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Closed loop supply chains with variable remanufacturability and the impact of subsidy and penalty by government

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**Closed loop supply chains with variable remanufacturability and the impact of subsidy
and penalty by government**

by

Lee Kil Jin

A thesis submitted to the graduate faculty
in partial fulfillment of the requirements for the degree of
MASTER OF SCIENCE

Major: Industrial Engineering

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2008

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ABSTRACT

In this thesis we model and analyze a series of closed loop supply chains to study the relationships among remanufacturability and profitability of each member of the supply chain. We also study the relationships among remanufacturability, government subsidy, and government penalty. In each model, we assume that the closed loop supply chain consists of a manufacturer who manufactures as well as collects used products and remanufactures and a retailer who in turn sells the products to her consumers. Also, we assume that the level of remanufacturability is a variable that is controllable via the level of investment of remanufacturing technology and equipment. Furthermore, in the case of government subsidy and penalty, we assume that the government provides a level of subsidy per remanufactured product and finances the subsidy by collecting an advance recovery fee per unit sold from the retailer. Throughout this thesis, we also assume that the manufacturer behaves as the leader and the retailer as the follower under a Stackelberg game framework and the model environment is captured in a static framework regarding manufacturing and remanufacturing. Numerous managerial insights and economic implications are obtained. For example, the manufacturer's profit may actually decrease with respect to the collection rate if the collection rate is low in the case of no government intervention in the form of the subsidy and penalty. Also, the government may be able to increase the total surplus consisting of all the profits of the supply chains and the consumer surplus by determining the appropriate level of the fee unit remanufactured subsidy and the fee unit sold advance recovery fee.

CHAPTER 1. INTRODUCTION

Remanufacturing is a production strategy to recover the residual value of used products by reusing components that are still functioning well. Remanufactured products can be obtained by collecting used products and replacing worn out components with new ones (Thierry et al. 1995). Also, Majumder and Groenevelt (2001) define remanufacturing as the process of disassembling used items, inspecting and repairing/reworking the components, and using these in new product manufacture. We define remanufacturability in our model as the fraction of used products that can be remanufactured and consider that the remanufacturability can be improved via fixed cost investment in technology.

An alternative approach to obtain environmental and economic studies is to increase the remanufacturability by government intervention. The European Union (EU) has adopted a Directive on Waste Electrical and Electronic Equipment (WEEE) such that, effective August 2005, EU member states must establish collection systems for electrical and electronics waste (Webster and Mitra 2007). There are State E-Waste Legislations in the U.S. The specific example in our model is State of Oregon E-Waste Legislation. It requires retailers to charge first in-state buyers of electronic devices an ARF (Advanced Recovery Fee) of up to \$10 to pay for the collection, reuse and recycling of products. The state DEQ (Department of Environmental Quality) will set the fee and deposit it into an Electronic Product Stewardship Account. The department is to monitor the cost of performing the required services and meeting performance goals, and may adjust the fee once a year.

The purpose of this study is to model and analyze the remanufacturability when the manufacturer collects used products directly from consumers in a manufacturer-retailer (MR)

closed loop supply chain. Furthermore based on this framework (MR model), we introduce the government penalty to the consumer (ARF) and subsidy to the manufacturer for remanufacturing.

The specific research objectives are 1) To identify the conditions under which it is beneficial to have higher remanufacturability given to the collection rate, 2) To investigate how supply chain coordination between the manufacturer and the retailer impacts on the remanufacturability, prices, and profits, 3) To analyze the impact of environmental legislation that penalizes retail purchasing of products to the consumers and subsidizes remanufacturing to the manufacturer.

For this study, we focused on the closed loop supply chain consisting of the manufacturer who collects used products from the consumers and retailer who in turns sells the manufacturer's products to the consumers. As the result of this paper, we found that the cost savings from remanufacturing and collection rate from the market condition have a critical impact on the manufacturer's decision. Also, we considered the government intervention (the government penalty and subsidy system) based on the closed loop supply chain and assumed that the total amount of penalty from the consumers is equal to the total amount of subsidy for remanufacturing as the non-profit organization. We showed that under the government revenue neutrality when the government increases the subsidy for remanufacturing to the manufacturer, the manufacturer's profit will be decreased due to the penalty to the consumers. Thus the lump-sum transfer money incentive may be provided to the manufacturer to increase the manufacturer's profit as well as the remanufacturability.

The rest of this thesis is organized as follows. In Chapter 2 we show how our model is derived and different from previous literature. Chapter 3 presents the assumptions, notations

and derivation of the manufacturer-retailer (MR model) and the centrally coordinated (CC model). We solve the equilibrium of the models and compare the equilibrium solutions of MR model with the solutions with CC model. Then by numerical example we discuss some managerial insights. In chapter 4 we propose the manufacturer-retailer with government (MRG model) and the centrally coordinated with government (CCG model) where the government imposes ARF to the consumers at the point of retail purchase and subsidizes the fee for remanufacturing. Also, we solve the equilibrium of the models and compare the equilibrium solutions of MRG model with the solutions with CCG model. Then by numerical example we discuss some managerial insights. In chapter 5 we show the total surplus that the government finds the optimal value of subsidy that maximizes the government's objective as total surplus maximization. Chapter 6 concludes our findings and suggests future research.

CHAPTER 2. LITERATURE REVIEW

Product remanufacturing after its end-of life has received much research attention in the recent years. There is extensive literature regarding the relationship among manufacturer, retailer, and collector, analyzing how it influences the retail price, collection rate, remanufacturing, and channel profit.

Majumder and Groenvelt (2001) study a two-period horizontal competition model for remanufacturing in which an OEM (Original Equipment Manufacturer) who manufactures new products and also remanufactures competes with a local remanufacturer under different allocation mechanisms for returns. Also, they proposed a model where, even though the consumers cannot tell if a product is new or remanufactured by the OEM, they value products sold by the OEM more than those sold by the local remanufacturer. They find that an increase in the fraction available for remanufacturing does not necessarily increase OEM profit. Thus, regulation and taxation incentives may be provided to the OEM to increase remanufacturing activities.

Ferguson and Totay (2005) develop a two-period model with a monopolist manufacturer in the first period and duopoly of manufacturer and remanufacturer in the second period. They show the internal cannibalization effect of remanufactured products on an OEM's new product sales. They also analyze a collection strategy wherein the OEM chooses to collect the used product (with no intention of remanufacturing) to deter entrant. Further, when remanufacturing costs are low and the manufacturer also remanufactures, the manufacturer sells more in the first period to increase returns in the second period in order to benefit from remanufacturing.

Debo et al. (2005) consider an industry with a manufacturer who may also sell remanufactured product and potentially multiple independent remanufacturers. They identify conditions under which a monopolist will invest in technologies to make a product suitable for remanufacturing so as to produce both new and remanufactured product, and they identify how these conditions change when independent remanufacturers may enter the market to compete with the manufacturer.

Savaskan et al. (2004) address the problem of choosing a suitable channel structure for the collection of end-of-life returns from customers. They assume that customer cannot distinguish a new product from a remanufactured product, i.e., ink-cartridges and single-use cameras. They consider a manufacturer with three options: (1) undertake the collection effort himself, (2) provide suitable rewards to the retailer to undertake the collection efforts, (3) subcontract the collection effort to a third party. They also explore the implication of these different reverse channel structures on pricing decisions and supply chain profits. Their research shows that agencies closer to the customer are most effective in the collection effort for the manufacturer.

Savaskan and Van Wassenhove (2006) analyze the trade-offs between centralization and decentralization of the product collection activity. The manufacturer is the Stackelberg leader, and two retailers compete on price in close loop frame work. In the centralized system, the manufacturer collects used products directly from the consumers (e.g., as in print and copy cartridge) whereas in the decentralized system, the two retailers collect the product returns (e.g., as in a single-use cameras and cellular phones). The decentralization of product collection activities result in incentives for retailers to reduce their margins with the expectation of compensation through buyback payments for returned products.

Jung and Hwang (2007) study a remanufacturing in a reverse logistics chain with one OEM and one remanufacturer under take-back requirement. The environmental legislative pressure like EPR (Extended Producer Responsibility) requires firms to take back the used products from end-users for remanufacturing so that they consider the penalty charged per unit, in case that the obligatory take-back quota is breached. They show that the remanufacturers can be a potential competitor for the OEMs by cannibalizing the sales of the OEM's new product but the OEMs have the advantage of being free of take-back quota when remanufactures are active in collecting end-of-use products.

State in the United States concerns specific to the landfill disposal or incineration of e-waste are largely due to its increasing volume and often bulky nature; hazardous constituents, such as lead and mercury, it may contain; its high cost of recycling; and the inability of interested stakeholders, such as electronics retailers and manufacturers, to reach consensus on how to voluntarily implement a national e-waste management system. Luther (2007) shows how states respond to this concern by enacting their own e-waste management laws and the overview of enacted state legislation. Also the paper explains a mechanism to fund the program (the consumer pays model which is represented as ARF and the producer pays model which is represented as EPR).

This paper builds on the above models and analyses. We assume a single period game-theoretic model and no difference between new and remanufactured product as in Savaskan et al. (2004). However, our focus is on the level of the remanufacturability. We propose a manufacturer and retailer model that the manufacturer considers his remanufacturability instead of focusing on the collection of the used products. Furthermore we show the role of government that collects fee (ARF) from the consumers and subsidizes it

to the manufacturer for remanufacturing. To our knowledge, this linkage (penalty to consumer and subsidy for remanufacturing) has not been captured in prior literature. Because the ARF in our model is just beginning to go into effect and may become increasingly prevalent, our insights into the impact of these environmental laws are relevant for the government policy-makers and managers.

CHAPTER 3. MANUFACTURER-RETAILER MODEL (MR)

Savaskan et al. (2004) address the problem of choosing a suitable channel structure for collection of end-of-life returns from consumers. In this chapter we propose a manufacturer-retailer model that the manufacturer considers his remanufacturability instead of focusing on the collection of the used products. An important feature of our model is the option for the manufacturer to invest in remanufacturability, the fraction of products that may be economically remanufactured. Investment in remanufacturing equipment and processes may facilitate the remanufacturing of product subassemblies that are too difficult or expensive to remanufacture. Sundin and Bras (2005) studied remanufacturing process of several companies and found that cleaning and inspection were often the most time consuming steps of the remanufacturing process. Automation of these steps was recommended to reduce the processing time. Recently Xerox replaced its traditional cleaning technology with carbon dioxide blasting which lead to reduced cleaning times as well as improved part recovery rates (Xerox 2005).

In this paper we investigate manufacturer-retailer closed loop supply chain model with variable remanufacturability. The key research objectives under this circumstance are,

- 1) How the collection rate impacts on the remanufacturability?
- 2) How supply chain competition and coordination between the manufacturer and retailer impact on the remanufacturability, prices, and profits?

In order to answer these questions, we set up a steady-state model to analyze the manufacturer-retailer closed loop supply chain.

3.1 Model Assumptions and Notations

This thesis considers a steady-state model with two players: the manufacturer and the retailer. We formulate and analyze a manufacturer-retailer closed loop supply chain model that the manufacturer collects used products from consumers and remanufactures. For example, Xerox Corporation provides prepaid mailboxes so that the consumers can return their used copy or print cartridges. The manufacturer-retailer (MR) model is illustrated in Figure 1.

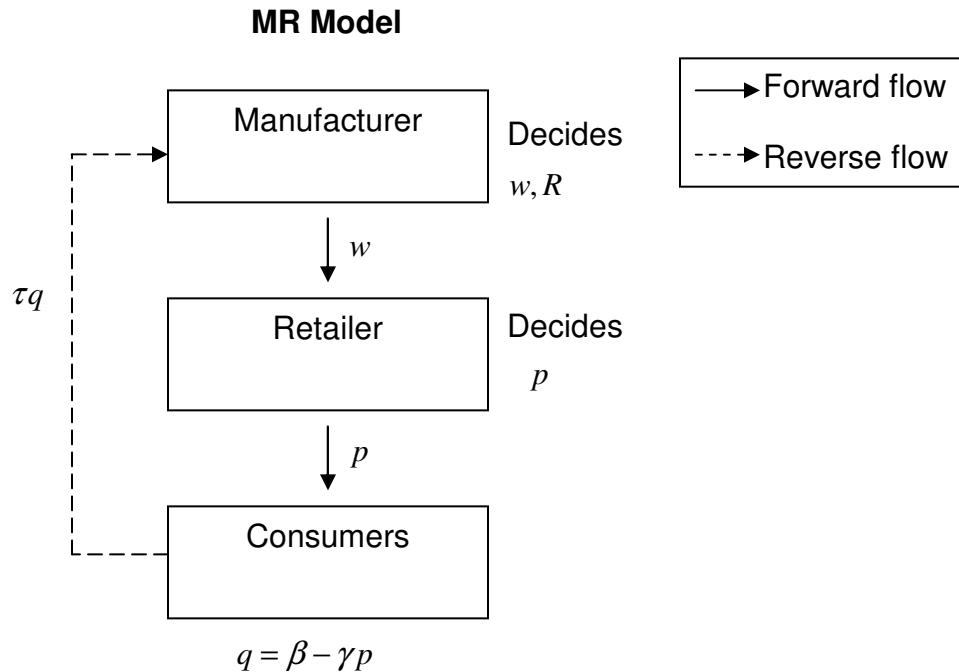


Figure 1. Illustration of Manufacturer-Retailer Model

MR model consists of a manufacturer who manufactures as well as remanufactures his products and a retailer who in turn sells the manufacturer's products to her consumers. The retailer charges a price p per unit to consumers so that the retailer faces a demand of $q = \beta - \gamma p$ units of product where β and γ are parameters which denote the maximum demand for products and the decrease in demand for a unit increase in price p respectively.

Linear demand functions have been widely used in supply chain literature (McGuire and Staelin 2008, Choi 1991, Savaskan et al. 2004).

We formulate this closed loop supply chain as a Stackelberg game with the manufacturer as the leader and the retailer as the follower. The Stackelberg game is appropriate for modeling a dominant supply chain member as it typically results in a higher profit to the leader due to the advantage of choosing his strategies first. Notations used in this chapter are explained below:

w : The wholesale price decided by the manufacturer (dollars per product);

p : The retail price decided by the retailer (dollars per product);

c_m : The manufacturing cost for each new product (dollars per new product);

c_r : The remanufacturing cost for each remanufactured product (dollars per remanufactured product);

Δ : The cost savings (dollars per product), $\Delta = c_m - c_r$;

τ : The collection rate of used products from consumers ($0 < \tau < 1$);

R : The remanufacturability decided by the manufacturer, the fraction of the used products that can be economically remanufactured ($0 < R^* < 1$);

c_c : The collection cost (dollars per product);

k : The scaling parameter for remanufacturing;

q : The demand function without government's subsidy and penalty ($q = \beta - \gamma p$);

π_R^{MR} : The retailer's profit in MR model;

π_M^{MR} : The manufacturer's profit in MR model;

π_C^{MR} : The total channel profit in MR model ($\pi_C^{MR} = \pi_R^{MR} + \pi_M^{MR}$);

π_C^{CC} : The centrally coordinated planner's profit in CC model;

In order to model the manufacturer-retailer closed loop supply chain with the variable remanufacturability, we made the following assumptions.

Assumption 1: The closed loop supply chain decisions are considered in a single period (steady-state) setting.

The planning horizon is a single period representing the effective operation period of the remanufacturing technology. Savaskan et al. (2004) assume the previous existence of the product in the market. Those products sold in the previous periods can be returned to the manufacturer for reuse. The price p , wholesale price w , and remanufacturability R in our model are all decided at the start of the single period and are held constant thereafter.

Assumption 2: No difference between the quality of the manufactured and remanufactured products.

The example of this assumption depends on the nature of the product. For instance, the assumption is reasonable for single use cameras or copy machines that are remanufactured to extremely high standards.

Assumption 3: Producing a new product by using a used product is less costly than manufacturing a new one.

c_m is the manufacturing cost dollars per new product while c_r is the remanufacturing cost per remanufactured product. Given that production cost savings Δ , the difference from manufacturing cost to remanufacturing cost, is the primary economic motive for remanufacturing, we assume that the remanufacturing cost is lesser than manufacturing cost

per product by a fixed amount Δ ($\Delta = c_m - c_r$). From this assumption, the average unit cost of manufacturing is $c = c_m(1 - R\tau) + c_r R\tau$. It can be rewritten as $c = c_m - \Delta R\tau$.

Assumption 4: We assume that the optimal remanufacturability R^* in our model satisfies $0 \leq R^* \leq 1$ and the optimal demands and the profits are greater than zero.

We also assume that the manufacturer can increase the remanufacturability R by investing $I(R)$ in improved used product testing and remanufacturing process technologies. It is assumed that $I(R)$ is convex and increasing function of R implying that increasing investments are required to obtain a fixed increment in remanufacturability. This assumption is reasonable because, in practice extremely high investments will be required if every returned product is to be remanufactured. In our model, we will use a specific quadratic investment function $I(R) = kR^2$. The scaling parameter k defined in the remanufacturability cost function is assumed to be sufficiently large, such that $R^* \leq 1$. The use of specific functional form enables us to gain some insights and perform sensitivity analysis. The quadratic investment function is often used to represent an investment with diminishing results in closed form expressions for most optimal quantities (Savaskan and Wassenhove 2006).

Assumption 5: The manufacturer has sufficient channel power over the retailer to act as a Stackelberg leader.

The manufacturer uses his foresight about the retailer's reaction function in his decision making. The Stackelberg structure for the solution of similar games has been widely used in the supply chain literature (Savaskan et al. 2004, Dowrick 1986).

Assumption 6: Both the manufacturer and the retailer have access to the supply chain information consisting of the manufacturing cost c_m , remanufacturing cost c_r , demand function q and collection rate τ .

3.2 Derivation of MR Model

We consider the MR model that the manufacturer collects the used products directly from consumers. For instance, Xerox Corporation provides prepaid mailboxes so that consumers can easily return their used copy or print cartridge to Xerox. The green remanufacturing program saves the company 40%-65% in manufacturing costs through reuse of parts and materials (Ginsburg 2001). In this model, manufacturer collects the used products and then remanufactures them. We set up the closed loop supply chain as a Stackelberg game where the manufacturer is leader and makes his decisions first while the retailer is the follower makes her decisions later.

Being the leader, the manufacturer anticipates the retailer's reaction function and determines the optimal remanufacturability R and wholesale price w that maximize his profit. To solve the Stackelberg game, we first optimize the retailer's profit π_R^{MR} and determine her reaction function $p(w)$ to a given w . The retailer's profit maximization given the wholesale price w is formulated by (3.1).

$$\underset{p}{\text{Max}} \pi_R^{MR} = (p - w)(\beta - \gamma p) \quad (3.1)$$

In the Stackelberg game, the manufacturer takes the retailer's reaction function into consideration for his price decision. The concavity of the follower's objective implies that

her response function is single valued and is a sufficient condition for the existence of the Stackelberg equilibrium.

From (3.1),

$$\frac{\partial^2 \pi_R^{MR}}{\partial p^2} = -2\gamma \quad (3.2)$$

$\frac{\partial^2 \pi_R^{MR}}{\partial p^2} = -2\gamma < 0$. Therefore π_R^{MR} is a concave function of the price w implying that

the retailer's best response function is single valued.

The retailer's reaction function given wholesale price w can be derived from the first order necessary condition of (3.1).

$$\frac{\partial \pi_R^{MR}}{\partial p} = \beta - p\gamma - (p - w)\gamma = 0 \quad (3.3)$$

Solving (3.3), the retailer's best response function $p(w)$ is as provided by (3.4).

$$p(w) = \frac{\beta + w\gamma}{2\gamma} \quad (3.4)$$

The next step in solving the Stackelberg game is to determine the wholesale price w and the remanufacturability R that maximize the manufacturer's profit π_M^{MR} while considering the retailer's best response function $p(w)$. The manufacturer's profit maximization problem is formulated by (3.5). The c_m is manufacturing cost for new products, $\Delta R\tau$ is cost savings per remanufactured product after collection, $c_c\tau$ is collection cost per collected product, and kR^2 is quadratic investment cost for remanufacturing.

$$\text{Max}_{w,R} \pi_M^{MR} = (w - c_m + \Delta R\tau - c_c\tau)(\beta - \gamma p(w)) - kR^2 \quad (3.5)$$

After substituting the retailer's best response function $p(w)$ from (3.4) into (3.5), the equilibrium price w and remanufacturability R are found by equating the first derivatives of the manufacturer's profit (3.6) and (3.7) to zero.

$$\frac{\partial \pi_M^{MR}}{\partial w} = \frac{1}{2}(\beta - wr) - \frac{1}{2}\gamma(w + R\Delta\tau - \tau c_c - c_m) \quad (3.6)$$

$$\frac{\partial \pi_M^{MR}}{\partial R} = \frac{1}{2}(\beta - w\gamma)\Delta\tau - 2Rk \quad (3.7)$$

We can check if the first order necessary condition satisfying point is optimal by checking the second order sufficient condition evaluated at such a point, that, is checking the Hessian matrix of π_M^{MR} .

The Hessian matrix is

$$H = \begin{bmatrix} h11 & h12 \\ h21 & h22 \end{bmatrix} = \begin{bmatrix} \frac{\partial^2 \pi_M^{MR}}{\partial w^2} & \frac{\partial^2 \pi_M^{MR}}{\partial w \partial R} \\ \frac{\partial^2 \pi_M^{MR}}{\partial R \partial w} & \frac{\partial^2 \pi_M^{MR}}{\partial R^2} \end{bmatrix} = \begin{bmatrix} -\gamma & -\frac{1}{2}\gamma\Delta\tau \\ -\frac{1}{2}\gamma\Delta\tau & -2k \end{bmatrix} \quad (3.8)$$

To meet the second order sufficient condition, we should have

$$h11 = -\gamma < 0 \quad (3.9)$$

which is true.

Also, to meet the second order sufficient condition, we should have

$h11 * h22 - h12 * h21 > 0$, ie.,

$$h11 * h22 - h12 * h21 = \left(2\gamma\left(k - \frac{1}{8}\gamma\tau^2\Delta^2\right) \right) \quad (3.10)$$

In order to satisfy the second order sufficient condition for a maximum, we assume that (3.11) is true.

$$k > \frac{1}{8} \gamma \tau^2 \Delta^2 \quad (3.11)$$

The optimal w and R can be found by solving the first order condition (3.6) and (3.7) as shown in (3.12) and (3.13).

$$w = \frac{\beta \gamma \Delta^2 \tau^2 - 4(\beta + \gamma \tau c_c + \gamma c_m)k}{\gamma(\gamma \Delta^2 \tau^2 - 8k)} \quad (3.12)$$

$$R = \frac{\Delta \tau (\beta - \gamma \tau c_c - \gamma c_m)}{8k - \gamma \Delta^2 \tau^2} \quad (3.13)$$

We can get the equilibrium for optimal p by substituting (3.12) into (3.4).

$$p = \frac{\beta \gamma \Delta^2 \tau^2 - 2(3\beta + \gamma \tau c_c + \gamma c_m)k}{\gamma(\gamma \Delta^2 \tau^2 - 8k)} \quad (3.15)$$

Finally we can obtain the optimal demand q , remanufacturer's profit π_R^{MR} , and manufacturer's profit π_M^{MR} by using optimal w , R , and p . The equilibrium solutions in MR model are summarized in Table 1.

Table 1. The optimal equilibriums of MR model

R^{MR*}	$\frac{\Delta\tau(\beta - \gamma\tau c_c - \gamma c_m)}{8k - \gamma\Delta^2\tau^2}$
w^{MR*}	$\frac{\beta\gamma\Delta^2\tau^2 - 4(\beta + \gamma\tau c_c + \gamma c_m)k}{\gamma(\gamma\Delta^2\tau^2 - 8k)}$
p^{MR*}	$\frac{\beta\gamma\Delta^2\tau^2 - 2(3\beta + \gamma\tau c_c + \gamma c_m)k}{\gamma(\gamma\Delta^2\tau^2 - 8k)}$
q^{MR*}	$\frac{2(\beta - \gamma(\tau c_c + c_m))k}{8k - \gamma\Delta^2\tau^2}$
π_R^{MR*}	$\frac{4(\beta - \gamma(\tau c_c + c_m))^2 k^2}{\gamma(8k - \gamma\Delta^2\tau^2)^2}$
π_M^{MR*}	$\frac{(\beta - \gamma(\tau c_c + c_m))^2 k}{\gamma(8k - \gamma\Delta^2\tau^2)}$
π_C^{MR*}	$\frac{4(\beta - \gamma(\tau c_c + c_m))^2 k(12k - \gamma\Delta^2\tau^2)}{\gamma(8k - \gamma\Delta^2\tau^2)^2}$

3.3 The Centrally Coordinated Model (CC)

In the previous section, we discussed MR model such as manufacturer-retailer closed loop supply chain. In this section, we assume that the manufacturer and the retailer are centrally coordinated by a central planner with the objective of maximizing total supply chain profit. It is obvious that in a decentralized decision-making environment which is addressed as MR model in our thesis, the equilibrium outcome will not be optimal from a system's perspective (Debo et al. 2002). Some degree of coordination is necessary in order to

align the incentives of the individual decision makers with the objective to achieve greater efficiency from the point of view of the overall supply chain.

The CC model is illustrated in Figure 2. The CC model has only one player as monopoly model: the central planner who manufactures as well as remanufactures the products and then sells the products to the consumers. The central planner charges a price p per unit to consumers so that the central planner's demand function is same as the retailer's demand function in MR model. Also, the central planner considers the remanufacturability R that can be economically remanufactured after collection of the used products from consumers.

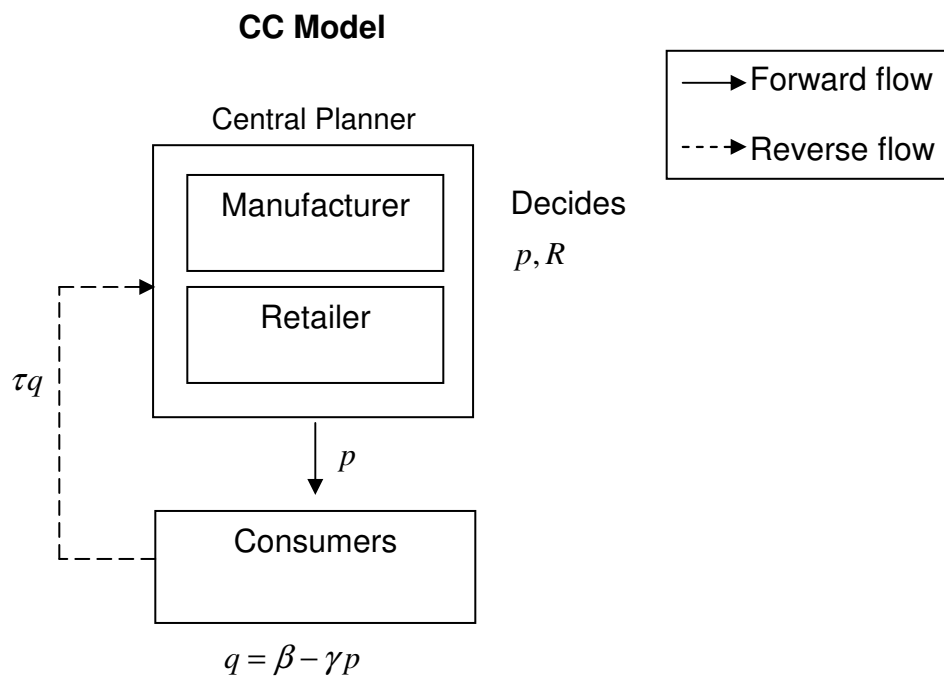


Figure 2. Illustration of Centrally Coordinated Model

The CC model provides a benchmark scenario to compare the MR model with respect to the supply chain profits and the reverse channel performance such as remanufacturability R .

The central planner's profit maximization is formulated by (3.16).

$$\text{Max}_{p,R} \pi_c^{CC} = (p - c_m + \Delta R \tau - c_c \tau)(\beta - \gamma p) - kR^2 \quad (3.16)$$

The optimal p and R in the central planner can be derived from the first order necessary conditions of (3.16).

$$\frac{\partial \pi_c^{CC}}{\partial p} = \beta - p\gamma - \gamma(p + R\Delta\tau - \tau c_c - c_m) = 0 \quad (3.17)$$

$$\frac{\partial \pi_c^{CC}}{\partial R} = (\beta - p\gamma)\Delta\tau - 2Rk = 0 \quad (3.18)$$

We can also check if the first order necessary condition in CC model satisfying point is optimal by checking the second order sufficient condition evaluated at such a point that, is checking the Hessian matrix of π_c^{CC} .

The Hessian matrix is

$$H = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} \frac{\partial^2 \pi_c^{CC}}{\partial p^2} & \frac{\partial^2 \pi_c^{CC}}{\partial p \partial R} \\ \frac{\partial^2 \pi_c^{CC}}{\partial R \partial p} & \frac{\partial^2 \pi_c^{CC}}{\partial R^2} \end{bmatrix} = \begin{bmatrix} -2\gamma & -\gamma\tau\Delta \\ -\gamma\tau\Delta & -2k \end{bmatrix} \quad (3.19)$$

To meet the second order sufficient condition, we should have

$$h_{11} = -2\gamma < 0 \quad (3.20)$$

which is true.

Also, to meet the second order sufficient condition, we should have

$h_{11} * h_{22} - h_{12} * h_{21} > 0$, i.e.,

$$h_{11} * h_{22} - h_{12} * h_{21} = \left(4\gamma \left(k - \frac{1}{4} \gamma \tau^2 \Delta^2 \right) \right) \quad (3.21)$$

In order to satisfy the second order sufficient condition for a maximum, we assume that (3.22) is true.

$$k > \frac{1}{4} \gamma \tau^2 \Delta^2 \quad (3.22)$$

The optimal p and R can be found by solving the first order condition (3.17) and (3.18) as shown in (3.23) and (3.24).

$$p = \frac{\beta \gamma \Delta^2 \tau^2 - 2(\beta + \gamma \tau c_c + \gamma c_m)k}{\gamma(\gamma \Delta^2 \tau^2 - 4k)} \quad (3.23)$$

$$R = \frac{\Delta \tau (\beta - \gamma \tau c_c - \gamma c_m)}{4k - \gamma \Delta^2 \tau^2} \quad (3.24)$$

Finally we can obtain the optimal demand q and central planner's profit π_C^{CC} by using optimal p and R . The equilibrium solutions in CC model are summarized in Table 2.

Table 2. The optimal equilibriums of CC model

R^{CC*}	$\frac{\Delta \tau (\beta - \gamma \tau c_c - \gamma c_m)}{4k - \gamma \Delta^2 \tau^2}$
p^{CC*}	$\frac{\beta \gamma \Delta^2 \tau^2 - 2(\beta + \gamma \tau c_c + \gamma c_m)k}{\gamma(\gamma \Delta^2 \tau^2 - 4k)}$
q^{CC*}	$\frac{2(\beta - \gamma(\tau c_c + c_m))k}{4k - \gamma \Delta^2 \tau^2}$
π_C^{CC*}	$\frac{(\beta - \gamma(\tau c_c + c_m))^2 k}{\gamma(4k - \gamma \Delta^2 \tau^2)^2}$

3.4 Comparison of MR and CC Model

One of our research objectives is to investigate how the supply chain competition and coordination between manufacturer and retailer impact on the remanufacturability, prices, and profits. In this section, we compare the equilibrium solutions of MR model with the equilibrium solutions of CC model.

3.4.1 Comparison of Remanufacturability

In this section, we will show how the closed loop supply chain can increase the remanufacturability by coordinating the manufacturer and the retailer. The remanufacturability R in our model is important to the manufacturer, retailer, and central planner because it may increase their profits due to the cost savings. We will compare R^{MR*} with R^{CC*} .

The optimal remanufacturability in MR and CC model are summarized in Table 3.

Table 3. The optimal remanufacturability in MR and CC model

R^{MR*}	$\frac{\Delta\tau(\beta - \gamma\tau c_c - \gamma c_m)}{8k - \gamma\Delta^2\tau^2}$
R^{CC*}	$\frac{\Delta\tau(\beta - \gamma\tau c_c - \gamma c_m)}{4k - \gamma\Delta^2\tau^2}$

The difference of optimal remanufacturability from MR to CC is shown in (3.25)

$$R^{MR*} - R^{CC*} = \left(-\frac{4\Delta\tau(\beta - \gamma\tau c_c - \gamma c_m)k}{(8k - \gamma\Delta^2\tau^2)(4k - \gamma\Delta^2\tau^2)} \right) \quad (3.25)$$

We assumed that the scaling parameter k is sufficiently large, such that $R^* \leq 1$ so that the denominator $(8k - \gamma\Delta^2\tau^2)(4k - \gamma\Delta^2\tau^2)$ in (3.25) is always greater than zero. If

$\beta > \gamma(\tau c_c + c_m)$, then $R^{MR*} - R^{CC*} < 0$. Under this condition ($\beta > \gamma(\tau c_c + c_m)$), the remanufacturability in CC model is always greater than the remanufacturability in MR model. In other words, we can increase the remanufacturability R by coordinating a manufacturer and a retailer.

3.4.2 Comparison of Price

In this section, we will show how the CC model impacts on the price by comparing each optimal price. We will compare p^{MR*} with p^{CC*} .

The optimal price in MR and CC model are summarized in Table 4.

Table 4. The optimal price in MR and CC model

p^{MR*}	$\frac{\beta\gamma\Delta^2\tau^2 - 2(3\beta + \gamma\tau c_c + \gamma c_m)k}{\gamma(\gamma\Delta^2\tau^2 - 8k)}$
p^{CC*}	$\frac{2(\beta - \gamma(\tau c_c + c_m))k}{4k - \gamma\Delta^2\tau^2}$

The difference of optimal price from MR to CC is shown in (3.26)

$$p^{MR*} - p^{CC*} = \left(\frac{8(\beta - \gamma\tau c_c - \gamma c_m)k^2}{\gamma(8k - \gamma\Delta^2\tau^2)(4k - \gamma\Delta^2\tau^2)} \right) \quad (3.26)$$

The denominator $\gamma(8k - \gamma\Delta^2\tau^2)(4k - \gamma\Delta^2\tau^2)$ in (3.26) is also greater than zero. If $\beta > \gamma(\tau c_c + c_m)$, then $p^{MR*} - p^{CC*} > 0$. Under this condition ($\beta > \gamma(\tau c_c + c_m)$), the price in MR model is always greater than the price in CC model. In other words, we can reduce the retail price p by coordinating a manufacturer and a retailer so that the consumers' demand will be increased.

3.4.3 Comparison of Demand

In this section, we will check if the demand of CC model is greater than the demand of MR model. We showed that the retail price p in CC model is less than the retail price in MR model. It leads to the increased demand quantities in CC model by the linear demand function ($q = \beta - \gamma p$). We will compare q^{MR*} with q^{CC*} .

The optimal demand in MR and CC model are summarized in Table 5.

Table 5. The optimal demand in MR and CC model

q^{MR*}	$\frac{2(\beta - \gamma(\tau c_c + c_m))k}{8k - \gamma\Delta^2\tau^2}$
q^{CC*}	$\frac{2(\beta - \gamma(\tau c_c + c_m))k}{4k - \gamma\Delta^2\tau^2}$

The difference of optimal demand from MR to CC is shown in (3.27)

$$q^{MR*} - q^{CC*} = \left(-\frac{8(\beta - \gamma\tau c_c - \gamma c_m)k^2}{(8k - \gamma\Delta^2\tau^2)(4k - \gamma\Delta^2\tau^2)} \right) \quad (3.27)$$

The denominator $(8k - \gamma\Delta^2\tau^2)(4k - \gamma\Delta^2\tau^2)$ in (3.27) is also greater than zero. If $\beta > \gamma(\tau c_c + c_m)$, then $q^{MR*} - q^{CC*} < 0$. Under this condition ($\beta > \gamma(\tau c_c + c_m)$), the demand in CC model is always greater than the demand in MR model. In other words, we can increase the consumers' demand q by coordinating a manufacturer and a retailer.

3.4.4 Comparison of Channel Profit

The CC model provides a benchmark scenario to compare the MR model with respect to the closed loop supply chain profits. The benefits to the centrally coordinated model, in terms of an increased remanufacturability as well as an increased ability to buy the product

(greater demand) will lead to the higher profit for CC model. We will compare π_C^{MR*} with π_C^{CC*} .

The optimal channel profit in MR and CC model are summarized in Table 6.

Table 6. The optimal channel profit in MR and CC model

π_C^{MR*}	$\frac{4(\beta - \gamma(\tau c_c + c_m))^2 k(12k - \gamma\Delta^2\tau^2)}{\gamma(8k - \gamma\Delta^2\tau^2)^2}$
π_C^{CC*}	$\frac{(\beta - \gamma(\tau c_c + c_m))^2 k}{\gamma(4k - \gamma\Delta^2\tau^2)^2}$

The difference of optimal channel profit from MR to CC is shown in (3.28)

$$\pi_C^{MR*} - \pi_C^{CC*} = \left(\frac{16(\beta - \gamma\tau c_c - \gamma c_m)^2 k^3}{\gamma(\gamma\Delta^2\tau^2 - 8k)^2(\gamma\Delta^2\tau^2 - 4k)} \right) \quad (3.28)$$

The numerator $16(\beta - \gamma\tau c_c - \gamma c_m)^2 k^3$ in (3.28) is greater than zero if k is positive.

The denominator $\gamma(\gamma\Delta^2\tau^2 - 8k)^2(\gamma\Delta^2\tau^2 - 4k)$ is less than zero because k was assumed to be sufficiently large so that $\pi_C^{MR*} - \pi_C^{CC*} < 0$. Thus, the profit in CC model is always greater than the profit in MR model.

In conclusion, we can increase the remanufacturability as well as the channel profit by coordinating a manufacturer and a retailer. Furthermore the decreased retail price in CC model encourages consumers to buy more products.

3.5 Numerical Examples

We now provide numerical examples to illustrate the analytical insights and make further observations comparing MR and CC model equilibriums. This numerical example

was performed to determine the effects that key parameters had on the equilibrium of remanufacturability, wholesale price, retail price, and profits. The parameters, which include cost savings Δ and collection rate τ , lead the discussion for the analysis of MR and CC model. The examples used for the sensitive analysis are shown in Appendix D. The example data are:

$\beta = 1000$, $\gamma = 5$, $\Delta = \$10/ \text{product}$, $\tau = 0.5$, $c_c = \$2/ \text{product}$, $c_m = \$20/ \text{product}$, and $k = \$2000$.

3.5.1 Variation with Cost Savings (Δ)

In this section, the cost savings Δ in MR and CC model was varied from 5 to 15 while all other parameters were held constant. The remanufacturability of MR and CC model are shown in Figure 3 when the cost savings Δ increases from 5 to 15.

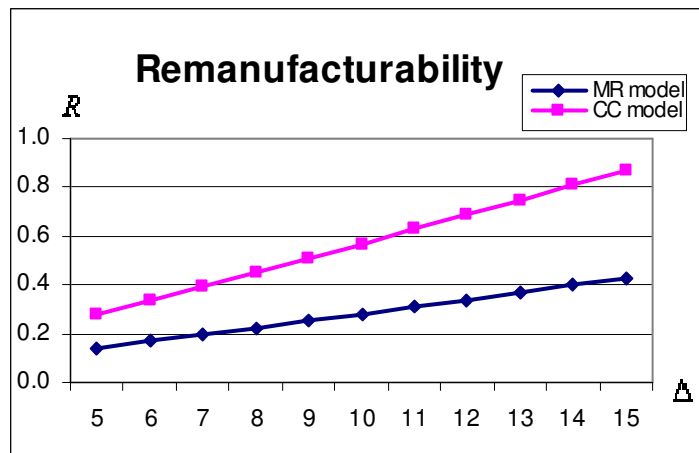


Figure 3. Remanufacturability with variation of Δ

From Figure 3, we can observe that R is increasing with Δ in both MR and CC model. In addition, the difference between the remanufacturability of the MR and CC model is the greatest when Δ is large. This observation can be expected from (3.25): the difference between R^{MR^*} and R^{CC^*} becomes a larger as Δ increases.



Figure 4. Retail price with variation of Δ

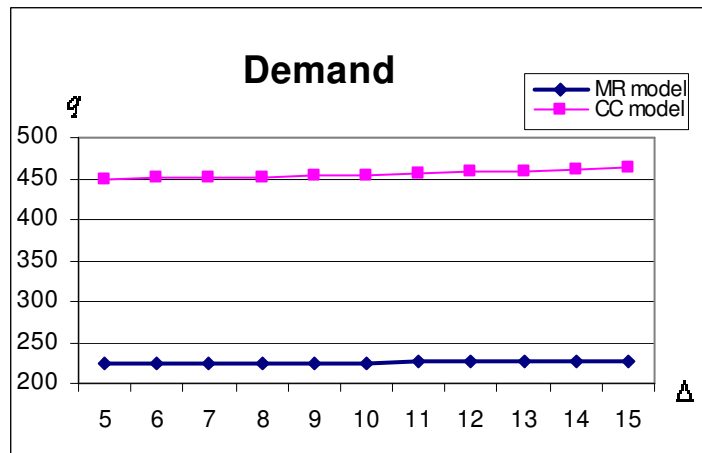


Figure 5. Demand with variation of Δ

Figure 4 and 5 show the retail price and demand with variation of Δ in MR and CC model. We can see the retail price in MR model is always greater than the retail price in CC model so that it leads to the higher demand in CC model than the demand in MR model. Also, the retail price in both MR and CC model decreases as Δ increases. From (3.26), the difference between p^{MR*} and p^{CC*} becomes a larger as Δ increases.

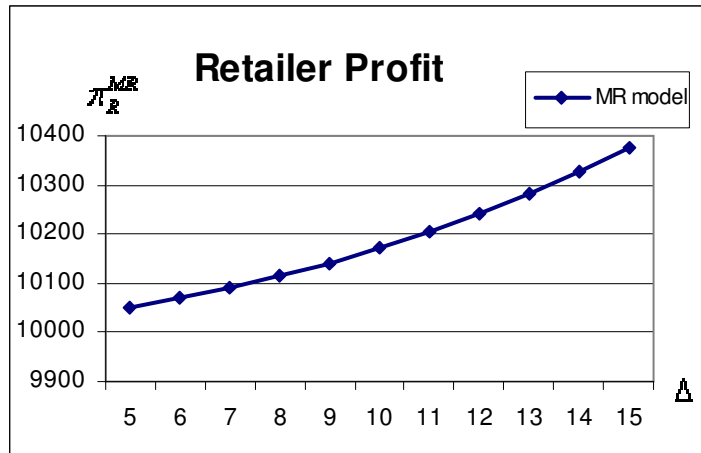


Figure 6. Retailer profit with variation of Δ

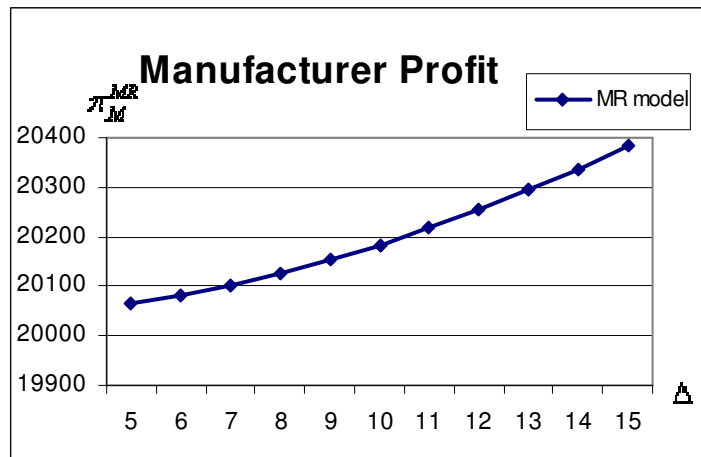


Figure 7. Manufacturer profit with variation of Δ

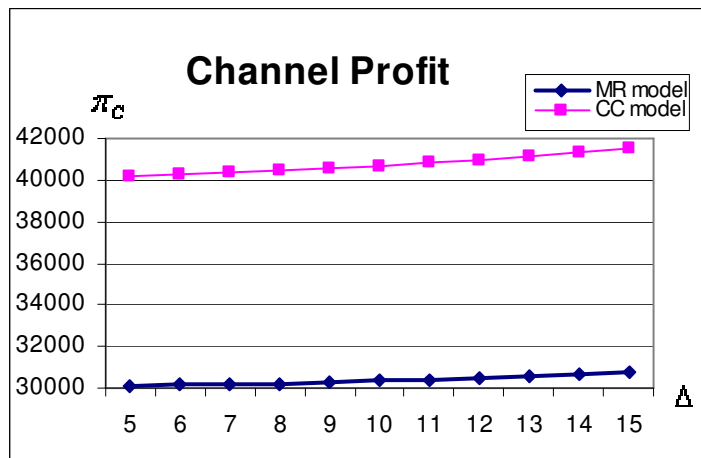


Figure 8. Channel profit with variation of Δ

Figure 6, 7, and 8 show the retailer, manufacturer, and total channel profit with variation of Δ in MR and CC model. We can observe that the profit of both the retailer and manufacturer is nonlinearly increasing with Δ . It implies that the additional cost savings in Δ would be profitable for both the manufacturer and retailer to a more number of products resulting in a more incentive to increase the remanufacturability. Moreover, we can observe that the total channel profit (the central planner profit) in CC model is always greater than the total channel profit (the manufacturer and retailer profit) in MR model. This observation can be also expected from (3.28): the difference between π_C^{MR*} and π_C^{CC*} becomes a larger as Δ increases.

As we have observed through this numerical example (variation of Δ):

- 1) The remanufacturability increases as cost savings increases.
- 2) We can increase the remanufacturability as well as the total channel profit by coordinating the manufacturer and the retailer.

3.5.2 Variation with Collection Cost (τ)

In previous section, we varied the cost savings Δ from 5 to 15. From this section, we investigate how the collection rate τ impacts on the remanufacturability, retail price, and profits. Now we vary the collection rate τ from 0 to 1 when the cost savings Δ is fixed at 10 in both MR and CC model. The remanufacturability of MR and CC model are shown in Figure 9 when the collection cost τ increases from 0 to 1.

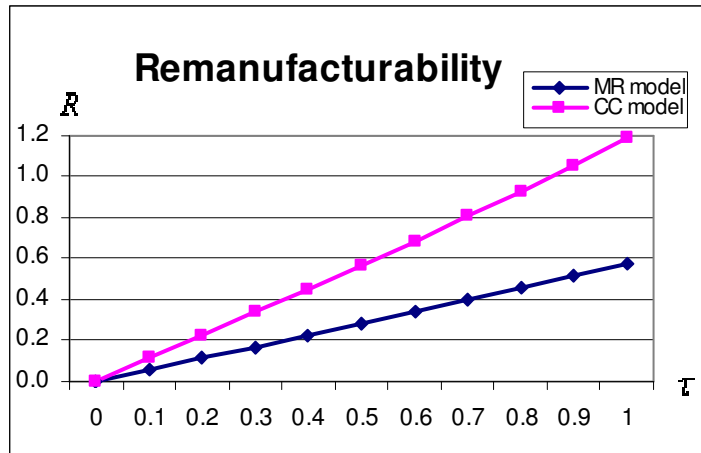


Figure 9. Remanufacturability with variation of τ

From Figure 9, we can observe that R is increasing with τ in both MR and CC model. In addition, the difference between the remanufacturabilities of the MR and CC model is the greatest when τ is the largest ($= 1.0$).

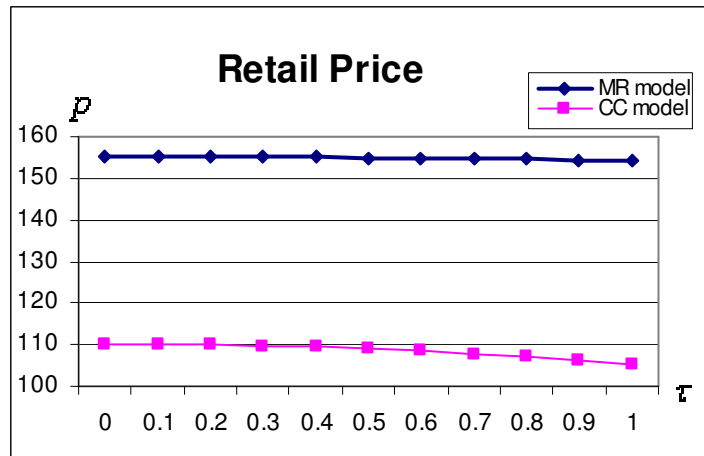


Figure 10. Retail price with variation of τ

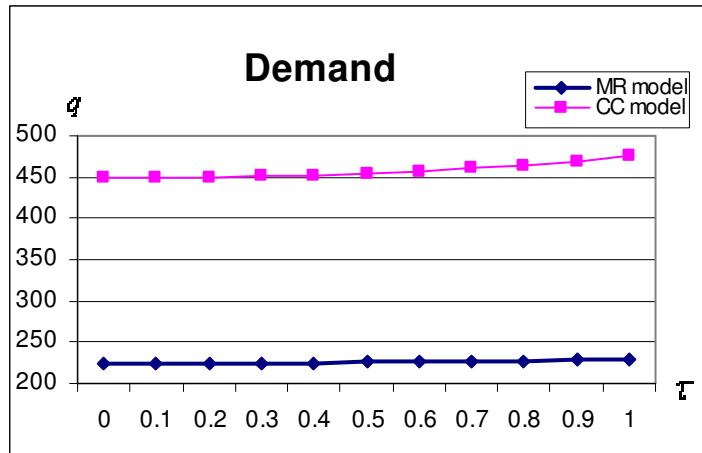


Figure 11. Demand with variation of τ

Figure 10 and 11 show the retail price and demand with variation of τ in MR and CC model. We can see the retail price in MR model is always greater than the retail price in CC model so that it leads to the higher demand in CC model than the demand in MR model. Also, the retail price in both MR and CC model decreases as τ increases. When τ is a maximum ($\tau=1.0$), the retail price is a minimum and the demand is a maximum in both models (See the tables in Appendix D).

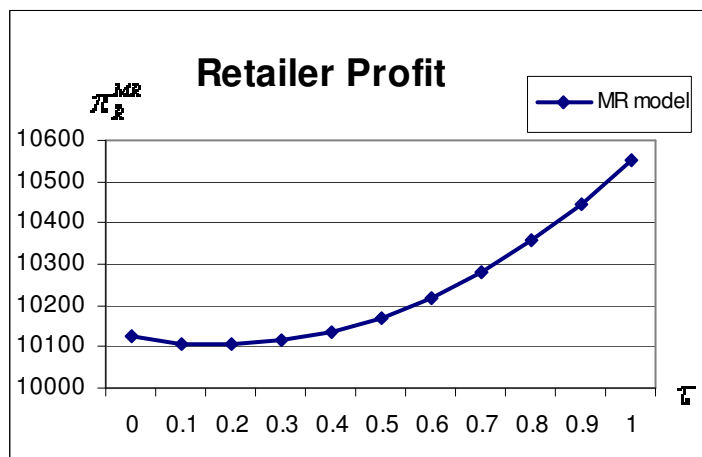


Figure 12. Retailer profit with variation of τ

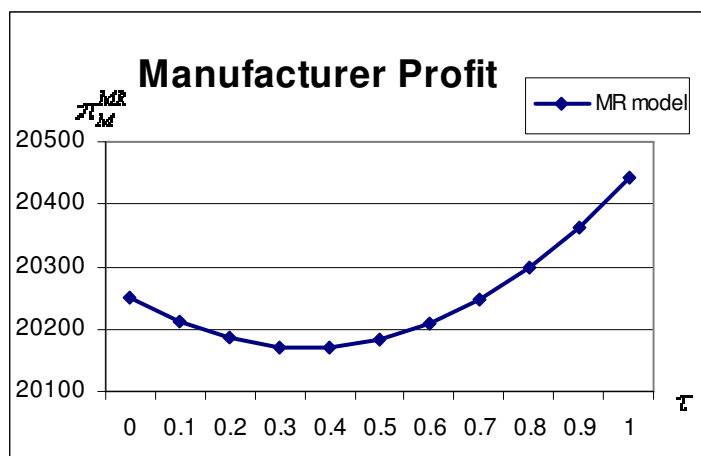


Figure 13. Manufacturer profit with variation of τ

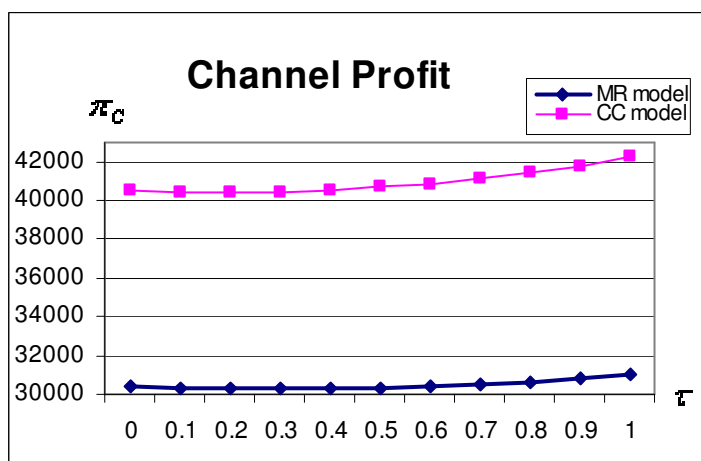


Figure 14. Channel profit with variation of τ

Figure 12, 13, and 14 show the retailer, manufacturer, and total channel profit with variation of τ in MR and CC model. From Figure 12 to 14, the profit of the retailer, manufacturer, and central planner is a maximum when the collection rate τ is one. However, the profit curve is convex when τ varies 0 to 1.

Now we consider when k (scaling parameter, $I(R) = kR^2$) is 1000 and 2000 in Figure 15. The manufacturer's profit when $k = 1000$ is greater than the profit when $k = 2000$ because he can reduce the investment cost for remanufacturability. Also, we can see when τ is small ($0 < \tau < 0.354$) and $k = 1000$, the manufacturer's profit with remanufacturing is

lower than the profit without remanufacturing because the total collection cost ($c_c \tau q$) and remanufacturability investment cost (kR^2) is higher than the total cost savings from remanufacturing ($\Delta R \tau q$).

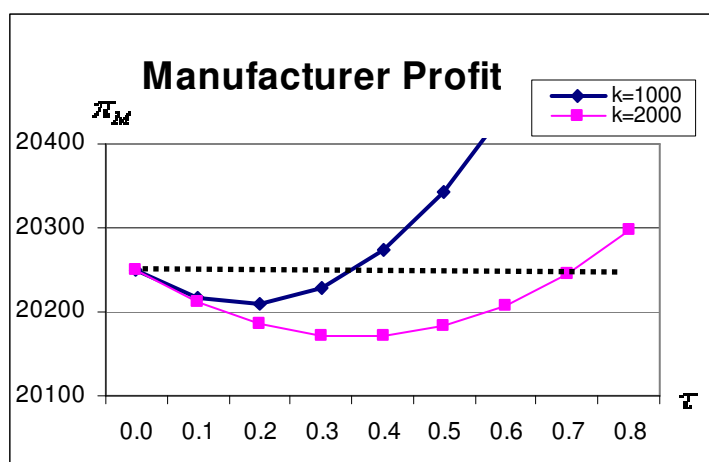


Figure 15. Manufacturer's profit by τ and k

When $\tau = 0$, there is no remanufacturing. When $k = 1000$ and $0 < \tau < 0.354$, without remanufacturing is more profitable than with remanufacturing. When $k = 1000$ and $0.345 < \tau \leq 1$, with remanufacturing is more profitable than without remanufacturing. In order that the remanufacturing is profitable, $\Delta R \tau q > (c_c \tau q + kR^2)$.

Table 7. Manufacturer's profit when $k = 1000$

β	γ	c_c	c_m	Δ	k	τ	R	w	p	q	π_M^{MR}	$c_c \tau q + kR^2$	$\Delta R \tau q$
1000	5	2	20	10	1000	0	0.00	110.0	155.0	225.0	20250.0	0.0	0.0
1000	5	2	20	10	1000	0.05	0.06	110.0	155.0	224.9	20230.7	25.7	6.3
1000	5	2	20	10	1000	0.1	0.11	110.0	155.0	224.9	20217.7	57.6	25.3
1000	5	2	20	10	1000	0.15	0.17	110.0	155.0	224.9	20211.0	95.9	56.9
1000	5	2	20	10	1000	0.2	0.23	110.0	155.0	225.1	20210.6	140.7	101.3
1000	5	2	20	10	1000	0.25	0.28	109.9	154.9	225.3	20216.6	191.9	158.6
1000	5	2	20	10	1000	0.3	0.34	109.8	154.9	225.5	20229.0	249.7	228.9
1000	5	2	20	10	1000	0.354	0.40	109.6	154.8	225.9	20249.6	319.8	319.7
1000	5	2	20	10	1000	0.4	0.45	109.5	154.7	226.3	20273.1	385.8	409.6
1000	5	2	20	10	1000	0.45	0.51	109.3	154.7	226.7	20305.0	464.3	520.6
1000	5	2	20	10	1000	0.5	0.57	109.1	154.5	227.3	20343.5	550.2	645.8

As we have observed through this numerical example (variation of τ):

- 1) The remanufacturability increases as collection rate increases.
- 2) When the collection rate is one, the retailer and manufacturer's profit is a maximum.
- 3) Under the condition $\Delta R\tau q > (c_e \tau q + kR^2)$, the remanufacturing is profitable. Otherwise, manufacturer will lose money by introducing remanufacturing activities.

CHAPTER 4. MANUFACTURER-RETAILER WITH GOVERNMENT MODEL (MRG)

In the previous chapter, we formulated and analyzed the manufacturer-retailer (MR) model that the manufacturer considers his remanufacturability. In this chapter, we will introduce the government penalty and subsidy system based on the MR model. This MRG (Manufacturer-Retailer with Government) model will show the impact of the subsidy and penalty system of government to the members (manufacturer and retailer) of the closed loop supply chain that are involved.

The subsidy fee will cover costs of recycling and/or reuse, transportation of product from first point of collection, and a reasonable, limited collection incentive payment to encourage collection by a variety of entities, including retailers, municipalities, non-profits, etc (Jackson 2003). In reality, due to the increasing consciousness on environmental sustainability, legislative pressure like ARF (Advanced Recovery Fee) requires consumers to pay at the point of retail purchase a recovery fee for recycling or remanufacturing. Also, the local government has policies to increase consumer's used products to be recycled, reused, or refurbished. To date, 12 states have enacted some form of e-waste management law (as many as 20 states proposed e-waste laws in 2006 and 2007). Most state laws and proposals have certain broad elements in common, such as specifying the electronic devices covered under the law; how a collection and recycling and/or remanufacturing program will be financed; collection and recycling criteria that must be met to minimize the impact to human health and the environment; and restrictions or requirements that products must meet to be sold in the state. For example, the e-waste legislation in State of Oregon requires retailers to charge first

in-state buyers of electronic devices an ARF of up to \$10 to pay for remanufacturing of products (INFORM 2007).

In this chapter, we will consider the ARF (Advanced Recovery Fee) system as environmental fee in our model. The retailer collects the fee at the point of retail purchase and remits the fee into the government. The ARF in this chapter is defined as the government collects the fee from consumers to subsidize manufacturer's remanufacturing of the collected products.

The objective of our research in this chapter is to investigate into following questions:

- 1) How the government collects the ARF from consumers and distributes it for remanufacturing?
- 2) How the government's penalty to consumers (ARF) and subsidy for remanufacturing to the manufacturer impact on the remanufacturability and profits?
- 3) How supply chain competition and coordination between manufacturer and retailer with government impacts on the remanufacturability, prices, and profits?

In order to answer these questions, we establish a closed-loop supply chain with the government penalty to consumers and subsidy for remanufacturing to the manufacturer.

4.1 Model Assumptions and Notations

In this chapter, we formulate and analyze the manufacturer-retailer with government subsidy and penalty model (MRG) that the government collects the fee (ARF) as environmental fee from consumers and subsidizes it for remanufacturing. The MRG model is illustrated in Figure 16.

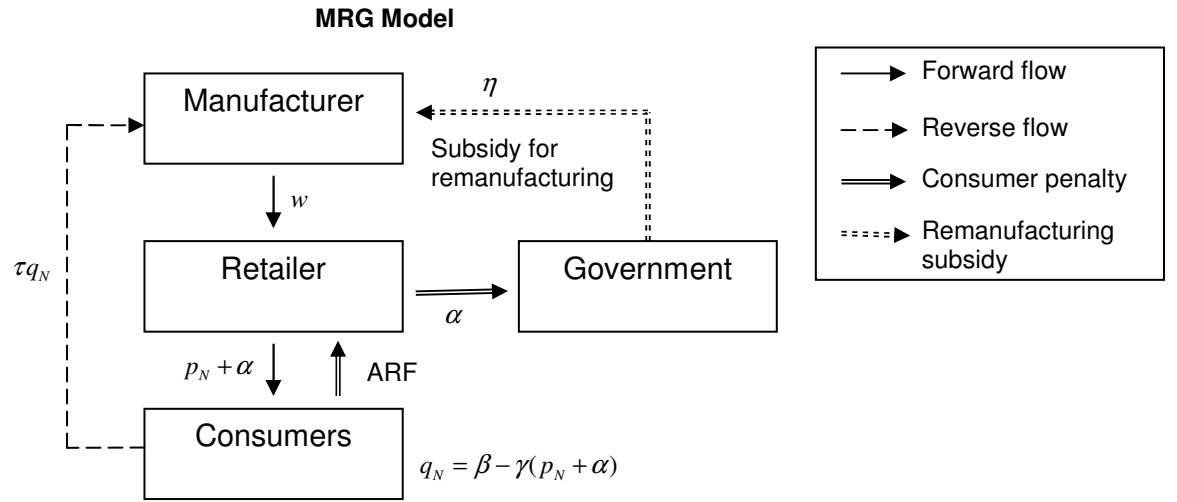


Figure 16. Illustration of Manufacturer-Retailer with Government Model

We formulate this MRG model as a Stackelberg game with the manufacturer as the leader and the retailer as the follower. The key assumptions and notations in this model are the same as those in the MR model in chapter 3. However in order to consider the government penalty and subsidy system, we use additional notations and assumptions for the government. Additional notations used in this chapter are explained below:

α : ARF (Advanced Recovery Fee), the retailer collects the fee from consumers at the point of retail purchases (dollars per product) and remits it to the government;

η : The subsidy for remanufacturing, the government subsidizes the fee collected from the consumers into the manufacturer for remanufacturing (dollars per remanufactured product).

$p_N + \alpha$: The price that the consumers pay after the government's penalty and subsidy;

p_N : The new retail price after the government's penalty and subsidy, the retailer receives p_N ;

q_N : The consumer demand function after the government's penalty and subsidy,

$(q_N = \beta - \gamma(p_N + \alpha))$;

π_R^{MRG} : The retailer's profit in MRG model;

π_M^{MRG} : The manufacturer's profit in MRG model;

π_C^{MRG} : The total channel profit in MRG model ($\pi_C^{MRG} = \pi_R^{MRG} + \pi_M^{MRG}$);

π_C^{CCG} : The centrally coordinated planner's profit in CCG model;

In order to model the closed loop supply chain with the government subsidy for remanufacturing, we made the following additional assumptions.

Assumption 7: The total collected fee (ARF) is balanced with the total remanufacturing subsidy in this model. It means the government revenue neutrality through the penalty and subsidy system.

The fee collected from the consumers will be used to increase the remanufacturability in this model. The government collects ARF from the consumers as αq_N and pays it to the manufacturer for remanufacturing as $\eta R \tau q_N$. We assume that the government has a policy that the total amount of ARF is balanced with the total amount of subsidy. It means that the amount of the fee collected from the consumers is equal to the amount of subsidy for remanufacturing. This assumption is shown in (4.1). However, we assume that the retailer and manufacturer do not know this government policy. If this policy is opened to the manufacturer, the subsidy will not impact on his remanufacturability because he knows that the fee collected from the consumers is his profit before setting up the remanufacturability.

$$\alpha q_N = \eta R \tau q_N \quad (4.1)$$

Assumption 8: The demand shift is caused by ARF.

The original demand function without ARF consideration is $q = \beta - \gamma p$ in MR model. It presents a downward sloping linear demand function. It is shown that the vertical interaction between the channel members and the optimality of the channel strategies depend

on the convexity of the demand function (Lee and Staelin 1997). The demand shift caused by the ARF is shown in (4.2).

$$q_N = \beta - \gamma(p_N + \alpha) \quad (4.2)$$

The derivation of the demand shift caused by the ARF is given in the Appendix A.

4.2 Derivation of MRG Model

In the manufacturer-retailer with government intervention model (MRG), the government collects ARF from consumers and subsidizes it to the manufacturer for remanufacturing. Based on this environmental fee consideration, we set up the closed-loop supply chain as a Stackelberg game where the manufacturer is leader and makes his decisions first while the retailer is the follower makes her decisions later.

Being the leader, the manufacturer anticipates the retailer's reaction function and determines the optimal wholesale price that maximizes his profit. To solve the Stackelberg game, we first optimize the retailer's profit π_R^{MRG} and determine her reaction function $p_N(w)$ to a given w . The retailer's profit maximization given the wholesale price w is formulated by (4.3).

$$\underset{p_N}{Max} \pi_R^{MRG} = (p_N - w)(\beta - \gamma(p_N + \alpha)) \quad (4.3)$$

In the Stackelberg game, the manufacturer takes the retailer's reaction function into consideration for his price decision. The concavity of the follower's objective implies that her response function is single valued and is a sufficient condition for the existence of the Stackelberg equilibrium.

From (4.3),

$$\frac{\partial^2 \pi_R^{MRG}}{\partial p_N^2} = -2\gamma \quad (4.4)$$

$\frac{\partial^2 \pi_R^{MRG}}{\partial p_N^2} = -2\gamma < 0$. Therefore π_R^{MRG} is a concave function of the price w implying

that the retailer's best response function is single valued.

The retailer's reaction function given wholesale price w can be derived from the first order necessary conditions of (4.3).

$$\frac{\partial \pi_R^{MRG}}{\partial p_N} = \beta - (p_N - w)\gamma - (p_N + \alpha)\gamma = 0 \quad (4.5)$$

Solving (4.5), the retailer's best response function $p_N(w)$ is as provided by (4.6).

$$p_N(w) = \frac{\beta + (w - \alpha)\gamma}{2\gamma} \quad (4.6)$$

The next step in solving the Stackelberg game is to determine the wholesale price w and the remanufacturability R that maximize the manufacturer's profit π_M^{MRG} while considering the retailer's best response function $p_N(w)$. The manufacturer's profit maximization problem is formulated by (4.7). The $\eta R \tau (\beta - \gamma p_N((w) + \alpha))$ is the government subsidy to the manufacturer for remanufacturing.

$$\begin{aligned} \text{Max}_{w,R} \pi_M^{MRG} = \\ (w - c_m + \Delta R \tau - c_c \tau) (\beta - \gamma (p_N(w) + \alpha)) - kR^2 + \eta R \tau (\beta - \gamma (p_N(w) + \alpha)) \end{aligned} \quad (4.7)$$

After substituting the retailer's best response function $p_N(w)$ from (4.6) into (4.7), the equilibrium price w and remanufacturability R are found by equating the first derivatives of the manufacturer's profit (4.8) and (4.9) to zero.

$$\frac{\partial \pi_M^{MRG}}{\partial w} = \frac{1}{2}(\beta - \gamma(2w + \alpha + R(\Delta + \eta)\tau) + \gamma\tau c_c + \gamma c_m) \quad (4.8)$$

$$\frac{\partial \pi_M^{MRG}}{\partial R} = \frac{1}{2}((\beta - (w + \alpha)\gamma)(\Delta + \eta)\tau - 4Rk) \quad (4.9)$$

We can check if the first order necessary condition satisfying point is optimal by checking the second order sufficient condition evaluated at such a point, that is checking the Hessian matrix of π_M^{MRG} .

The Hessian matrix is

$$H = \begin{bmatrix} h11 & h12 \\ h21 & h22 \end{bmatrix} = \begin{bmatrix} \frac{\partial^2 \pi_M^{MRG}}{\partial w^2} & \frac{\partial^2 \pi_M^{MRG}}{\partial w \partial R} \\ \frac{\partial^2 \pi_M^{MRG}}{\partial R \partial w} & \frac{\partial^2 \pi_M^{MRG}}{\partial R^2} \end{bmatrix} = \begin{bmatrix} -\gamma & -\frac{1}{2}\gamma\tau(\Delta + \eta) \\ -\frac{1}{2}\gamma\tau(\Delta + \eta) & -2k \end{bmatrix} \quad (4.10)$$

To meet the second order sufficient condition, we should have

$$h11 = -\gamma < 0 \quad (4.11)$$

which is true.

Also, to meet the second order sufficient condition, we should have

$h11 * h22 - h12 * h21 > 0$, i.e.,

$$h11 * h22 - h12 * h21 = \left(2\gamma(k - \frac{1}{8}\gamma\tau^2(\Delta + \eta)^2) \right) \quad (4.12)$$

In order to satisfy the second order sufficient condition for a maximum, we assume that (4.13) is true.

$$k > \frac{1}{8}\gamma\tau^2(\Delta + \eta)^2 \quad (4.13)$$

The optimal w and R can be found by solving the first order condition (4.8) and (4.9) as shown in (4.14) and (4.15).

$$w = \frac{\gamma(\beta - \alpha\gamma)(\Delta + \eta)^2 \tau^2 - 4(\beta - \alpha\gamma + \gamma\tau c_c + \gamma c_m)k}{\gamma(\gamma(\Delta + \eta)^2 \tau^2 - 8k)} \quad (4.14)$$

$$R = \frac{(\Delta + \eta)\tau(\beta - \alpha\gamma - \gamma\tau c_c - \gamma c_m)}{8k - \gamma(\Delta + \eta)^2 \tau^2} \quad (4.15)$$

We can get the equilibrium for optimal p_N by substituting (4.14) into (4.6).

$$p_N = \frac{\gamma(\beta - \alpha\gamma)(\Delta + \eta)^2 \tau^2 - 2(3\beta - 3\alpha\gamma + \gamma\tau c_c + \gamma c_m)k}{\gamma(\gamma(\Delta + \eta)^2 \tau^2 - 8k)} \quad (4.16)$$

Finally we can obtain the optimal demand q_N , π_R^{MRG} , and π_M^{MRG} by using optimal w ,

R , and p_N . The equilibrium solutions in MRG model are summarized in Table 8.

Table 8. The optimal equilibriums of MRG model

R^{MRG*}	$\frac{(\Delta + \eta)\tau(\beta - \alpha\gamma - \gamma\tau c_c - \gamma c_m)}{8k - \gamma(\Delta + \eta)^2 \tau^2}$
w^{MRG*}	$\frac{\gamma(\beta - \alpha\gamma)(\Delta + \eta)^2 \tau^2 - 4(\beta - \alpha\gamma + \gamma\tau c_c + \gamma c_m)k}{\gamma(\gamma(\Delta + \eta)^2 \tau^2 - 8k)}$
p_N^{MRG*}	$\frac{\gamma(\beta - \alpha\gamma)(\Delta + \eta)^2 \tau^2 - 2(3\beta - 3\alpha\gamma + \gamma\tau c_c + \gamma c_m)k}{\gamma(\gamma(\Delta + \eta)^2 \tau^2 - 8k)}$
q_N^{MRG*}	$\frac{2(\beta - \alpha\gamma - \gamma(\tau c_c + c_m))k}{8k - \gamma(\Delta + \eta)^2 \tau^2}$
π_R^{MRG*}	$\frac{4(\beta - \alpha\gamma - \gamma(\tau c_c + c_m))^2 k^2}{\gamma(8k - \gamma(\Delta + \eta)^2 \tau^2)^2}$
π_M^{MRG*}	$\frac{(\beta - \alpha\gamma - \gamma(\tau c_c + c_m))^2 k}{\gamma(8k - \gamma(\Delta + \eta)^2 \tau^2)}$
π_C^{MRG*}	$\frac{(\beta - \alpha\gamma - \gamma(\tau c_c + c_m))^2 k(12k - \gamma(\Delta + \eta)^2 \tau^2)}{\gamma(8k - \gamma(\Delta + \eta)^2 \tau^2)^2}$

4.3 The Centrally Coordinated with Government Model (CCG)

In the previous section, we discussed MRG model such as manufacturer-retailer with government closed loop supply chain. In this section, we assume that the manufacturer and the retailer are centrally coordinated by a central planner with the objective of maximizing total supply chain profits. The CCG model is illustrated in Figure 17.

The central planner charges a price p_N per unit to consumers so that the central planner's demand function is same as the retailer's demand function in MRG model. Also, the central planner considers the remanufacturability R that can be economically remanufactured after collection of the used products from consumers.

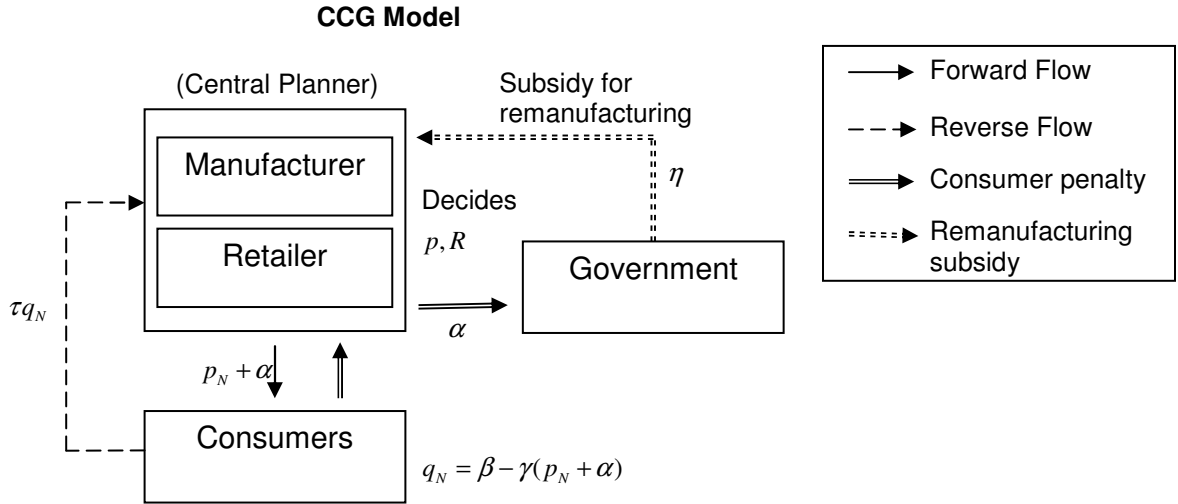


Figure 17. Illustration of Centrally Coordinated with Government Model

The CCG model provides a benchmark scenario to compare the MRG model with respect to the supply chain profits and the reverse channel performance such as remanufacturability R .

The central planner's profit maximization is formulated by (4.17).

$$\text{Max}_{p_N, R} \pi_C^{\text{CCG}} = (p_N - c_m + \Delta R \tau - c_c \tau)(\beta - \gamma(p_N + \alpha)) - kR^2 + \eta R \tau (\beta - \gamma(p_N + \alpha)) \quad (4.17)$$

The optimal p_N and R in the central planner can be derived from the first order necessary conditions of (4.17).

$$\frac{\partial \pi_C^{CCG}}{\partial p_N} = \beta - (p_N + \alpha)\gamma - R\gamma\eta\tau - \gamma(p_N + R\Delta\tau - \tau c_c - c_m) = 0 \quad (4.18)$$

$$\frac{\partial \pi_C^{CCG}}{\partial R} = (\beta - (p_N + \alpha)\gamma)\Delta\tau + (\beta - (p_N + \alpha)\gamma)\eta\tau - 2Rk = 0 \quad (4.19)$$

We can also check if the first order necessary condition in CCG model satisfying point is optimal by checking the second order sufficient condition evaluated at such a point that, is checking the Hessian matrix of π_C^{CCG} .

The Hessian matrix is

$$H = \begin{bmatrix} h11 & h12 \\ h21 & h22 \end{bmatrix} = \begin{bmatrix} \frac{\partial^2 \pi_C^{CCG}}{\partial p_N^2} & \frac{\partial^2 \pi_C^{CCG}}{\partial p_N \partial R} \\ \frac{\partial^2 \pi_C^{CCG}}{\partial R \partial p_N} & \frac{\partial^2 \pi_C^{CCG}}{\partial R^2} \end{bmatrix} = \begin{bmatrix} -2\gamma & -\gamma\tau(\Delta + \eta) \\ -\gamma\tau(\Delta + \eta) & -2k \end{bmatrix} \quad (4.20)$$

To meet the second order sufficient condition, we should have

$$h11 = -2\gamma < 0 \quad (4.21)$$

which is true.

Also, to meet the second order sufficient condition, we should have

$h11 * h22 - h12 * h21 > 0$, ie.,

$$h11 * h22 - h12 * h21 = \left(4\gamma(k - \frac{1}{4}\gamma\tau^2(\Delta + \eta)^2) \right) \quad (4.22)$$

In order to satisfy the second order sufficient condition for a maximum, we assume that (4.23) is true.

$$k > \frac{1}{4} \gamma \tau^2 (\Delta + \eta)^2 \quad (4.23)$$

The optimal p_N and R can be found by solving the first order condition (4.18) and (4.19) as shown in (4.24) and (4.25).

$$p_N = \frac{\gamma(\beta - \alpha\gamma)(\Delta + \eta)^2 \tau^2 - 2(\beta - \alpha\gamma + \gamma\tau c_c + \gamma c_m)k}{\gamma(\gamma(\Delta + \eta)^2 \tau^2 - 4k)} \quad (4.24)$$

$$R = \frac{(\Delta + \eta)\tau(\beta - \alpha\gamma - \gamma\tau c_c - \gamma c_m)}{4k - \gamma(\Delta + \eta)^2 \tau^2} \quad (4.25)$$

Finally we can obtain the optimal demand q_N and central planner's profit π_C^{CCG} by using optimal p_N and R . The equilibrium solutions in CC model are summarized in Table 9.

Table 9. The optimal equilibriums of CCG model

R^{CCG*}	$\frac{(\Delta + \eta)\tau(\beta - \alpha\gamma - \gamma\tau c_c - \gamma c_m)}{4k - \gamma(\Delta + \eta)^2 \tau^2}$
p_N^{CCG*}	$\frac{\gamma(\beta - \alpha\gamma)(\Delta + \eta)^2 \tau^2 - 2(\beta - \alpha\gamma + \gamma\tau c_c + \gamma c_m)k}{\gamma(\gamma(\Delta + \eta)^2 \tau^2 - 4k)}$
q_N^{CCG*}	$\frac{2(\beta - \alpha\gamma - \gamma(\tau c_c + c_m))k}{4k - \gamma(\Delta + \eta)^2 \tau^2}$
π_C^{CCG*}	$\frac{(\beta - \alpha\gamma - \gamma(\tau c_c + c_m))^2 k}{\gamma(4k - \gamma(\Delta + \eta)^2 \tau^2)}$

4.4 Comparison of MRG and CCG Model

One of our research objectives is to investigate how the supply chain competition and coordination between manufacturer and retailer with government impact on the

remanufacturability, prices, and profits. In this section, we compare the equilibrium solutions of MRG model with the equilibrium solutions of CCG model.

4.4.1 Comparison of Remanufacturability

In this section, we will show how the closed loop supply chain with government can increase the remanufacturability by coordinating the manufacturer and the retailer. The remanufacturability R in our model is important to the manufacturer, retailer, and central planner because it may increase their profits due to the cost savings. We will compare R^{MRG^*} with R^{CCG^*} .

The optimal remanufacturability in MRG and CCG model are summarized in Table 10.

Table 10. The optimal remanufacturability in MRG and CCG model

R^{MRG^*}	$\frac{(\Delta + \eta)\tau(\beta - \alpha\gamma - \gamma\tau c_c - \gamma c_m)}{8k - \gamma(\Delta + \eta)^2\tau^2}$
R^{CCG^*}	$\frac{(\Delta + \eta)\tau(\beta - \alpha\gamma - \gamma\tau c_c - \gamma c_m)}{4k - \gamma(\Delta + \eta)^2\tau^2}$

The difference of optimal remanufacturability from MRG to CCG is shown in (4.26).

$$R^{MRG^*} - R^{CCG^*} = \left(-\frac{4(\Delta + \eta)\tau(\beta - \alpha\gamma - \gamma\tau c_c - \gamma c_m)k}{(8k - \gamma(\Delta + \eta)^2\tau^2)(4k - \gamma(\Delta + \eta)^2\tau^2)} \right) \quad (4.26)$$

We assumed that the scaling parameter k is sufficiently large, such that $R^* \leq 1$ so that the denominator $(8k - \gamma(\Delta + \eta)^2\tau^2)(4k - \gamma(\Delta + \eta)^2\tau^2)$ in (4.26) is always greater than zero. If $\beta > \gamma(\alpha + \tau c_c + c_m)$, then $R^{MRG^*} - R^{CCG^*} < 0$. Under this condition ($\beta > \gamma(\alpha + \tau c_c + c_m)$), the remanufacturability in CCG model is always greater than the

remanufacturability in MRG model. In other words, we can increase the remanufacturability R by coordinating a manufacturer and a retailer under the government penalty and subsidy system.

4.4.2 Comparison of Price

In this section, we will show how the CCG model impacts on the price by comparing each optimal price. We will compare p_N^{MRG*} with p_N^{CCG*} .

The optimal price in MRG and CCG model are summarized in Table 11.

Table 11. The optimal price in MRG and CCG model

p_N^{MRG*}	$\frac{\gamma(\beta - \alpha\gamma)(\Delta + \eta)^2 \tau^2 - 2(3\beta - 3\alpha\gamma + \gamma\tau c_c + \gamma c_m)k}{\gamma(\gamma(\Delta + \eta)^2 \tau^2 - 8k)}$
p_N^{CCG*}	$\frac{\gamma(\beta - \alpha\gamma)(\Delta + \eta)^2 \tau^2 - 2(\beta - \alpha\gamma + \gamma\tau c_c + \gamma c_m)k}{\gamma(\gamma(\Delta + \eta)^2 \tau^2 - 4k)}$

The difference of optimal price from MRG to CCG is shown in (4.27)

$$p_N^{MRG*} - p_N^{CCG*} = \left(\frac{8(\beta - \alpha\gamma - \gamma\tau c_c - \gamma c_m)k^2}{\gamma(\gamma^2(\Delta + \eta)^4 \tau^4 + 4k(8k - 3\gamma(\Delta + \eta)^2 \tau^2))} \right) \quad (4.27)$$

The denominator $\gamma(\gamma^2(\Delta + \eta)^4 \tau^4 + 4k(8k - 3\gamma(\Delta + \eta)^2 \tau^2))$ in (4.27) is also greater than zero. If $\beta > \gamma(\alpha + \tau c_c + c_m)$, then $p_N^{MRG*} - p_N^{CCG*} > 0$. Under this condition ($\beta > \gamma(\alpha + \tau c_c + c_m)$), the price in MRG model is always greater than the price in CCG model. In other words, we can reduce the retail price p_N by coordinating a manufacturer and a retailer under the government intervention so that the consumers' demand in CCG will be increased.

4.4.3 Comparison of Demand

In this section, we will check if the demand of CCG model is greater than the demand of MRG model. We showed that the retail price p_N in CCG model is less than the retail price in MRG model. It leads to the increased demand quantities in CCG model by the linear demand function ($q_N = \beta - \gamma(p_N + \alpha)$). We will compare q_N^{MRG*} with q_N^{CCG*} .

The optimal demand in MRG and CCG model are summarized in Table 12.

Table 12. The optimal demand in MRG and CCG model

q_N^{MRG*}	$\frac{2(\beta - \alpha\gamma - \gamma(\tau c_c + c_m))k}{8k - \gamma(\Delta + \eta)^2 \tau^2}$
q_N^{CCG*}	$\frac{2(\beta - \alpha\gamma - \gamma(\tau c_c + c_m))k}{4k - \gamma(\Delta + \eta)^2 \tau^2}$

The difference of optimal demand from MRG to CCG is shown in (4.28)

$$q_N^{MRG*} - q_N^{CCG*} = \left(-\frac{8(\beta - \alpha\gamma - \gamma(\tau c_c + c_m))k^2}{(8k - \gamma(\Delta + \eta)^2 \tau^2)(4k - \gamma(\Delta + \eta)^2 \tau^2)} \right) \quad (4.28)$$

The denominator $(8k - \gamma(\Delta + \eta)^2 \tau^2)(4k - \gamma(\Delta + \eta)^2 \tau^2)$ in (4.28) is also greater than zero. If $\beta > \gamma(\alpha + \tau c_c + c_m)$, then $q_N^{MRG*} - q_N^{CCG*} < 0$. Under this condition ($\beta > \gamma(\alpha + \tau c_c + c_m)$), the demand in CCG model is always greater than the demand in MRG model. In other words, we can increase the consumers' demand q_N by coordinating a manufacturer and a retailer under the government intervention.

4.4.4 Comparison of Channel Profit

The CCG model provides a benchmark scenario to compare the MRG model with respect to the closed loop supply chain profits. The benefits to the centrally coordinated

model, in terms of an increased remanufacturability as well as an increased ability to buy the product (greater demand) will lead to the higher profit for CCG model. We will compare

$\pi_C^{MRG^*}$ with $\pi_C^{CCG^*}$.

The optimal channel profit in MRG and CCG model are summarized in Table 13.

Table 13. The optimal channel profit in MRG and CCG model

$\pi_C^{MRG^*}$	$\frac{(\beta - \alpha\gamma - \gamma(\tau c_c + c_m))^2 k (12k - \gamma(\Delta + \eta)^2 \tau^2)}{\gamma(8k - \gamma(\Delta + \eta)^2 \tau^2)^2}$
$\pi_C^{CCG^*}$	$\frac{(\beta - \alpha\gamma - \gamma(\tau c_c + c_m))^2 k}{\gamma(4k - \gamma(\Delta + \eta)^2 \tau^2)}$

The difference of optimal channel profit from MR to CC is shown in (4.29)

$$\pi_C^{MRG^*} - \pi_C^{CCG^*} = \left(\frac{16(\beta - \alpha\gamma - \gamma\tau c_c - \gamma c_m)^2 k^3}{\gamma(\gamma(\Delta + \eta)^2 \tau^2 - 8k)^2 (\gamma(\Delta + \eta)^2 \tau^2 - 4k)} \right) \quad (4.29)$$

The numerator $16(\beta - \alpha\gamma - \gamma\tau c_c - \gamma c_m)^2 k^3$ in (4.29) is greater than zero if k is positive. The denominator $\gamma(\gamma(\Delta + \eta)^2 \tau^2 - 8k)^2 (\gamma(\Delta + \eta)^2 \tau^2 - 4k)$ is less than zero because k was assumed to be sufficiently large so that $\pi_C^{MRG^*} - \pi_C^{CCG^*} < 0$. Thus, the profit in CCG model is always greater than the profit in MRG model.

In conclusion, we can increase the remanufacturability as well as the channel profit by coordinating a manufacturer and a retailer under the government environmental fee consideration (the penalty to consumers and the subsidy to the manufacturer for remanufacturing). Furthermore the decreased retail price in CCG model encourages consumers to buy more products like CC model in the chapter 3.

4.5 Numerical Examples

We will show the numerical examples in both MRG and CCG model to illustrate the impact of the government subsidy and penalty system. In this chapter the numerical example was performed to determine the effects that key parameter had on equilibrium of remanufacturability, wholesale and retail price, and profits. The parameter, which include government subsidy for remanufacturing, leads the discussion for the analysis of MRG and CCG model. The example data are the same as those in the chapter 3 and the examples used for the sensitive analysis are shown in Appendix E.

4.5.1 Variation with Subsidy (η)

The government subsidy η for remanufacturing in both MRG and CCG model was varied 0 to 20 while all other parameters were held constant. Figure 18 shows optimal remanufacturability when the government subsidy η varies. In both MRG and CCG model as η increases, the remanufacturability R increases because η has directly influences on the remanufacturability so that the manufacturer or the central planner will set up R as high. Thus we can observe that the government may prefer to the higher η to increase the remanufacturability which can be considered as environmental improvement. In addition, the difference between the remanufacturability of the MRG and CCG model is the greatest when η is large. We can observe that the difference between R^{MRG^*} and R^{CCG^*} becomes a larger as η increases through Figure 18.

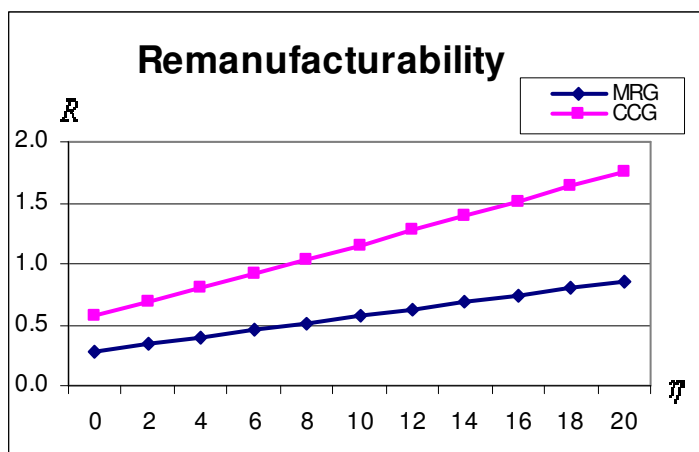


Figure 18. Remanufacturability with variation of η

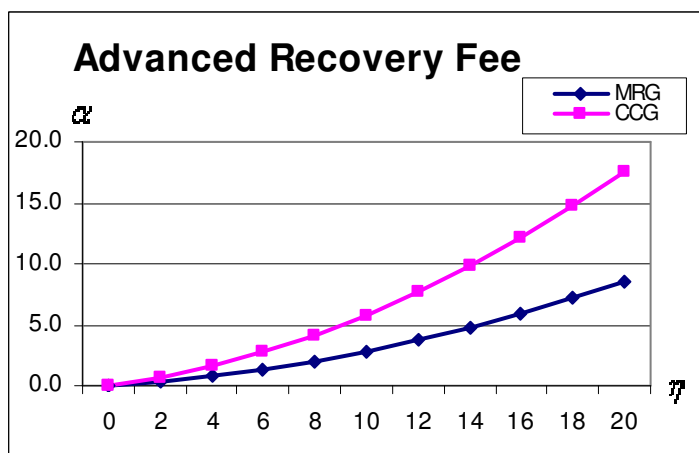


Figure 19. ARF with variation of η

From Assumption 7, we assumed the government revenue neutrality through the penalty and subsidy system. Figure 19 shows the relationship between η and α . For example, we found that the equilibrium value of R is 0.86 given to the collection rate $\tau = 0.5$ in MRG model through the manufacturer profit maximization objective. If the government put this value into the government revenue neutrality constraint from (4.1), we can find $\alpha = 8.59$ when $\eta = 20$.

Figure 20 and 21 show wholesale price and retail price when η varies from 0 to 20 in MRG model. We can observe that the wholesale price decreases as η increases because the

government subsidy for remanufacturing affects to the cost savings so that the manufacturer can reduce the total cost of manufacturing. The retailer also has decreasing retail price as η increases because the increased η leads to the increased α . We can observe that the retailer prefers to reduce the retail price more as α increases. Also, the Figure 21 shows that the difference between $p_N^{MRG^*}$ and $p_N^{CCG^*}$ becomes a larger as η increases.

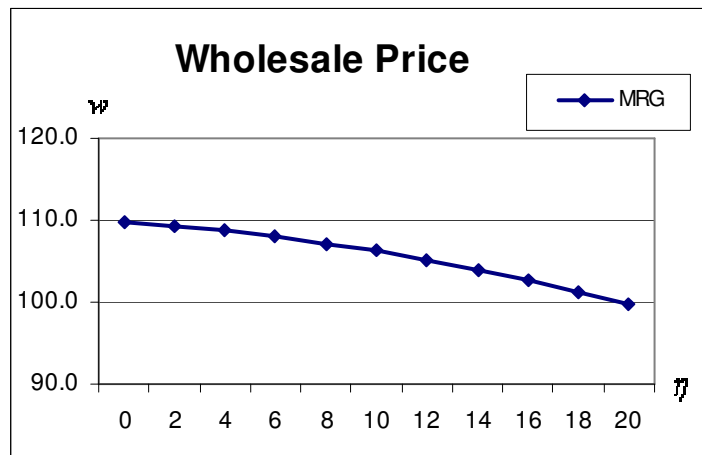


Figure 20. Wholesale price with variation of η

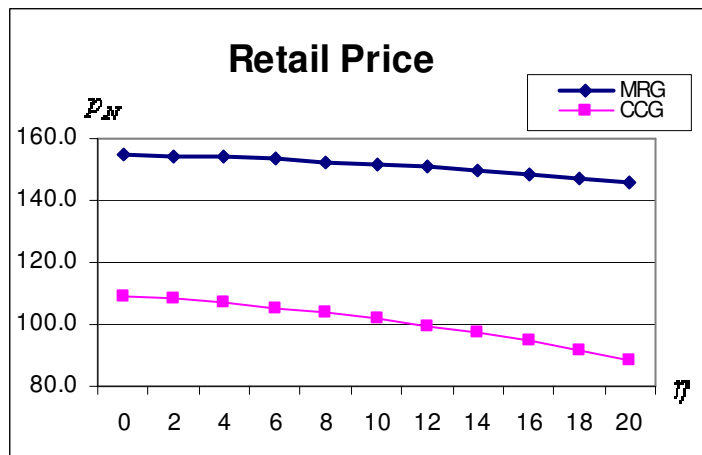


Figure 21. Retail price with variation of η

Figure 22 and 23 show the final retail price $p_N + \alpha$ (retailer's equilibrium price with the advanced recovery fee) and the consumers' demand. It shows interesting results that the

final price decreases even if the government increases α . The reason is that the retailer or the central planner may reduce her price more in order to keep the demand as the penalty to the consumer increases. For this reason, the consumer demand is slightly increased.

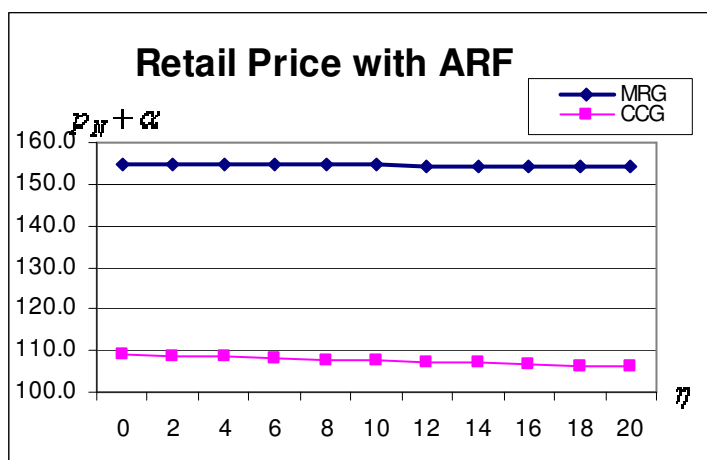


Figure 22. Retail price with ARF with variation of η

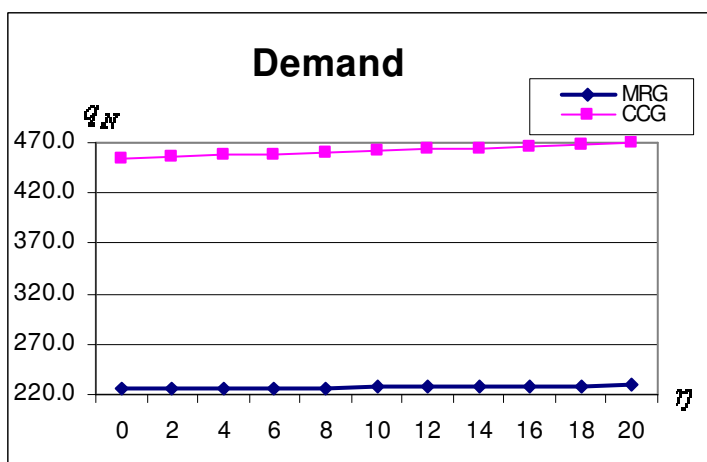


Figure 23. Demand with variation of η

Next we can observe that the manufacturer's profit decreases as η increases in Figure 24. This is opposite result in that the government subsidy for remanufacturing to the manufacturer has a negative impact on the manufacturer's profit because the subsidy comes from the consumers. The primary reason is the government revenue neutrality. In the aspect of manufacturer's profit we can see that the profit loss by the consumer penalty α is greater

than the profit gain by the remanufacturing subsidy η . In contrast to the manufacturer, we can also observe that the profit gain by remanufacturing subsidy is greater than the loss by the consumer penalty so that the retailer earns more money.

We prove that the manufacturer's profit decreases as the government subsidy increases. The manufacturer's profit in MRG model is shown in (4.30).

$$\pi_M^{MRG} = \frac{(\beta - \alpha\gamma - \gamma(\tau c_c + c_m))^2 k}{\gamma(8k - \gamma(\Delta + \eta)^2 \tau^2)} \quad (4.30)$$

From (4.1),

$$\alpha = \frac{\eta(\Delta + \eta)\tau^2(\beta - \gamma\tau c_c - \gamma c_m)}{8k - \gamma\Delta(\Delta + \eta)\tau^2} \quad (4.31)$$

By substituting (4.31) into (4.30), we have

$$\pi_M^{MRG} = \frac{(\beta - \gamma\tau c_c - \gamma c_m)^2 (8k - \gamma(\Delta + \eta)^2 \tau^2) k}{\gamma(\gamma\Delta(\Delta + \eta)\tau^2 - 8k)^2} \quad (4.32)$$

$$\frac{\partial \pi_M^{MRG}}{\partial \eta} = \frac{16\eta\tau^2(\beta - \gamma\tau c_c - \gamma c_m)^2 k^2}{(\gamma\Delta(\Delta + \eta)\tau^2 - 8k)^3} \quad (4.33)$$

From (4.33) we can observe that the manufacturer profit decreases as the government subsidy increases $(\partial \pi_M^{MRG} / \partial \eta) < 0$.

We prove that the retailer's profit increases as the government subsidy increases. The retailer's profit in MRG model is shown in (4.34).

$$\pi_R^{MRG} = \frac{4(\beta - \alpha\gamma - \gamma(\tau c_c + c_m))^2 k^2}{\gamma(8k - \gamma(\Delta + \eta)^2 \tau^2)^2} \quad (4.34)$$

By substituting (4.31) into (4.34), we have

$$\pi_R^{MRG} = \frac{4(\beta - \gamma\tau c_c - \gamma c_m)^2 k^2}{\gamma(\gamma\Delta(\Delta + \eta)\tau^2 - 8k)^2} \quad (4.35)$$

$$\frac{\partial \pi_R^{MRG}}{\partial \eta} = -\frac{8\Delta\tau^2(\beta - \gamma\tau c_c - \gamma c_m)^2 k^2}{(\gamma\Delta(\Delta + \eta)\tau^2 - 8k)^3} \quad (4.36)$$

From (4.36) we can observe that the retailer profit increases as the government subsidy increases $(\partial \pi_R^{MRG} / \partial \eta) > 0$. We provide more numerical examples including specific numbers by comparing manufacturer's profit when $\eta = 0$ with manufacturer's profit when $\eta = 6$ in Appendix C.

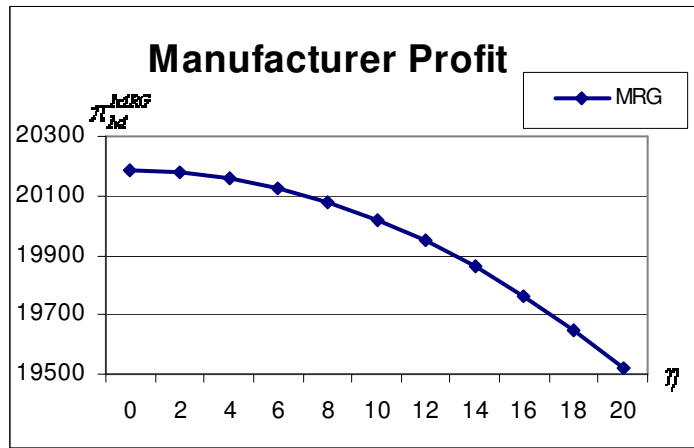


Figure 24. Manufacturer's profit with variation of η

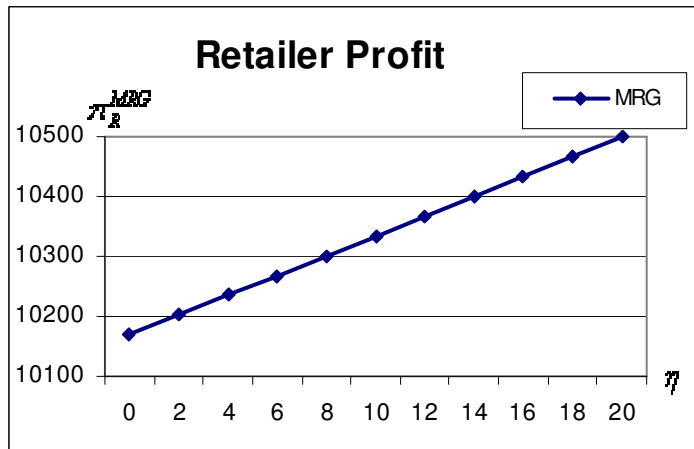


Figure 25. Retailer's profit with variation of η

Figure 26 shows that the channel profit in MRG and CCG model. It is shown that the total channel profit (the central planner profit) in CCG model is always greater than the total channel profit (the manufacturer and retailer profit) in MRG model. Figure 26 shows that the difference between π_C^{MRG*} and π_C^{CCG*} becomes a smaller as η increases.

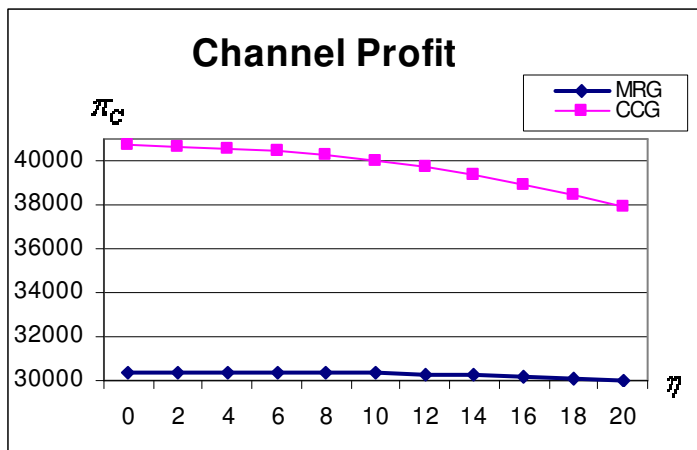


Figure 26. Channel profit with variation of η

As we have observed through this numerical example (variation of η):

- 1) The remanufacturability increases as the government subsidy for remanufacturing increases.
- 2) Under the government penalty and subsidy system, we can increase the remanufacturability as well as the total channel profit by coordinating the manufacturer and the retailer.
- 3) When the government increase η (the subsidy for remanufacturing to the manufacturer), the manufacturer's profit will be decreased due to α (the penalty to the consumers) because the profit loss by α is greater than the profit gain by η . We showed that $(\partial \pi_M^{MRG} / \partial \eta) < 0$.

CHAPTER 5. TOTAL SURPLUS (TS)

In the previous chapter, we formulated and analyzed the manufacturer-retailer with government (MRG) model that the government imposes ARF to the consumers and subsidizes it to the manufacturer for remanufacturing when the manufacturer considers his remanufacturability. In this chapter, we introduce the government's objective such as total surplus maximization. The government in this chapter is considered as policy-maker. The government will take the best response functions of the retailer and manufacturer and find the optimal value of η that maximizes the government's objective function. As a non-profit organization, the government's objective is to maximize total surplus (social welfare) of the members of the supply chain. In order to show the efficiency of total surplus with government's penalty and subsidy, we first derive total surplus without government's penalty and subsidy. The total surplus can be described as a summation of individual surplus (Tomaru 2006, Hinloopen 1997). Additional notations used in this chapter are explained below:

CS^{MR} : The consumer surplus in MR model;

CS^{CC} : The consumer surplus in CC model;

CS^{MRG} : The consumer surplus in MRG model;

CS^{CCG} : The consumer surplus in CCG model;

TS^{MR} : The government's total surplus in MR model;

TS^{CC} : The government's total surplus in CC model;

TS^{MRG} : The government's total surplus in MRG model;

TS^{CCG} : The government's total surplus in CCG model;

5.1 Total Surplus in MR Model

In this section, we will consider the total surplus including the manufacturer, retailer and consumers. Following this we have that

$$TS^{MR} = \pi_R^{MR} + \pi_M^{MR} + CS^{MR} \quad (5.1)$$

In (5.1), CS^{MR} represents the consumer surplus that is defined as the difference between consumer benefit and cost: the difference between the total value consumers receive from consuming a particular product and the total amount they pay for it. So CS^{MR} can be expressed as

$$CS^{MR} = \left(\int_0^q (p) dq \right) - pq = \frac{(q)^2}{2\gamma} = \frac{(q^{MR*})^2}{2\gamma} \quad (5.2)$$

The derivation and graphical expression of consumer surplus in MR and MRG model are given in the Appendix B.

By substituting the q^{MR*} from Table 1 into (5.2), we have

$$CS^{MR} = \frac{2(\beta - \gamma(\tau c_c + c_m))^2 k^2}{\gamma(8k - \gamma\Delta^2\tau^2)^2} \quad (5.3)$$

By substituting the π_R^{MR} , π_M^{MR} , and CS^{MR} into the (5.1), we have the total surplus in MR model.

$$TS^{MR} = \frac{(\beta - \gamma(\tau c_c + c_m))^2 k(14k - \gamma\Delta^2\tau^2)}{\gamma(8k - \gamma\Delta^2\tau^2)^2} \quad (5.4)$$

5.2 Total Surplus in CC Model

In this section, we will consider the total surplus including the central planner and consumers. Following this we have that

$$TS^{CC} = \pi_c^{CC} + CS^{CC} \quad (5.5)$$

The CS^{CC} can be expressed as

$$CS^{CC} = \left(\int_0^q (p) dq \right) - pq = \frac{(q)^2}{2\gamma} = \frac{(q^{CC*})^2}{2\gamma} \quad (5.6)$$

By substituting the q^{CC*} from Table 2 into (5.6), we have

$$CS^{CC} = \frac{2(\beta - \gamma(\tau c_c + c_m))^2 k^2}{\gamma(4k - \gamma\Delta^2\tau^2)^2} \quad (5.7)$$

By substituting the π_c^{CC} and CS^{CC} into the (5.5), we have the total surplus in CC model.

$$TS^{CC} = \frac{(\beta - \gamma(\tau c_c + c_m))^2 k(6k - \gamma\Delta^2\tau^2)}{\gamma(4k - \gamma\Delta^2\tau^2)^2} \quad (5.8)$$

5.3 Total Surplus in MRG Model

In this section, we will consider the government's total surplus to find optimal value of subsidy for remanufacturing that maximizes the objective function in (5.9) in MRG model.

Following this we have that

$$Max_{\eta} TS^{MRG} = \pi_R^{MRG} + \pi_M^{MRG} + CS^{MRG} \quad (5.9)$$

The CS^{MRG} can be expressed as

$$CS^{MRG} = \left(\int_0^{q_N} p_N dq \right) - p_N q_N = \frac{(q_N)^2}{2\gamma} = \frac{(q_N^{MRG*})^2}{2\gamma} \quad (5.10)$$

By substituting the q_N^{MRG*} from Table 8 into (5.10), we have

$$CS^{MRG} = \frac{2(\beta - \alpha\gamma - \gamma(\tau c_c + c_m))^2 k^2}{\gamma(8k - \gamma(\Delta + \eta)^2 \tau^2)^2} \quad (5.11)$$

By substituting the equilibrium solution of π_R^{MRG} , π_M^{MRG} , and CS^{MRG} into (5.9), we can obtain the government's total surplus maximization objective. The government maximization function of TS (Total Surplus) is shown in (5.12).

$$Max_{\eta} TS^{MRG} = \frac{(\beta - \alpha\gamma - \gamma(\tau c_c + c_m))^2 k(14k - \gamma(\Delta + \eta)^2 \tau^2)}{\gamma(8k - \gamma(\Delta + \eta)^2 \tau^2)^2} \quad (5.12)$$

As stated in Assumption 7, we try to see the effect of government intervention where money that is taken at a retail purchase, is returned at remanufacturing to the manufacturer.

Taking that, we notice that the government faces a constraint $\alpha q_N = \eta R \tau q_N$ from (4.1).

By substituting R^{MRG*} from Table 8 into (4.1), we can get the relationship between α and η .

$$\alpha = \frac{\eta(\Delta + \eta)\tau^2(\beta - \gamma\tau c_c - \gamma c_m)}{8k - \gamma\Delta(\Delta + \eta)\tau^2} \quad (5.13)$$

By substituting (5.13) into (5.12),

$$Max_{\eta} TS^{MRG} = \frac{(\beta - \gamma(\tau c_c + c_m))^2 k(14k - \gamma(\Delta + \eta)^2 \tau^2)}{\gamma(8k - \gamma\Delta(\Delta + \eta)^2 \tau^2)^2} \quad (5.14)$$

From (5.14),

$$\frac{\partial^2 TS^{MRG}}{\partial \eta^2} = -\frac{4\tau^2(-\beta + \gamma\tau c_c + \gamma c_m)^2 k^2(32k - \gamma\Delta(13\Delta - 8\eta))}{(\gamma\Delta(\Delta + \eta)\tau^2 - 8k)^4} \quad (5.15)$$

As we assumed that k is sufficiently large so we have $\frac{\partial^2 TS^{MRG}}{\partial \eta^2} < 0$. Therefore TS^{MRG} is a concave function on the subsidy η implying that the government's subsidy is single valued. The first order condition for maximizing the government's objective is given by (5.16).

$$\frac{\partial TS^{MRG}}{\partial \eta} = \frac{4(3\Delta - 4\eta)\tau^2(\beta - \gamma\tau c_c - \gamma c_m)^2 k^2}{(8k - \gamma\Delta(\Delta + \eta)\tau^2)^3} = 0 \quad (5.16)$$

Solving (5.16) the government's optimal subsidy in MRG model is as provided by (5.17).

$$\eta^{MRG*} = \frac{3}{4}\Delta \quad (5.17)$$

By substituting η^{MRG*} from (5.17) into the (5.13), we have α^{MRG*} .

$$\alpha^{MRG*} = \frac{21\Delta^2\tau^2(\beta - \gamma\tau c_c - \gamma c_m)}{4(32k - 7\gamma\Delta^2\tau^2)} \quad (5.18)$$

By substituting η^{MRG*} into the (5.14), we can obtain TS^{MRG*} .

$$TS^{MRG*} = \frac{7(\beta - \gamma\tau c_c - \gamma c_m)^2 k}{\gamma(32k - 7\gamma\Delta^2\tau^2)} \quad (5.19)$$

5.4 Total Surplus in CCG Model

In this section, we will consider the government's total surplus to find optimal value of subsidy for remanufacturing that maximizes the objective function in (5.20) in CCG model.

Following this we have that

$$MaxTS_{\eta}^{CCG} = \pi_c^{CCG} + CS^{CCG} \quad (5.20)$$

The CS^{CCG} in (5.20) can be expressed as

$$CS^{CCG} = \left(\int_0^{q_N} p_N dq \right) - p_N q_N = \frac{(q_N)^2}{2\gamma} = \frac{(q_N^{CCG*})^2}{2\gamma} \quad (5.21)$$

By substituting the q_N^{CCG*} from Table 9 into (5.21), we have

$$CS^{CCG} = \frac{2(\beta - \alpha\gamma - \gamma(\tau c_c + c_m))^2 k^2}{\gamma(4k - \gamma(\Delta + \eta)^2 \tau^2)^2} \quad (5.22)$$

By substituting the equilibrium solution of π_c^{CCG} and CS^{CCG} into (5.20), we can obtain the government's total surplus maximization objective in CCG model. The government maximization function of TS (Total Surplus) is shown in (5.23).

$$MaxTS_{\eta}^{CCG} = \frac{(\beta - \alpha\gamma - \gamma(\tau c_c + c_m))^2 k(6k - \gamma(\Delta + \eta)^2 \tau^2)}{\gamma(4k - \gamma(\Delta + \eta)^2 \tau^2)^2} \quad (5.23)$$

As stated in Assumption 7, we try to see the effect of government intervention where money that is taken at a retail purchase, is returned at remanufacturing to the manufacturer. Taking that, we notice that the government faces a constraint $\alpha q_N = \eta R \tau q_N$ from (4.1).

By substituting R^{CCG*} from Table 9 into (4.1), we can get the relationship between α and η .

$$\alpha = \frac{\eta(\Delta + \eta)\tau^2(\beta - \gamma\tau c_c - \gamma c_m)}{4k - \gamma\Delta(\Delta + \eta)\tau^2} \quad (5.24)$$

By substituting (5.24) into (5.23),

$$MaxTS_{\eta}^{CCG} = \frac{(\beta - \gamma(\tau c_c + c_m))^2 k(6k - \gamma(\Delta + \eta)^2 \tau^2)}{\gamma(4k - \gamma\Delta(\Delta + \eta)^2 \tau^2)^2} \quad (5.25)$$

From (5.25),

$$\frac{\partial^2 TS^{CCG}}{\partial \eta^2} = -\frac{4\tau^2(-\beta + \gamma\tau c_c + \gamma c_m)^2 k^2 (8k - \gamma\Delta(5\Delta - 4\eta))}{(\gamma\Delta(\Delta + \eta)\tau^2 - 4k)^4} \quad (5.26)$$

As we assumed that k is sufficiently large so we have $\frac{\partial^2 TS^{CCG}}{\partial \eta^2} < 0$. Therefore

TS^{CCG} is a concave function on the subsidy η implying that the government's subsidy is single valued. The first order condition for maximizing the government's objective is given by (5.27).

$$\frac{\partial TS^{CCG}}{\partial \eta} = \frac{4(\Delta - 2\eta)\tau^2(\beta - \gamma\tau c_c - \gamma c_m)^2 k^2}{(4k - \gamma\Delta(\Delta + \eta)\tau^2)^3} = 0 \quad (5.27)$$

Solving (5.27) the government's optimal subsidy in CCG model is as provided by (5.28).

$$\eta^{CCG*} = \frac{1}{2}\Delta \quad (5.28)$$

By substituting η^{CCG*} from (5.28) into the (5.24), we have α^{CCG*} .

$$\alpha^{CCG*} = \frac{3\Delta^2\tau^2(\beta - \gamma\tau c_c - \gamma c_m)}{16k - 6\gamma\Delta^2\tau^2} \quad (5.29)$$

By substituting η^{CCG*} into the (5.25), we can obtain TS^{CCG*} .

$$TS^{CCG*} = \frac{3(\beta - \gamma\tau c_c - \gamma c_m)^2 k}{\gamma(8k - 3\gamma\Delta^2\tau^2)} \quad (5.30)$$

The TS (Total Surplus) in each model is summarized in Table 14.

Table 14. The optimal TS (Total Surplus)

TS^{MR*}	$\frac{(\beta - \gamma(\tau c_c + c_m))^2 k(14k - \gamma\Delta^2\tau^2)}{\gamma(8k - \gamma\Delta^2\tau^2)^2}$
TS^{CC*}	$\frac{(\beta - \gamma(\tau c_c + c_m))^2 k(6k - \gamma\Delta^2\tau^2)}{\gamma(4k - \gamma\Delta^2\tau^2)^2}$
TS^{MRG*}	$\frac{7(\beta - \gamma\tau c_c - \gamma c_m)^2 k}{\gamma(32k - 7\gamma\Delta^2\tau^2)}$
TS^{CCG*}	$\frac{3(\beta - \gamma\tau c_c - \gamma c_m)^2 k}{\gamma(8k - 3\gamma\Delta^2\tau^2)}$

5.5 Numerical Examples

We will show the numerical examples in both MRG and CCG model to illustrate the total surplus of the government. Also, the government subsidy η for remanufacturing in MRG and CCG model was varied 0 to 20 while all other parameters were held constant. The total surplus in MR and CC has the value where the government's penalty and subsidy are zero. Figure 27 shows the total surplus in MRG model when the government subsidy η varies. We can get the optimal value of η ($\eta^{MRG*} = 7.5$). Figure 28 shows the total surplus in CCG model and we also can get the optimal value of η ($\eta^{CCG*} = 5.0$). Through this numerical example, we can see that the government can improve the total surplus by introducing the optimal value of subsidy and penalty that maximize the total surplus.

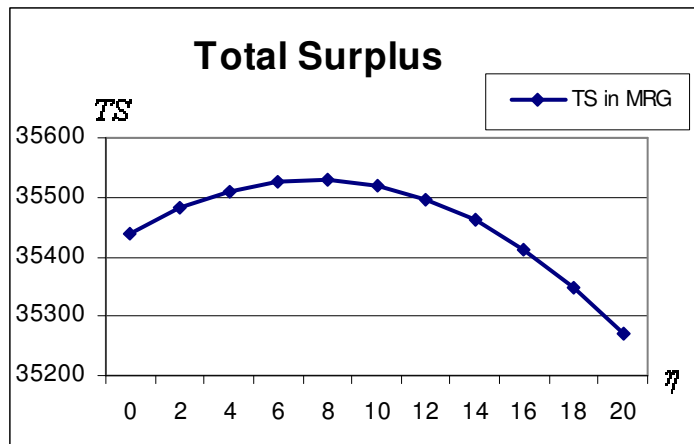


Figure 27. Total surplus with variation of η in MRG

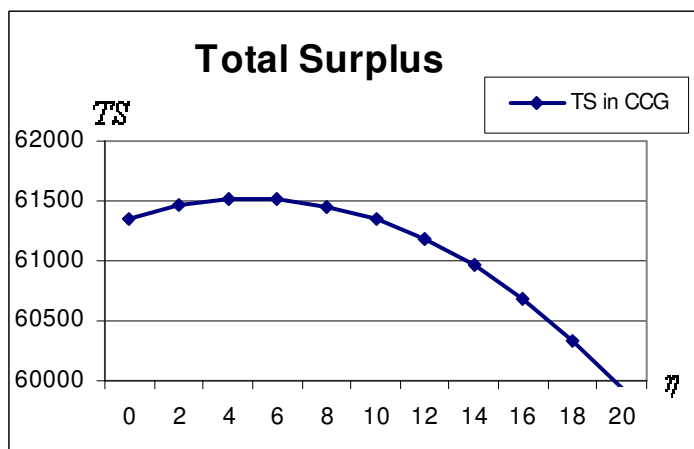


Figure 28. Total surplus with variation of η in CCG

However, we checked that the manufacturer and central planner's profits decrease as the subsidy increases from the Figure 24 and 26. Even if the government subsidizes the fee collected from consumers into the manufacturer or the central planner for remanufacturing, the manufacturer and central planner's profit will be decreased due to the penalty to the consumers. For this reason, we have another research question: how we can increase the manufacturer or central planner's profit to compensate the profit loss by the government penalty and subsidy system?

Table 15. Government's lump-sum transfer money system

MRG Model					CCG Model			
η	π_R^{MRG}	π_M^{MRG}	CS^{MRG}	TS^{MRG}	η	π_C^{MRG}	CS^{CCG}	TS^{CCG}
0	10171.12	20183.31	5085.56	35440.0	0	40686.98	20666.40	61353.4
7.5	10293.22	20092.13	5146.16	35530.6	5.0	40520.33	20998.39	61518.7
	+121.20	-91.17	60.60	+90.63		-166.65	+331.9	+165.33

Table 15 shows the lump-sum transfer money system in both MRG and CCG model.

Andrei (1985) assume that the lump-sum money T can be collected, and that the government does not care about the distribution of income between the firm and consumers so that the lump-sum transfers do not affect welfare. The lump-sum transfer money T in (5.33) and (5.34) does not affect the government's total surplus and the equilibrium of the manufacturer and central planner so that we can increase the manufacturer and central planner's profits to compensate the profit loss by the government penalty and subsidy system

$$\begin{aligned} \underset{w,R}{Max} TS^{MRG} &= (w - c_m + \Delta\tau R - c_c\tau)(\beta - \gamma(p_N(w) + \alpha)) \\ &\quad - kR^2 + \eta R\tau(\beta - \gamma(p_N(w) + \alpha)) + T \end{aligned} \quad (5.33)$$

$$\begin{aligned} \underset{p,R}{Max} TS^{CCG} &= (p_N - c_m + \Delta R\tau - c_c\tau)(\beta - \gamma(p_N + \alpha)) \\ &\quad - kR^2 + \eta R\tau(\beta - \gamma(p_N + \alpha)) + T \end{aligned} \quad (5.34)$$

As we have observed through this numerical example (TS with variation of η):

- 1) When the government increase η (the subsidy for remanufacturing to the manufacturer), the manufacturer and central planner's profit will be decreased due to α (the penalty to the consumers). The government penalty and subsidy system leads to the higher remanufacturability and lower manufacturer and central planner's

profits. Thus the lump-sum transfer money incentive may be provided to the manufacturer or the central planner to increase his profits as well as the remanufacturabilities.

- 2) The government may be able to increase the total surplus consisting of all the profits of the supply chains and the consumer surplus by determining the appropriate level of the fee unit remanufactured subsidy and the fee unit sold advance recovery fee.

CHAPTER 6. CONCLUSION AND FUTURE RESEARCH

6.1 Conclusions

The purpose of this study is to model and analyze the remanufacturability when the manufacturer collects used products directly from consumers in a manufacturer-retailer (MR) closed loop supply chain. We formulated the MR model when the collection rate is a parameter as a Stackelberg game, performed analysis of the equilibrium and compared the MR model equilibrium with the CC model equilibrium. Assuming linear demand function, we found that lack of coordination in the MR model is resulting in a higher price and lower remanufacturability compared to the CC model. We also showed that the remanufacturability increases as cost savings from remanufacturing increases. In addition, we found that the total cost savings from remanufacturing should be greater than the total collection cost and remanufacturability investment cost in order that the remanufacturing is profitable.

Furthermore, based on these frameworks (MR and CC model) we introduce the government penalty to the consumer (ARF) and subsidy to the manufacturer for remanufacturing. We found that the remanufacturability increases as the government subsidy for remanufacturing increases. We showed that under the government revenue neutrality when the government increases the subsidy for remanufacturing to the manufacturer, the manufacturer's profit is decreased due to the penalty to the consumers because the profit loss by the consumer penalty is greater than the profit gain by remanufacturing subsidy. In other words, the remanufacturing subsidy which is subsidized from the consumer penalty, it leads to the higher remanufacturability but lower manufacturer's profit. Thus the lump-sum transfer money incentive may be provided to the manufacturer to increase the manufacturer's

profit. The lump-sum transfer money may compensate the manufacturer's profit loss by the government penalty and subsidy system.

Last, we showed the total surplus that the government finds the optimal value of subsidy and penalty that maximizes the government's objective as total surplus maximization. This paper found that the total surplus without the government environmental legislation is improved by introducing the optimal value of subsidy and penalty.

6.2 Discussion and Future Research

The MR and MRG model can be extended in many ways.

6.2.1 Collection Rate τ as a Decision Variable

The manufacturer can increase the collection rate by investing more in collection, such as advertising or providing higher incentives for each returned product. If we can model the collection cost as a decision variable with a function of collection rate, we would be able to find the optimal collection rate for the manufacturer under which the manufacturer achieves maximized profit. This would give us a more comprehensive understanding of manufacturer's recovery strategies considering both remanufacturability and collection rate.

6.2.2 Difference between New and Remanufactured Product

We assumed that there is no difference between the quality of the manufactured and remanufactured product like single use camera and copy machine in our model. However, in a cell phones, tires, computers, automotive parts market customers can tell the difference between a new product and a remanufactured product (Ferguson and Toktay 2004).

6.2.3 Environmental Fee Consideration (Registration Fee)

In this thesis we address the Advanced Recovery Fee (ARF) and the remanufacturing subsidy by government. However, there exist several different kinds of penalty and subsidy system like Manufacturer Registration Fee in State of Maryland. To be specific, it requires manufacturers to register with the Department of the Environment on and submit an initial \$5,000 registration fee. It applies to manufacturers that manufactured an average of more than 1,000 computers per year in the immediately preceding 3-year period. The registration fee will be paid into the State Recycling Trust Fund. The funds will be used to provide grants to counties to develop and implement local recycling plans. If we can extend our model in such a way that the government imposes penalty to the manufacturer for the product sold, we are able to provide some managerial insights for various government penalty and subsidy systems.

APPENDIX A. DEMAND SHIFT CAUSED BY ARF

Consider the generic equation of the demand line:

$$q = \beta - \gamma p \quad (\beta, \gamma > 0) \quad (\text{A.1})$$

This can be rewritten as inverse demand function:

$$p = \frac{\beta}{\gamma} - \frac{1}{\gamma} q \quad (\text{A.2})$$

which is the line that we graph in the usual diagram with p , the variable on the vertical axis, expressed as a function of q . See Figure 29. Now consider the effect of α (ARF) from consumers to buy this good. This is equivalent to a shift to the left (or downward) of the demand line, exactly equal to the amount of α . That is:

$$p = \left(\frac{\beta}{\gamma} - \alpha\right) - \frac{1}{\gamma} q \quad (\text{A.3})$$

In the usual economics version, equation (A.3) can be rewritten as:

$$q = (\beta - \gamma\alpha) - \gamma p = \beta - \gamma(p + \alpha) \quad (\text{A.4})$$

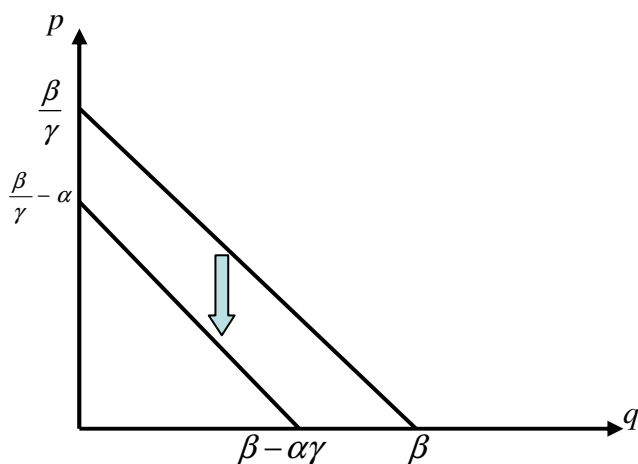


Figure 29. The effect of ARF on the original demand line

APPENDIX B. CONSUMER SURPLUS

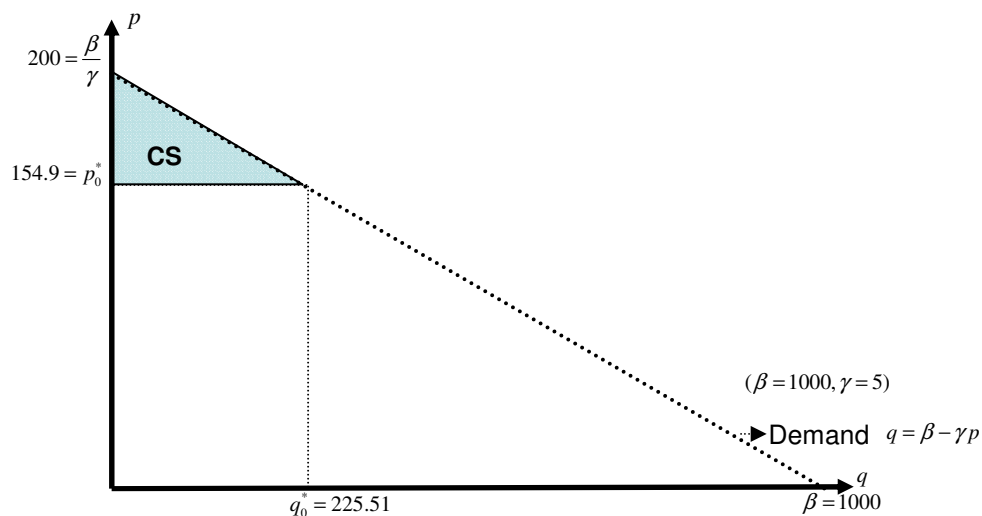


Figure 30. The consumer surplus without government

This is CS (Consumer Surplus) without the government penalty and subsidy. The derivation of consumer surplus in this model is

$$\begin{aligned}
 \text{When } p^* = p_0, \quad q^* = q_0 \quad q_0 &= \beta - \gamma p_0 \quad p_0 = \frac{\beta}{\gamma} - \frac{q_0}{\gamma} \\
 CS &= \frac{1}{2} q_0 \times \left(\frac{\beta}{\gamma} - p_0 \right) = \frac{1}{2} q_0 \times \left(\frac{\beta}{\gamma} - \left(\frac{\beta}{\gamma} - \frac{q_0}{\gamma} \right) \right) = \frac{(q_0)^2}{2\gamma} = \frac{(225.51)^2}{2 \times 5} = 5085.25
 \end{aligned}
 \tag{B.1}$$

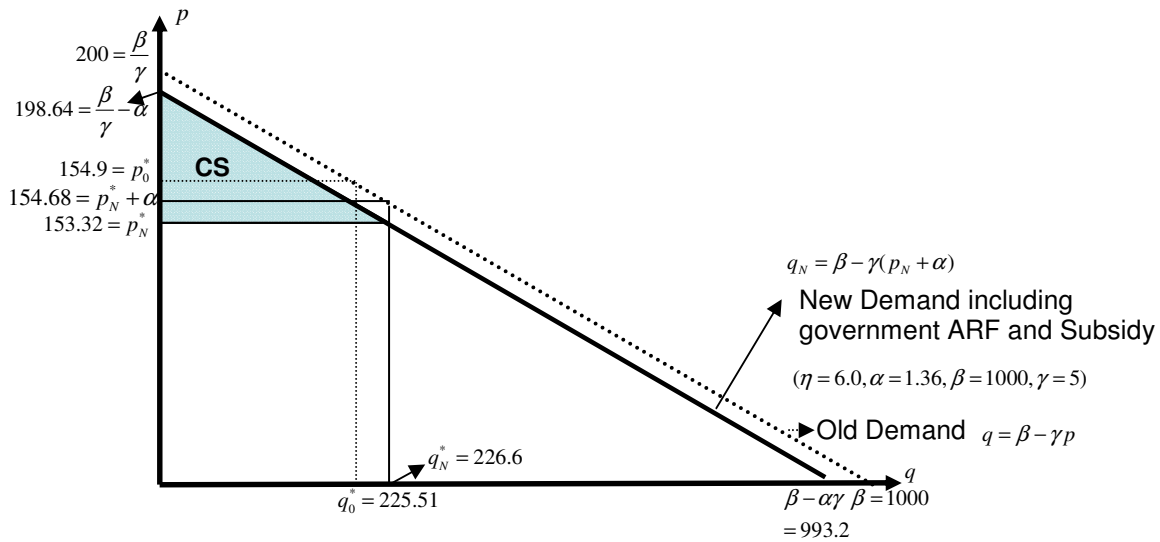


Figure 31. The consumer surplus with government penalty and subsidy

The dotted line is old demand function ($q = \beta - \gamma p$) when there is no government penalty and subsidy system.

The CS in the Figure 31 is Consumer Surplus with the government penalty and subsidy. The derivation of consumer surplus by graphical expression in this model is

$$\begin{aligned}
 CS &= \frac{1}{2} q_N \times \left(\frac{\beta}{\gamma} - \alpha - p_N \right) = \frac{1}{2} q_N \times \left(\frac{\beta}{\gamma} - \alpha - \left(\frac{\beta}{\gamma} - \alpha - \frac{q_N}{\gamma} \right) \right) \\
 &= \frac{(q_N)^2}{2\gamma} = \frac{(226.6)^2}{2 \times 5} = 5134.75
 \end{aligned} \tag{B.2}$$

APPENDIX C. THE EFFECT OF THE SUBSIDY AND PENALTY

Table 16. Manufacturer's profit with both subsidy and penalty

Both Subsidy and Penalty								
η	α	R	w	p_N	$P_N + \alpha$	q_N	π_M^{MRG}	Difference
0	0	0.28	109.80	154.90	154.90	225.51	20183.31	-58.21
6	1.36	0.45	108.01	153.32	154.68	226.58	20125.10	

Table 17. Manufacturer's profit with only subsidy

Only Subsidy								
η	α	R	w	p_N	$P_N + \alpha$	q_N	π_M^{MRG}	Difference
0	0	0.28	109.80	154.90	154.90	225.51	20183.31	+251.00
6	0	0.46	108.70	154.30	154.30	228.30	20434.31	

Table 18. Manufacturer's profit with only penalty

Only Penalty								
η	α	R	w	p_N	$P_N + \alpha$	q_N	π_M^{MRG}	Difference
0	0	0.28	109.80	154.90	154.90	225.51	20183.31	-305.42
0	1.36	0.28	109.10	153.90	155.20	223.80	19877.89	

APPENDIX D. NUMERICAL EXAMPLES: MR AND CC MODEL

Table 19. Numerical example of MR model

β	γ	c_c	c_m	Δ	τ	k	R	w	p	q	π_R^{MR}	π_M^{MR}	π_C^{MR}
1000	5	2	20	5	0.5	2000	0.14	110.3	155.2	224.2	10052.0	20064.8	30116.9
1000	5	2	20	6	0.5	2000	0.17	110.2	155.1	224.4	10069.4	20082.1	30151.5
1000	5	2	20	7	0.5	2000	0.20	110.2	155.1	224.6	10089.9	20102.6	30192.5
1000	5	2	20	8	0.5	2000	0.22	110.1	155.0	224.9	10113.7	20126.3	30240.0
1000	5	2	20	9	0.5	2000	0.25	109.9	155.0	225.2	10140.8	20153.2	30293.9
1000	5	2	20	10	0.5	2000	0.28	109.8	154.9	225.5	10171.1	20183.3	30354.4
1000	5	2	20	11	0.5	2000	0.31	109.6	154.8	225.9	10204.8	20216.7	30421.6
1000	5	2	20	12	0.5	2000	0.34	109.5	154.7	226.3	10242.0	20253.5	30495.4
1000	5	2	20	13	0.5	2000	0.37	109.3	154.7	226.7	10282.5	20293.6	30576.1
1000	5	2	20	14	0.5	2000	0.40	109.1	154.6	227.2	10326.6	20337.0	30663.7
1000	5	2	20	15	0.5	2000	0.43	108.9	154.4	227.8	10374.3	20383.9	30758.3
1000	5	2	20	10	0	2000	0.00	110.0	155.0	225.0	10125.0	20250.0	30375.0
1000	5	2	20	10	0.1	2000	0.06	110.1	155.0	224.8	10108.8	20211.3	30320.2
1000	5	2	20	10	0.2	2000	0.11	110.1	155.0	224.8	10105.3	20185.3	30290.6
1000	5	2	20	10	0.3	2000	0.17	110.0	155.0	224.9	10114.4	20172.0	30286.4
1000	5	2	20	10	0.4	2000	0.23	109.9	155.0	225.1	10136.3	20171.3	30307.6
1000	5	2	20	10	0.5	2000	0.28	109.8	154.9	225.5	10171.1	20183.3	30354.4
1000	5	2	20	10	0.6	2000	0.34	109.6	154.8	226.0	10219.1	20208.2	30427.3
1000	5	2	20	10	0.7	2000	0.40	109.3	154.7	226.7	10280.5	20246.2	30526.8
1000	5	2	20	10	0.8	2000	0.46	109.0	154.5	227.6	10355.9	20297.6	30653.4
1000	5	2	20	10	0.9	2000	0.51	108.6	154.3	228.5	10445.6	20362.4	30808.1
1000	5	2	20	10	1	2000	0.57	108.1	154.1	229.7	10550.3	20441.3	30991.6

Table 20. Numerical example of CC model

β	γ	c_c	c_m	Δ	τ	k	R	p	q	π_C^{CC}
1000	5	2	20	5	0.5	2000	0.28	110.1	449.3	40208.3
1000	5	2	20	6	0.5	2000	0.34	110.0	450.0	40277.8
1000	5	2	20	7	0.5	2000	0.39	109.8	451.0	40360.3
1000	5	2	20	8	0.5	2000	0.45	109.6	452.0	40455.8
1000	5	2	20	9	0.5	2000	0.51	109.4	453.2	40564.6
1000	5	2	20	10	0.5	2000	0.57	109.1	454.6	40687.0
1000	5	2	20	11	0.5	2000	0.63	108.8	456.1	40823.1
1000	5	2	20	12	0.5	2000	0.69	108.4	457.8	40973.1
1000	5	2	20	13	0.5	2000	0.75	108.1	459.6	41137.5
1000	5	2	20	14	0.5	2000	0.81	107.7	461.6	41316.6
1000	5	2	20	15	0.5	2000	0.87	107.2	463.8	41510.6
1000	5	2	20	10	0	2000	0.00	110.0	450.0	40500.0
1000	5	2	20	10	0.1	2000	0.11	110.0	449.8	40435.3
1000	5	2	20	10	0.2	2000	0.23	110.0	450.1	40421.3
1000	5	2	20	10	0.3	2000	0.34	109.8	451.0	40458.0
1000	5	2	20	10	0.4	2000	0.45	109.5	452.5	40546.3
1000	5	2	20	10	0.5	2000	0.57	109.1	454.6	40687.0
1000	5	2	20	10	0.6	2000	0.69	108.5	457.3	40881.6
1000	5	2	20	10	0.7	2000	0.81	107.9	460.6	41132.1
1000	5	2	20	10	0.8	2000	0.93	107.1	464.6	41440.8
1000	5	2	20	10	0.9	2000	1.06	106.1	469.3	41810.7
1000	5	2	20	10	1	2000	1.19	105.1	474.7	42245.3

APPENDIX E. NUMERICAL EXAMPLES: MRG AND CCG MODEL

Table 21. Numerical example of MRG model

β	γ	c_c	c_m	Δ	τ	k	η	α	R	w	p_N	$p_N + \alpha$	q_N	π_R^{MRG}	π_M^{MRG}	π_C^{MRG}
1000	5	2	20	10	0.5	2000	0.0	0.00	0.28	109.8	154.9	154.9	225.5	10171.1	20183.3	30354.4
1000	5	2	20	10	0.5	2000	2.0	0.34	0.34	109.3	154.5	154.8	225.9	10203.2	20176.9	30380.1
1000	5	2	20	10	0.5	2000	4.0	0.79	0.40	108.7	154.0	154.8	226.2	10235.5	20157.5	30393.0
1000	5	2	20	10	0.5	2000	6.0	1.36	0.45	108.0	153.3	154.7	226.6	10267.9	20125.1	30393.0
1000	5	2	20	10	0.5	2000	8.0	2.04	0.51	107.2	152.6	154.6	226.9	10300.5	20079.5	30380.0
1000	5	2	20	10	0.5	2000	10.0	2.84	0.57	106.2	151.7	154.5	227.3	10333.2	20020.6	30353.8
1000	5	2	20	10	0.5	2000	12.0	3.76	0.63	105.2	150.7	154.5	227.7	10366.1	19948.2	30314.3
1000	5	2	20	10	0.5	2000	14.0	4.79	0.68	104.0	149.6	154.4	228.0	10399.1	19862.3	30261.4
1000	5	2	20	10	0.5	2000	16.0	5.94	0.74	102.7	148.4	154.3	228.4	10432.3	19762.7	30195.0
1000	5	2	20	10	0.5	2000	18.0	7.21	0.80	101.3	147.0	154.2	228.8	10465.7	19649.3	30115.0
1000	5	2	20	10	0.5	2000	20.0	8.59	0.86	99.8	145.6	154.2	229.1	10499.2	19521.9	30021.1

Table 22. Numerical example of CCG model

β	γ	c_c	c_m	Δ	τ	k	η	α	R	P_N	$P_N + \alpha$	q_N	π_C^{CCG}
1000	5	2	20	10	0.5	2000	0.0	0.00	0.57	109.08	109.08	454.6	40687.0
1000	5	2	20	10	0.5	2000	2.0	0.68	0.68	108.11	108.79	456.1	40660.6
1000	5	2	20	10	0.5	2000	4.0	1.60	0.80	106.90	108.50	457.5	40580.7
1000	5	2	20	10	0.5	2000	6.0	2.75	0.92	105.45	108.21	459.0	40446.2
1000	5	2	20	10	0.5	2000	8.0	4.14	1.04	103.77	107.91	460.5	40256.2
1000	5	2	20	10	0.5	2000	10.0	5.77	1.15	101.84	107.61	461.9	40009.6
1000	5	2	20	10	0.5	2000	12.0	7.65	1.27	99.67	107.31	463.4	39705.2
1000	5	2	20	10	0.5	2000	14.0	9.76	1.39	97.25	107.01	464.9	39342.0
1000	5	2	20	10	0.5	2000	16.0	12.13	1.52	94.58	106.71	466.4	38918.8
1000	5	2	20	10	0.5	2000	18.0	14.74	1.64	91.66	106.41	468.0	38434.4
1000	5	2	20	10	0.5	2000	20.0	17.61	1.76	88.49	106.10	469.5	37887.8

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