ESTIMATION OF EDDY CURRENT PROBABILITY OF DETECTION (POD) USING FINITE ELEMENT METHOD

S. N. Rajesh, L. Udpa, and S. S. Udpa
Center for Nondestructive Evaluation
Iowa State University
Ames, IA 50011

INTRODUCTION

A considerable amount of attention has been focused in recent years towards the development of probability of detection (POD) models for a variety of nondestructive evaluation (NDE) methods. Interest in these models is motivated by a desire to quantify the variability introduced in the measurements during the process of testing. As an example, sources of noise in eddy current methods of NDE include those caused by variations in liftoff, material properties such as conductivity and permeability, probe canting angle, scan format, surface roughness and measurement noise. Fig. 1 pictorially depicts some of the factors that contribute to variations in the measured signal.

Numerical models have been used extensively to model physical processes underlying NDE phenomena. Such models can predict the transducer response for a given specimen geometry, defect configuration and test conditions. These models, however, are deterministic in nature and do not take into account the perturbations associated with the inspection and testing carried out in the field. This has limited the utility of deterministic models to practitioners in general since a considerable difference can exist between the nominal value of the transducer response predicted by the model and the actual measurement under realistic conditions.

Fig. 1. Measurement variabilities in eddy current NDT.
This paper presents a more comprehensive POD model [1,2] for eddy current NDE. Eddy current methods of nondestructive testing are used widely in industry to inspect a variety of nonferromagnetic and ferromagnetic materials. The development of a POD model is therefore of significant interest. The model incorporates several sources of variabilities characterized by a multivariate Gaussian distribution and employs the finite element method to predict the signal distribution.

The paper demonstrates the use of a finite element model within a probabilistic framework to predict the spread of the measured signal for eddy current nondestructive methods. Using the signal distributions for various flaw sizes the probability of detection curves for defects of varying widths have been computed. Results demonstrating the value and utility of the approach are presented. In contrast to experimental POD models [3], the cost of generating such curves is very low and complex defect shapes can be handled very easily. The results are also operator independent.

**PRINCIPLES OF EDDY CURRENT INSPECTION**

The eddy current NDE technique [4] is used extensively in several industries in large part due to a number of advantages associated with the technique. The advantages include noncontact nature and high sensitivity to defects especially to those that are close to the surface of the specimen. The physical principles of the eddy current method are illustrated in Fig. 2. When a coil excited by an alternating current source is brought close to a conducting material, the primary field set by the coil induces eddy currents in the material. The eddy currents in turn generate an opposing secondary field. When the test object is nonmagnetic the opposing secondary field results in a reduction of the net flux linkages of the coil and the inductance of the coil is thereby reduced. The resistance measured at the terminals of the coil is also altered to account for the eddy current losses within the material. The presence of a defect or inhomogeneity in the material causes a redistribution of the eddy currents, thereby changing the complex impedance of the probe coil. Changes in the coil impedance caused by defects in the material are often represented as trajectories in the impedance plane and used for defect characterization.

From considerations of the operating frequencies and dimensions of the experimental set-up, the underlying electromagnetic phenomenon is quasi-static in nature. The corresponding governing equations for the fields and currents are

\[ \nabla^2 \vec{E} = \mu \sigma \frac{\partial \vec{E}}{\partial t} \]  
\[ \nabla^2 \vec{H} = \mu \sigma \frac{\partial \vec{H}}{\partial t} \]  

![Fig. 2. Physical principles underlying the eddy current method.](image-url)
For a single frequency sinusoidal excitation, the one-dimensional equation (3) can be written in the phasor form as

\[
\frac{\partial^2 \vec{J}}{\partial x^2} = j\omega \mu \sigma \vec{J}
\]

(4)

Generally, the eddy current probe impedance measurements are influenced by several factors such as lift-off variations, permeability and conductivity, temperature variations, probe canting angle, scan format and measurement noise. In order to quantify the detectability of the eddy current method, a model that can predict flaw signals in the presence of the variabilities of measurement conditions is needed. The finite element model for eddy current NDT developed by Lord and Palanisamy [5] is used here in a probabilistic framework for generating the distribution of the eddy current signals.

THE FINITE ELEMENT MODEL

The governing diffusion equation describing the eddy current phenomenon under steady state excitation is given by

\[
\nabla \times \left( \frac{1}{\mu} \nabla \times \vec{A} \right) = \vec{J}_s - j\omega \sigma \vec{A}
\]

(5)

where \( \vec{J}_s \) is the source current density, \( \vec{A} \) the magnetic vector potential, \( \mu \) and \( \sigma \) are the permeability and conductivity of the material and \( \omega \) is the excitation frequency. For homogeneous media, equation (5) reduces to

\[
\frac{1}{\mu} \nabla^2 \vec{A} = j\omega \sigma \vec{A} - \vec{J}_s
\]

(6)

The numerical model based on the finite element analysis solves the axisymmetric form of the governing differential equation

\[
\left( \frac{1}{\mu} \frac{\partial^2 \vec{A}}{\partial r^2} + \frac{1}{r} \frac{\partial \vec{A}}{\partial r} + \frac{1}{r} \frac{\partial^2 \vec{A}}{\partial z^2} \right) = j\omega \sigma \vec{A} - \vec{J}_s
\]

(7)

Applying principles of variational calculus, the differential equation is solved by minimizing the energy functional given by

\[
F = \int \int_R \left[ \frac{1}{2\mu} \left\{ \frac{\partial \vec{A}}{\partial z} \right\}^2 + \frac{\partial \vec{A}}{\partial r} + \vec{A} \right] \cdot \nabla \left. \frac{\partial \vec{A}}{\partial z} \right| + \frac{j\omega \sigma}{2} \left[ \vec{A}^2 - \frac{\vec{J}_s \vec{A}}{r} \right] \right| \, r \, dr \, dz
\]

(8)

The major aspects of finite element modeling for electromagnetic NDT problems are described in a number of references [6,7]. The region of interest is subdivided into a finite number of triangular elements connected to each other at a discrete set of nodal points. The variation of the continuous field quantity is approximated by a polynomial in such a way that the approximated function is continuous across the interelement boundaries. The nodal point values are determined by minimizing the functional in equation (8), which yields a set of linear equations in the unknown nodal point values. Since the unknown value at each node is expressed in terms of the values at the adjoining nodes the resulting matrix is sparse, banded, symmetric and diagonally dominant. These attributes make the numerical computation robust and stable.
The eddy current probe impedance measurements are influenced by several factors as depicted in Fig. 1. In order to quantify the detectability of the eddy current method, a POD model [8] that can predict flaw signals, in the presence of these perturbations is required. Fig. 3 shows a block diagram of a POD model used for assessing the detectability.

The approach consists of simulating the measurement model repeatedly with random perturbations in the measurement conditions derived from an appropriate distribution. The model predicts signals that are distributed about a mean value. The probability density functions (pdfs) of the peak amplitude of the signals without a flaw \( p(y/x_0) \) and in the presence of flaw \( p(y/x_1) \) are shown in Fig. 4. A value \( T \) of the peak amplitude is then selected as the threshold level such that signals whose magnitude exceed \( T \) are interpreted as flaw signals and signals with peak amplitude below \( T \) are interpreted as absence of a flaw in the inspected component. This classification could result in two types of errors which are of significance, namely false alarm and false acceptance.

For a selected value of \( T \), the parameters of interest such as the probability of detection, probability of false alarm and probability of false acceptance are given by equations (9 -11).

The probability of detection can be computed from the conditional density function of the flaw signal given by
POD = \int_Y^\infty p(y/x_I) \, dy \quad (9)

Similarly, the probability of false alarm (PFA) is given by

\[ PFA = \int_T^\infty p(y/x_0) \, dy \quad (10) \]

and the probability of false acceptance which is simply \((1 - POD)\) is given by

\[ POFA = \int_0^T p(y/x_1) \, dy. \quad (11) \]

The degree of overlap between the on-flaw and off-flaw signal distribution is in general, a function of the flaw size. The overlap area increases with decreasing flaw dimensions resulting in a reduction of the probability of detection of flaws for a given threshold \(T\). As can be seen in Fig. 4 a low value of the threshold would result in a high probability of detection. However, this would yield a high false alarm rate. On the contrary, a high value of threshold would result in a low probability of detection but a low false alarm rate. A plot of POD vs PFA, explained in the next section, at different values of threshold gives the relative operating characteristics (ROC) of the system for a given flaw.

The total probability of error in the signal classification can be modeled as a weighted sum of PFA and POFA given by

\[
\text{probability of error} = a \int_T^\infty p(y/x_1) \, dy + (1-a) \int_0^T p(y/x_0) \, dy \quad (12)
\]

where \(a\) is the prior probability of a flaw being present in the region of interest.

Since the signal classification error is a function of \(T\), the threshold value can be chosen so that the error is minimized. The threshold value can also be derived using other criteria. One such criterion could be to choose a threshold to set the probability of false alarm at a preset value. In these cases thresholds are chosen from the noise pdf to set the PFA to a value \(\theta_1\). Alternatively the threshold may be selected on the basis of a fixed value of probability of false acceptance of a critical flaw.

**SIMULATION RESULTS**

An eddy current inspection system consisting of an absolute eddy current probe scanning the surface of an aluminum plate was modeled. The physical dimensions of the probe geometry are as summarized in Table 1.

In the first test, the individual variations in liftoff, were considered using a univariate Gaussian distribution. The effect of these variations was observed by perturbing the measurement model using random variates derived from this distribution. The corresponding model predictions were used to generate conditional pdfs of the measurement variable in the
Table 1. Physical dimensions of the probe.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Inside diameter (mm)</td>
<td>1.07</td>
</tr>
<tr>
<td>Outside diameter (mm)</td>
<td>2.62</td>
</tr>
<tr>
<td>Mean coil radius (mm)</td>
<td>0.92</td>
</tr>
<tr>
<td>Coil length (mm)</td>
<td>2.93</td>
</tr>
<tr>
<td>Lift-off height (mm)</td>
<td>0.56</td>
</tr>
<tr>
<td>Number of turns (mm)</td>
<td>235</td>
</tr>
</tbody>
</table>

presence and absence of the flaw. Similar pdfs for different flaws of varying widths and a constant depth of 0.25 mm were generated. The threshold, $T_1$, was selected such that the probability of false alarm, which is independent of the flaw size, is 0.05. A threshold value of 21.64 Ohms was obtained and used for computing the POD curve.

The second test involved the study of the simultaneous effects of variations in liftoff, material conductivity and surface roughness, modeled by a multivariate normal. As explained in test 1 the pdfs in the presence and absence of flaws were generated by perturbing the measurement model using the random variates derived from this distribution. Such pdfs were generated for different flaws of varying widths and a constant depth of 0.25 mm. As before, a threshold was selected such that the probability of false alarm is 0.05. A threshold value of 21.68 Ohms was obtained and used to compute the POD curve. This POD vs flaw width curves for tests 1 and 2 are shown in Fig. 5. The POD curve for test 2, where all the three variabilities are used to perturb the measurement model is lower compared to the curve for test 1. For instance, a flaw of width 0.258 mm has a POD of 0.81 in test 1 due to the influence of liftoff. However in test 2 the POD is reduced to 0.77 due to the effects of liftoff as well as material conductivity and surface roughness.

![ Fig. 5. POD curves showing the influence of the various variabilities acting on the test system. ](image)
The third test was concerned with the relative operating characteristics (ROC) of the system. A flaw of dimensions 0.1 mm width and 0.25 mm depth was selected. The pdfs, of the measurement variable, in the presence and absence of the flaw was generated by perturbing the measurement model. Different values of threshold were selected and the corresponding POD and PFA values computed. The plot of the POD vs PFA, for different threshold values, shown in Fig. 6 constitute the ROC of the system for the flaw. The ROC curve enables one to determine the POD of the flaw if a particular threshold is selected for a fixed value of the probability of false alarm. As an example if the threshold is selected such that the PFA is equal to 0.2, the corresponding probability of detecting the flaw would be 0.45.

In the final test, nine axisymmetric flaws of different dimensions were modeled and the pdfs in the absence and presence of the flaws generated. The flaw 'e' was selected as the critical flaw and the threshold selected such that the probability of detecting the critical flaw is 0.95. The threshold value was set at 21.68 Ohms and the corresponding POD and PFA values for the nine flaws were computed. The flaw dimensions and the corresponding POD and PFA values are as summarized in Table 2.

**CONCLUSIONS**

The feasibility of using a finite element model in a probabilistic framework for estimating the probability of detecting a flaw has been investigated. The model based POD estimation procedure complement the experimental POD models. Theoretical POD models allow complex defect shapes to be handled easily at low cost. Future work involves incorporation of the POD model into a CAD framework.

**ACKNOWLEDGMENTS**

This work was supported by the FAA-Center for Aviation Systems Reliability, operated by the Ames Laboratory, USDOE, for the Federal Aviation Administration under Contract No. W-7405-ENG-82 with Iowa State University.
Table 2. POD and PFA values for the 9 different flaws with the threshold selected such that the probability of detection of the critical flaw 'e' is 0.95.

<table>
<thead>
<tr>
<th>Flaw</th>
<th>Dimensions Width x depth (mm)</th>
<th>POD</th>
<th>PFA</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>0.125 x 0.5</td>
<td>0.04</td>
<td>0</td>
</tr>
<tr>
<td>b</td>
<td>0.125 x 1.0</td>
<td>0.04</td>
<td>0</td>
</tr>
<tr>
<td>c</td>
<td>0.125 x 1.5</td>
<td>0.04</td>
<td>0</td>
</tr>
<tr>
<td>d</td>
<td>0.25 x 0.25</td>
<td>0.871</td>
<td>0</td>
</tr>
<tr>
<td>e</td>
<td>0.25 x 1.0</td>
<td>0.95</td>
<td>0</td>
</tr>
<tr>
<td>f</td>
<td>0.25 x 2.5</td>
<td>0.9528</td>
<td>0</td>
</tr>
<tr>
<td>g</td>
<td>0.25 x 1.25</td>
<td>0.951</td>
<td>0</td>
</tr>
<tr>
<td>h</td>
<td>0.50 x 1.25</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>i</td>
<td>1.25 x 1.25</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

REFERENCES