EDDY-CURRENT FLAW RECONSTRUCTION BY INVERTING FIELD DATA

C H Travis, J R Bowler and M C Jones

Department of Physics
University of Surrey
Surrey GU2 5XH,
United Kingdom.

EDDY CURRENT INVERSION

In eddy-current reconstruction the aim is to approach a full characterisation of defects using either probe responses or electromagnetic field data to infer the flaw geometry. The process of inverting the data is inherently ill-conditioned therefore a good deal of care is needed in designing an algorithm giving stable results in the presence of noise[1]. We have developed a new general purpose numerical inversion method, applicable to a wide range of problems in nondestructive evaluation and to eddy-current flaw reconstruction in particular. Here we shall apply the procedure to solve an inverse source problem in eddy-current testing, using limited *a priori* knowledge of the flaw to ensure that the source distribution is uniquely defined.

Suppose we know the magnetic field above the plane surface of a conductor resulting from the scattering of eddy currents by a subsurface crack. If the crack has a negligible opening and acts as a surface barrier to eddy-currents, its effect on the field is the equivalent of a surface layer of current dipoles [2]. Because the dipole moment extends over the whole crack surface and is zero elsewhere, finding the dipole distribution from the scattered magnetic field reveals the size, shape and orientation of the flaw. In general the dipole density associated with a given magnetic field outside the conductor is not unique, therefore one cannot in general, determine the crack geometry from the external magnetic field at a particular frequency, without additional information. However by assuming the crack lies in a known plane, the extent of the dipole distribution in the plane may be found and hence the flaw geometry inferred.

SCALAR POTENTIAL

In formulating the inverse problem we neglect displacement current. The magnetic field in air ($z>0$) scattered by a flaw in the conductor ($z<0$) is expressed in terms of the transverse electric (TE) Hertz potential $W(\bar{r})$ as [3]

$$\bar{H}(\bar{r}) = \nabla\times\nabla\times\bar{a}_z W(\bar{r}), \qquad (1)$$

$\bar{a}_z$ being the unit vector normal to the surface of the conductor. In source free regions the Hertz potential satisfies the Laplace equation. Thus

$$\nabla^2 W(\bar{r}) = 0 . \qquad (2)$$

Assuming the z-component of the magnetic field is known in a plane $z=z_o$ in air ($z_o>0$), then $W(\bar{r})$ may be related to the field sample in this plane through the two-dimensional transform of the z-component of equation (1). Writing the two-dimensional Fourier transform of the potential as

$$\tilde{W}(z) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} W(x,y,z) e^{-(iux+ivy)}\, dxdy, \qquad (3)$$

where the tilde implies a dependence on the Fourier-space variables u and v, we find

$$s^2\, \tilde{W}(z) = \tilde{H}_z(u,v,z_o)\, e^{-s(z-z_o)} . \qquad (4)$$

With s as the positive root of $s=(u^2+v^2)^{1/2}$, the potential vanishes as $z\to\infty$ and (2) is satisfied.

Using the Green's function method we get an integral relationship between the source distribution on the crack and $W(\bar{r})$. For a flaw giving rise to an effective volume source $P(\bar{r})$ in a half space conductor, the TE Hertz potential representing the scattered field is given by

$$W(\bar{r}) = \int_{\text{flaw}} [\nabla'\times\bar{a}_z U(\bar{r},\bar{r}')].\bar{P}(\bar{r}')\, d\bar{r}', \qquad (5)$$

where $\bar{P}(\bar{r})$ is the current dipole density of the flaw and $U(\bar{r},\bar{r}')$ is a half-space Green's function transforming an electric source distribution in the conductor ($z'<0$) into the TE Hertz potential in air ($z>0$). The derivation of the Green's function for a source inside the conductor is similar to the analysis leading to scalar Green's functions for external sources [3] and gives

$$\nabla\times\bar{a}_z\, U(\bar{r},\bar{r}') = \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{i(v\bar{a}_x - u\bar{a}_y)}{s^2(s+\gamma)} .$$

$$e^{-sz+\gamma z'+iu(x-x')+iv(y-y')}\, dudv, \qquad (6)$$

where $\gamma = (u^2 + v^2 - i\omega\mu_o\sigma)^{1/2}$, taking the root with a positive real part.

For the case of a crack of negligible opening in the plane $x=0$, the dipole distribution induced on the flaw has the form [2]

$$\bar{P}(\bar{r}) = p(y,z)\delta(x)\bar{a}_x \qquad (7)$$

where $p(y,z)$ is the surface current dipole density excited on the crack by the induced current in the conductor. With this form for the dipole distribution, the two-dimensional Fourier transform of (5) may be combined with (4) to give

$$\tilde{H}_z(z)= \int \frac{iv}{(s+\gamma)} e^{-sz+\gamma z'}\tilde{p}(v,z')\,dz', \qquad (8)$$

where the integral extends over the flaw z-coordinate.

Given the magnetic field normal to the surface of the conductor in some sample plane $z=z_0$, (8) is used determine an approximation of the dipole density. Note that (7) introduces prior knowledge in that it restricts the distribution we seek to a predefined plane.

NUMERICAL INVERSION

Discretizing equation (8) for K sample points of constant spacing δz, along z' we obtain

$$H_n^{(m)}= \delta z \sum_{k=0}^{K-1} \frac{iv^{(m)}}{s_n^{(m)}+ \gamma_n^{(m)}} \exp\{-s_n^{(m)}z_0+\gamma_n^{(m)}z_k\}\ p(v^{(m)},z_k) \qquad (9)$$

$$m=1,2..M$$

where we have written $H_n^{(m)}$ for $\tilde{H}_z(u_n,v^{(m)},z)$, $\gamma_n^{(m)}$ for $\gamma(u_n,v^{(m)})$ and similarly for $s_n^{(m)}$. (8) is a modified Laplace transform; therefore retrieving the dipole distribution $p(v^{(m)},z^k)$ is essentially a problem of inverting M complex Laplace Transforms (m=1..M). Each inversion consists of finding $\tilde{p}(v^{(m)},z)$ from the values of $\tilde{H}_z(u_n,v^{(m)},z)$ at constant $v^{(m)}$. Referring to Fig. 1 we see how the Fourier transform of the surface potential relates to the dipole distribution across the crack for the m^{th} inversion.

Difficulty will be encountered in inverting the system (9) because of the highly ill-conditioned nature of the matrix kernel and also because we wish to be able to tolerate observational noise in the data. Among the methods that have been used to invert the Laplace transform and other ill-conditioned systems is singular value decomposition. However here we use an alternative method of bidiagonalization previously applied to the reconstruction of bandlimited noisy data in Fourier-space problems[1].

As a consequence of the extreme ill-conditioning of the Laplace transform we need not fully decompose its matrix representation. This is because the elements of the bidiagonalized matrix fall off rapidly down the two non-zero diagonals. Thus only a few components of the bidiagonalized system need be included to get the best reconstruction possible. The remaining components are intractable due to the noise level imposed on the pseudo observations. For a full description of the technique of bidiagonalization see Jones and Travis [1].

RESULTS

In order to test the effectiveness of the inversion algorithm we use the discrete operator of equation (9) to generate pseudo-data from a known source. The inversion algorithm is then applied to the pseudo-data and the results compared with the original source distribution.

Fig. 2a-c shows the real and imaginary parts at each stage of generating the pseudo-data. Fig. 2a shows the dipole distribution on an idealized rectangular crack calculated using boundary integral techniques[2]. Fig. 2b shows the dipole distribution after it has been Fourier transformed with respect to the y-coordinate and 2c shows the two-dimensional Fourier transform of the normal component of the magnetic field found using equation (8).

Various amounts of noise have been added to $\tilde{H}_z$ and then the dipole density reconstructed using the inversion algorithm. Noise levels are

given by the noise variance as a percentage of the variance of the imaginary part of the observation $\tilde{H}_z$. Fig. 2d shows the reconstruction obtained at low noise (10^{-9}%). Fig. 2e shows the same reconstruction after inverse Fourier transforming $\tilde{p}(v,z)$ in the v direction and is to be compared with the original dipole distribution shown in Fig. 2a.

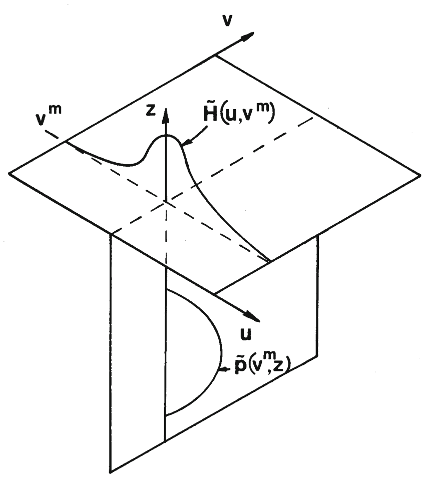


Fig. 1 Schematic relationship between the dipole density and the normal component of the magnetic field.

A comparison of the original dipole density and the reconstruction is shown in Fig. 3 for a number of different noise samples each of 0.5%. The results show that a consistant estimate of $\tilde{p}(v^{(m)},z_k)$ was found for different noise samples having the same variance. Reconstructions in two dimensions with experimental noise levels were carried out to obtain $\tilde{p}_{rec}$ These are built up from the results of 32 inversion. Fig. 4a shows the real and imaginary parts of the pseudo observation $\tilde{H}_z$ (u,v,z) with 0.05% noise. Fig. 4b shows the corresponding reconstruction $p(y,z)_{rec}$ Figs. 4c and 4d show the same for 5.0% noise.

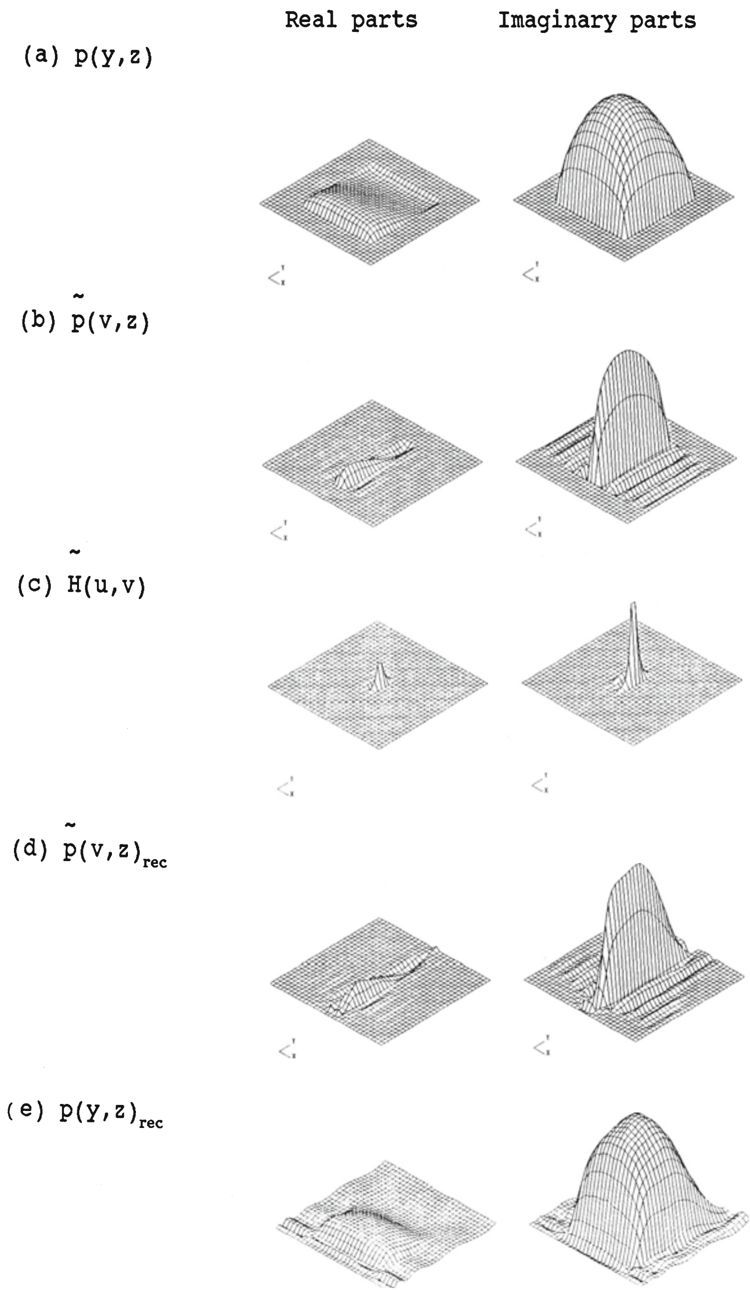
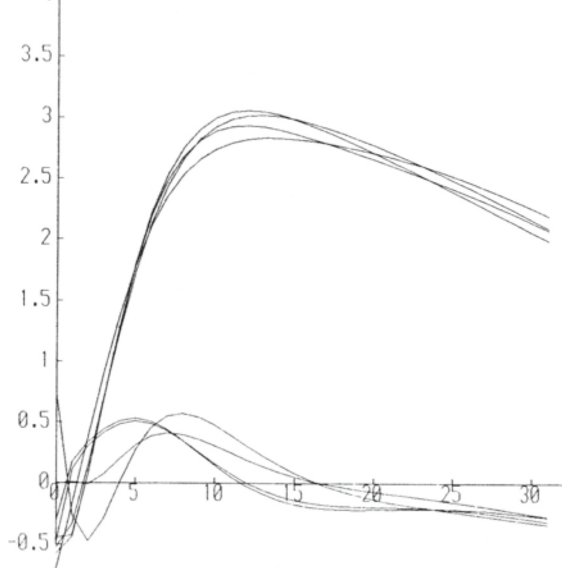


Fig. 2 Generation of pseudo-data and reconstruction with low noise.

a



b

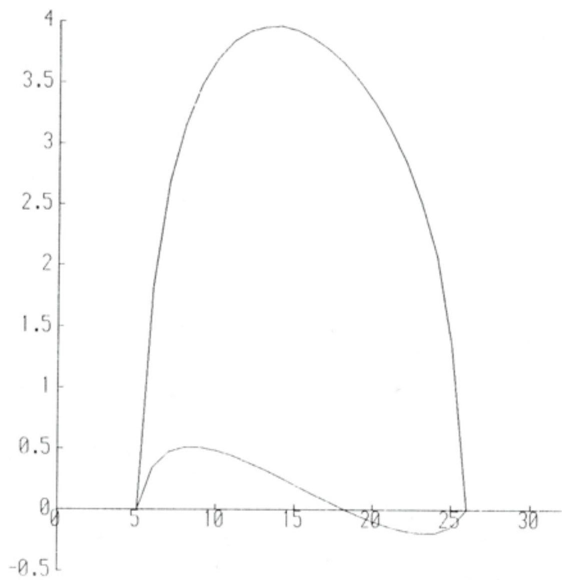
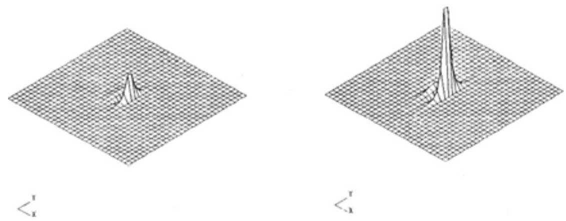


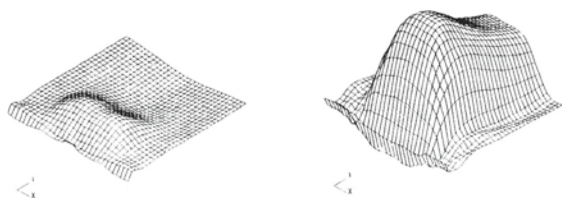
Fig.3 (a) Reconstructions of $\tilde{p}(v^{(16)},z)$ using four different noise samples each of 0.5% (b) The original $\tilde{p}(v^{(16)},z)$

Real parts Imaginary parts

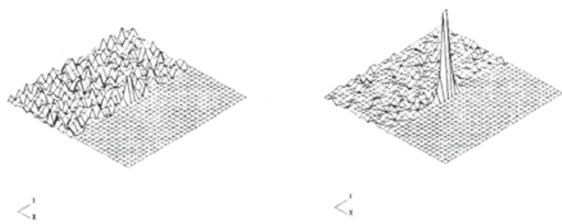
(a) $\tilde{H}(u,v)$ (0.05% noise)



(b) $p(y,z)_{rec}$(0.05% noise)



(c) $\tilde{H}(u,v)$ (5.0% noise)



(d) $p(y,z)_{rec}$ (5.0% noise)

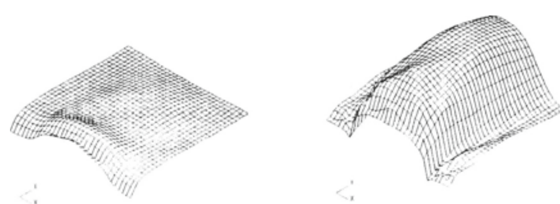


Fig. 4 Pseudo-observation with (a) 0.05% noise and (c) 5.0% noise along with the corresponding reconstructions (b) and (d).

CONCLUSION AND COMMENTS

We have used a new inversion algorithm to determine the effective electrical source strength of a crack in a conductor carrying eddy-currents. The dipole density on a sub-surface crack has been calculated from synthetically generated field data on the assumption that the crack lies in a known plane. At this stage of development no prior knowledge about the noise or signal covariance has been built into the algorithm but it is possible to incorporate this information. The basic technique is quite general and may be extended to determine crack profiles using impedance data.

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REFERENCES

1. M. C Jones and C H Travis, Bidiagonalization for linear inverse problems, J. Op. Soc. Am., **A15**, 660, May 1988.
2. J. R. Bowler, Eddy current calculations using half-space Green's functions, J. Appl. Phys., **61** (3) 833, 1987.
3. J. R. Bowler, Eddy current calculations for a flawed conducting half-space, Review of Progess in Quantitative NDE (Plenum, New York, 1986) Vol.5A pp149-155.