GENERALIZED LAMB MODES IN FLUID-SATURATED POROUS PLATE

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INTRODUCTION

Since analysis by Rayleigh [1] and Lamb [2], the vibration modes for an elastic homogeneous infinite solid thin plate are well understood. These so-called "Lamb modes" result from a pure compressional wave and pure shear wave. Similarly, excitation of "leaky Lamb modes" in elastic plates immersed in a fluid, caused by incident acoustic waves, has been extensively described theoretically [3-7] and experimentally [8,9]. Results are generally presented as dispersion curves which relate the phase velocity of the mode to the product of frequency and plate thickness.

The purpose of this work is to extend this formalism to the particular case of fluid-filled porous thin plates. These structures, first studied by Biot [10,11], are two-phase materials made of a solid continuous matrix and connected pores filled with a fluid. One fundamental aspect of acoustic wave propagation in fluid saturated porous solids is the existence of a "slow" compressional wave, in addition to the classical "fast" compressional wave and the shear wave.

In the present study we direct our attention to significant modifications in the dispersion curves due to the propagation of the slow wave. Numerical procedure is carried out to obtain the dispersive behavior of different Lamb modes in water-saturated porous plates. Comparison of theoretical data with and without the slow wave is presented. The computed predictions are then compared to a series of measurements. Unusual behavior of the transmitted and the reflected S_1 mode is demonstrated. The main features of the dispersion curves are discussed for various values of the product f.d (ultrasonic frequency . thickness of the plate).

THEORY

The geometry of the problem is shown in Fig. 1. Lamb wave propagation in a water-saturated porous plate is considered. The porous plate is macroscopically homogeneous, isotropic, and infinitely large, so potential functions for fast and slow compressional waves and shear waves can be used to compute dispersive behavior of different Lamb modes.

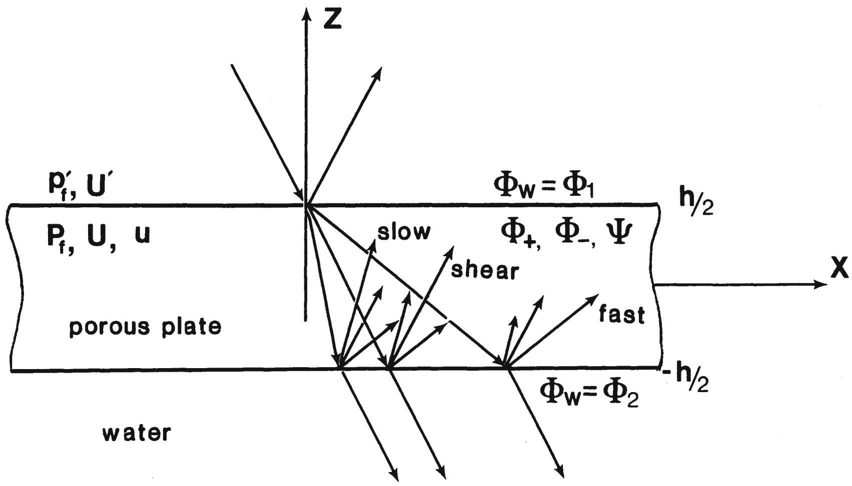


Fig. 1. Scheme of the coordinate system for Lamb wave propagation in a fluid-saturated porous plate. Φ_+, Φ_-, and Ψ are potential functions for fast, slow, and shear wave in the porous plate, and Φ_W is the potential of compressional wave in the fluid.

Boundary conditions at upper and lower interfaces ($z = \pm h/2$) of the porous plate are adopted from Ref. 12 using the same notations:

1. Continuity of stress at the plate surfaces (two equations)

$$\sigma_{iz} + s_{iz} = P_{f'}\,\delta_{iz} \quad i = x, z \tag{1}$$

2. Conservation of the fluid volume

$$\phi U_z + (1 - \phi) u_z = U'_z \tag{2}$$

3. Proportionality between discontinuity in pressure and relative velocity of fluid to solid in porous medium

$$P_f - P_{f'} = T\phi(U_z - u_z) \tag{3}$$

where T is called surface flow impedance. In the following discussion we only consider an open pore boundary where T = 0.

By using standard procedures described in Ref. 14, expressions for the Lamb modes can be calculated. The numerical procedure of computing Lamb waves in porous plate is briefly shown by a block diagram in Fig. 2.

RESULTS

Velocity dispersion curves for water-saturated porous materials (Sample 15) are shown in Fig. 3 as a function of frequency times plate thickness. The phase velocity C_L, chosen as the variable, is deduced from the incident angle θ by Snell's law:

$$C_L = C_{f'}/\sin\theta \tag{4}$$

To emphasize the role of the slow wave, similar plots using the same data but without (Fig. 3(a)) and with (Fig. 3(b)) slow wave are presented. The most significant and distinctive features between these two sets of curves are observed for the modes S_0 and S_1. Instead of having asymptotic behavior for decreasing phase velocity, the mode S_1 on the plot in Fig. 3(b)

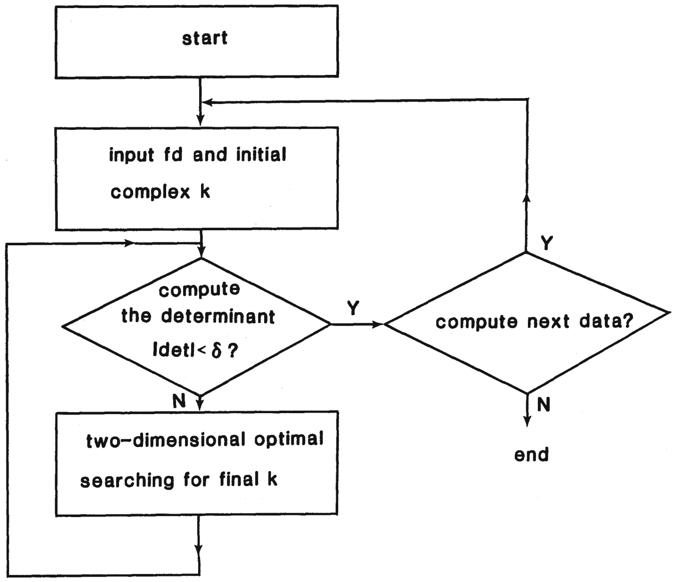


Fig. 2. Block diagram of numerical procedure.

exhibits a change in direction. Such unusual behavior has been observed before by Nayfeh and Chimenti [14] for fiber reinforced composites with solid density close to the density of the fluid. Also, because of the existence of a slow wave, its asymptotic value of mode S_0 is the slow wave velocity (800 ms^{-1}) instead of the Rayleigh wave velocity.

For higher modes, good agreement between the two sets of curves shows that the slow wave plays an important role only in the low frequency range.

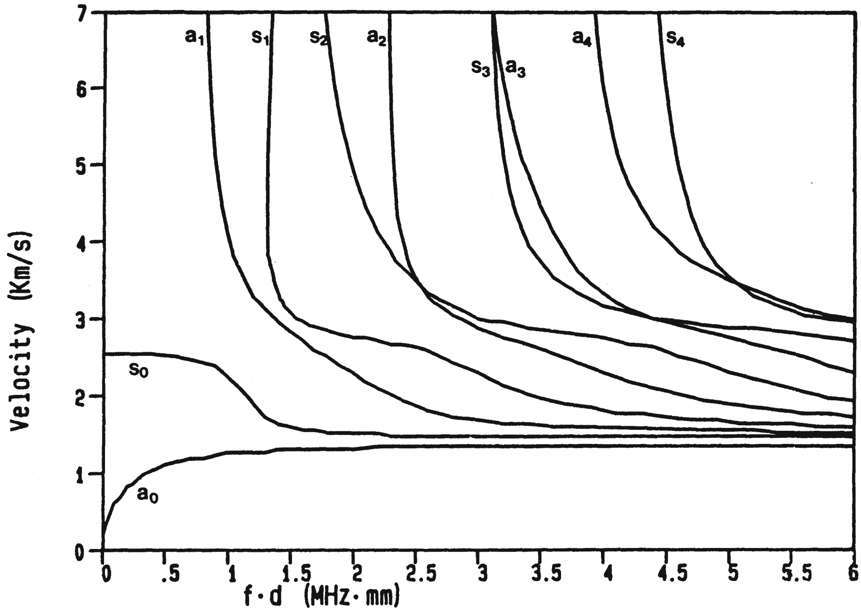
MEASUREMENTS

Experimental Procedure

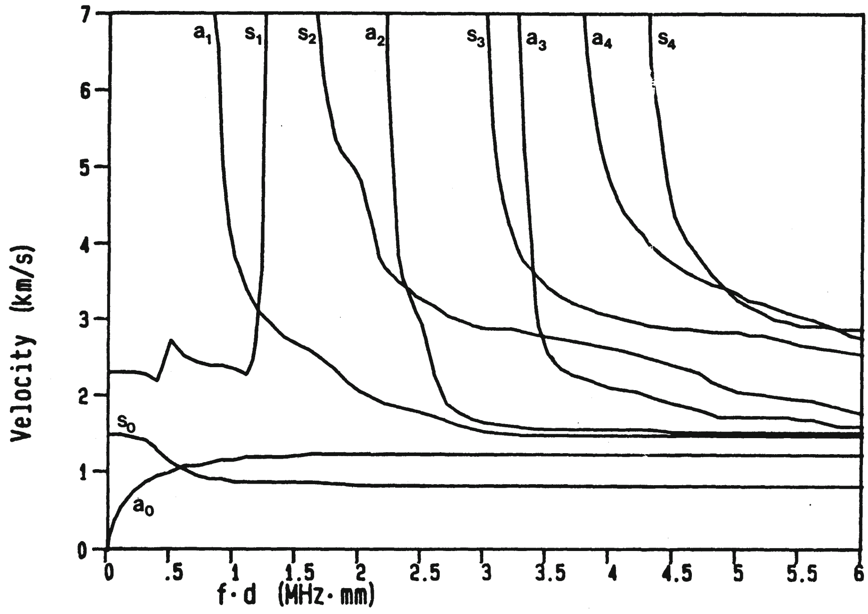
Leaky Lamb waves in fluid-saturated porous plates are obtained by using two different techniques as shown in Fig. 4. The first one (Fig. 4(a)) is a two-transducer immersion technique. Excitation of plate modes gives destructive interference between the specular reflected wave and the leaky plate modes radiating in the same direction. As a consequence, the reflected spectrum exhibits minima for frequencies at which these modes are excited. The second arrangement (Fig. 4(b)) is based on a single transducer and reflector immersion method. In this latter case, the leaky wave is only present on the back face of the plate when a plate mode is propagating. Hence plate modes are identified as maxima in the received spectrum. In order to get a higher sensitivity, a second transducer, acting as a receiver, can be substituted for the reflector.

Both methods give leaky Lamb wave phase velocity dispersion data. They have been described and discussed in detail by P. B. Nagy, et. al [15] previously. Although in the single transducer and reflector technique alignment is simpler, the two transducers' pitch-catch arrangement has better accuracy and sensitivity.

The diagram in Fig. 5 shows mode conversion and refraction at different interfaces for a particular case where the shear velocity in the plate is



(a)



(b)

Fig. 3. Comparison of dispersion curves for leaky Lamb waves in a porous thin plate (a) without slow wave and (b) with slow wave. Material properties are listed in Table I for Sample PG15.

lower than the bulk velocity in the fluid. Paths [1] [2] [3] correspond to the direct transmission of the compressional fast, shear, and compressional slow velocities, respectively. For clarity, the reflected waves are not shown but obviously have to be included in the problem (see Fig. 1).

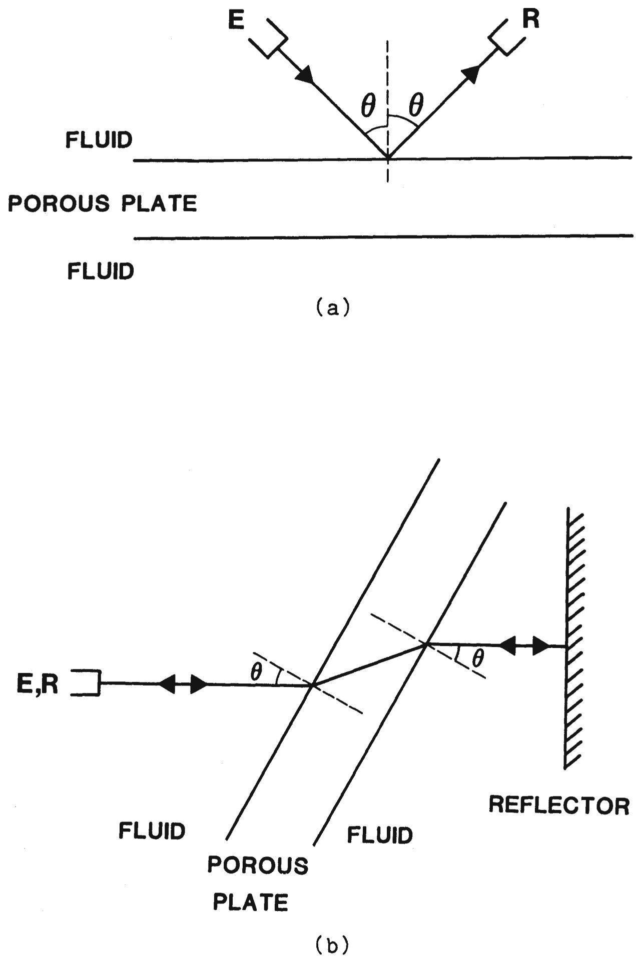


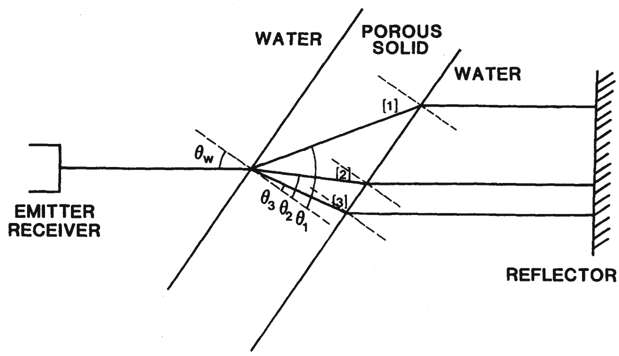
Fig. 4. Experimental arrangement: (a) pitch-catch for reflection measurement and (b) pulse-echo for transmission measurement.

The equipment includes a pulser which excites a broadband transducer. The received signal is amplified and sent to a digital oscilloscope for storage and signal processing, i.e. time gating, time domain averaging, fast Fourier transforming, decovolving, and smoothing of the normalized power spectrum.

Sample Description and Preparation

The experimental data are obtained from different materials in order to outline the influence of the compressional slow wave on the dispersion waves. Table I gives the relevant parameters of the solid plates which have been investigated. The slow wave has been observed only on three of them (PG15, PG55, PSa1). The plexiglass (PL1) sample has also been included as a reference for a solid homogeneous medium with a shear velocity lower than the bulk velocity in the fluid.

As observed by Plona's first experiment [16], slow wave measurements are only obtained with artificial samples made of glass beads fused by sintering (PG15, PG55) synthetically bonded natural sand grains (PS1).



(1) = MODE CONVERTED FAST COMPRESSIONAL WAVE

(2) = MODE CONVERTED SHEAR WAVE

(3) = MODE CONVERTED SLOW COMPRESSIONAL WAVE

$C_{SLOW} < C_{SHEAR} < C_{WATER} < C_{FAST}$

Fig. 5. Refraction and mode conversion in porous plate where the velocity of shear wave is lower than that of compressional wave in water.

TABLE I. Geometric and Physical Parameters of the Porous Samples.

SAMPLE	MATERIAL	MAXIMUM PORE DIAMETER (μm)	POROSITY %	DENSITY OF THE SOLID	FLUID SATURATED SAMPLE V_{fast} (ms^{-1})	V_{shear} (ms^{-1})	V_{slow} (ms^{-1})	DRY SAMPLE V_{Long}	V_{Shear}
PG15	Glass	15	30	2.48	2851	1516	822	3582	2217
PG55	Glass	55	30	2.48	2926	1560	810	2960	1724
PST3	Steel	7	26	7.90	3450	1980		2885	2060
PSal	Sand		37	1.90	3270	1970	1010		
PL1	Plexiglas				2310	1152			

The porous structure we used consisted of thousands of tiny precisely sized spherical particles bonded together to form a uniform three-dimensional shape. Saturation with water is attempted using a vacuum pump to first empty the air content from the pores. Then water is entered into the vacuum tank to fill the open pores. An additional few hours time delay is allowed so the immersed sample expels the remaining gas.

RESULTS AND DISCUSSION

Preliminary experiment is performed to verify the basic assumption used in our model: the theory applies to materials with shear velocity close to the velocity of the bulk wave in water.

To demonstrate the validity of our theoretical model for materials with a shear velocity close to or smaller than the velocity of bulk wave in water, experimental and computed values of the velocities have been plotted versus the product frequency times thickness for a plexiglass sample (see Table I). As shown in Fig. 6, excellent agreement between the two sets of data is obtained.

Experimental measurement of generalized leaky Lamb waves in a water-saturated thin porous plate (Sample PG15) is presented in Fig. 7. The solid lines are the computed values and the discrete squares are the experimental results. The most important modification of the dispersion curve for the porous plate with slow waves is indicated in the low frequency range, so the diagram is limited to f.d $\leq$ 3.2 MHz mm.

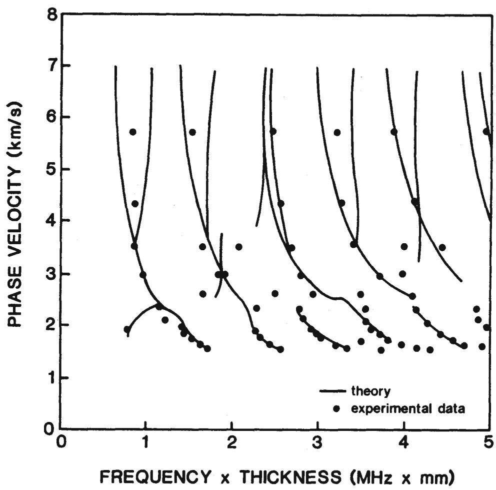


Fig. 6. Comparison between theory and experiment for a plexiglass plate with shear velocity lower than that of compressional wave in water.

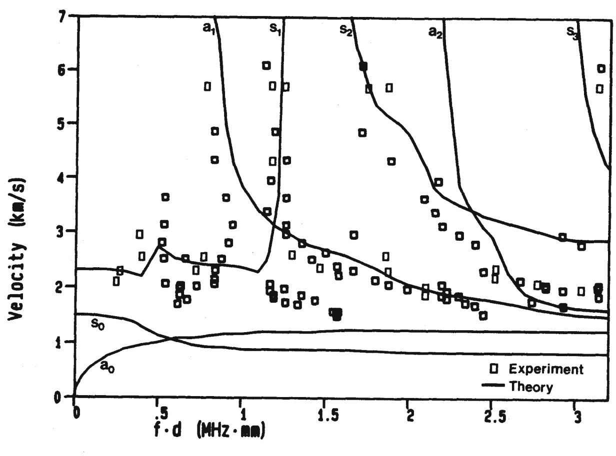


Fig. 7. Comparison between theoretical predictions for leaky Lamb modes in porous plate (Sample PG15) and experimental data measured from reflected spectrum minima.

The particular behavior of the S_1 is confirmed by the experimental data. However, the scattering of some experimental values around the curves of interest make their identification more difficult. Although there are many factors affecting the measured dispersion curve which could lead to the poor agreement between theory and experiment, the main reason is that the reflection and transmission measurements cannot be translated to the dispersion curve [17] for leaky Lamb modes as was earlier assumed [6]. Calculations of reflection and transmission coefficients through fluid-filled porous plates will be presented at a later time.

CONCLUSION

Preliminary study on generalized leaky Lamb waves in water-saturated porous thin plates has been performed. A theoretical model has been developed by including a potential function for slow wave. Numerical solutions have been obtained showing the modification of the velocity dispersion curves due to the existence of the slow wave. The validity of the theory has been demonstrated in the case of a non-porous solid having a shear wave velocity close to or below the bulk velocity in water. The existence of a second compressional wave in the porous materials used has been verified. Theoretical and experimental dispersion curves show unusual behavior for the S_1 mode. A reverse direction instead of an asymptotic trend is observed for this mode. On the other hand, most other modes look rather same as for many usual solid homogeneous isotropic plates. Scattering of experimental data due to inhomogeneities, low density, and high attenuation of the porous medium is the main limitation to better identification of the modes.

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