BACKSCATTER FROM A SPHERICAL INCLUSION WITH COMPLIANT INTERPHASE CHARACTERISTICS

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INTRODUCTION

In studies of scattering by an inclusion it is generally assumed that the inclusion is perfectly bonded to the surrounding matrix material. The actual bond between two materials is, however, generally effected by a thin layer, which may be called an interphase, rather than an interface. It is well known that the mechanical behavior of such an interphase may significantly influence the overall mechanical behavior of a solid containing inclusions.

In this paper it is investigated to what extent an interphase affects the scattered field generated by an incident ultrasonic wave. Both a completely intact but compliant interphase, and an interphase which does not transmit tractions over part of the area between the inclusion and the matrix, have been considered.

The interphase is generally very thin. In this paper, it is assumed that the radial and the tangential tractions are continuous across the interphase, but the displacements may be discontinuous from inclusion to matrix. The tractions are assumed to be proportional to the corresponding displacement discontinuities. The proportionality constants characterize the stiffness and strength of the interphase. On the basis of this interphase model, which corresponds to a distribution of springs between the inclusion and the matrix, a rigorous analysis has been carried out of the backscattered field generated by an incident longitudinal wave. Within the context of the present model the case of a partially defective interphase, i.e., the case of a crack over part of the surface between the inclusion and the matrix, is easily included by setting the spring constants identically zero over the cracked surface.

The analysis has been carried out by deriving a set of singular integral equations for the tractions and displacements across the interphase. These equations have been solved by the boundary element method, and the scattered field has subsequently been obtained by the use of the elastodynamic integral representation.

An analysis for a similar configuration, but by the use of the null field approach, and for a completely intact interphase, has been presented by Datta, Olsson and Boström [1].

FORMULATION

Let $\bar{\lambda}$ and $\bar{\mu}$ be Lamé's elastic constants, and $\bar{\rho}$ the mass density of the inclusion, and let λ, μ, ρ be the corresponding quantities of the surrounding matrix material, as shown in Fig. 1. In this figure, $\underset{\sim}{u}^I$ and $\underset{\sim}{u}^S$ are the incident and the scattered wave, respectively. The total wave field is defined as $\underset{\sim}{u} = \underset{\sim}{u}^I + \underset{\sim}{u}^S$. In the following, the time-harmonic factor $\exp(-i\omega t)$ has been suppressed, and the upper bar-notation is used for quantities related to the inclusion.

Boundary Integral Equations for the Matrix and the Inclusion

The boundary integral equation for the matrix material takes the form

$$C_{ij}(\underset{\sim}{x})u_j(\underset{\sim}{x}) = \int_S U_{ij}(\underset{\sim}{x},\underset{\sim}{y})t_j(\underset{\sim}{y})dS_y - \int_S T_{ij}(\underset{\sim}{x},\underset{\sim}{y})u_j(\underset{\sim}{y})dS_y + u_i^I(\underset{\sim}{x}), \quad \underset{\sim}{x} \in S, \tag{1}$$

where S is the interphase boundary at the matrix side and $U_{ij}(\underset{\sim}{x},\underset{\sim}{y})$ is the fundamental solution for 3D time-harmonic elastodynamics:

$$U_{ij}(\underset{\sim}{x},\underset{\sim}{y}) = \frac{1}{4\pi\mu}\left[\frac{e^{ik_T r}}{r}\delta_{ij} + \frac{1}{k_T^2}\frac{\partial}{\partial x_i}\frac{\partial}{\partial x_j}\left\{\frac{e^{ik_T r}}{r} - \frac{e^{ik_L r}}{r}\right\}\right], \tag{2}$$

while $T_{ij}(\underset{\sim}{x},\underset{\sim}{y})$ is the corresponding traction. The boundary integral equation for the inclusion is of the form

$$\bar{C}_{ij}(\underset{\sim}{x})\bar{u}_j(\underset{\sim}{x}) = \int_{\bar{S}} \bar{U}_{ij}(\underset{\sim}{x},\underset{\sim}{y})\bar{t}_j(\underset{\sim}{y})dS_y - \int_{\bar{S}} \bar{T}_{ij}(\underset{\sim}{x},\underset{\sim}{y})\bar{u}_j(\underset{\sim}{y})dS_y, \quad \underset{\sim}{x} \in \bar{S}, \tag{3}$$

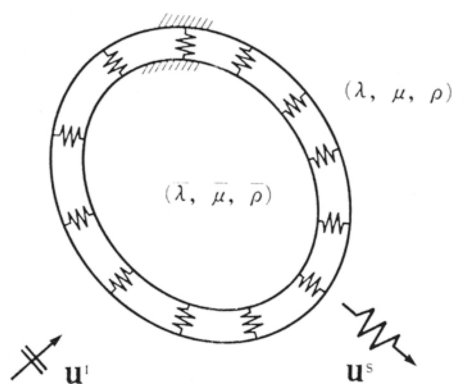

Fig. 1 Scattering by an inclusion with a compliant interphase.

where $\bar{S}$ is the interphase boundary at the inclusion side. In matrix form Eq.(1) becomes after the discretization:

$$\underset{\approx}{H}\,\underset{\sim}{u} = \underset{\approx}{G}\,\underset{\sim}{t} + \underset{\sim}{u}^{I} \tag{4}$$

Next we operate with $\underset{\approx}{A}\underset{\approx}{G}^{-1}$ on both sides of Eq.(4), to obtain

$$\underset{\approx}{K}\underset{\sim}{u} = \underset{\approx}{F} + \underset{\approx}{A}\underset{\approx}{G}^{-1}\underset{\sim}{u}^{I} \quad , \tag{5}$$

where $\underset{\approx}{K}$ and $\underset{\sim}{F}$ are defined as

$$\underset{\approx}{K} = \underset{\approx}{A}\,\underset{\approx}{G}^{-1}\,\underset{\approx}{H},\ \underset{\sim}{F} = \underset{\approx}{A}\,\underset{\sim}{t} \quad . \tag{6a,b}$$

Here it has been assumed that in taking $\underset{\approx}{G}^{-1}$, the frequency does not coincide with an eigenfrequency of the inclusion. The matrix $\underset{\approx}{A}$ has the following diagonal form

$$\underset{\approx}{A} = \begin{bmatrix} \underset{\approx}{A}_1 & & & & \underset{\approx}{0} \\ & \underset{\approx}{A}_2 & & & \\ & & \ddots\ \underset{\approx}{A}_\alpha & & \\ & & & \ddots & \\ \underset{\approx}{0} & & & & \underset{\approx}{A}_M \end{bmatrix} \quad . \tag{7a}$$

Here the subscript M denotes the total number of boundary elements, and the α-th sub-matrix $\underset{\approx}{A}_\alpha$ has the form

$$\underset{\approx}{A}_\alpha = \begin{bmatrix} A_\alpha & 0 & 0 \\ 0 & A_\alpha & 0 \\ 0 & 0 & A_\alpha \end{bmatrix} \quad , \tag{7b}$$

where A_α is the area of the α-th element.

In the same way, Eq.(3) for the inclusion may be reduced to

$$\bar{\underset{\approx}{K}}\,\bar{\underset{\sim}{u}} = \bar{\underset{\approx}{F}} \quad , \tag{8}$$

where

$$\bar{\underset{\approx}{K}} = \bar{\underset{\approx}{A}}\,\bar{\underset{\approx}{G}}^{-1}\,\bar{\underset{\approx}{H}},\quad \bar{\underset{\sim}{F}} = \bar{\underset{\approx}{A}}\,\bar{\underset{\sim}{t}} \quad . \tag{9a,b}$$

Spring Equations to represent the Interphase

The interphase between the inclusion and the surrounding matrix is now modelled by a continuous distribution of springs, see, Fig. 1. The springs are assumed to be linear and they have three components. The system of equations defining the spring connections can be written down in the following form

$$\begin{bmatrix} \underset{\approx}{S} & -\underset{\approx}{S} \\ -\underset{\approx}{S} & \underset{\approx}{S} \end{bmatrix} \begin{Bmatrix} \bar{\underset{\sim}{u}}^{SP} \\ \underset{\sim}{u}^{SP} \end{Bmatrix} = \begin{Bmatrix} \bar{\underset{\sim}{F}}^{SP} \\ \underset{\sim}{F}^{SP} \end{Bmatrix} \quad , \tag{10}$$

where $\underset{\approx}{S}$ is the spring constant matrix, which is defined as

$$\underset{\approx}{S} = \begin{bmatrix} \underset{\approx}{R}_1 & & & \underset{\approx}{0} \\ & \underset{\approx}{R}_2 & & \\ & & \ddots\ \underset{\approx}{R}_\alpha & \\ \underset{\approx}{0} & & & \ddots\ \underset{\approx}{R}_M \end{bmatrix} \quad . \tag{11a}$$

In Eq.(10), $\bar{\underset{\sim}{u}}^{SP}$ and $\bar{\underset{\sim}{F}}^{SP}$ ($\underset{\sim}{u}^{SP}$ and $\underset{\sim}{F}^{SP}$) are the displacements and forces on the springs at the inclusion (matrix) side. The superscript SP designates quantities in the springs. In Eq.(11a), the α-th sub-matrix $\underset{\approx}{R}_\alpha$ is the spring constant matrix of the α-th element; it has the form

$$\underset{\approx}{R}_\alpha = \begin{bmatrix} R_{1\underline{\alpha}}A_{\underline{\alpha}} & 0 & 0 \\ 0 & R_{2\underline{\alpha}}A_{\underline{\alpha}} & 0 \\ 0 & 0 & R_{3\underline{\alpha}}A_{\underline{\alpha}} \end{bmatrix} , \tag{11b}$$

where $R_{1\alpha}$, $R_{2\alpha}$, and $R_{3\alpha}$ are spring constants per unit area in the x_1, x_2, and x_3 directions, respectively. The under-bar for the subscript α means that no summation is implied.

In Eqs.(10) and (11) the spring layer is assumed to be of zero thickness, and hence $\bar{A}_\alpha$ is equal to A_α. This means that $\bar{\underset{\approx}{R}}_\alpha = \underset{\approx}{R}_\alpha$ and $\bar{\underset{\approx}{S}} = \underset{\approx}{S}$. The zero thickness assumption also implies that the effect of inertia of the interphase is neglected.

Interaction Conditions and the System of Integral Equations

For the eight vector quantities, $\underset{\sim}{u}$ and $\underset{\sim}{F}$ in Eq.(5), $\bar{\underset{\sim}{u}}$ and $\bar{\underset{\sim}{F}}$ in Eq.(8), and $\underset{\sim}{u}^{SP}$, $\underset{\sim}{F}^{SP}$, $\bar{\underset{\sim}{u}}^{SP}$ and $\bar{\underset{\sim}{F}}^{SP}$ in Eq.(10), the following interaction conditions hold

$$\bar{\underset{\sim}{u}}^{SP} = \bar{\underset{\sim}{u}} \quad , \quad \underset{\sim}{u}^{SP} = \underset{\sim}{u} \tag{12a,b}$$

$$\bar{\underset{\sim}{F}}^{SP} + \bar{\underset{\sim}{F}} = \underset{\sim}{0} \quad , \quad \underset{\sim}{F}^{SP} + \underset{\sim}{F} = \underset{\sim}{0} \quad . \tag{12c,d}$$

From Eqs.(5), (8), (10), and (12), we obtain the following system of integral equations

$$\begin{bmatrix} \bar{\underset{\approx}{K}} + \underset{\approx}{S} & -\underset{\approx}{S} \\ -\underset{\approx}{S} & \underset{\approx}{K} + \underset{\approx}{S} \end{bmatrix} \begin{Bmatrix} \bar{\underset{\sim}{u}} \\ \underset{\sim}{u} \end{Bmatrix} = \begin{Bmatrix} \underset{\sim}{0} \\ \underset{\approx}{A}\,\underset{\approx}{G}^{-1}\underset{\sim}{u}^I \end{Bmatrix} \quad . \tag{13}$$

For the given incident field, $\underset{\sim}{u}^I$, this system can be solved for $\bar{\underset{\sim}{u}}$ at the inclusion side and $\underset{\sim}{u}$ at the matrix side. We can subsequently obtain $\bar{\underset{\sim}{F}}$ and $\underset{\sim}{F}$ from Eqs.(8) and (5). The scattered field can then be calculated from the original integral representation.

PARTIAL DEBONDING OF THE INTERPHASE

It should be noted that the spring constants per unit area in

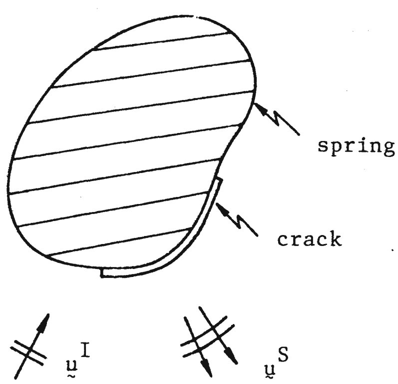

Fig. 2 Inclusion with a crack over part of the interphase.

Eq.(11b), $R_{i\alpha}$ ($i = 1,2,3$; $\alpha = 1,2, \cdots M$), may change for each direction i($= 1,2,3$) and they may also change in each element $\alpha(=1,2,\cdots M)$. As a special case, we can put $R_{i\alpha} \equiv 0$ ($i = 1,2,3$; $\alpha = k+1, \cdots k+N$), where N is less than M. This is a model for partial debonding of the interphase. Part of the boundary (N elements) is a crack-type surface, while in the remaining part (M-N elements) contact between inclusion and matrix is maintained by the spring connection, as shown in Fig. 2.

Numerical Examples

We restrict our attention to scattering by a sphere of radius d. The incident wave is chosen to be a plane longitudinal wave which travels in the x_3-direction: $\underset{\sim}{u}^I(\underset{\sim}{x}) = \exp(ik_L x_3)\underset{\sim}{e}_3$. In the following calculation, the non-dimensional wave number $k_L d$ was fixed as $k_L d = 1.0$, and the material properties were taken as $\bar{c}_L/c_L = 1/\sqrt{2}$, $\bar{\nu} = \nu = 1/4$, $\bar{\rho}/\rho = 1$.

To display general trends for the present spring model, the absolute values of the total displacement on the both sides of the interphase are shown in Fig. 3, for the case of spring contact over the whole boundary. The spring constants per unit area are $R_r/R_o = R_\theta/R_o = R_\phi/R_o = 0.1$ for all elements, where $R_o = \rho c_L^2/d$

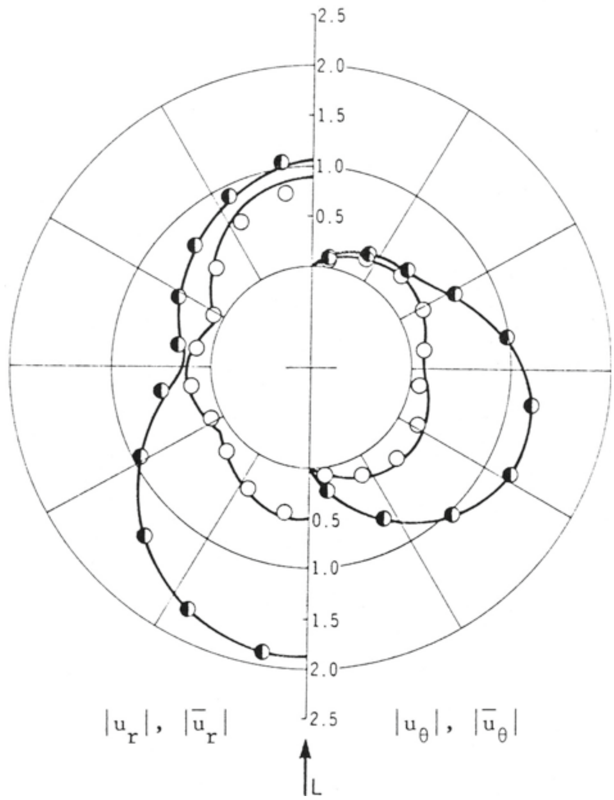

Fig. 3 Total displacements at the inclusion and matrix sides of the interphase, $R_r/R_o = R_\theta/R_o = R_\phi/R_o = 0.1$.

(for simplicity, we write R_r instead of $R_{r\alpha}$). The left hand side shows polar plots of the radial displacements $|u_r|$ on the matrix side (half black circle) and $|\bar{u}_r|$ on the inclusion side (open circle). The right hand side shows the tangential components $|u_\theta|$ and $|\bar{u}_\theta|$. For this case of complete spring contact, we can obtain the exact solution by a slight extension of Pao and Mow's expressions [2]. The solid lines of Fig. 3 show these exact solutions.

Figure 4 shows the components of the scattered field for the boundary displacements, where $|u_r^S|$ and $|\bar{u}_r^S|$ are plotted on the left-hand side, and $|u_\theta^S|$ and $|\bar{u}_\theta^S|$ on the right-hand side. The scattered field is defined as $u^S = u - u^I$. For this calculation, the spring constants R_r/R_o, $R_\theta/\bar{R}_o$ and $\bar{R}_\phi/R_o$ were set equal to zero in the lower half ($x_3 \leq 0$) of the interphase. This corresponds to the case that the lower half of the interphase is crack surface. Over the upper half of the spherical interphase, $R_r/R_o = R_\theta/R_o = R_\phi/R_o = 100$ for all elements.

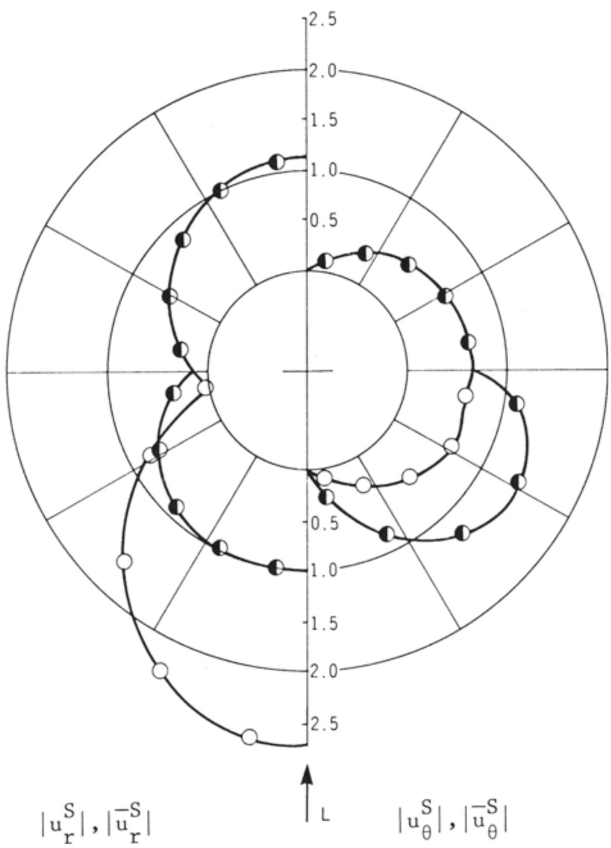

Fig. 4 Scattered displacements at the inclusion and matrix sides of the interphase, at the insonified side of the inclusion; spring constants over the shadow side: $R_r/R_o = R_\theta/R_o = R_\phi/R_o = 100$.

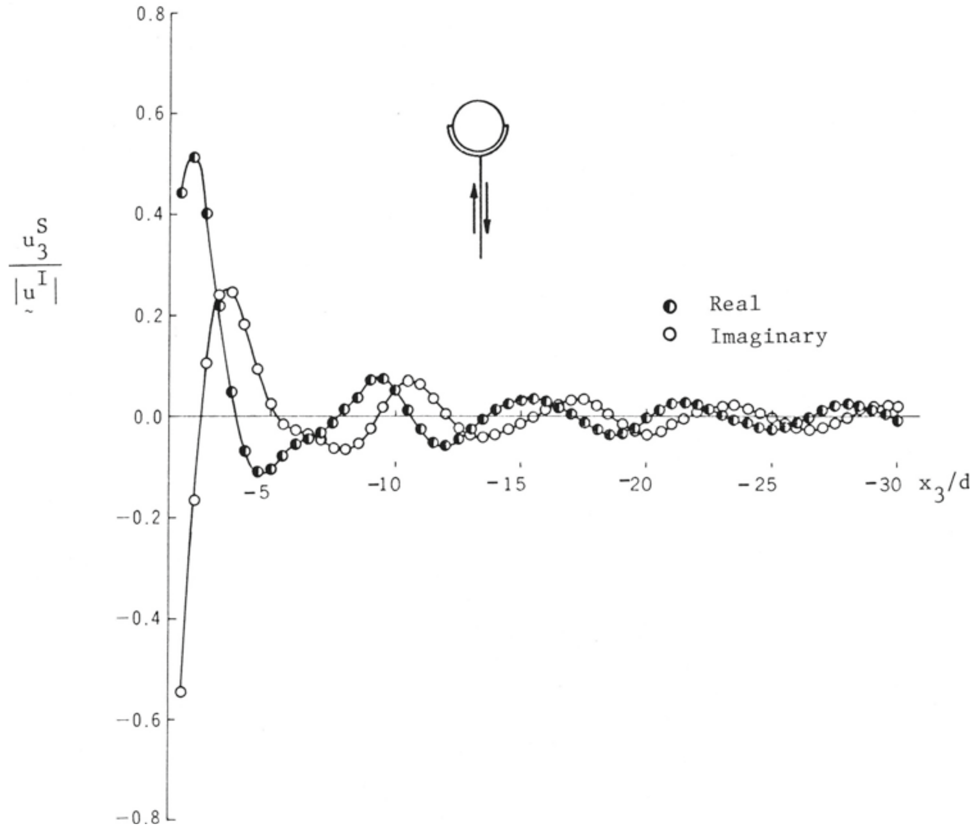

Fig. 5 Real and imaginary parts of the backscattered displacement field for the case of a crack at the insonified side of the inclusion; spring constants over shadow side: $R_r/R_o = R_\theta/R_o = R_\phi/R_o = 100$.

Figures 5 and 6 show the real and imaginary parts of the back scattered field, $u_3^S = u_3 - u_3^I$, along the x_3-axis. Figure 5 is for the case of a crack over the lower half of the interphase ($x_3 \leq 0$), and Fig. 6 is for the case of a crack over the upper half ($x_3 \geq 0$). Over the half of the spherical interphase which remains intact, the spring constants are $R_r/R_o = R_\theta/R_o = R_\phi/R_o = 100$ for both figures. Because of the symmetry with respect to the x_3 - axis, the backscattered field on the x_3-axis has only a u_3^S component, since $u_2^S = u_1^S \equiv 0$ along the x_3-axis. The backscattered displacement u_3^S has been plotted at a distance of 30d from the front face of the inclusion. It is noted that a comparison of the real and imaginary parts of the scattered wave forms of Fig. 5 and Fig. 6, shows a phase shift in the amount of about one half wave length.

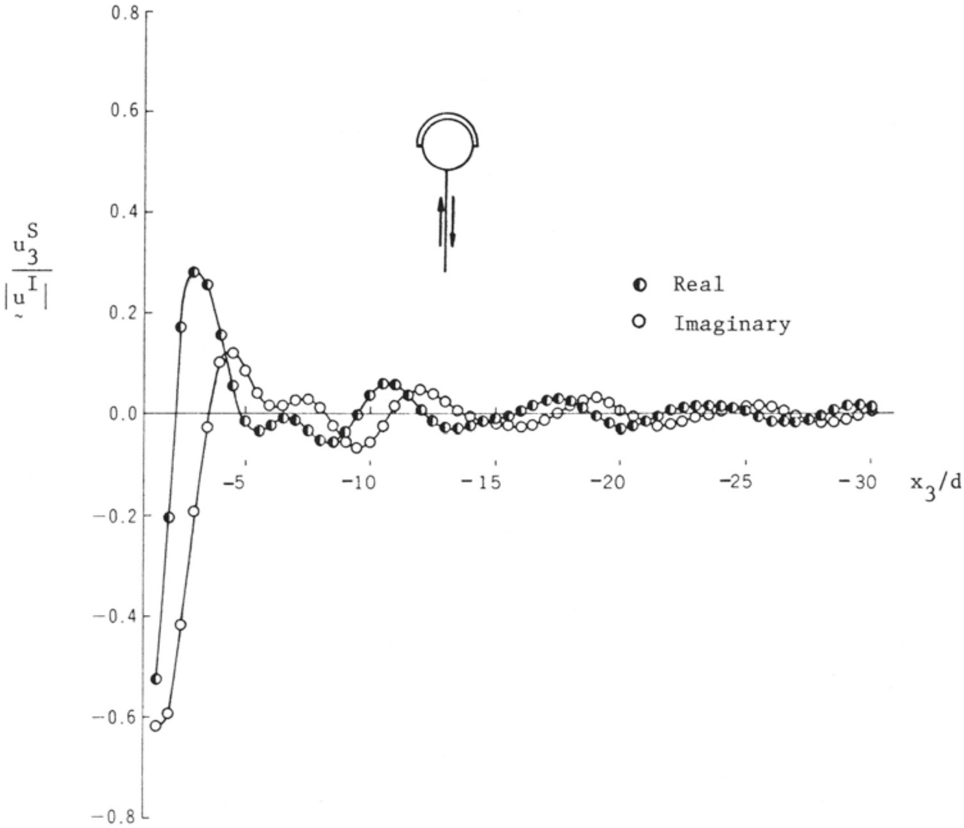

Fig. 6 Real and imaginary parts of the backscattered displacement fields for the case of a crack at the shadow side; spring constants at the insonified side: $R_r/R_o = R_\theta/R_o = R_\phi/R_o = 100$.

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