EDDY CURRENT DETECTION OF SHORT CRACKS IN SHEETS

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INTRODUCTION

Eddy-current NDI of through-thickness cracks in plates is characterized by a number of competing length scales. In the case of a thin plate, i.e. plate thickness $h \ll$ skin-depth δ, there are three important length parameters: the crack length $2c$, the average coil radius $\bar{r}$ (or some other measure of probe size) and the intrinsic length scale for induction in thin plates [1], $\beta^{-1} = \delta^2/h$. In this paper we consider the case where $c \ll \beta^{-1}$, which we term the 'short' crack limit, and derive a closed form expression for the change in coil impedance ΔZ due to the crack using the first two terms of an asymptotic expansion for $\beta c \ll 1$. The short crack limit defined here applies irrespective of the probe size, and can be satisfied even if the crack is long relative to the skin-depth $c \ll \delta$ provided that h/δ is sufficiently small. This represents a definite advance over the usual small defect theory [2] which requires $c \ll \delta$ and $c \ll \bar{r}$ as well as the thin plate condition $h \ll \delta$.

THIN-PLATE THEORY

The interaction of induced currents with cracks in thin plates has been studied at several levels of approximation [3-6]. The present analysis is based on the thin-plate theory recently proposed by Burke and Rose [1]. According to this model, the plate is replaced by an equivalent current sheet and the current scattered by the crack is simulated by a distribution of 'sources' along the crack. The sources in this case are generalized current vortices, which are more complicated than conventional hydrodynamic vortices because they must incorporate the feedback between the 2D current flow and the 3D induced magnetic field in order to satisfy Faraday's law.

A typical geometry for eddy-current NDI is shown in Fig. 1, where a probe-coil, carrying a driving current $I \exp(i\omega t)$, is scanned across a conducting plate containing a through crack. Following [1], the coil-impedance change ΔZ due to the crack is given for any coil position by the one-dimensional integral

$$\Delta Z = i\omega/I^2 \int_{crack} ds\ D(s)\Gamma^U(s) \tag{1}$$

involving the vortex density D(s) along the crack and the Hertz potential for the uncracked sheet Γ^u. Here, Γ^u and the surface current density $K^U(x,y)$ in the uncracked sheet are related by

$$\underset{\sim}{K}^U(x,y) = -i\omega\sigma h\ \mathrm{curl}(\hat{z}\Gamma^U) \tag{2}$$

where σ is the conductivity of the sheet and an implicit time dependence of the form $\exp(i\omega t)$ is assumed.

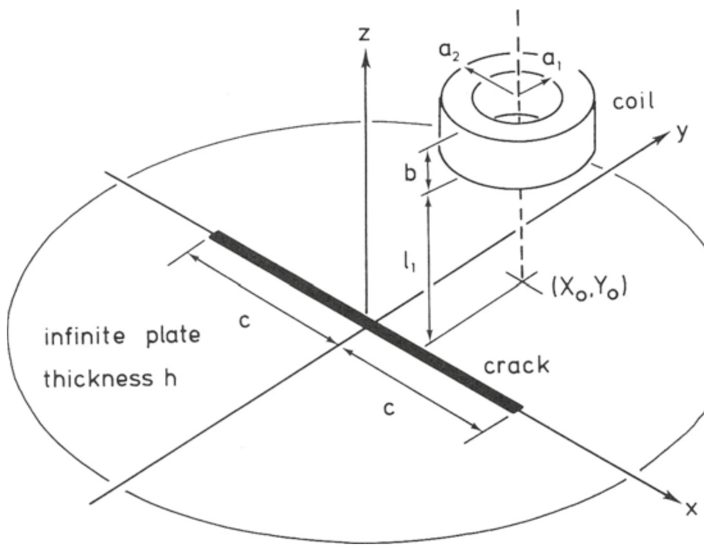

Fig. 1. Eddy-current induction by a coil in an infinite plate containing a straight crack, showing the circular coil configuration used for the experimental measurements of ΔZ and the notation used for the coil parameters.

For an infinite sheet containing a straight crack along the segment $|x| < c$, $y = 0$ of the x-axis, as in Fig. 1, it is convenient to normalize all lengths relative to c. Then, the vortex density D(x/c) can be obtained by solving the following one-dimensional singular integral equation [1] for the range $-1 < u = x/c < 1$:

$$\frac{1}{2\pi}\int_{-1}^{+1} dv\ \frac{D(v)}{u - v} - \frac{i\eta}{2\pi}\int_{-1}^{+1} dv\ D(v)F(u - v) = -K_y^U(cu,0) \tag{3a}$$

where the kernel F has the integral representation

$$F(v) = \int_0^\infty d\xi\ \frac{J_1(\xi v)}{\xi + i\eta},\quad \eta \equiv \beta c \tag{3b}$$

and the normal component of surface current density incident on the crack on the right-hand side of (3) is regarded as known. It can be seen from (3) that η is the basic nondimensional parameter describing the induction process in the cracked plate.

As discussed in [1], it is not possible to solve (3) analytically for arbitrary η and a numerical solution must be obtained using Gauss-Chebyshev quadrature. However, for the short cracks $\eta << 1$ under consideration in this paper, a solution to (3) can be obtained using a regular perturbation expansion, which leads to an asymptotic expansion for ΔZ, correct to order η, derived in the following sections.

CALCULATION OF ΔZ TO ZEROTH ORDER

The calculation of ΔZ to zeroth order in η is presented first. Retaining only the leading term, (3) becomes

$$\frac{1}{2\pi}\int_{-1}^{+1} dv \, \frac{D^0(v)}{u - v} = -K_y^U(cu,0) \tag{4}$$

which can be inverted analytically to give [7]

$$D^0(u) = -\frac{2}{\pi}\int_{-1}^{+1} dv \, \frac{(1 - v^2)^{1/2}}{(1 - u^2)^{1/2}} \frac{K_y^U(cv,0)}{v - u} \tag{5}$$

To evaluate D^0 and the corresponding impedance change ΔZ^0, it is convenient to represent Γ^U and K^U on the interval $-1 < x/c < 1$ using the Chebyshev polynomials T_n and U_n:

$$\Gamma^U(x) = \mu_0 I \sum_{n=1}^{\infty} b_n T_n(x/c) \tag{6a}$$

$$K_y^U(x) = 2i\beta I/c \sum_{n=1}^{\infty} n b_n U_{n-1}(x/c) \tag{6b}$$

where (6b) follows from (6a) using (2) and the relationship $T_n = n\, U_{n-1}$ between Chebyshev polynomials of the first and second kind. Substituting (6b) into (5) and evaluating the (standard) integral, one obtains

$$D^0(u) = 4i\beta I/c \sum_{n=1}^{\infty} n b_n T_n(u)\,(1-u^2)^{-1/2} \tag{7}$$

Substituting (6a) and (7) into (1), and using the orthogonality relationship between Chebyshev polynomials, it follows that

$$\Delta Z^0 = -2\pi\mu_0\omega\beta \sum_{n=1}^{\infty} n b_n^2 \tag{8}$$

Thus the calculation of ΔZ has been reduced to evaluating the coefficients b_n in the Chebyshev expansion of Γ^u for the uncracked plate. For a uniform incident current density, only the term $n = 1$ enters the summation and (8) is equivalent to the result obtained using Burrows' small-defect approximation [1,2]. Seen in this light, (8) could be regarded as an extension of Burrows' procedure to a non-uniform incident field and larger skin-depth.

CALCULATION OF ΔZ TO FIRST ORDER

Building on the results of the previous section, the calculation of ΔZ can now be extended to include terms involving η. Retaining only first-order terms in η, the integral equation (3) can be written in the form

$$\frac{1}{2\pi}\int_{-1}^{+1}dv\,\frac{D(v)}{u-v} = \frac{i\eta}{\pi}\int_{-1}^{+1}dv\,D(v) - K_y^U(cu,0) \tag{9}$$

and a solution $D^1(u)$ correct to $O(\eta)$ can be obtained by the method of successive approximations [8]. Approximating $D(v)$ on the right hand side of (9) by $D^0(v)$ from (7), equation (9) then has the same form as (4) and can be inverted analytically to give

$$D^1(u) = D^0(u) - 8\beta\eta I/(c\pi)^2 \sum_{n=1}^{\infty} b_n \int_{-1}^{+1}dv\,\frac{(1-v^2)}{(1-u^2)^{1/2}}\,\frac{U_{n-1}(v)}{v-u} \tag{10}$$

Substituting (10) into (1) and carrying out the double integration (by changing the order of integration), one obtains the impedance change

$$\Delta Z^1 = -2\pi\mu_0\omega\beta\left(\sum_{n=1}^{\infty} nb_n^2 - 4i\eta/\pi^2 \sum_{n,m=1}^{\infty} b_n b_m F_{nm}\right) \tag{11a}$$

where the combinatorial function F_{nm} is defined as follows,

$$F_{nm} = \begin{cases} 4nm/\{[(m-n)^2-1][m+n)^2-1]\}, & \text{if } |m-n| \text{ is even;} \\ 0, & \text{otherwise.} \end{cases} \tag{11b}$$

As with the zeroth order approximation, the calculation of ΔZ^1 simply requires a calculation of the coefficients b_n in the expansion of Γ^u for the uncracked plate.

RESULTS AND DISCUSSION

The validity of the short-crack approximation was examined by comparing the predictions for ΔZ given by (8) and (11) against both exact calculations [1] and experimental measurements, using the geometry of Figure 1. The results of this comparison are shown in Figures 2 and 3, where the change in coil resistance ΔR and change in coil inductance ΔL are given as a function of coil center position X_0 for scans along two through-cracks in a thin aluminum sheet. In Fig. 2, the crack length $2c = 61$ mm is large compared to the mean coil radius $\bar{r} = 13.7$ mm, while for Fig. 3, the crack length $2c = 10.7$ mm less than $\bar{r}$. The value of the test frequency in each case (958 Hz for the longer crack and 5461 Hz for the shorter crack) was chosen so that η could be fixed at 0.4. For the aluminum sheets used here ($h = 0.100$ mm, $\rho = 2.88$ μΩcm), the skin-depths were 2.76 mm for the calculations involving the longer crack and 1.16 mm for the shorter crack, so that in both cases the skin-depth was small compared with the crack length. These choices of frequency also ensured that the basic thin-plate condition $h/\delta << 1$ was satisfied. The coil consisted of 1910 turns (with $a_1 = 9.33$ mm, $a_2 = 18.04$ mm and $b = 10.05$ mm) and the coil liftoff l_1 was 1.90 mm. The zeroth-order and first-order approximations to ΔZ were calculated by truncating the infinite series in (8) and (11) after 24 terms for $2c = 61$ mm and 4 terms for $2c = 10.7$ mm, giving an estimated accuracy of 0.5%. The exact calculations were performed using the Gauss-Chebyshev procedure outlined in [1] and the experimental values of ΔZ were determined using a low-frequency impedance analyzer (HP-4192A).

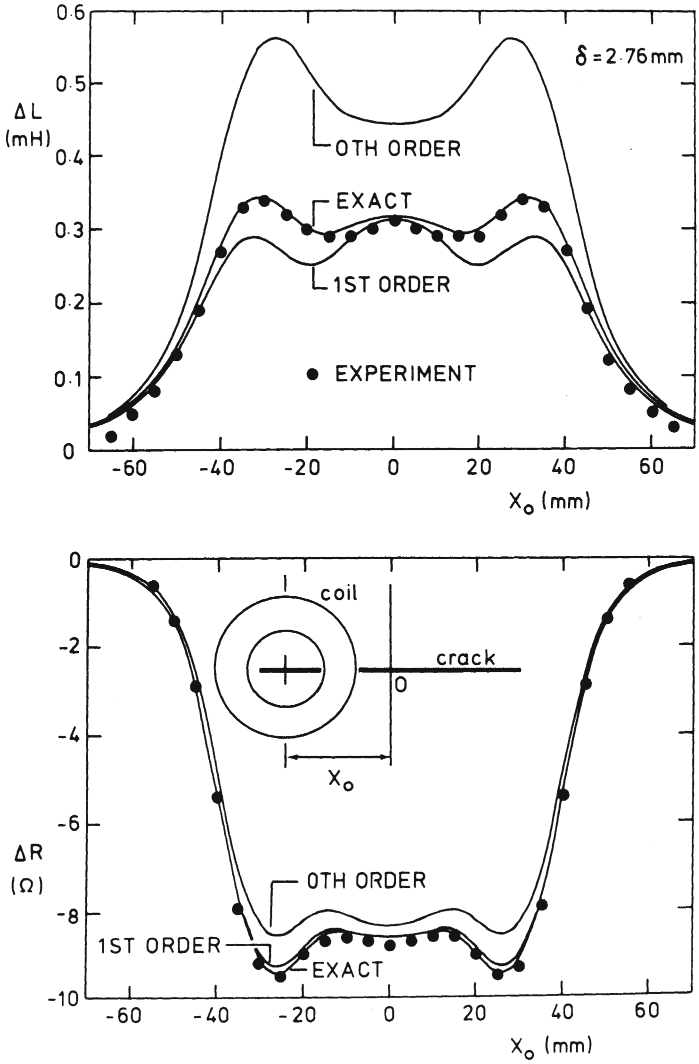

Fig. 2 Change in coil inductance ΔL and resistance ΔR as a function of coil center position X_0 for a crack length 2c = 61 mm and fixed frequency of 958 Hz. Theoretical results obtained using the 0th order approximation (equation 8), the 1st order approximation (equation 11) and the exact theory are shown by the solid curves. The experimental results are shown by the solid circles. Here, δ = 2.76 mm, η = 0.4 and the crack is large compared with the mean coil radius, $c/\bar{r}$ = 2.25.

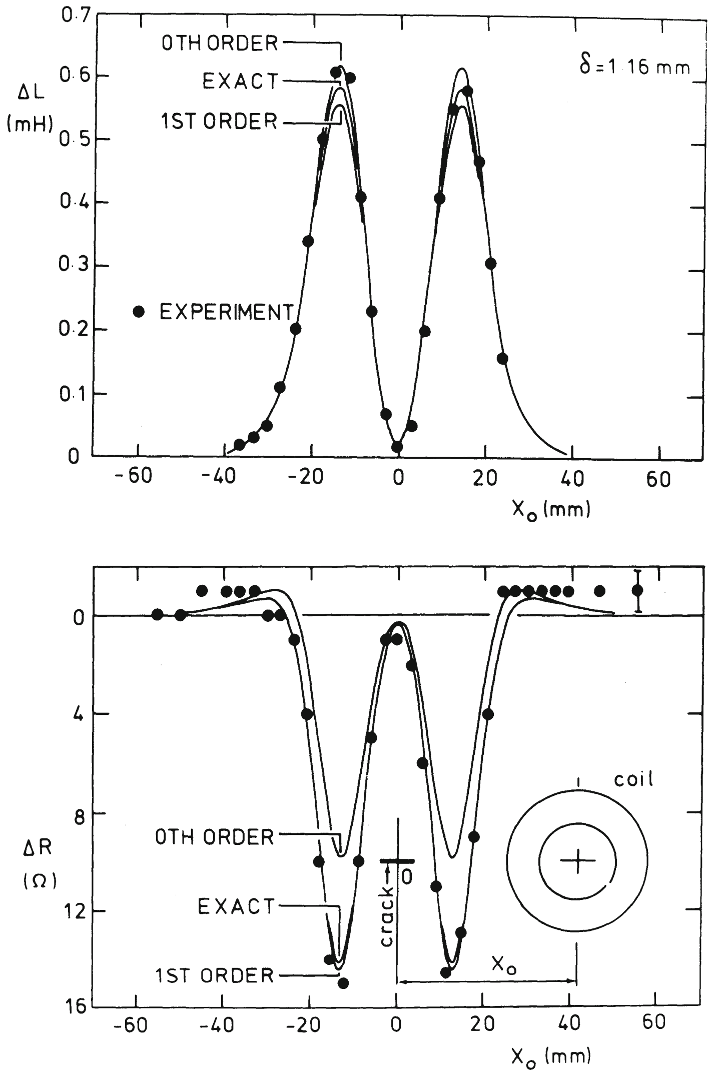

Fig. 3. Change in coil inductance ΔL and resistance ΔR as a function of coil center position X_0 for a crack length 2c = 10.7 mm and fixed frequency of 5461 Hz. Theoretical results obtained using the 0th order approximation (equation 8), the 1st order approximation (equation 11) and the exact theory are shown by the solid curves. The experimental results are shown by the solid circles. Here, δ = 1.16 mm, η = 0.4 and the crack is small compared with the mean coil radius $c/\bar{r}$ = 0.39.

As shown in Figs. 2 and 3, the first-order approximation given by (11) is in good agreement with both the exact theory and experiment. The best agreement is observed for ΔR values, where the first-order approximation agrees with the exact results to better than 2%. The worst discrepancy, of order 15%, is found in the calculation of ΔL when the coil is centered near the tip of the longer crack. Thus, the short crack approximation performs well even at the relatively large value of $\eta = 0.4$ used in these tests. Additional calculations indicate that the equation (11) becomes successively more accurate as η is decreased.

Although these tests were performed using a circular coil, the short crack approximation for ΔZ given by (11) applies to coils of arbitrary geometry and orientation, the details of the coil entering via calculation of the coefficients b_n. Such a general result is rare in eddy-current NDI, even given the restrictions on η and h/δ for which the present approximation is valid.

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