

THE SCATTERING RESPONSE
OF A FLAT-BOTTOM HOLE

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INTRODUCTION

The flat-bottom hole is one of the oldest reference/calibration standards in the field of ultrasonic nondestructive evaluation (NDE). It has been used for both calibration of ultrasonic test equipment sensitivity and for the generation of distance-amplitude correction (DAC) curves [1]. Flat-bottom holes are also useful for equivalent flaw sizing applications since they can represent the response, at normal incidence, of ideal "perfect" scatterers, such as flat cracks.

In spite of the applicability of flat-bottom holes for these purposes, there has been very little work done on modeling the ultrasonic scattering properties of such reflectors. Krautkramer used a small flaw, far-field approximation to obtain the amplitude response of ideal disc-shaped reflectors at normal incidence, as part of the distance-gain-size (DGS) system [3]. However, in many practical cases the hole may be larger and closer to the transducer than this simple theory allows. Because of this fact, the theoretically-based parts of the DGS curve are usually extended into these regimes [2], via empirical results.

Here, we will develop a new model for the scattering of a flat-bottom hole that does go significantly beyond currently available results. Specifically, we will derive approximate analytical expressions for the average pressure received by a contact compressional wave transducer, operating in pulse-echo mode, from a flat-bottom hole at normal incidence (Fig. 1). The center of the hole is assumed to be aligned with the central axis of the transducer. This theory will be shown to reduce to results predicted by the measurement model of Thompson and Gray [4] when the radius of the hole is small, and to the Krautkramer model [2], [3], when both the radius of the hole is small and the hole is in the far-field of the transducer. Numerical results will also be obtained to illustrate the scattering response, as a function of frequency, predicted by this new model.

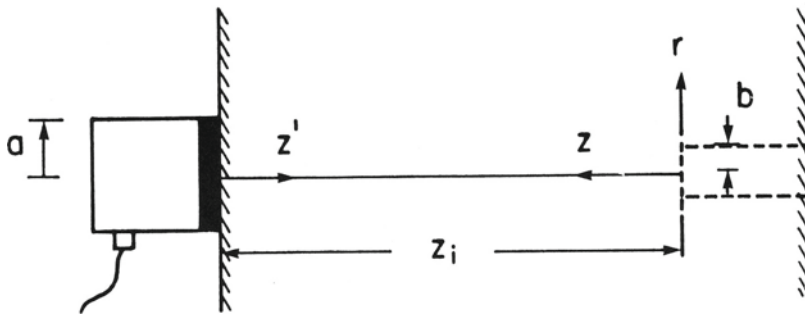


Fig. 1. Transducer and flat-bottom hole geometries.

ELEMENTS OF THE MODEL

Ultrasonic scattering problems are difficult to solve even for simple scattering shapes. Thus, few exact solutions are available and even these must often involve extensive numerical evaluation. The success of many scattering models, therefore, often lies in their judicious use of simplifying assumptions. The flat-bottom hole scattering model developed here also depends on several key approximations. These are: (1) the shear strength of the medium is neglected, i.e. the model replaces the elastic solid by an equivalent fluid medium. This is an assumption often employed in solving ultrasonic scattering problems [5]. Recently Sedov and Schmerr have examined the errors involved in making this simplification [6], [7], [8] for transducer modeling problems; (2) the radiated wavefield incident on the hole from the transducer is represented by an approximate solution originally developed by Schoch [9]; (3) the interaction of the incident waves with the hole is treated via a Kirchhoff-like approximation of the boundary conditions; (4) the scattered waves received back at the transducer are obtained via a combination of exact integrations and the method of stationary phase.

In the next sections we will describe briefly how these assumptions are employed and combined to obtain our scattering model. First, we will consider the incident waves generated by the transducer.

The Schoch Solution

If the ultrasonic compressional wave transducer is modeled as a baffled piston source radiating into a fluid medium, an exact solution is available for the radiated wavefield [10]. Unfortunately, the complexity of this solution prohibits its use in analytical models of the type we wish to construct here. Therefore, we need an appropriate approximate model of the transducer wavefield that can adequately represent the waves incident on a hole of radius b with $b \leq a$, where a is the radius of the transducer (Fig. 2). Schoch [9] developed such a solution over forty years ago. For example, the velocity produced by a baffled piston source, v_z^{inc} , is given in the Schoch approximation by (Fig. 1):

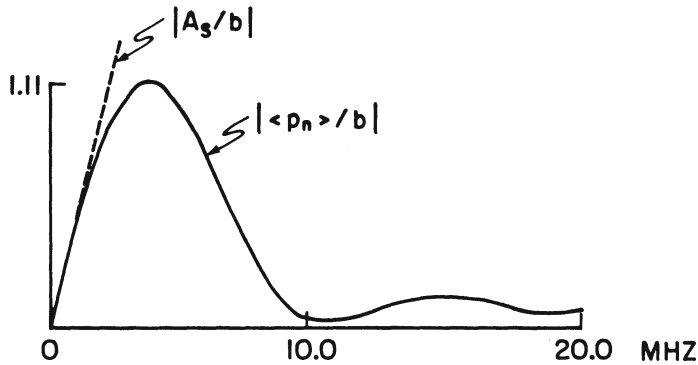


Fig. 2. Magnitude of the normalized average pressure $|\langle p_n \rangle / b|$ _____ and magnitude of the normalized scattering amplitude $|A_s / b|$ ---- versus frequency for a 6 mm radius compressional wave transducer and a 1 mm radius flat-bottom hole in steel where $a/z_1 = 0.20$.

$$v_z^{inc} = -v_o [\exp(ikz_1) - \cos \alpha J_0(kr \sin \alpha) \exp(ik(a^2 + z_1^2)^{1/2})] \quad (1)$$

where $k = \omega/c$ is the wavenumber with ω and c the circular frequency and wavespeed, respectively. Also $\alpha = \sin^{-1}(a/(a^2 + z_1^2)^{1/2})$ and v_o is the velocity of the piston source.

Although Eq. (1) is formally valid only for $ka \gg 1$ and $r \ll a$, as in many asymptotic solutions its range of applicability may be much wider than might be expected from these limits. Sedov and Schmerr [6] have examined similar approximations in a elastodynamic transducer model and have shown that they are indeed small for all $r \leq a$ in many practical situations. Thus, we feel that the Schoch solution can be used as an approximate but realistic model of the waves incident on a hole of radius $b < a$ where b need not necessarily be much smaller than the transducer radius.

The Kirchhoff Approximation

The Schoch solution only gives the waves incident on the flat-bottom hole. To obtain the pulse-echo response of this scatterer, we need to also calculate the pressure in the scattered waves. Ordinarily, this would involve satisfying zero stress boundary conditions on both the face and sides of the hole--a difficult problem to solve even for this simple geometry. However, if we assume that the velocity on the flat face of the hole is the same as that produced when a wave strikes an infinite planar stress-free surface and the sides of the hole are undisturbed, this solution process can be reduced to simply one of integral calculation. It should be noted that since the effects of the sides of the hole are neglected in this Kirchhoff-like approximation, our results will also apply to a thin circular reflector, such as a flat crack, on the axis of the transducer.

Following this procedure, we then have on the face of the hole the total velocity field, $v_z(r, 0, \omega)$ given by

$$v_z(r, 0, \omega) = 2v_z^{inc} \quad (2)$$

Using the integral representation theorem, one can obtain the pressure in the scattered waves as [5]

$$p^{sc}(r, z, \omega) = (-i\omega\rho/2\pi) \int_{\xi=0}^{\infty} \int_{r_0=0}^b \int_{\phi_0=0}^{2\pi} \{ \xi r_0 J_0(\xi r') v_z^{inc}(r_0, z_i, 0, \omega) \cdot \exp(-(\xi^2 - k^2)^{1/2} z) d\phi_0 dr_0 d\xi / (\xi^2 - k^2)^{1/2} \} \quad (3)$$

where $r' = (r^2 + r_0^2 - 2rr_0 \cos \phi_0)^{1/2}$ and ρ is the density of the medium. If we set $z = z_i$ in Eq. (3) and average this pressure over the face of the transducer (acting as a receiver), then the average received pressure, $\langle p \rangle$, is given by

$$\langle p \rangle = (1/\pi a^2) \int_0^a p^{sc}(r, z_i, \omega) 2\pi r dr \quad (4)$$

Since the r_0, ϕ_0 , and r integrations can all be performed exactly, we obtain

$$\langle p \rangle = (2i\omega\rho v_0 b/a) \int_0^{\infty} \{ J_1(b\xi) \exp(ikz_i) / \xi - (\cos\alpha/2\pi) \exp(ik(a^2 + z_i^2)^{1/2}) \cdot \int_0^{2\pi} [J_1(bK)/K] dx \} J_1(\xi a) \exp(-(\xi^2 - k^2)^{1/2} z_i) d\xi / (\xi^2 - k^2)^{1/2} \quad (5)$$

where $K = (k^2 \sin^2 \alpha + \xi^2 + 2k\xi \sin \alpha \cos x)^{1/2}$

High Frequency Asymptotic Solution

Equation (5) is the formal solution to our scattering problem. It represents the average pressure received by the transducer via the Schoch and Kirchhoff approximations for the incident and scattered waves, respectively. Because of the infinite ξ -integration this solution is not directly useful. If we again assume $ka \gg 1$, however, then we can apply high frequency asymptotics to Eq. (5) in the same manner as was done in the elastodynamic transducer models considered by Sedov and Schmerr [6], [7]. In this limit, the x -integration can also be performed, so we obtain explicitly

$$\langle p \rangle = -(\rho v_0 c b^2 / a^2) \{ \exp(ikz_i) [\exp(ikz_i) - 2\cos\alpha J_1(kb \sin\alpha) \cdot \exp(ik(a^2 + z_i^2)^{1/2}) / (kb \sin\alpha)] - \exp(ik(a^2 + z_i^2)^{1/2}) \cdot [2J_1(kb \sin\alpha) \exp(ikz_i) / (kb \sin\alpha) - \cos\alpha (J_0^2(kb \sin\alpha) + J_1^2(kb \sin\alpha)) \exp(ik(a^2 + z_i^2)^{1/2})] \} \quad (6)$$

THE SOLUTION

It is interesting to compare the results of our scattering model with some of the more restrictive solutions that have appeared previously in the literature. In particular, we will consider the following three special cases: case (a):

$$ka^4/8z_i^3 \ll 1, a^2/z_i^2 \ll 1 \quad (7)$$

If we place the above conditions on our solution in Eq. (6), we obtain the reduced form

$$\langle p \rangle = -(\rho v_0 c b^2 / a^2) \exp(2ikz_i) \{ 1 - [4J_1(kba/z_i) / (kba/z_i)] \cdot \exp(ika^2/2z_i) + [J_0^2(kba/z_i) + J_1^2(kba/z_i)] \exp(ika^2/z_i) \} \quad (8)$$

To our knowledge, Eq. (8) has not appeared in the literature before. We present it here because in many practical situations the inequalities in Eq. (7) are very well satisfied. Thus, in those cases one can use the simpler form of Eq. (8) in place of Eq. (6). case (b):

$$k\alpha^4/8z_i^3 \ll 1, a^2/z_i^2 \ll 1, kba/z_i \ll 1 \quad (9)$$

We can consider this case to represent the response of a "small" flaw (third inequality) not "too" close to the transducer (second inequality), and where the frequency is not "too" high (first inequality). Under these circumstances we find that

$$\langle p \rangle = -(\rho v_o c b^2/a^2)[1 - \exp(ika^2/2z_i)]^2 \exp(2ikz_i) \quad (10)$$

Eq. (10) can be rewritten in the form

$$\langle p \rangle = (\rho v_o c) \exp(ikz_i) C(z_i, \omega, a) A_s(\omega, b) [2 \exp(ikz_i) C(z_i, \omega, a) / (-ika^2)] \quad (11)$$

where

$$C = [1 - \exp(ika^2/2z_i)] \quad (12)$$

is a diffraction coefficient for the transducer [4] and

$$A_s = ikb^2/2 \quad (13)$$

is the far-field pulse-echo scattering amplitude predicted by the Kirchhoff approximation for a circular reflector of radius b . Equation (11) is identical in form to that given by Thompson and Gray's measurement model [4] which is valid for small flaws on the axis of a transducer. case (c):

$$ka^2/2z_i \ll 1, a^2/z_i^2 \ll 1, kba/z_i \ll 1 \quad (14)$$

Physically, this case corresponds to a small flaw far enough away from the transducer to be in the "spherical-wave" spreading region [5]. Here we find Eq. (6) reduces to

$$\langle p \rangle = (\rho v_o c) \exp(2ikz_i) (A_b/\lambda z_i) (A_t/\lambda z_i) \quad (15)$$

where $A_b = \pi b^2$ and $A_t = \pi a^2$ are the cross-sectional areas of the hole and transducer, respectively, and λ is the wavelength. Equation (15) is identical to that found by Krautkramer [3]. So, this model does indeed agree with and extend that simple solution.

NUMERICAL RESULTS

If we normalize the solution obtained by Krautkramer (Eq. (15)) appropriately we can recover the far-field scattering amplitude, A_s , of the hole. Specifically, if we take

$$\langle p_n \rangle = 2z_i^2 \langle p \rangle \exp(-2ikz_i) / (-ika^2 \rho v_o c) \quad (16)$$

then from Eq. (15) we see that

$$\langle p_n \rangle = A_s = ikb^2/2 \quad (17)$$

If we similarly normalize our original result, (Eq. (6)), or our first approximation of this result, Eq. (8), then the normalized pressure can be written as

$$\langle p_n \rangle = A_s(\omega, b) D(\omega, b, a, z_i) \quad (18)$$

where D describes the deviation from the small flaw, far-field results of Krautkramer. For example, from Eq. (8) we obtain

$$D(\omega, b, a, z_i) = -(4z_i^2/k^2 a^4) \{1 - [4J_1(kba/z_i)/(kba/z_i)] \exp(ika^2/2z_i) + [J_0^2(kba/z_i) + J_1^2(kba/z_i)] \exp(ika^2/z_i)\} \quad (19)$$

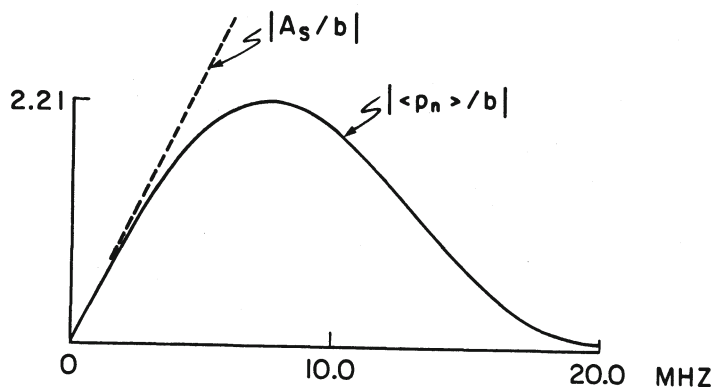


Fig. 3. Magnitude of the normalized average pressure $|\langle p_n \rangle / b|$ _____ and magnitude of the normalized scattering amplitude $|A_s / b|$ ---- versus frequency for a 6 mm radius compressional wave transducer and a 1 mm radius flat-bottom hole in steel where $a/z_1 = 0.10$.

To see how important these deviations are from the Krautkramer solution, we have considered the response of a 6 mm radius compressional wave transducer from a 1 mm radius hole in steel (Figs. 2, 3). In Fig. 2, we have taken $a/z_1 = 0.20$ so that the hole is at a distance equal to five times the transducer radius. Plotting the magnitudes of $\langle p_n \rangle / b$ and A_s / b (both of which are dimensionless) versus frequency, we see that the deviation is non-negligible for all frequencies above approximately 1.5 MHz. In Fig. 3, the same transducer, hole and material parameters were used but the hole was placed at a distance equal to ten times the transducer radius. In this case the Krautkramer model is good to approximately 2.5 MHz. Thus, the deviations predicted by this model should be significant in many practical situations. An experimental program is currently being developed to verify these predictions.

SUMMARY AND CONCLUSIONS

We have developed an approximate analytical model of the pulse-echo scattering response of a flat-bottom hole which is significantly more general than existing models. This model should be useful in a number of modern quantitative ultrasonic NDE applications. A similar model for immersion testing is possible and is currently under development.

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