3-D ULTRASONIC SCATTERING BY SURFACE-BREAKING CRACKS

D. E. Budreck
Dept. of Engineering Science and Mechanics, and
Dept. of Mathematics
Iowa State University
Ames, IA 50011

J. D. Achenbach
Center for Quality Engineering and Failure Prevention
Catalysis Building, Third Floor
2137 North Sheridan Road
Evanston, IL 60208

INTRODUCTION

Ultrasonic scattering by cracks which break the traction-free sur-face of an engineering structure is of considerable interest in the field of nondestructive evaluation, especially when the crack mouth is inacces-sible to direct observation. Considerable effort has been expended in obtaining solutions to the two-dimensional problem, from which much has been learned. A representative cross-section of work on the two-dimen-sional problem includes papers by Mendelsohn, et al. [1], Achenbach and Norris [2], Datta and Shah [3], Zhang and Achenbach [4], and Roberts [5]. However, if the crack dimensions are comparable to the width of the interrogating ultrasonic beam, the full three-dimensional problem must be considered. To this end, Budreck and Achenbach [6] investigated the scattering of elastic waves from a 3-D normal edge crack of arbitrary shape.

In this paper we employ the method of Ref. [6] to investigate a fundamental problem in quantitative NDE: ultrasonic scattering from a semicircular normal edge crack. The problem is formulated and solved in the frequency domain by boundary integral equation methods, where the full-space Green's function is used for the solution of the half-space problem. Consequently, the surface of the half-space is one of the sur-faces of the system of boundary integral equations. The geometrical decay of both body waves and Rayleigh waves along this traction-free surface allows, however, for a discretization of the surface over a finite region. It will be demonstrated that this proves to be a viable method of solution for low/intermediate frequency scattering.

We begin by stating the scattering problem to be considered, which allows introduction of the relevant verbiage and notational formalisms. Next, the solution method is outlined, as is the subsequent use of far-field asymptotic expansions of the Green's functions required for the calculation of scattering amplitudes. Finally, numerical results are presented for the case of scattering from a semicircular normal edge crack with radius a (see Fig. 1).

The crack is insonified by a 45° T-wave with a dimensionless wavenumber of $k_T a = 1$. Far-field L- and T-wave scattering amplitudes are plotted as a function of the observation angle ψ.

PROBLEM STATEMENT

In what follows, Roman super/subscripts take on values 1-3, over which the summation convention is assumed throughout. Greek indices are reserved for all exceptions to this rule. The Greek indices γ and δ can take on only the values 1 and 3, and will <u>never</u> be summed.

We begin by considering a homogeneous, isotropic, linearly elastic half-space, $V_{1/2}$. The traction-free surface $S^{(1)}$ of $V_{1/2}$ coincides with the Cartesian (x_2, x_3)-plane and has outward normal $n^{(1)} = (-1, 0, 0)$. Furthermore, $V_{1/2}$ has a bounded internal boundary, $S^+ U S^-$, which breaks the surface $S^{(1)}$ of $V_{1/2}$ and extends in the x_1-direction. Here S^+ and S^- are two coplanar traction-free surfaces that are infinitesimally close, and have outward normals $\underset{\sim}{n}^+ = (0, 0, -1)$ and $\underset{\sim}{n}^- = (0, 0, +1)$, respectively. This surface-breaking crack $S^+ U S^-$ will in the future be referred to by its planar surface $S^{(3)}$, with normal $\underset{\sim}{n}^{(3)} = (0, 0, -1)$.
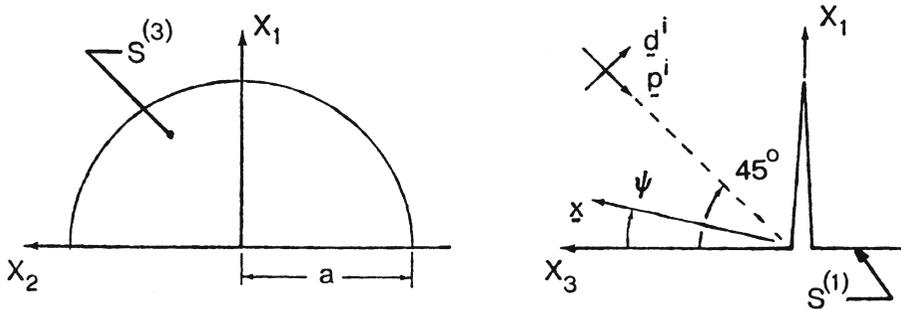

Figure 1.   Scattering geometry of semicircular edge crack with radius a. Crack is insonified by a transverse wavefield incident under an angle of 45°; ψ denotes the angle of observation.

Further consider a time-harmonic ($e^{-i\omega t}$) displacement field propagating in the $\underset{\sim}{p}^i$ direction, such that $p_1^i \leq 0$. This incoming field will give rise to a reflected field at $S^{(1)}$, such that the sum of the incoming and reflected fields has spatial dependence defined by $u_i^{in}(\underset{\sim}{x})$. This field is incident on the crack $S^{(3)}$. The crack generates a scattered field $u_i^{sc}(\underset{\sim}{x})$, such that the total displacement field in $V_{1/2}$ is given by

$$u_i(\underset{\sim}{x}) = u_i^{in}(\underset{\sim}{x}) + u_i^{sc}(\underset{\sim}{x}), \quad x \ \varepsilon \ V_{1/2}. \tag{1}$$

An application of the divergence theorem to the Betti-Rayleigh reciprocal relation for the scattered elastodynamic field and the __full-space__ elastodynamic Green's function gives the following representation integral for the scattered displacement field:

$$u_k^{sc}(\underset{\sim}{x}) \equiv I_k^{(3)}(\underset{\sim}{x}) + I_k^{(1)}(\underset{\sim}{x}), \quad \underset{\sim}{x} \in V_{\frac{1}{2}}, \tag{2a}$$

where

$$I_k^{(\delta)}(\underset{\sim}{x}) \equiv \int_{S^{(\delta)}} D_k^{\delta m}(\underset{\sim}{x}-\underset{\sim}{y}) \phi_m^{(\delta)}(\underset{\sim}{y}) dS(\underset{\sim}{y}), \tag{2b}$$

where

$$\phi_m^{(3)}(\underset{\sim}{y}) \equiv \Delta u_m^{sc}(\underset{\sim}{y}), \quad \phi_m^{(1)}(\underset{\sim}{y}) \equiv u_m^{sc}(\underset{\sim}{y}). \tag{2c,d}$$

This result is valid provided $u_k^{sc}(\underset{\sim}{x})$ satisfies the usual radiation condition for $|\underset{\sim}{x}| \to \infty$, $\underset{\sim}{x} \in V_{\frac{1}{2}}$. In Equation (2b) the summation over δ is __not__ implied. The crack-opening displacement $\Delta u_m(\underset{\sim}{y})$ of (2c) is defined on $S^{(3)}$ and given by

$$\Delta u_m^{sc}(\underset{\sim}{y}) \equiv {}^{+}u_m^{sc}(\underset{\sim}{y}) - {}^{-}u_m^{sc}(\underset{\sim}{y}), \tag{3}$$

where ${}^{+}u_m^{sc}(\underset{\sim}{y})$ and ${}^{-}u_m^{sc}(\underset{\sim}{y})$ define the scattered displacement field evaluated on the crack faces S^+ and S^-, respectively. The kernel of the integral operator in Equation (2b) is the full-space stress Green's function $D_m^{ij}(\underset{\sim}{x}-\underset{\sim}{y})$. The explicit functional form of D_m^{ij}, as well as its Taylor series expansion in $|\underset{\sim}{x}-\underset{\sim}{y}|$, can be found in Ref. [7].

An alternate representation integral for the scattered displacement field results from the application of the divergence theorem to the Betti-Rayleigh reciprocal relation for the scattered elastodynamic field and the __half__-space elastodynamic Green's function:

$$u_k^{sc}(\underset{\sim}{x}) \equiv {}_{\frac{1}{2}}I_k^{(3)}(\underset{\sim}{x}), \quad \underset{\sim}{x} \in V_{\frac{1}{2}}, \tag{4a}$$

where

$${}_{\frac{1}{2}}I_k^{(3)}(\underset{\sim}{x}) \equiv \int_{S^{(3)}} {}_{\frac{1}{2}}D_k^{3m}(\underset{\sim}{x},\underset{\sim}{y}) \Delta u_m(\underset{\sim}{y}) dS(\underset{\sim}{y}). \tag{4b}$$

The kernel of the integral operator in Equation (4b) is the half-space stress Green's function ${}_{\frac{1}{2}}D_k^{3m}(\underset{\sim}{x},\underset{\sim}{y})$; see Ref. [6].

Notice that in each of the representation integrals (2) and (4), the surfaces of integration comprise those surfaces that have boundary conditions not satisfied by the Green's functions in the integrand. Thus an integration over $S^{(3)} \cup S^{(1)}$ has been reduced to an integration over $S^{(3)}$, but at the expense of introducing a more complex Green's

function, which must take into account the traction-free boundary condition on $S^{(1)}$.

The problem statement is as follows: given a crack-plane $S^{(3)}$ and an incident field $u_i^{in}(\underset{\sim}{x})$, the resulting scattered field $u_i^{sc}(\underset{\sim}{x})$ is to be obtained. By virtue of Equation (4a,b) the problem can be reduced to finding the crack-opening displacement $\Delta u_m^{sc}(\underset{\sim}{y})$ caused by the incident field $u_i^{in}(\underset{\sim}{x})$. The two quantities are connected by the traction-free boundary condition, which implies

$$\sigma_{3j}^{sc}(\underset{\sim}{x}) = -\sigma_{3j}^{in}(\underset{\sim}{x}), \quad \underset{\sim}{x} \ \varepsilon \ S^{(3)}. \tag{5}$$

SOLUTION METHOD

The crack-opening displacement Δu_m^{sc} will be obtained through the solution of a discretized integral equation derived from the representation integral (2), where the decay of Rayleigh waves emanating from the crack-mouth and propagating along $S^{(1)}$ will allow a discretization over a finite area of $S^{(1)}$. Solution by this method will require, in addition to Equation (5), the boundary condition

$$\sigma_{1j}^{sc}(\underset{\sim}{x}) = -\sigma_{1j}^{in}(\underset{\sim}{x}) \equiv 0, \quad \underset{\sim}{x} \ \varepsilon \ S^{(1)}. \tag{6}$$

Then, upon insertion of the far-field asymptotic expansion of $_{1\!\!2}D_k^{3m}$, the representation integral (4a,b) will be used to calculate the scattered far-field at various points of interest.

The derivation of the discretized integral equation to be solved for the crack-opening displacement Δu_m^{sc} can be found in Ref. [6]. Essentially, the surfaces $S^{(\delta)}$ of Equation (2b) are first discretized into $N^{(\delta)}$ elements,

$$S^{(\delta)} \rightarrow \sum_{q=I(\delta)}^{J(\delta)} S_q^{(\delta)}, \tag{7}$$

where $S_q^{(\delta)}$ denotes the qth element lying on the surface $S^{(\delta)}$, and where $I(3) = 1$, $J(3) = N^{(3)}$, $I(1) = N^{(3)} + 1$, and $J(1) = N^{(3)} + N^{(1)}$.

Next, the unknowns given by Equations (2c,d) are removed from the integrals over each element by assuming that they can be evaluated (approximated) anywhere on the element simply by extrapolating their value at the element's centroid. This is accomplished by assuming (a) a dependence that emulates the static solution over those elements that share a border with the curved crack edge, and (b) a constant approximation over all remaining elements. The advantage of assumption (a) is that it is asymptotically correct near the curved crack edge, and thus a fine discretization that would be required if assumption (b) had been

employed over those same elements is avoided. The difference in crack-opening shape where the crack-edge intersects the surface of the half-space is a very local effect, and is not accounted for in this analysis.

Subsequent to both discretization and the approximations described above, but prior to taking the limit $\underset{\sim}{x} \to \underset{\sim}{x}_p$ ($\underset{\sim}{x}_p \in S^{(\delta)}$ is the centroid of element $S_p^{(\delta)}$), the representation integral is regularized (see Ref. [6]) by isolating the potentially hypersingular integrals, which are then evaluated exactly. Subsequent to the limit $\underset{\sim}{x} \to \underset{\sim}{x}_p$, there results the discretized integral equation

$$-\sigma_{\gamma j}^{in}(\underset{\sim}{x}_p) = {}^D J_{\gamma j}^{(3)}(\underset{\sim}{x}_p) + {}^D J_{\gamma j}^{(1)}(\underset{\sim}{x}_p), \quad \underset{\sim}{x}_p \in S_p^{(\gamma)}, \tag{8}$$

where

$$^D J_{\gamma j}^{(\delta)}(\underset{\sim}{x}) \equiv \sum_{q=I(\delta)}^{J(\delta)} \left[H_{\gamma j m}^q(\underset{\sim}{x}) \right]^{(\delta)} \phi_m^{(\delta)}(\underset{\sim}{y}_q), \tag{9}$$

where for $\gamma \neq \delta$

$$\left[H_{\gamma j m}^q(\underset{\sim}{x}) \right]^{(\delta)} \equiv C_{\gamma j k l} \int_{S_q^{(\delta)}} D_{k,l}^{\delta m}(\underset{\sim}{x}-\underset{\sim}{y}) f_m^{(\delta)}(\underset{\sim}{y}) dS(\underset{\sim}{y}), \tag{10}$$

which are all regular integrals that can be performed numerically. For $\gamma = \delta$

$$\left[H_{\gamma j m}^q(\underset{\sim}{x}) \right]^{(\delta)} = \left[{}^N H_{\gamma j m}^q(\underset{\sim}{x}) \right]^{(\delta)} + \left[{}^E H_{\gamma j m}^q(\underset{\sim}{x}) \right]^{(\delta)}, \tag{11}$$

where the first term on the right-hand side of Eq. (11) are regular integrals that can be performed numerically, and the following term contains those integrals which during the process of regularization have been evaluated exactly. In Eq. (10) the function $f_m^{(\delta)}$ denotes the element shape function corresponding to assumptions (a) and (b) discussed just after Eq. (7).

Due to the symmetry of the scattering geometry, it can be shown that the system of Equations (8) decouples in such a way that a crack under normal loading gives rise to all components of surface displacement on $S^{(1)}$, but only the normal component of crack-opening displacement on $S^{(3)}$. Numerical results will be presented for this case by considering a normal edge crack insonified by a T-wave approaching the half-space surface at an angle of 45°.

SCATTERED FAR-FIELD

Upon insertion of the far-field half-space stress Green's function (see Ref. [6]) into the representation integral (4a,b), there results

$$^{\alpha}u_k^{sc}(\underset{\sim}{x}) \sim {}_{\frac{1}{2}}^{\alpha}I_k^{(3)}(\underset{\sim}{x}), \quad \underset{\sim}{x} \, \varepsilon \, V_{\frac{1}{2}}, \quad |\underset{\sim}{x}| \to \infty, \tag{12a}$$

where

$$^{\alpha}_{\frac{1}{2}}I_k^{(3)} \equiv \int_{S(3)} {}_{\frac{1}{2}}^{\alpha}D_k^{3m}(\underset{\sim}{x},\underset{\sim}{y}) \Delta u_m(\underset{\sim}{y}) dS(\underset{\sim}{y}), \tag{12b}$$

where the superscript $\alpha = L,T$ serves to indicate an outgoing L-, T-wave observed at the far-field position $\underset{\sim}{x}$.

The form of Eq. (12) which reveals the scattering amplitude U_k^{α} is

$$^{\alpha}u_k^{sc}(\underset{\sim}{x}) \sim U_k^{\alpha}(\psi) \frac{e^{ik_{\alpha}|\underset{\sim}{x}|}}{4\pi|\underset{\sim}{x}|}, \quad |\underset{\sim}{x}| \to \infty, \tag{13}$$

where here $\underset{\sim}{x}$ has been restricted to lie in the (x_1,x_3)-plane, and ψ is the angle of observation in that plane such that $\underset{\sim}{x} \cdot \underset{\sim}{i}_3/|\underset{\sim}{x}| = \cos\psi$. Here i_3 denotes a unit vector in the x_3-direction.

NUMERICAL RESULTS

In this section numerical results are presented for scattering from the semicircular surface-breaking crack depicted in Fig. 1. The crack, which lies in the (x_1,x_2)-plane and breaks the surface $S^{(1)}$ of the half-space, is insonified by a T-wave, whose propagation vector and displacement direction are in the (x_1,x_3)-plane. For simplicity, all calculations of the scattered field will also be made in the (x_1,x_3)-plane, and for an observation angle ψ measured from the positive x_3-axis.

The solutions of Eq. (8) yield the crack-opening displacements over the $S_q^{(3)}$ elements, and the three components of the displacement on the $S_q^{(1)}$ elements. Since the crack-opening displacement is all that is required by the representation integral (4), the acid test which Eq. (8) must pass is that the crack-opening displacement converge with successively larger regions of discretization of the surface $S^{(1)}$. This convergence will be demonstrated in the sequel by comparing far-field results obtained with discretizations of $S^{(1)}$ over circular regions with radii 2a and 3a, respectively.

In the case that the incoming T-wave is incident under an angle of 45°, there is no reflected P-wave, and the total stress field incident on $S^{(3)}$ is

$$\sigma_{33}^{in}(\underset{\sim}{x}) = 2\mu\sin(k_T x_1/\sqrt{2}), \quad \underset{\sim}{x} \; \epsilon \; S^{(3)}, \qquad (14)$$

which serves as input to Eq. (8), together with Eq. (6). The resulting crack-opening displacement, together with the representation integral (12), allows calculation of the far-field scattering amplitudes U_K^α given by Eq. (13). The displacement amplitudes of the far-field scattered L- and T-waves are plotted as functions of the observation angle ψ in Fig.
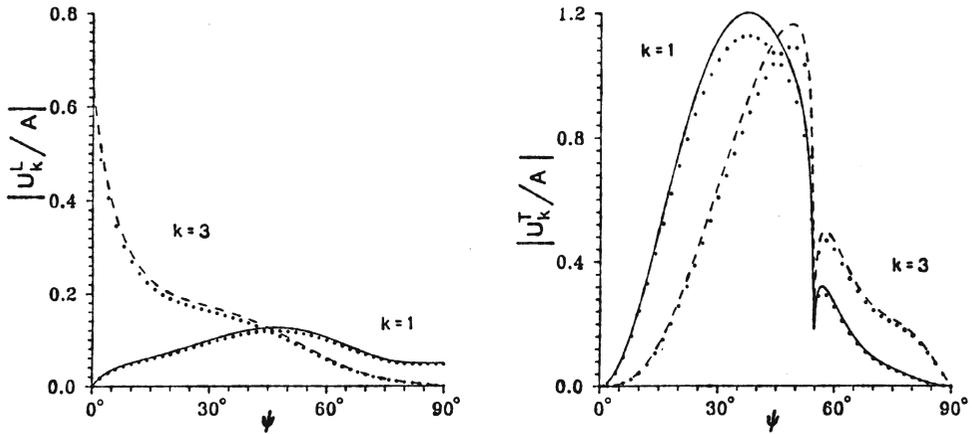

Figure 2. Dimensionless scattered L- and T-wave amplitude spectra due to incident field represented by Eq. (14).

2. Here the incident field was taken with $k_T a = 1$, and Poisson's ratio was taken to be $\nu = 0.25$. The plotted amplitudes were nondimensionalized through division by the surface area of the crack, A. A solid line (dashed line) corresponds to the amplitude of the x_1- (x_3-) component of the displacement field. Both solid and dashed lines correspond to a discretization of $S^{(1)}$ over a circular region with radius 3a. The points below these lines correspond to a discretized region of radius 2a. The close proximity of the two results shows the convergence of the crack-opening displacement with an increased discretization of $S^{(1)}$.

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