greater would be useful to determine the exact radius of influence of the well and a the relationship of time vs. drawdown during the later part of the test. This may also help to more clearly define at which point steady state is encountered in the aquifer.

The author plans to develop an aquifer test analysis model which will use the methods outlined by Neuman (1974, 1975) and Boulton (1954) for analysis of data obtained during the testing of an unconfined aquifer. The data obtained during the testing described in this paper will be reanalyzed to determine the differences it may have on the aquifer parameter determination. This research may help to determine the most effective technique to analyze an aquifer composed of this type of material. The data obtained from the partially penetrating wells will be very useful to evaluate the aquifer parameters.

It may also be beneficial to perform other tests using different flow rates to evaluate the differences in the aquifer parameters determined from these tests. This will allow a check of the validity of the solutions presented in this dissertation. The solutions should, if they are correct, allow the prediction of the drawdown results from these tests. Although some confounding factors may be present, the prediction of the data obtained at different flow rates is necessary to validate these results.

Since the water level at the site varies throughout the year, the opportunity is present to evaluate the effect the initial saturated thickness exerts on the aquifer test results. This may help to determine how the material properties of hydraulic conductivity and specific yield vary with depth.

The impact of pumping in the upper aquifer on the lower confining layer is also of interest to the author. The installation of several deeper wells into the confining layer would allow the determination of the impact of pumping the upper aquifer on the water in the lower confining layer. Neuman and Witherspoon (1972) have developed the governing equations for the determination of the hydraulic parameters of the confining layer.

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