2004

Supply chain management of differentiated agricultural products under imperfect information

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Supply chain management of differentiated agricultural products under imperfect information

by

Miguel Alberto Carriquiry

A dissertation submitted to the graduate faculty
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

Major: Agricultural Economics

Program of Study Committee:
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Ames, Iowa

2004
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ACKNOWLEDGEMENTS

I thank my wife, my daughter and my parents for their unconditional support, and continuous encouragement during my years of graduate school. Thanks go also to my sisters, extended family and friends.

I would also like to thank my major professor, Bruce Babcock, for his immense patience, dedication, and his always helpful suggestions, comments, and advice.

I also thank Alicia Carriquiry, Wolfgang Kliemann and Gabriel Camaño for all their support, and for making themselves available each and every time I needed their help and advice.

Last but not least, I would like to thank Dermot Hayes, Brent Hueth, Helen Jensen, and Michael Larsen for serving in my Committee, and to the faculty and staff of the Center for Agricultural and Rural Development and the Economics Department for the help I received during my graduate studies.
CHAPTER 1. GENERAL INTRODUCTION

1.1 Introduction

The keys to success in commodity agricultural production are market expansion to obtain scale economies, specialization, and adoption of cost reducing technologies. Producers and processors of commodities could only remain viable following the prescribed strategy of producing and marketing their goods as inexpensively as possible. The sustained race for cost reductions led to an adoption spiral of increasingly investment-intensive high-volume production technologies. As an alternative, some market participants not able to cope with the costs of technological upgrade started to look for different production and marketing models. Other entrepreneurs both at the production and processing stages foresaw the potential of marketing differentiated (value added) agricultural products, the most common avenue to success identified by industrial firms (Hennessy, Miranowski, and Babcock, 2004). These factors, coupled with an increase in consumer demand for specialty products, contributed to the tremendous (and sustained) growth of value added agriculture in the past decade.¹

Rising standards of living, and increasing health and environmental awareness are frequently cited as forces driving the demand for a variety of high quality products, highly processed products, and information about on farm activities. New production technologies and information systems available make it possible to supply the differentiated product that consumers are increasingly demanding.

Participants in a supply chain of agricultural value added products face two significant challenges. First, many of the costly distinctive traits being desired by consumers are difficult (if not impossible) to observe even after consumption.² In order for markets for these classes of goods to arise, firms touting the quality of the product need to be trusted. Hence, maintaining an excellent reputation is essential for firms to keep their customers good will. A complicating factor, addressed in this dissertation, is that in some circumstances

¹ For example, Dimitri and Greene (2002) reported that the retail industry for organic food has grown at a rate of 20% per year since 1990.
² That is, they fall into what Nelson (1970) and Darby and Karni labeled as experience and credence attributes. The former refers to attributes that can be observed after consumption (for example toughness of a steak), whereas for the latter consumption does not provide information about the quality of the product (e.g. free range eggs, dolphin safe tuna, organic beef).
delivered quality can only be imperfectly learned and/or affected stochastically by producers. For example, even after using the best genetics available and following best management practices to obtain tender beef, it is possible that the resulting beef is not tender. Also, elevators cannot be absolutely certain that the grain they handle contain the claimed traits. The list of examples is large. Secondly, production is conducted in an environment of yield uncertainty, making it impossible for producers and processors to predict how much of the input (and of what quality) will be available in any given season. Value added products may require the usage of specialized facilities, especially if they perishable, or if they have to follow a segregation protocol (e.g. some specialty grains). At least partial information about the volumes of the product that will be available for processing is needed, if facilities are going to be operated efficiently. The current discussion makes clear that production, processing, and marketing of some value added products require tighter coordination mechanisms than those afforded by open market transactions.3

The food supply chain has reacted to the changes in consumer demand, by beginning to address the challenges just outlined. Contracting has been introduced to govern relationships within a supply chain. This provide a higher degree of coordination between different links of the chain, allowing downstream decision makers to have a more accurate idea of the amount of produce that will be available from their suppliers at any given point in time, which has obvious managerial implications. The choice between contracting and open market transactions has been the focus of much research. Much less is known about the rationale for co-existence of both markets, particularly in agriculture, which is somewhat surprising given the widespread presence of this phenomenon.4 Models where both markets co-exist have been presented for the electricity sector (e.g. Newbery, 1998). In agriculture, the first paper recognizing this co-existence explicitly is due to Xia and Sexton (2004). The prevalence of contracting in some sectors (such as poultry production), and the increase in its relative importance in other areas (e.g. specialty grains), may indicate that once contracts are introduced, eventually all production in that industry will be conducted under these

3 Perhaps not surprisingly, many value added supply chains have chosen contracting as the way to govern their transactions. But see Chapter 2 and the references therein for more reasons advocated in the literature for contract usage.
4 For examples, see Chapter 2.
arrangements. Some worry that if this is true, producers would not have the choice to remain independent, since they will not have access to markets, and will be forced to accept unfavorable contracts. More broadly, if reasonable conditions under which co-existence arises in equilibrium can not be found, co-existence could be seen just as a transition stage between the open market to contracting. In this sense, the steps to be followed by industries where contracting has just begun could be gleaned by observing other sectors more advanced in the transition.

Additionally, some chains expend resources to control and/or learn about the quality of the products they deliver to their customers. In this context, a wide variety of privately and/or publicly developed and run quality assurance systems, with different degrees of stringency and accuracy have been introduced. For example, several dairy companies claiming that their products do not contain recombinant bovine somatotropin (rbST, a genetically engineered hormone used to enhance milk production), rely on information from suppliers about how their milk was produced. There is currently no test able to distinguish rbST milk from milk obtained from rbST treated cows (Carriquiry, Babcock, and Carbone, 2003). Other companies, such as Laura’s Lean Beef or Niman Ranch, have much more stringent quality controls (including site visits and product testing) in place (Carriquiry, Babcock, and Carbone, 2003). The choice and design of the optimal quality assurance system (with its associated stringency and accuracy) is contingent, in general, on the technology available, on the relevant economic environment, and on the objective pursued by the decision maker. In summary, some groups of producers and processors are trying to move away from the commodity paradigm and participate in a market for value added products, facing and addressing the particular challenges presented by this type of markets and supply chains. An analysis of the issues introduced is the focus of this dissertation.

1.2 Dissertation organization

In this section I provide a brief outline of the chapters that follow. Chapter 2 introduces a theoretical model designed to study alternative mechanisms for the procurement of an agricultural product. More specifically, the focus is on the possibility of co-existence of

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This is especially relevant for agricultural industries, where the buyer side is usually concentrated, whereas the supply side (farmers) is atomistic.
two procurement channels, namely contract and spot markets. For the model proposed, co-existence obtains as equilibrium for a wide variety of parameterizations. Hence, the mere introduction of contracting in an industry does not imply that spot markets will eventually disappear in that sector. Further, fundamental economic factors that affect the relative size of each market are analyzed. This provides information to policymakers about how to direct their policies if they decide to affect the market balance and favor one market over the other.

Chapter 3 models the choice of the optimal quality assurance system (QAS) by a supply chain participant (an input processor), providing rationales for the wide variety of QAS observed in the real world. The QASs can be seen as efforts by processors to obtain information and control (stochastically) the quality of the product they deliver. The variables under the control of the processor are the degree of “stringency” or assurance in his/her quality control system over procurement of agricultural output when there exists uncertainty about quality, and the amount of the output to procure, both under a profit maximization goal. Downstream customers choose the amount of product to buy from the processors. Incentives for implementing QASs are given by reputational mechanisms, in the sense that processors will face a positive demand only if a product of substandard quality was not discovered by its customers in the past. The effects of different market structures (monopoly and duopoly), natures of reputations (firm specific and industry-wide), and quality discoverability (from credence to experience attributes) on the choice of the quality assurance system are explored analytically and numerically. Then, firms’ profits and consumer welfare are compared for each scenario. Concluding remarks close the chapter.

Chapter 4 investigates the fundamental economic factors to consider in the development of quality certification programs for red meats. The focus of the chapter is on beef tenderness, but the framework applies broadly. A decision framework is proposed to determine optimal thresholds for certification based on the results of tenderness measurements. That is, a product certification, as opposed to a process certification is analyzed. Identification of thresholds for certification of tenderness has been an elusive task. A review of the meat and animal science literature points to the lack of clearly defined objectives as the source of much confusion. In the framework analyzed, the decision maker receives a signal of the tenderness of a particular beef cut and has to decide whether to certify
it as tender and sell it for a premium, or not to certify it and obtain a commodity price for it. The analysis reveals that the optimal threshold depends on the objective pursued, the distribution of quality available in the cattle population of interest, the distribution of tastes in the consumer target population, the degree of accuracy with which tenderness can be measured, and the value that the decision maker assigns to different costumers. The framework is then put to work, using data on Warner-Bratzler shear force (the most accurate method currently available to measure tenderness) and consumer’s perceptions.

General conclusions and a discussion of possible extensions are provided in the last chapter.
CHAPTER 2. CAN SPOT AND CONTRACT MARKETS CO-EXIST IN AGRICULTURE?

2.1 Introduction

A growing proportion of agricultural production is being raised and sold under contract arrangements. The literature is rich in reasons for the increasing use of contracts (see for example Barkema, 1993; Drabenstott, 1994; Dimitri and Jeaenicke, 2001; Featherstone and Sherrick, 1992; Sykuta and Parcell 2003; Hennessy 1996; Hueth and Ligon 1999; Hennessy and Lawrence 1999). The move to production under contracts has some concerned about the viability of remaining spot markets and about the degree to which farmer welfare is negatively impacted by market power of processors (Smith 2001; Hayenga et al. 2002).

While a lot of research has been devoted to the spot market versus contracts question (starting with the work of Coase [1937]), much less is known about the motivations that lead to the co-existence of both spot and contract markets. It is somewhat surprising that agricultural economists have not explored the rationales for co-existence thoroughly since many important sectors exhibit this feature.\(^1\) The first paper that explicitly models co-existence of spot and contract markets in agriculture is by Xia and Sexton (2004).\(^2\) The point made by these authors is that “top-of-the-market-pricing” (TOMP) clauses in cattle procurement result in reduced competition, when buyers can also influence the base price. The intuition is that as buyers have committed to buy output at a price tied to a spot price to be determined later on, they have incentives to compete less aggressively in that market. In their setting, due to externalities or coordination problems among themselves, sellers can be induced to sign contracts that are not in their collective best interests with little or no financial incentive.

In this paper, we study how fundamental economic factors (prices, production variability, competitive environment, and costs) influence the relative profitability of contract and spot production for farmers and processors. Our objective is to gain insight into how

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\(^1\) Some examples are hogs, corn, soybeans, wheat, and cattle.

\(^2\) For an example of a model where co-existence may result in equilibrium in a non-agricultural setting, see Newbery (1998). This author models the British electricity market focusing on the use of contracts as entry-deterring devices.
these factors help determine the incentives for participation in contract and spot markets. For the sake of concreteness, this article focuses on co-existence of spot and contract markets in the context of specialty grain production. However, the economic forces at work apply broadly (to any industry where the same group of buyers and sellers participate in spot and contract markets, and the buying side is concentrated).

2.2 Specialty grain production and marketing

Increasing proportions of agricultural products being produced and sold under contracts have been reported not only on livestock (Lawrence and Hayenga 2002; Hayenga et al. 2000) and fruits and vegetables, but also on some grains and oilseeds. For example, the National Agricultural Statistics Services (NASS, 2003) reports that 10.4 percent of all U.S. corn production, 8.6 percent of the soybeans, and 4.8 percent of the wheat was sold through marketing contracts in 2001. The value of the three crops marketed under contracts was about $2.25, $1.25, and $0.25 billions for corn, soybeans, and wheat respectively. However, the proportion of specialty grains and oilseeds planted under contracts is much larger. Just to mention a few examples, the U.S. Grain Council reports that about 60-70 percent of waxy corn and 60-65 percent of white corn are grown under contracts, with the remaining area produced speculatively for the spot market. The interest in planting and marketing specialty corn and soybean is growing (Good et al. (2000)). These authors conducted a survey of grain handling firms in Illinois. In 1998, these firms obtained 85 percent, 87 percent, 59 percent, and 96 percent of the volumes handled of high oil, white, yellow food grade, and waxy corn respectively from contracts with farmers. The remaining product was procured mostly through spot markets. For soybeans, the surveyed firms reported that 97 percent, 80 percent, and 85 percent of the volumes handled of STS (Dupont herbicide-tolerant), tofu, and non-GMO soybeans were obtained through contracts with farmers. Thus, the firms surveyed procure most of the input (for both types of crops) through contracts, using existing spot markets as residual suppliers.

Marketing contracts are most often used for procurement (NASS 2003; Boland et al. 1999). The market price plus a premium accounted for 60 percent of the contracts reported in a survey of specialty corn producers conducted by Ginder et al. (2000). A premium is paid over a reference price (yellow no. 2 corn), conditional on the crop meeting certain
quality specifications. Yield drags, higher variable costs of production (specially transportation and handling), and additional management time required are among the reasons that producers command premiums to plant specialty crops (Fulton, Pritchett, and Pederson, 2003).\(^3\)

The situation modeled in this paper is similar to that in Xia and Sexton (2004) in the sense that we model an oligopsonistic industry that contracts with upstream price taking input providers and the market for processed products (downstream) is perfectly competitive. The TOPM clause is also assumed to be used in our model. However, we depart from this setting by modeling a situation where the pricing arrangement embedded in the contract is not affected by the behavior of buyers in the spot market. That is, buyers do not have power in the market to influence the reference price to which the contract price is pegged,\(^4\) although they can influence price in the cash market for the commodity procured. Four other distinctive features of our model are as follows; a) we assume a harsher type of competition (in prices) in the spot market; b) we include uncertainty as a factor inherent in agricultural activity; c) we recognize that at any given point in time or growing season, there is only so much of an homogeneous output coming from a fixed pool of producers (i.e. supply in the cash market is fixed, influenced by decisions made possibly months in advance\(^5\)); and d) processors are constrained in the amount of the agricultural produce they can process at any point in time. There are concessions, of course, that one has to make to model a more realistic setting. First, we lose some tractability, in the sense that we are not able to obtain analytic solutions for some cases. Second, an ad hoc assumption is needed to close the model in one of the possible scenarios (more on this to follow).

The situation we examine is fairly typical in agriculture. Processors offer farmers contracts to purchase all production on a specified number of acres at a price pegged to a yet to be realized market price. In other words, the pricing arrangement embedded in the contract

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\(^3\) Within value enhanced corn, average premiums were highest for white corn and lowest for hard endosperm varieties over the 1996-2001 period (Stewart, 2003).

\(^4\) For example, most specialty grain contracts use commodity prices as references. Good, Bender, and Lowell (2000) found that virtually all specialty grain handlers (corn and soybeans) in Illinois pay premiums based on cash or futures markets for commodities. Thus, it is reasonable to assume they can not manipulate the base price.

\(^5\) This is also recognized by Weninger and Reinhom (n.d.), and by Sexton and Zhang (1996).
is TOMP, or “market price plus a premium”. Each processor has a target amount of production to procure. Procurement in excess of this target—for example, when per-acre yields are very high—can be sold at some salvage price, which is assumed throughout to be the commodity price for the product. When contract supply is low, the processor can turn to the spot market to make up the difference, but only if farmers have planted for the spot market. The price in the spot market clears the ex post (after random yields are realized) processor demand with the fixed supply of the product. Thus, stochastic yields lead to stochastic spot market prices. We account for such economic factors as the price the processor receives for output, salvage values of excess supply for the processor and the farmer, the farm-level cost of production, the number of farmers and processors, and the amount of yield variability. We find that purely financial reasons can explain the preference of contract and/or spot market procurement.

We begin by providing an overview of the problem to be addressed here and then present a formal presentation of a model that captures the profit incentives of processors and farmers. Numerical simulations show how the Nash solution reacts to changes in key economic parameters. These simulation results allow us to determine the key factors affecting the preference of farmers and processors for contract and spot production.

2.3 Overview of the problem

Suppose there are $M$ input buyers (processors) in a geographic region who can enter into grower contracts that specify that the processor will purchase at a guaranteed premium over a yet to be determined market price all the production that comes off of contracted acres. There are $N$ growers. Each of the $M$ processors has a capacity constraint $Q$ that limits the amount of delivered input that can be used. Output technology for the processor is a fixed proportions technology $q = k(A_c y + x_s)$, where $q$ is output, $A_c$ is acreage contracted by each processor, $x_s$ is the amount bought on the spot market, and $y$ is the per-acre yield on all

---

6 Note that we are not making any claim about the efficiency of this contract (observed) relative to other mechanisms. Incentives that shape the form of the contract (e.g., related to quality assurance, etc) are taken as given and not modeled formally.

7 We motivate the target amount of input to procure as given by the capacity constraint. However, that target could also be the result of processor’s commitments with downstream customers. For example, Good, Bender, and Lowell (2000) report that 66 percent of all firms handling white corn in Illinois contract with sellers based on acres, and 61 percent contract with buyers based on bushels. These percentages are 80 percent and 48 percent for waxy corn, and 87 percent, and 47 percent for STS soybeans.
spot and contract acres in a given year. At no additional cost of generality, the conversion factor between raw input and output $k$ is set to one (by choosing units of measurement appropriately). A problem arises because per-acre yield on the contract acres is stochastic so that the total quantity of produced input from contracted acres will vary from the amount needed by the processor to achieve capacity production. Any excess production can be sold by the processor for a salvage price. Farmers have the option to plant additional acres without a contract. The amount of production depends on the expected spot price they will receive.

The $ex post$ input price in the spot market equals the commodity price if there is excess supply of the input. If, $ex post$, there is excess demand of the input in the spot market, then the spot price of the input (given that there are sufficient buyers) will be bid up to the point where profits for the processors equal zero. Under certain excess demand conditions, there is no equilibrium price that can be obtained. That is, there is no general solution to the problem of a limited number of buyers bidding for a fixed supply of the input. To get around this problem, we assume, for now, that the spot price in such excess demand conditions is midway between the excess demand and the excess supply prices.

The processor offers to identical risk-neutral farmers a premium, $\delta$, and a total number of contract acres, $A_c$. If this contract price exceeds or meets the opportunity cost of land, then the processor will find an excess demand for the contract acres and will not have a problem finding takers for the contracts. We assume throughout that farmers will not plant for the spot market if they know they will be obtaining the commodity price for their output. The processor’s capacity constraint and the number of contract acres determines the probability that production will be less than that needed to run at capacity. This creates the possibility that the processor will find it profitable to buy in the spot market. If this probability is high enough, this creates an incentive for farmers to produce for the spot market.

The processor can control the profitability of farmers growing for the spot market through the choice of $A_c$. That is, increases in $A_c$ decrease the profitability because there

---

8 Sexton (2000) discusses the fixed-proportions assumption in models of agricultural markets.
9 This is consistent with the survey findings previously reported.
will be less total spot demand. There is a spot market supply curve $A_i = g(A_c)$ with $g_{A_i} < 0$ that captures farmers’ willingness to plant for the spot market.

Then, each processor $i$ has a demand for contract acreage function that results from the profit-maximization problem that depends on the number of spot acres and the number of acres contracted by its rivals: $A_{c,i} = h\left(A_{c,-i}, A_s\right)$. Vector $A_{c,-i}$ contains the acreage contracted by other processors in the industry. Presumably, $h_{A_{c,i}} < 0; h_{A_s} < 0$ because an increase in the demand for contracted acres by the processor’s rivals increases the premium required to entice farmers to take contracts, and increases in spot market acreage decrease the payoff from additional contract acres. All of the above is common knowledge for both farmers and processors.

Farmers and processors face a two-stage optimization problem. In the first stage processors decide simultaneously and independently the premium to pay for the input and how many contract acres to offer. The farmer’s choices are whether to take the contracts offered and how many acres to plant for the spot market. Contracts cannot be reoffered. Both decision makers face spot price uncertainty caused by supply uncertainty from random yields. They would also face uncertainty in the contract price, since the premium is tied to a yet to be determined cash price, and premiums usually reflect quality differences. To keep things simple, however, we assume that the reference price and quality are fixed at their expected value. In the second stage, processors compete to buy the input they need if it turns out that the contracted input is not enough to work at capacity. The optimization problem is solved using backward induction. The next section formalizes the problem described so far. First, we analyze the optimization problem of processors; then, we analyze the problem farmers face.

### 2.4 The model

#### 2.4.1 The processor problem

The second stage of the processor problem occurs after harvest, so yield uncertainty has been resolved. *Ex post*, processors face a perfectly inelastic supply of the homogeneous input, given by the total acres planted for the spot market multiplied by its yield.
After observing yield \( y \), processors decide whether or not to try to buy more input. The second-stage (ex post) demand for each processor is \( x_s = (Q - A_c y) \),\(^{10}\) which can be negative if it turns out that the processors get more input than they need.

The price in the spot market, and hence the allocation of the rent among farmers and processors, will depend on the ex post relative “bargaining” power. Spot price is determined by demand and supply in the spot market, both of which are determined by planting and contracting decisions made in the first period. In this stage, processors bid simultaneously and independently to purchase the amount of input they need (if any) to work at capacity. Processors are engaging in a Bertrand type of competition with each other. After seeing the bids of each processor, farmers decide whether to sell their production to one of the processors (and to which one) or not to sell at all and obtain the salvage value for the production. For the allocation of the input, we assume that farmers will sell it first to the processor offering the highest price. This processor is able to buy all the input he or she needs (if less than the aggregate supply); then, farmers offer the input remaining to the second-highest bid, and so on. In case of a tie, the input is distributed evenly across processors (in the cases where there is excess demand). In short, we are using a “highest offer first” allocation rule (see Weninger and Reinhorn, and footnote 18).

Ex post, processors will find themselves in one of four possible scenarios regarding their demand for additional production. The first situation examined is when yields are high enough so that processors exceed their capacity constraint with contracted production. This occurs when \( y \geq \frac{Q}{A_c} \). Any surplus production on contract acres will be sold at the going salvage or commodity price. In this case, profits for the processor are

\[
\pi_1 = \left( p - (r_i + \delta) \right) A_c y - r_i (Q - A_c y),
\]

or

\[
\pi_1 = \left( p - (r_i + \delta) \right) A_c y + (p - r_i) (Q - A_c y),
\]

---

\(^{10}\) The resulting demand is rectangular. Processors will demand \( x_s \) as long as the spot price does not exceed their marginal valuation of the input.
where $p$ is the price of output (net of processing costs)$^{11}$, and $r_i + \delta$ is the per-unit price of contracted acreage. Here $x_i = (Q - A_i, y) \leq 0$, and the firm is getting some money back for the excess input. Note, however, that processors lose money on each bushel contracted in excess of their target. They would have contracted less acres, had they known that such a realization of $y$ was going to occur. This situation happens with probability 

$$Pr(y \geq \frac{Q}{A_c}) = Pr(y \geq v) = 1 - F(v),$$

where $v = \frac{Q}{A_c}$, and $F(\cdot)$ is the cumulative distribution function of yield.

The second scenario occurs when ex post demand by processors is positive but there is still excess supply in aggregate. That is, $\sum_{i=1}^{M} (Q - A_i, y) \leq \bar{A}_s, y$, where $\bar{A}_s$ is total acreage planted for the spot market by all farmers. Thus $\bar{A}_s, y$ is ex post aggregate, fixed supply in the spot market. The range of yields for this situation is given by

$$\frac{MQ}{(\bar{A}_s + \sum_{i=1}^{M} A_i)} \leq y \leq \frac{Q}{A_c}.$$

Because there is still aggregate excess supply, an offer by processors of $r_s = r_i$ for spot production (where $r_i$ is the salvage value for spot production) constitutes a pure strategy Nash equilibrium$^{12}$. This is what Sexton and Zhang (1996) observed in the market for California iceberg lettuce. $^{13}$ Processors do not need to bid the price up to get all the input they need to work at capacity.

Again processors can work at capacity, and profits are

$$\pi_z = p Q - (r_i + \delta) A_i, y - r_i (Q - A_i, y),$$

$^{11}$ $p = P - C$, where $P$ denotes the output price (taken as given by the processor) and $C$ represents the per-unit processing costs.

$^{12}$ One could argue that there is not much strategic interaction in this scenario and that simple supply and demand analysis would yield the same outcome, since it is optimal for a processor to bid the reservation price of farmers no matter what other processors do. However, we can also say that bidding $r_i$ is a dominant strategy for all processors and we can analyze all scenarios using the same framework. This situation would be representative of what happened on the 1999 crop year for white corn. A combination of higher-than-normal yields, combined with a substantial increase in the number of acres planted by speculators (or "wildcatters") led to excess supply conditions (Boland et al., 1999).

$^{13}$ However, Sexton and Zhang's approach differs in that they treated ex post supply and demand as being exogenous.
or

$$
\pi_2 = (p - (r_i + \delta)) A_c y + (p - r_i)(Q - A_c y).
$$

This case happens with probability

$$
\Pr\left(\frac{MQ}{\bar{A}_s + \sum_{i=1}^{M} A_{ci}} \leq y \leq \frac{Q}{A_c}\right) = \Pr(u \leq y \leq v) = F(v) - F(u),
$$

where $v$ is defined as above, and $u = \frac{MQ}{\bar{A}_s + \sum_{i=1}^{M} A_{ci}}$.

The third scenario is when there is excess demand \textit{ex post} but only one processor would not have enough production to run at capacity. One would think that in an excess demand condition, processors would bid up the spot price to the point in which all their rents are dissipated. But if only one processor does not have enough input to run at capacity, then there is an incentive for this processor to strategically underbid his or her rivals for the residual supply. This is the case in which a Nash equilibrium in pure strategies fails, in general, to exist.\(^{14}\) Edgeworth cycles may arise in this case (following a loose dynamic argument). For example, the price may rise as the processors try to increase their share in the input market, until the profit of doing so is lower than the one resulting from offering the reservation price of the farmers (or commodity price) and keeping the residual supply. Once the price is at the reservation price of the farmers (or low enough), processors will find it profitable to bid \(c\) above their rivals and work at capacity.

This case implies

$$
\frac{(M-1)Q}{\bar{A}_s + \sum_{i=1}^{M-1} A_{ci}} \leq y \leq \frac{MQ}{\bar{A}_s + \sum_{i=1}^{M} A_{ci}}.
$$

Because we are assuming a symmetric solution in which processors get an even distribution of the fixed supply (i.e., each processor is able to buy \(\frac{A_c y}{M}\) in the spot market), processors will not be able to work at capacity. Profits in this case are

\(^{14}\) Kreps and Scheinkman (1983), Tasnadi (1999), and Levitan and Shubik (1972) addressed this problem for capacity-constrained duopolists. Weninger and Reinhor (n.d.), studying a more closely related problem (oligopsonists facing a fixed supply), concluded that a pure strategy Nash equilibrium for this problem exists if the number of buyers is sufficiently large and the price space is discrete.
\[ \pi_s = \left( p - (r_i + \delta) \right) A_s y + (p - r_i) \frac{\bar{A}}{M} y, \]

where \( r_i \) is the resulting input price in the spot market.

In this case we cannot know with certainty what \( r_s \) will be. The Nash equilibrium for this situation will be in mixed strategies, implying that the payoff function will not be continuous. To close the model, we set it equal to the average of the marginal valuations of the processors and the farmers. This case happens with probability

\[
\Pr \left( \frac{(M-1)Q}{\bar{A} + \sum_{i=1}^{M-1} A_{ci}} \leq y \leq \frac{MQ}{\bar{A} + \sum_{i=1}^{M-1} A_{ci}} \right) = \Pr \left( s \leq y \leq u \right) = F(u) - F(s),
\]

where \( u \) is defined as above and \( s = \frac{(M-1)Q}{\bar{A} + \sum_{i=1}^{M-1} A_{ci}} \).

The fourth and final scenario is when yield is so low (given the areas contracted and planted for the spot) that at least one processor would be left out of the market if that processor tries to underbid his or her rivals. For example, the second-to-last processor finds some supply but it is not enough to work at capacity. In this case, there is no room for strategic underbidding by the last processor. If the last processor tries to underbid his or her rivals, he or she will be left out of the market. This will happen if \( \sum_{i=1}^{M-1} (Q - A_{ci}) \geq \bar{A}, y \), or, equivalently, \( y \leq \frac{(M-1)Q}{\bar{A} + \sum_{i=1}^{M-1} A_{ci}} \). The pure strategy Nash equilibrium here is for processors to offer their marginal valuation \( (p) \) for the input. Processors will again split the input evenly and will not be able to work at capacity. Profits in this case are

\[ \pi_4 = p A_s y + p \frac{\bar{A}}{M} y - (r_i + \delta) A_s y - p \frac{\bar{A}}{M} = \left( p - (r_i + \delta) \right) A_s y. \]

This will occur with probability

\[
\Pr \left( y \leq \frac{(M-1)Q}{\bar{A} + \sum_{i=1}^{M-1} A_{ci}} \right) = \Pr \left( y \leq s \right) = F(s),
\]

where \( s \) is defined as above.

\(^{15}\) The mixed strategy equilibrium for \( M=2 \) is presented in Appendix A.
2.4.1.1 Processor expected profits

Now we are ready to write the first-stage objective function of a representative processor. Bearing the rules that will arise in the second stage in mind, processors independently and simultaneously make a decision as to how many contracts to offer and price premiums to pay in order to maximize their expected payoff. That is, each processor chooses \( A_{ci} \) and \( \delta \) to maximize its expected profits, which are defined as

\[
E\left( \pi_i \left| A_{ci}, \overline{A}_c \right. \right) = E\left( \left( p - (r_i + \delta) \right) A_{ci} y + \left( p - r_i \right) \left( Q - A_{ci} y \right) \right) \Pr(y \geq u) \\
+ E\left( \left( p - (r_i + \delta) \right) A_{ci} y + \left( p - (p + r_i) / 2 \right) \overline{A}_c y \right) \Pr(s \leq y \leq u) \\
+ E\left( \left( p - (r_i + \delta) \right) A_{ci} y \right) \Pr(y \leq s)
\]

subject to the constraint that the contracts need to be accepted by farmers. This objective function can be rewritten as follows:

\[
E\left( \pi_i \left| A_{ci}, \overline{A}_c \right. \right) = (p - r_i) \left( A_{ci} \int y f(y) + Q(1 - F(u)) + \frac{\overline{A}_c}{2M} \int y dF(y) \right) - \delta A_{ci} E(y)
\] (2.1)

where \( f(y) \), \( 0 \leq y \leq y_m \) is the density function for the random yield and \( E(y) \) is the expected value of the yield. Formally, the problem is

\[
\max_{A_{ci}, \delta \geq 0} E\left( \pi_i \left| A_{ci}, \overline{A}_c \right. \right)
\]

subject to

\[
(r_i + \delta) E(y) \geq C'(a_s + a_c).
\]

The marginal cost for farmers of planting an acre of the specialty crop is \( C'(a_s + a_c) \).

The number of acres planted for the cash market and contract market by a farmer are \( a_s \) and \( a_c \) respectively. The farmer's problem will be presented subsequently. The constraint indicates that the expected marginal revenue of a contracted acre can not be lower than the marginal cost of planting that extra acre. In other words, the premium and number of acres offered to a farmer have to be consistent with his or her supply schedule if the contract is going to be accepted\(^\text{16}\). The constraint has to be binding at any optimum for this problem (if

\(^\text{16}\) Here we are interpreting the supply function as giving the minimum price per unit at which firms are willing to sell any given quantity.)
any positive number of contracts is offered). Otherwise, processors can reduce the financial incentive offered and still entice acceptance of the contracts by farmers. This observation allows us to subsume the constraint into the objective function, and perform the optimization only with respect to the number of contracts offered.

The first-order condition for a maximum is found by differentiating equation (2.1) with respect to $A_c$ using the Leibnitz rule. After imposing symmetry and using some algebra, we get

$$\frac{\partial E \left[ \pi | A_{c-1}, A_i \right]}{\partial A_j} = (p - r_i) \left( \int y dF(y) + \frac{\bar{A}_i}{2M} \left( \frac{f(u)u^2}{M} + \frac{f(s)s^3}{(M-1)} \right) - E(y) \left( \frac{\partial \delta}{\partial A_c} A_c + \delta \right) \right) \leq 0 \quad (2.2)$$

with equality if $A_{ci} > 0$, $i = 1, \ldots, M$. Second order sufficient conditions are presented in Appendix B.

Processors take into account that they can affect the probability of being in each of the situations described. They realize, for example, that if they increase the contracted area, it is less likely that they will have to buy in the spot market. Of course, the magnitude of the marginal effect each processor has decreases as the number of processors increase. Before exploring the ramifications of this assumption, we will first examine the problem of farmers.

### 2.4.2 Farmer decisions

Farmers have rational expectations, share common beliefs, and take contract area and contract price as given in solving their optimization problem. They decide whether to take a processor’s offer of acreage and price and whether to plant for the spot market. If they are indifferent between taking and rejecting the contracts, they are assumed to accept the processors offer. Assume a large number $N$ of identical farmers. The large number assumption is not crucial. The important assumption is that farmers take prices and aggregate acreages as given. Thus, they do not act as if they can affect the probability of ex post spot market demand. Here, $a_{cj}$ is the acreage contracted by farmer $j$, $a_{yj}$ is the area planted for the

---

17 This can be written as $\delta \left( A_{ci} \right) = \frac{C'(a_y + a_c)}{E(y)} - r_i$, with primes indicating first derivatives.

18 These assumptions regarding the farmer’s problem closely follow Newbery (1998). Xia and Sexton (2004), stress the importance of considering rational agents on both sides of the market when modeling procurement of raw product inputs.
spot market by farmer \( j \), and \( C(a_y + a_y) \) is the cost of production. Because it is assumed that farmers are identical, the subscript \( j \) will be dropped. The expected profit of a farmer thus can be written as

\[
E(\pi^F) = E((r_i + \delta) a_y + r_i a_y - C(a_y + a_y)).
\]

Again, four different scenarios may arise in the spot market. These situations are the same as the ones described in the processor problem and for that reason we will only state here what farmers expect, that is, their payoff function and their probability of being in each case. In the first scenario, there is no demand in the spot market. Here, farmers do not receive any bid for their output and therefore the value of the crop equals the commodity price. The profit function for this scenario is

\[
\pi_i^F = (r_i + \delta) a_y + r_i a_y - C(a_y + a_y),
\]

which, as before, occurs with probability \( \Pr\left(y \geq \frac{Q}{A_y}\right) = \Pr(y \geq \nu) = 1 - F(\nu) \).

In the second scenario, there is demand for the input in the spot market. However, aggregate supply exceeds aggregate demand. Profits and probability of occurrence for this scenario are

\[
\pi_2^F = (r_i + \delta) a_y + r_i a_y - C(a_y + a_y)
\]

and

\[
\Pr\left(\frac{MQ}{A_y + \sum_{i=1}^{M} A_{ci}} \leq y \leq \frac{Q}{A_{ci}}\right) = \Pr(u \leq y \leq v) = F(v) - F(u).
\]

The third scenario corresponds to the one where only \((M-1)\) processors can work at capacity. Farmer profits and probability of occurrence for this scenario are

\[
\pi_3^F = (r_i + \delta) a_y + r_i a_y - C(a_y + a_y)
\]

and

\[
\Pr\left(\frac{(M-1)Q}{A_y + \sum_{i=1}^{M-1} A_{ci}} \leq y \leq \frac{MQ}{A_y + \sum_{i=1}^{M} A_{ci}}\right) = \Pr(s \leq y \leq u) = F(u) - F(s).
\]
And the last scenario is as before, where at least one processor would be left out of the market if that processor tries to underbid his or her rivals so that the spot price is bid up to \( \rho \), the processor’s marginal valuation for the input. Profits and probability of occurrence for this case are

\[
\pi^F_s = (r + \delta) a c y + p a c y - C(a_c + a_s)
\]

and

\[
\Pr\left( y \leq \frac{(M-1)Q}{A_s + \sum_{i=1}^{M-1} A_{si}} \right) = \Pr(y \leq s) = F(s).
\]

Now we can write the expected profit of farmers as

\[
E(\pi^F) = (r + \delta) a_c E(y) + a_s \left( \int_{y}^{\infty} r y f(y) dy + \int_{y}^{\infty} y f(y) dy + \int_{y}^{\infty} py dy \right) - C(a_c + a_s) \tag{2.3}
\]

The first-order condition for a maximum is obtained by differentiating equation (2.3) with respect to \( a_s \):

\[
\frac{\partial E(\pi^F)}{\partial a_s} = r \int_{y}^{\infty} y f(y) dy + r_s \int_{y}^{\infty} y f(y) dy + p \int_{y}^{\infty} y f(y) dy - C'(a_c + a_s) \leq 0, \tag{2.4}
\]

with equality if \( a_s > 0 \). Second order conditions trivially hold for any convex cost function.

Note that farmers take the aggregate amount planted for the spot as given. Hence, we do not use the Leibnitz rule here. Farmers do not realize they can change the probability of being in the scenarios described (they take \( A_s \) as given). This is because individual farmer output is assumed to be too small to affect the aggregate spot supply. Recall that farmers are assumed to have rational expectations. Farmers believe the aggregate acreage for the spot will be \( A_s \), and in equilibrium their expectations are realized.

### 2.5 Equilibrium

In this section, we characterize the equilibrium acreages for both the farmer and processor problems. An equilibrium for this model, is conformed by two main components corresponding to the processor’s and farmer’s problem. A pure strategy Nash equilibrium for the processor’s game is a number of contracted acres \( A'_i, i = 1, \ldots, M \) such that no processor can benefit from unilateral deviations, for a given number of acres planted for the spot.
market. That is to say that in equilibrium, a processor cannot increase its profit by unilaterally choosing to contract a different number of acres. Farmers take the number of contract acres and the price offered as given, and, based also on their beliefs regarding the aggregate spot acreage, they choose the number of acres to plant for the spot market in order to maximize profits. It cannot be overemphasized that farmers’ decisions are the result of an optimization problem and do not arise from strategic interactions with processors or other farmers. However, the discussion will proceed as if farmers were “reacting” optimally to processors’ offers, and Nash equilibrium will be used in a loose way to refer to the equilibrium of the model.

Three additional conditions have to hold in equilibrium. First, beliefs regarding aggregate spot acreage are confirmed: \( \bar{A}_s = Na_s \). Second, aggregate demand and supply for contracts are equalized: \( Na_c = MA_c \). Finally, farmers’ profits are nonnegative. In short, any equilibrium has to satisfy equations (2.5) and (2.6), which obtain from imposing the equilibrium conditions to equations (2.2) and (2.4), respectively:

\[
(p-r_i)
\left(\int ydF(y) + \frac{\bar{A}_s}{2MQ}\left(\frac{f(u)u^3}{M} + \frac{f(s)s^3}{(M-1)}\right) - E(y)\left(\frac{\partial \delta \cdot \partial a_c \cdot \partial A_{ci} + \delta}{\partial A_{ci}}\right)\right) \leq 0, 
\]

with equality if \( A_c > 0 \);

\[
r_i \int yf(y)dy + r_i \int yf(y)dy + p \int yf(y)dy - C^1\left(\frac{A_c}{N} + a_s\right) \leq 0, 
\]

with equality if \( a_s > 0 \), for \( v = \frac{Q}{A_c} \), \( u = \frac{MQ}{(Na_c + MA_c)} \), \( s = \frac{(M-1)Q}{(Na_c + (M-1)A_c)} \), and equation (2.3) \( \geq 0 \).

The equilibrium for a particular environment can be predicted using of the best-response functions of processors and farmers, which are given \( A_{ij} = h(\bar{A}_i) \) and \( a_{ij} = z(A_i) \) (for \( z(A_c) = g(A_c)/N \)) respectively, previously introduced in the problem overview section. These functions are defined implicitly by equations (2.5) and (2.6), respectively. The equilibrium is determined by the intersection of the best-response functions or, equivalently,

\[\text{Keep in mind that when reaction functions for farmers are mentioned, those are not "true" reaction functions.}\]
by solving equations (2.5) and (2.6) simultaneously for $A_c^*$ and $a_{ij}^*$. Unfortunately, we cannot obtain closed solutions for this model. However, numeric techniques can be used to characterize the predicted Nash equilibrium as well as responses to changes in the environment. In what follows, we provide a characterization of the equilibrium for a particular calibration of the model (hereupon referred to as the benchmark case). The parameter values assumed for the benchmark case are shown in Table 2.1. The parameters were chosen in a way such that if the yield were to be fixed at its expected value (which we fix at $E(y) = 100$), there would be no acres planted for the spot market in equilibrium; that is, processors would be fully contracted. To see this, suppose that processors offer a number of contracts such that they will not obtain enough input to work at capacity. Since there is no uncertainty, both farmers and processors know at planting time whether the spot price will be $p$ or $r_1$, given a positive expected aggregate spot acreage. If the cash market price will be $p$, farmers will refuse to take any contract that offers $r_1 + \delta < p$, leaving zero profits for processors who in turn will not want to offer those contracts (they would be better off by being fully contracted and not participating in a cash market). If the cash price will be $r_1$, individual farmers will not plant for that market, and expectations will not be confirmed. If, on the other hand, processors offer a number of contracts such that they will not need any extra input, expected aggregate spot acreage is zero, farmers know that the cash market price will be $r_1$, they will take the contracts and confirm expectations by not planting for the spot market.

The problem for each processor reduces to the classical symmetric Cournot game with $M$ players. That is max $\left\{ \left( p - r_1 - \delta \right) A_c E(y) \right\}$, subject to $(r_1 + \delta)E(y) \geq C'(a_{ij})$, where $a_{ij} = (1/N)\left( \sum_{i=1}^{M} A_{ci} \right)$, $j = 1, ..., N$. Using the cost function in what follows, the solution is easily checked to be $A_c^* = N\left( pE(y) - d \right)/b(M + 1)$. Plugging in the parameters from Table 2.1, we find that $A_c^* = 50$, implying that processors can work at capacity with the contracted

---

20 Note that there has to be excess demand in order for the spot market price to be between $p$ and $r_1$. This situation is ruled out in the absence of uncertainty.
acres (or that they are fully contracted). Note that the parameters presented in Table 2.1 are also consistent with the observation that producers will not plant for the spot market if they will obtain the commodity price with certainty.

Table 2.1. Parameterization for the benchmark case

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p$</td>
<td>4</td>
</tr>
<tr>
<td>$N$</td>
<td>10</td>
</tr>
<tr>
<td>$M$</td>
<td>5</td>
</tr>
<tr>
<td>$r_l$</td>
<td>1</td>
</tr>
<tr>
<td>$Q$</td>
<td>5000</td>
</tr>
<tr>
<td>$y_m$</td>
<td>200</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>1.5</td>
</tr>
<tr>
<td>$\beta$</td>
<td>1.5</td>
</tr>
<tr>
<td>$d$</td>
<td>100</td>
</tr>
<tr>
<td>$b$</td>
<td>10</td>
</tr>
</tbody>
</table>

To solve the model, we need to assume a probability distribution for the random yield and a functional form for the farmers’ cost function. The probability distribution for the random yield is assumed to be a three-parameter beta distribution, commonly used to model yield risk (see Goodwin and Ker, 2002), which has the following density function:

$$f(y) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} \frac{y^{\alpha-1}(y_m - y)^{\beta-1}}{y_m^{\alpha+\beta-1}}, \ 0 \leq y \leq y_m.$$ 

The parameters of the density function presented in Table 2.1 imply that $E(y) = 100$.

We will assume that the functional form of the cost function is the following:

$$C(a_s + a_c) = d(a_s + a_c) + \frac{b}{2}(a_s + a_c)^2.$$ 

The first term captures the factors of production that can be easily obtained to increase the area planted, whereas the quadratic term represents the increasing costs associated with less easily adjustable factors. An example of such a factor is the managerial capacity of the farmer. Note that an implication of the functional form assumed to describe farmers’ costs is that increasing the number of acres planted for the spot market increases the marginal costs of contract production, shifting contract supply to the left. That is, higher premiums will be needed to entice farmers to take a given number of contracts when they also plant for the spot market. This specification effectively links the supply for contract and spot production. The structure of the model implies that in any co-existence equilibrium, expected marginal revenues per contract and spot acres are equalized (this is also true in Newbery (1998)).
Figure 2.1 shows aggregate best-response functions for farmers and processors. Since all processors are identical, the aggregate best-response function is obtained by multiplying the individual function by the number of processors. The same rule applies to the best-response function for farmers.

![Figure 2.1](image.png)

Figure 2.1. Farmer and processor best-response functions for the benchmark case

The equilibrium for the model is where both curves intersect. The curve labeled as “processors” indicates the equilibrium aggregate number of contracts offered (and hence taken) in the game played between processors for every aggregate spot area. The curve labeled as “farmers” denotes the aggregate acreage planted for the spot market, consistent with any given contract offer by processors. Equilibrium for the model occurs, of course, where both curves intersect. At that point, the processors’ best-response functions intersect, and beliefs about aggregate spot acreage are confirmed.

Note that the introduction of yield uncertainty results in a quite different equilibrium. Each processor reduced the number of contracts offered (from 50 to 27 acres), and room was created for production for the open market (39 acres in aggregate), as a complement to the contract market. Recognizing the presence of uncertainty as a fact of agricultural production, yield a very different outcome than that in a deterministic model. In particular, co-existence of contract and spot production results in equilibrium.

The intuition behind this result is simple. With uncertainty, the possibility of over-buying the input arises. The more contract acres are offered, the higher the likelihood that a
fraction of the input cannot be processed and has to be sold at a loss at the ongoing commodity price. As processors offer fewer contracts the probability that they will need additional input (and the amount needed) *ex-post* to work at capacity increases, along with the expected spot price.

Introduction of uncertainty benefits processors and harms farmers in this setting. Yield randomness allows processors to reduce the number of contracts offered (and hence the premiums needed to get farmers to accept the contracts), while creating an expected spot price that induces farmers to plant for the cash market. Profits for farmers decrease from $3125 to $1509, whereas processors' profits increase from $2500 to $3826. Introduction of uncertainty allows processors to lower the contract premiums from $\delta^* = 2.5$ to $\delta^* = 1.74$, where the bar on $\delta^*$ indicates that yield is fixed.

As mentioned before, there is a growing concern that spot markets are thinning and that farmers get more exposed to the market power of processors. The model presented allows us to comment on those concerns. Suppose for a moment that processors are effectively able to discourage production for the spot market.\footnote{Either they are able to commit not to buy or collude implicitly not to bid aggressively on the open market.} In that case, farmers can be enticed to take contracts offering lower premiums. This clearly benefits processors and harms farmers (profits now are predicted to be $4048$ and $1382$ respectively). However, this is not an equilibrium, if processors compete *ex-post* as noted in the model\footnote{Though not presented, if the marginal costs of contract and spot production were independent, processors strictly prefer co-existence of the two markets. This result is driven by the fact that the spot market acts as a complement to the contract market without increasing the costs of contract production.}. Although comforting, the previous argument points to the need for authorities to be vigilant concerning the behavior of spot buyers.\footnote{Note also that processors' profits decrease and farmers' profits increase when spot markets are viable. This may provide yet another rationale for processors to integrate backwards into farm production, as observed for example, in the hog industry. See, for example Hennessy (1996), Murray (1995), and Perry (1978) for other rationales.}

### 2.6 Equilibrium responses to changes in economic parameters

In this section, we examine how changes in production uncertainty, the price of the processed good, the reference price, the number of processors, and the number of farmers affect the optimal choices of farmers and processors. The first variable we examine is production uncertainty. The effects of a mean-preserving spread in yields are illustrated by
solving the model for coefficients of variation (CV) of yields ranging from 0.1 to 1. The parameters of the yield distribution for the benchmark case represent a CV of 0.5.

Figure 2.2(a) depicts the Nash equilibria for the model for the different amounts of yield uncertainty. As Figure 2.2(a) makes clear, increasing (decreasing) the level of production uncertainty confirms the reduction (increase) in the number of acres contracted. The same intuition holds. A mean-preserving spread in yield risk increases the expected excess supply of input, conditional on yields being high enough to cover all input demand. This effect tends to decrease contract acres because processors will want to reduce the amount of excess supply they have to sell at a loss. But, a mean-preserving spread in risk also increases the yield shortfall conditional on being low enough to generate positive ex post demand. However, processors know that if they marginally reduce their offer of contract acres, farmers will marginally increase their supply of spot acres. This substitution, which reduces the risk of a yield shortfall, is observed for low levels of yield uncertainty. As processors keep lowering their contract offers, farmers stop increasing their spot acreage. We speculate that this effect is due to a decrease in the premium offered with fewer contracts (see Figure 2.2(b)). Farmers’ expected marginal revenues from contracting decrease, and this slows down the expansion in spot area. Recall that by the structure of the model, farmers’ expected marginal revenues in spot and contract production have to be equalized in equilibrium.

![Figure 2.2](image_url)

Figure 2.2. Effects of yield uncertainty (cv) on (a) the equilibrium acres in contract and spot markets and (b) contract premium
Overall, an increase in yield uncertainty will tend to increase the relative importance of spot market activity as processors try to avoid situations of excess supply from contracted acres. This suggests that spot markets will be more prevalent in situations where yield risk is relatively large.

Figure 2.3 shows how output profitability (measured by the per-unit margin) affects the set of Nash equilibrium. Higher profit margins result in more contracts offered and less area planted for the spot market. This result is very intuitive for processors—higher margins imply less willingness to operate at less than capacity. However, this result is less obvious for farmers. For the farmer, there are two forces acting in opposite directions. On the one hand, increases in the output price result in a higher return in the spot market in the case where processors have to bid their marginal valuation to get the input. On the other hand, this situation of excess demand is less likely to arise because of the reduced expected demand in the spot market. Figure 2.3 shows that the latter force dominates farmers’ decisions for low output price, but for higher prices the forces even out or the former force dominates slightly.

![Figure 2.3. Effect of output price (p) on equilibrium acreage](image)

When the output price is low enough, \( p = 2 \), the market for contracts disappear. Running at less than capacity is less expensive (in terms of foregone profits) for small margins. Also, adding premiums over the base price reduces the margin even further. Processors also recognize that if they decrease slightly the number of contracts offered, farmers will increase their spot acreage, responding to stronger \textit{ex-post} demand expectations.
Analogous results apply for changes in the commodity price. Increasing the reference price increases the expected spot price for any given contract acreage, providing incentives for farmers to plant more area for that market. Contracting becomes more expensive for processors, leading them to reduce the number of contracts offered.

Figure 2.4 shows the effects of an increase in the number of processors, holding per processor demand constant. This situation simulates the effects of an increase in total demand for the input, holding the number of farmers constant. An increase in demand increases the probability that processors will have to purchase in the spot market, thereby raising the input price. *Ceteris paribus*, this induces processors to offer more contract acres, to avoid a fierce

![Figure 2.4](image)

**Figure 2.4.** Equilibrium responses to changes in the number of processors (M) (per processor capacity held constant). Spot acres are per farmer. Contract acres are per processor

*ex-post* competition with other buyers. However, the increased likelihood of a strong spot demand also induces farmers to increase the area planted for the spot market, which is shown to occur in Figure 2.4. Additionally, with increasing marginal costs, it is more difficult for individual processors to find takers for any fixed number of contracts as the number of processing firms in the market increases. The forces just described act to reduce the number of contract acres offered by individual processors, though this is not enough to reduce aggregate contract offers. As a whole, the processing industry offers more contracts as it becomes larger (for a given pool of farmers) in an attempt to reduce competition in the spot
market. Thus, an increase in demand, holding the number of suppliers constant results in more contract acres.24

Similar results are found by holding total market demand constant but increasing the number of farmers. The effect of such an increase depends on the form of the farm-level marginal cost function. If marginal costs increase with output (as in our case), then an increase in the number of (identical) farmers will reduce the premium needed to entice farmers to take a fixed contract acreage as each farmer is offered fewer contracts. This creates an incentive for processors to expand the area procured under contract, which is what we observe in Figure 2.5. Also, for any given number of per-farm spot acres, supply for the cash market increases, leading to lower expected price, which creates incentives for individual farmers to reduce speculative production. However, this reduction is not as strong as to reduce the aggregate spot area.

![Figure 2.5. Equilibrium responses to changes in the number of farmers. Spot acres are per farmer. Contract acres are per processor](image)

24 Alternatively, if we increase the number of processors while holding aggregate capacity constant, the same pattern of reduction in contract offers by individual processors is observed, without increasing aggregate contracted area. Actually, the total number of contracted acres declines slightly. This scenario simulates the effects of a decrease in the market power of processors, as each holds a smaller share of industry capacity (hence their individual decisions have smaller marginal impacts). Growers also receive improved price premia in this setting, which is consistent with findings of Elbehri and Paarlberg (2003).
2.7 Alternative specification for the spot market supply

When product differentiation is absent (i.e. the input is of homogeneous quality), it is
sometimes asserted that firm’s strategic variables in the cash market (ex-post) are the
quantities to buy (Sexton, 2000). In those cases, Cournot’s equilibrium can be utilized. An
advantage of that approach over the situation modeled in the previous sections is that
problems of non-existence of Nash equilibrium in pure strategies arising from capacity
constraints and/or fixed supplies are sidestepped. However, for firms to be competing in
quantities an inverse supply function is required, implying that the cash market supply
function at any point in time is not perfectly inelastic. Firms can attract increasing quantities
from sellers simply by raising the price. The reasons for this may go in the line of inter-
temporal substitutions, search costs in part of sellers, or the like. Although easy to work with,
and mathematically tractable, this approach is not realistic for a large number of relevant
situations in agriculture. In particular, products that are perishable, have to be harvested on a
given time window, or are the result of decisions made in advance (sometimes months or
even years) escape this framework. For these situations, one may argue that producers have a
reservation price over which they will sell all their output (or model explicitly why more of
the input can be obtained ex-post by raising the price). This is certainly true if a fixed group
of ex-ante identical suppliers of a homogeneous product is assumed for the first-stage.

In this section we briefly consider the effects of assuming an upward sloping ex-post
supply function. We left unchanged the timing, and decisions for the first-stage of the game,
and model a different scenario for the second-stage. In particular, we assume here that the
slope of the supply function for the cash market is positive, and is influenced by the number
of acres planted in the first period. Further, we assume that the ex-post aggregate inverse
supply function in the open market has the following form \( r_s = r_i + \frac{S^i}{A_i} \), where \( S^i \) is the
amount of input supplied by farmers. Hence, farmers will supply increasing quantities of the
raw product as the price rises above their reservation price (or price in the market for
commodities). Intuitively, as the aggregate spot acreage increases, so does the amount of
input supplied at any given price. We can see this situation as if the farmers of the modeled
region were contributing to a pool of the input. If processors raise the price sufficiently high,
input crops from other regions would flow into the area of interest (or coming out of storage in case of non-perishable goods). Here the amount bought by processors in the cash market, is not necessarily constrained by the amount planted by farmers in the first stage. This specification may seem a little bit of a stretch, but is not more so than fixing the number of potential homogeneous producers, letting them put to run some biological process (growing a crop, finishing some pigs or cattle, etc), and when a cash market opens latter on assume that the supply is elastic (or not strongly inelastic).

Suppose first that processors are not capacity constrained. This is a standard Cournot game, where buyers compete in quantities. The problem of processor $i$ is to choose the amount of input $(s_i)$ to buy in order to maximize profits in the cash market, given the action of its rivals. Formally, $\max_{s_i} \left( p - r_i \right) s_i$. The market clears when $S^D = \sum_{i=1}^{M} s_i = S^*$, that is, aggregate supply and demand are equalized. A symmetric equilibrium for this model is easily shown to be $s_i^* = \frac{r_i + Mp}{M+1}$, $s^* = \frac{\tilde{A}_i y \left( p - r_i \right)}{M+1}$.

Moving to the first stage, processor $i$'s profit is given by

$$E_i \left( \pi_i \left| A_{-i}, \tilde{A}_i \right. \right) = \int y \left( p - r_i - \delta \right) A_{-i} y \left( p - r_i^* \right) s_i^* dF(y)$$

As before, processors choose the number of contract acres to offer and premiums to be paid, subject to the constraint that those quantities are consistent with the supply schedule of farmers $\left( (r_i + \delta) E(y) \geq C'(a_i + a_y) \right)$. The constraint is again subsumed into the objective function, leaving processors the choice of the number of acres to contract. The first order conditions are

$$\frac{\partial E_i \left( \pi_i \left| A_{-i}, \tilde{A}_i \right. \right)}{\partial A_{ci}} = \left( p - r_i - \delta - \frac{\partial \delta}{\partial a_c} \frac{\partial a_c}{\partial A_{ci}} A_{ci} \right) E(y) \leq 0, \ i = 1, ..., M, \tag{2.7}$$

with equality if $A_{ci}^* > 0$.

The first stage problem for farmers is again to choose whether to take the contracts and the number of acres to plant for the spot market. Since by construction, the number of contracts offered is consistent with the supply of contract acres, we can assume that contracts
will be taken and focus on the decision of the number of acres to plant without a contract. As before, farmers maximize

\[ \pi^F = \left( r^*_s a_s + \left( r_s + \delta a_c \right) E(y) - d \left( a_s + a_c \right) - \frac{b}{2} \left( a_s + a_c \right)^2 \right) \]

with respect to \( a_s \). This yields the following first order conditions

\[ \frac{\partial \pi^F}{\partial a_s} = r^*_s E(y) - d - b(a_s + a_c) \leq 0, \] with equality if \( a_s^* > 0 \) \hspace{1cm} (2.8)

Equations (2.7) and (2.8) are solved simultaneously by

\[ A^*_c = \frac{(p - r_s) E(y) N}{(M + 1) b} = 50 \]

\[ a_s^* = \frac{E(y) r_s - d}{b} = 0.25 \] Processors would be fully contracted in expectations, and there will be no area planted for the spot market. However, note that this result also holds in the presence of uncertainty (i.e. we did not fix yield at its expected value as in the previous case). Thus, only in the absence of uncertainty, with the proposed parameterization, both models yield the same prediction that spot markets do not have a space in equilibrium.

In addition, as we will show, when the capacity constraint is introduced both models yield different predictions. In particular, we obtain that introduction of a capacity constraint in the current model reduces the number of contracts offered (as in the fixed supply case), without increasing the number of acres planted for the spot.

Three ex-post scenarios (analogous to the ones modeled before) are introduced with a capacity constraint. Either processors over-procured the input, or they will have a positive demand in the cash market. In the latter case processors may turn to the spot market and buy enough input to work at capacity or in certain supply and demand conditions choose not to work at capacity.

The first scenario is exactly analogous to the previous model, and so is the payoff function and probability of occurrence, thus we do not repeat it here. In the second and third scenarios, processors will choose to participate in the spot market. Clearly, when processors have positive ex-post demand for the specialty grain, they will choose to work at capacity.

\[ ^{25} \] To have a positive number of spot acres, farmers would need to be willing to plant the specialty crop for the spot market, even when they know the price equals the commodity price. This is not likely to be the case, since the lack of sufficient incentives or premiums was the highest rated factor influencing the decision not to grow value enhanced corn in a survey of producers in the corn-belt (Stewart, 2003).
whenever the amount that they would procure without the constraint is larger than the
amount they would like to procure with the constraint. That is, when $M s^* > MQ \sum_{i=1}^{M} A_{ci} y$, or equivalently when, $y \geq \frac{M(M + 1)Q}{MA_y(p - r_i) + (M + 1)\sum_{i=1}^{M} A_{ci}} = t$. In this scenario, the cash price will be $r^*_s = r_i + \frac{MQ - \sum_{i=1}^{M} A_{ci} y}{A_y}$, and processor $i$'s profit

$$\pi_2 = (p - r_i - \delta) A_{ci} y + (p - r^*_s)(Q - A_{ci} y)$$

The last scenario (which occurs when $y \leq t$), is the same that would result if the capacity constraint does not bind. In this case $r_s = r^*_s$, and processors buy $s^*$ units of the input in the cash market. Processor $i$'s profit in scenario is

$$\pi_3 = (p - r_i - \delta) A_{ci} y + (p - r^*_s)s^*.$$  

Putting all together, the first state problem for processor $i$ is to maximize

$$E \left( \pi \middle| A_{c_{-i}}, A_i \right) = \int \left( (p - r_i - \delta) A_{ci} y + (p - r_i)(Q - A_{ci} y) \right) dF \left( y \right)$$

$$+ \int \left( (p - r_i - \delta) A_{ci} y + (p - r^*_s) \right) dF \left( y \right)$$

$$+ \int \left( (p - r_i - \delta) A_{ci} y + (p - r^*_s)s^* \right) dF \left( y \right)$$

with respect to $A_{ci}$. Taking first derivatives (using Leibnitz rule), and imposing symmetry yields after some algebra

$$\frac{\partial E \left( \pi \middle| A_{c_{-i}}, A_i \right)}{\partial A_{ci}} = \left( p - r_i - \delta - \frac{\partial \delta}{\partial a_e} A_{ci} \right) E \left( y \right)$$

$$- (p - r_i) \int y dF \left( y \right) + \left( \frac{M + 1}{A_y} \right) \int (Q - A_{ci} y) dF \left( y \right) \leq 0$$

with equality if $A_{ci} > 0$, $i = 1, ..., M$.

Farmers maximize profits by choosing $a_s$ in exactly the same fashion described above. The first order condition for farmer’s problem is

$$\frac{\partial \pi^F}{\partial a_s} = \delta \int y dF \left( y \right) + r^*_s \int y dF \left( y \right) + r^*_s \int y dF \left( y \right) - d - b (a_s + a_e) \leq 0$$
Imposing equilibrium conditions \( Na_s^* = \bar{A}_s \), and \( Na_c = MA' \), the solution for the system of equations is \( a_s^* = 0 \) and \( A_c^* = 33 \) acres. In this setting, processors will also choose not to be fully contracted in an expectations sense. However, contrasting with the fixed supply modeling approach, there is no role for the spot market as a complement for a contracts market in this scenario. In other words co-existence is not an equilibrium for this model. This result makes sense. Competition in quantities is known to be milder than competition in prices. When farmers know that processors will not be competing vigorously in the cash markets, they have weaker incentives to plant for that destination.

### 2.8 Concluding remarks

In this study we develop a simple theoretical model that, suitably parameterized, allows for the co-existence of contracts and spot markets in agriculture. The presentation here focused on the production of a specialty crop. However, suitable parameterizations would afford the study of a wide variety of agricultural production processes. Numerical simulations were conducted to study the impacts of fundamental economic factors on the equilibrium outcomes. Our results suggest that for a wide range of distinct parameters, participation in both markets constitutes a Nash equilibrium for the model. This result would indicate that the fact that a growing proportion of agricultural raw products are transacted by means other than cash markets does not necessarily imply that spot markets for these sectors will disappear altogether. There is a balance to be attained between the sizes of the markets. Because the model assumes that all the market participants are risk-neutral, the equilibrium outcomes are the results of purely financial considerations.

The predictions of the model make clear the substitutability between both systems of production (when the market structure remains unchanged). It is worth emphasizing, that co-existence of both markets only arose in the presence of uncertainty. For both specifications of the cash market supply function, specialization in contract markets is obtained absent uncertainty. This implies that because of its strong qualitative implications, uncertainty has to be explicitly accounted for in any modeling situation in which contracts and spot markets co-exist and production outcomes are subject to randomness. This is quite important in agriculture, for which biological uncertainty and weather variability (among other sources of randomness) play a major role.
The model demonstrates the antagonistic interests of farmers and processors concerning the relative size of the spot and contract markets. With increasing marginal costs of contracting (affected by spot acreage), processors would prefer specialization in contracting. Farmers prefer the equilibrium predicted by the model (co-existence). The fact that processors would like to be able to discourage production for the spot market makes clear that safeguards should be established to insure vigorous competition when cash markets arise. In the current setting, producers have the option of planting without a contract (a spot market is likely to arise). This constrains the behavior of processors since contract offers have to provide sufficient financial inducement, relative to expected spot prices, to entice farmers to take them.

The role of spot markets complementing contract production is illustrated. The term "complementing" is used here in the sense that spot serves as a residual market in which buyers can make up for any difference in input needs, relative to their target procurement levels. Farmers can plant (speculatively) additional acres for spot markets based on the expected spot price.

Although the model presented here may not capture many aspects of contracting decisions in agriculture, it is a reasonable starting point. We discuss the basic elements, and indicate points at which assumptions may yield different qualitative predictions. The model could be modified in several directions to tackle different complexities that commonly arise in the markets. Examples are the introduction of quality issues or transaction costs related to the contracted versus spot market procurement. A different direction in which this research could be extended is to explore the efficiency of the contract observed relative to other coordination mechanisms, and whether co-existence also attains for other contractual arrangements.
Appendix A. Mixed strategy equilibrium for scenario 3. \( M=2 \)

The argument presented here closely follows that of Levitan and Shubik (1972). A mixed-strategy equilibrium for this game is a pair of probability distributions over each player’s strategy space. These probability distributions must have the property that any strategy chosen by a player with positive probability must be optimal against the other player’s probability distribution.

The amount that firm \( i \) is able to procure as a function of both offered prices in general is

\[
x_{ii} = \begin{cases} 
\min \left\{ \max \left( \frac{A_{y} - (Q - A_{x}y)}{2}, 0 \right), (Q - A_{x}y) \right\} & \text{if } r_{i} < r_{j} \\
\frac{A_{y} - (Q - A_{x}y)}{2} & \text{if } r_{i} = r_{j} \\
\min \left\{ (Q - A_{x}y), \frac{A_{y}}{2} \right\} & \text{if } r_{i} > r_{j}
\end{cases}
\]

However, since this scenario arises when \( \frac{Q}{A_{i} + A_{j}} \leq y \leq \frac{2Q}{A_{i} + 2A_{j}} \), or \( (Q - A_{x}y) \leq A_{y} \leq 2(Q - A_{x}y) \), and \( (Q - A_{x}y) = x_{s} \) we can focus on the following case

\[
x_{ii} = \begin{cases} 
A_{y} - x_{s} & \text{if } r_{i} < r_{j} \\
\frac{A_{y}}{2} & \text{if } r_{i} = r_{j} \\
x_{s} & \text{if } r_{i} > r_{j}
\end{cases}
\]

Since both prices will coincide with zero probability, the expected amount procured in the spot can be written as \( E(x_{ii}) = (1 - \phi_{j}(r_{i})) \left( \frac{A_{y} - x_{s}}{2} \right) + \phi_{j}(r_{i}) x_{s} \), and expected profits, \( \pi_{i}(r) = (p - r) \left( (1 - \phi_{j}(r)) \left( \frac{A_{y} - x_{s}}{2} \right) + \phi_{j}(r) x_{s} \right) \). The cumulative distribution function of player \( j \) is \( \phi_{j}(r) \). From this we obtain \( \phi_{j} \) as

\[
\phi_{j}(r) = \frac{(A_{x} - x_{s})(p - r) - \pi_{i}(r)}{(p - r)(A_{x} - 2x_{s})}
\]

with support \( [x_{r}, \bar{x}] \subseteq [p, r_{i}] \). If processor \( i \) is willing to randomize, it must be the case that \( \pi_{i} \) is constant on \( [x_{r}, \bar{x}] \). If firm \( i \) offers \( r_{r} \), it will be overbid with probability one. Since it
will be the residual claimant for the input, it will be optimal to bid \( r_i \) (if firm \( i \) knows it will be overbid, its payoff function is monotonically decreasing in the offer price, indicating that it is optimal to bid the reservation price of sellers). To pin down \( \bar{r} \) we use the fact that \( \pi_i(r) = \pi_i(\bar{r}) = \pi_i(\bar{r}) \). It follows that \( (\bar{A}_i y - x_i)(p - r_i) = x_i (p - \bar{r}) \). Solving for \( \bar{r} \) we get

\[
\bar{r} = p - \frac{(\bar{A}_i y - x_i)(p - r_i)}{x_i}
\]

and \( \phi_j(r) = \frac{(\bar{A}_i y - x_i)(r_i - r)}{(p - r)(\bar{A}_i y - 2x_i)} \).
Appendix B. Second order sufficient conditions for the processor’s problem

\[
\frac{\partial^2 \pi}{\partial A_{ci}^2} = \left( \frac{\partial f(u)}{\partial u} \left( \frac{\partial u}{\partial A_{ci}} \right)^2 + f(u) \frac{\partial^2 u}{\partial A_{ci}^2} \right) \left[ u \left( \frac{A_{ci}}{2M} + A_{ci} \right) - Q \right] + f(u) \frac{\partial u}{\partial A_{ci}} \left( \frac{\partial A_{ci}}{2M} + A_{ci} \right) + 2u \right)
\]

\[- \left( \frac{\partial f(s)}{\partial s} \right)^2 - f(s) \frac{\partial^2 s}{\partial A_{ci}^2} + f(s) \left( \frac{\partial s}{\partial A_{ci}} \right)^2 \right) \frac{A_{ci}}{2M} \right)
\]

\[- \frac{E(y)}{(p - r_i)} \left( \frac{2\partial \delta}{\partial A_{ci}} + \frac{\partial^2 \delta}{\partial A_{ci}^2} \right) \leq 0 \]

\[i = 1, ..., M.\]
CHAPTER 3. MARKET STRUCTURE, REPUTATIONS, AND
THE CHOICE OF QUALITY ASSURANCE SYSTEMS

3.1 Introduction

New quality assurance systems (QASs) are being put in place to facilitate the flow of information about agricultural and food products. Incentives for growers and food manufacturers to adopt QASs include (a) increased consumer demand for knowledge about where their food came from and how it was produced; (b) opportunities for producer groups to capture a greater share of the consumer dollar by differentiating their products; (c) greater protection for food manufacturers and retailers against food safety liability; and (d) increased chances that agreed upon specifications are met in a framework of imperfect information.

Various QASs have been developed in the United States and elsewhere to facilitate the flow of information about products (Bredahl et al. (2001), Reardon and Farina (2001), Lawrence (2002), Carriquiry, Babcock, and Carbone (2003)). In the United States, some have been developed purely by the private sector, while others have relied on the U.S. Department of Agriculture (USDA) to set standards. But what constitutes a proper mix of public and private efforts in setting up QASs is an unsettled question. A better understanding of private sector incentives for setting up such systems will help clarify what role the public sector might have in establishing and enforcing standards. We contribute to this understanding by modeling the optimal degree of “stringency” or assurance in a processor’s quality control system over procurement of agricultural output when there exists uncertainty about quality. More specifically, we study the role of reputational mechanisms in providing incentives to implement QASs. We compare the resulting degrees of stringency across different market structures, and nature of reputations.

The role of reputations as a deception preventing device has been studied by Klein and Leffler (1981), Shapiro (1983), among others. These authors examine a situation where the resulting quality is completely determined by a producer’s investments. The only study (we are aware of) that considers the case where investments lead to higher expected quality (stochastically) is by Rob and Sekiguchi (2001), who consider the producer-consumer
interface. In their study, consumers “discipline” firms by switching to rival firms when quality is below certain tolerance levels, and only one firm is able to make sales. We depart from that study by exploring the effects of different market structures on the choice of investments in quality, as well as the nature of reputations. In our framework, potentially more than one firm makes positive sales, firms compete in quantities purchased (and sold), and the focus is on an arbitrary link of a supply chain.

We include both process control, where verification of certain production methods are to be followed, as well as control over a physical attribute of the input. The models predict that the degree of stringency depends on (a) whether the sought-after attribute is discoverable by consumers, (b) the price premium paid for the attribute, (c) the market structure, and (d) the nature of reputations.

The next section provides an overview of the related literature, a detailed discussion of the situation studied, and the model presentation and analysis. Numerical simulations are performed in Section 3.3 with a corresponding discussion of results. Section 3.4 concludes.

3.2 The model

The two decisions modeled are the profit maximizing rate of output and whether a buyer of an input should implement a QAS as a way to gain information about product quality that can be provided to its potential customers. If the buyer decides to implement a QAS, then the profit maximizing level of assurance is determined.\(^1\) We study a particular case that is becoming pervasive in the food industry (Caswell, Bredahl, and Hooker, 1998; Reardon and Farina, 2001; Northen, 2001; Fearne, Hornibrook, and Dedman, 2001), in which an input buyer requires its suppliers to implement a given system of assurance. To do so, the buyer has to be an important player in the market (Reardon and Farina, 2001). This would be the case of a quasi-voluntary system (Caswell, Bredahl, and Hooker, 1998; Noelke and Caswell, 2000). To keep the framework general, we are not specifying the “type” of quality assurance strategy that an input processor will follow. For example, assurance potentially can come from a system run by the buyer, from reliance on certification by a private or public third party, or both. The information obtained through the implementation of the system

\(^1\) Caswell, Bredahl, and Hooker (1998) provide an overview of the functions and effects of quality management systems (or metasystems as they call them) on the food industry. See also Henson and Hooker 2001.
allows the processing firm to better sort the input it buys, to gain a better idea of the actual quality of the inputs, and to be able to convey assurance to its customers about the quality of its product.

The topic of this paper is especially relevant in an environment where it is difficult to assert the quality of a particular product (both before and after the input is processed and consumed). That is, there is imperfect information about quality. Many food attributes can be thus classified (see, e.g., Caswell and Mojduszka, 1996; Antle, 1996; and Unnevehr and Jensen, 1996). Clearly, if quality is readily observable by both input buyers and consumers there is no need for a QAS in the procurement process.

Quality characteristics of agricultural products have an inherently high degree of heterogeneity (Ligon, 2002). This variability stems mainly from the randomness of the production environment (e.g., weather and biological uncertainty) and/or the heterogeneity of the practices employed by farmers. We aim to capture this variability, and the fact that quality can be only imperfectly assessed, by assuming that the processor believes it can be represented by a random vector $Q$. Specifically, let $Q$ denote the vector of the imperfectly observable array of quality attributes, and let $Q$ denote the set of possible quality attributes of an input. In general, $Q$ could be a set in many dimensions. However, for the sake of tractability, we will assume that only one quality attribute is of interest, is imperfectly observable, or differs across goods. In this case, the sample space of interest is one-dimensional (i.e., $Q \subseteq \mathbb{R}^1$). Further, we assume that the unconditional cumulative distribution function of $Q$ is $F_Q(q) = P_Q(Q \leq q)$ for all $q$. This distribution of quality would prevail in the absence of a QAS or in the open market.

Noelke and Caswell (2000) propose a model of quality management for credence attributes in a supply chain. Based on a literature review, the authors discuss benefits and costs of a voluntary, quasi-voluntary, and mandatory quality management system (QMS) for a firm within a supply chain. They then provide some comparative static results on how the behavior of the firm (given by the choice of a QMS) changes when other firms upstream or

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2 These are the types of quality attributes that Nelson (1970) and Darby and Karni (1973) labeled as “experience” and “credence,” respectively. The authors introduced the terms search, experience, and credence to refer to the moment in which consumers can gain information about product quality.

3 The quantity of $Q$ is assumed to increase with the quality of the underlying input.
downstream modify their QMS under different tort laws. In our model, the firm under consideration is the one that has influence on the behavior of firms upstream in the supply chain. We also make the probability that a firm is punished dependent on the ability of consumers to discern when quality is substandard. Hence, our model accommodates credence, experience, and mixtures of the two types of goods.

We define quality as meeting an agreed-upon standard. Every unit of the product meeting or surpassing this minimum standard will be considered of high quality. Caswell, Noelke, and Mojduszka (2002, p.58) also define quality in a supply chain as consistently delivering a product that meets or exceeds “defined sets of standards for extrinsic indicators and cues.” The task of the processors is to decide which QAS to implement (if any) to better infer the nature of the product being certified. In particular, the QAS will inform the processor about the proportion of the input that is likely to deliver a product that meets the minimum standard. Note however that the processor will not be able to discern the difference in quality of any given unit it actually buys. We assume that the quality of the processed output has a direct relationship to its input counterpart, an assumption that is equivalent to claiming that the processing technology cannot be used as a substitute for input quality, or that it does so only at prohibitively high costs. We assume also that there is a one to one correspondence between the amount of input bought and output sold by a processor. Hence, the production technology works in a Leontief fashion, and the decision on the output rate essentially determines how much of the agricultural input is needed.

The proposed approach accommodates both the case where the quality attribute or trait is the production method itself and the case where the process alters the probability distribution of quality (i.e., the costlier process increases the probability of obtaining a high-quality product). The former has an analog to a discrete attribute (the good was produced using a desired process or it was not), whereas quality in the latter case is a continuous random variable whose distribution is altered by the process followed. The interpretation of the continuous case is straightforward. Let $q^M$ be the minimum acceptable quality to be considered good quality (or with desirable properties for good performance at the processing stage, or to deliver a good eating experience). In this case $q^M$ divides the range of the
random variable $Q$ into two subsets defined in $\mathbb{R}^1$, namely, $q_L = \{q \in Q : q \leq q^M\}$ and $q_H = \{q \in Q : q > q^M\}$. Hence, $F(q^M) = P_q(Q \in q_L)$ would be the unconditional probability that the product is inferior or unacceptable. A product that is deemed inferior receives a lower price than that certified as being high quality. This interpretation is akin to attributes such as the tenderness of a steak, where $q^M$ would be some acceptable degree of tenderness, or to food safety, where $q^M$ would be interpreted as the count of pathogens$^4$ that makes a particular food item unsafe$^5$.

The discrete case can be analyzed in a similar way. Strictly speaking, here the input has or fails to have a particular attribute or was or was not produced following a value-adding (cost-increasing) production process. For example, milk used by some companies was produced with or without treating milking cows with genetically engineered hormones such as rbST (recombinant bovine somatotropin). Also, eggs can be produced using animal welfare$^6$ enhancing techniques (e.g., free-range production) or by conventional means. Poultry is or is not fed with animal protein. Crops can be grown conventionally or using environmentally friendly practices (e.g., minimum tillage).

However, to unify the analysis, we will differentiate by class of production practice. For example, in the case of animal welfare, we will consider a product to be of high quality if the production facilities meet a minimum set of amenities or if certain practices, such as forced molting of egg-laying hens, are avoided. Another example would be the intensity of the soil conservation practices employed. A different interpretation of $Q$ is needed to tackle this type of attribute. Let $Q$ be the set of all production practices "bought" by the input-processing firm. The natural variability here would come from the heterogeneity of

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$^4$ For this interpretation, we would need to reverse the claim that a larger $Q$ represents a higher quality. Pathogen counts above $q^M$ represent an unacceptable good.

$^5$ Antle (2001) provides a literature review of the economics of food safety. Segerson (1999) discusses whether reliance on voluntary approaches to food safety will provide adequate levels of safety in a competitive framework that allows for consumers and firms to be imperfectly informed about potential damages. Marette, Bureau, and Gozlan (2000) explore the provision of product safety and possible public regulation for search, experience, and credence attributes when the supply sector competes imperfectly, and hence can use prices as signals. However in their model, and in Daughety and Reinganum (1995) sellers know the actual quality of the product they offer in each period.

$^6$ See Blandford et al. 2002 for a classification of science-based definitions and measures of animal welfare.
farmers. Implicit here is the idea that the firms will buy from producers and certify the product if they believe the input was produced following a process that meets or surpasses some minimum standard (more on this to follow). Hence $Q$ would represent the set of all input procured from producers who have the capabilities needed to produce, and are believed to produce, following the desired processes. The clarification is important because having the capabilities does not necessarily mean that the process will be strictly followed under conditions of imperfect information\textsuperscript{7}. A problem of moral hazard arises here. Because production of the high-quality input is costlier than production for a commodity market, and there is a strictly positive probability that deviant behavior will not be discovered and penalized, suppliers will find it rational to deviate from perfect compliance\textsuperscript{8} \textsuperscript{9}. Hence, there is a strictly positive fraction of the output that will not be produced under the desired cost-increasing conditions. This fraction is again represented by $F_q(q^M)$.

The preceding discussion suggests that there are two different types of uncertainties associated with the attribute under consideration. In the discrete case, the uncertainty is mainly about the opportunism of the suppliers, whereas the continuous case also entails the uncertainty derived from the randomness associated with agricultural processes that generate a distribution of qualities for any given set of production practices. Hence, both symmetric and asymmetric informational imperfections may be present in this setting (see Antle 2001).

In terms of quality, the choice of the firm is on the QAS and the associated stringency of controls. Suppose there is a set of alternative systems denoted by $S = \{s \in S : s^O \leq s \leq s^U\}$, where $s = s^O$ represents the absence of quality verification (reliance on claims made by suppliers), and $s = s^U$ is a situation in which the quality of the product is perfectly revealed (e.g., perfect monitoring, vertical integration, or a good system of incentives). In the absence of a QAS, the processor rationally expects to obtain an input of “average” quality from the

\textsuperscript{7} Hayes and Lence (2002) provide the example of Parma Ham. They discuss that only ham produced within a certain region can be marketed as Parma Ham. The rationale for the restriction is that the weather in this region during the dry-curing process is what gives the ham its unique attributes. Nowadays, however, the dry-curing process is mainly carried on in modern, climate-controlled facilities.

\textsuperscript{8} There is a large literature showing that when certification is imperfect, some producers of low quality will apply and obtain certification. See De and Nabar 1991 and Mason and Sterbenz 1994.

\textsuperscript{9} Hennessy (1996) and Chalfant et al. (1999) showed that imperfect testing and grading lead to under-investment in quality-enhancing techniques by farmers. This is because producers of low quality impose an externality on producers of high quality.
market for raw materials. When \( s = s^U \) is chosen, note that the actual quality of a given unit of input will be perfectly revealed only on the discrete case; in the continuous case, even the most stringent system available still leaves the uncertainty derived from the randomness of the production environment and attributable to scientific ignorance over biological processes. However, in the continuous case, a more stringent system potentially has the double effect of increasing the proportion of compliers and the probability of being in the set \( q^H \).

In short, a processor procuring raw materials from certified suppliers (using the QAS indexed by \( s \)) expects to buy a fraction of good-quality input, denoted by \( \lambda(s) = 1 - F(q^M | s) \). Also, a fraction \( 1 - \lambda(s) = F_q(q^M | s) \) of the inferior input is expected because of the informational imperfections noted earlier. Here, we consider the QAS as aiding in the selection of which input to buy. All inputs bought (and hence certified) will be subjected to the production activity and sold to downstream customers as possessing the desired trait\(^\text{10}\). The case where \( s = 0 \) (absent an assurance effort) will result in the processor buying an input that is expected to have average market quality. To put it concisely, \( \lambda(0) = 1 - F(q^M | 0) = 1 - F(q^M) \). Alternatively, if the input were going to be bought anyway, the QAS would be functioning as a sorting device that allocates the input either to the commodity or to the high-quality market (or production process). In this case, the QAS tells the processor how much a unit of the input is worth to him, just as an imperfect test does (Hennessy 1996).

With this in place, we can focus on how to model the effects of a more stringent QAS on the overall quality of the product traded by the processor. For this purpose, we will use the concept of first-order stochastic dominance. Implementation of different levels of stringency switches the relevant distributions for quality as follows. For any \( s', s' \in S \) there is an associated conditional distribution for quality, namely, \( F_q(q | s') \) and \( F_q(q | s') \). In this context, increasing the level of stringency of the QAS, for example by moving from \( s' \) to \( s' \) where \( s' \leq s' \), leads to a first-order stochastically dominating shift on the distribution of quality. Therefore, we have that \( F_q(q | s') \geq F_q(q | s') \) for all \( q \in Q \). In particular, this implies

\(^{10}\) Firms would be participating in only one market.
that \( \lambda(s') = 1 - F(q^M | s') \geq 1 - F(q^M | s') = \lambda(s') \). This can be interpreted as reducing the probability of incurring type I (rejecting an input that is of good quality\(^{11}\)) and type II (certifying a product that is of low quality) errors. In short, systems that are more stringent increase the precision with which the actual quality of the input is asserted by processors. If we are willing to assume that \( F_\theta(q | s) \) is differentiable with respect to \( s \), the above implies that \( \partial \lambda(s) / \partial s \geq 0 \).

However, the implementation of a QAS does not come without a cost. The use of cost-increasing technologies has to be compensated for by processors. Complications arise, however, because the incentives of farmers and processors are not aligned, and the production practices used at the farm level are only imperfectly observed. Hence, processors may have to give up some information rent if they want to elicit production of high-quality inputs (or in the language of agency theory, a high level of effort from the principal’s suppliers or agents). In other words, there are costs for monitoring and/or providing the incentives (e.g., premiums or discounts) to discourage dishonest performance on the part of input suppliers. We seek to capture those costs\(^{12}\) through a cost function \( C(s, y) \), which satisfies \( \partial C(y, s) / \partial s > 0 \), where \( y \) is the per-period output rate.

The exposition from here on will be made in terms of processors and consumers. However, we could replace consumers by downstream firms to study issues within a supply chain. Consumers value both types of goods but value the high-quality good more and are willing to pay a premium for it. Because consumers can only assert the quality of a given product imperfectly, they must rely on the signals sent by the processors, in the form of quality certification.\(^{13}\) Hence, we assume that consumers are willing to pay a price premium for a good that comes from a quality-assured process.

\(^{11}\) Type I errors may be due to imperfections on the QAS (for example, due to incorrect monitoring.). This would increase the costs of procuring the input the processor needs.

\(^{12}\) Note that we are being vague about what is being represented by this cost function. It could be modeling search costs, monitoring costs, or compensation given to farmers to induce high levels of effort.

\(^{13}\) This does not mean that their message to customers has to coincide with the assurance they get from the suppliers. For example, a restaurant that sources beef that is assured to be from a certain breed (e.g., Angus) may want to claim that the steaks they serve are tender. Another example would be a branded product. A customer of a well-known upscale restaurant expects to get a tender steak, and the restaurant will try to buy only beef it can certify to have been fed in a certain way or again from a certain breed.
Putting all this together, the processor chooses the output rate, whether to participate in the market for quality-certified goods and what level of certainty to obtain from the QAS or to direct its product toward the market for commodities. Clearly, the processor will implement a QAS as long as the benefits outweigh the costs of doing so.

Participation in the market for high-quality goods, using the QAS indexed by \( s \) yields a per-period profit of \( \pi^{i,r}(y,s;a) = R(y;a) - C(y,s) \), where the revenue function \( R(y;a) \) potentially depends on the firm's rate of output \( y \) (this will later represent a vector containing the outputs of more than one firm), and the strength of consumer preference for high-quality goods \( a \). The superscript in the profit function represents the state of the world, where the processor \( i \) has a reputation (more on this to follow).

We could append a term to the profit function, representing the economic loss due to certifying a product that is of low quality. Several potential interpretations are possible for this loss. It could be the result of obtaining a bad reputation through some form of information dissemination or could occur just because the consumer will make future purchases from other processors. It also could be the result of legal action under the current tort law, applying mainly in the case of a food safety interpretation of the model. However, Caswell and Henson (1997) argue that this last effect is likely to be less important than the loss of reputation or market share. Therefore, the latter is the interpretation to which we adhere in this paper. The punishment for a processor that delivers a good of noticeable substandard quality is that it will lose consumer's trust, and hence will be unable to sell its output in the market for value-added products in future periods. Note however that the processor obtains the price for the certified commodity no matter what the actual quality might be, because customers cannot assert a priori whether the claims made by the processor are false. In other words, processors will be trusted until proven wrong. Consumers' trust is what defines the states of the world in this model. For a given processor, demand is state contingent, where the states of the world reflect whether he/she is trusted by consumers or not. For this sort of punishment mechanism to have an impact on a firm's decisions, modeling more than one period is required (Klein and Leffler (1981)).
On the other hand, participation in the market for commodities or non-differentiated products yields a quasi-rent of \( \pi' = 0 \) independent of processors' reputation, since the market for the commodity is perfectly competitive.

### 3.2.1 Optimal choice of quality assurance systems

We are now in a position to examine how fundamental characteristics of the economic environment influence decisions about the implementation of a QAS and the relative profitability of the competing markets or options, paying special attention to market structure and different forms of punishment. Clearly, QASs will be observed if

\[
E(\Pi_{r}^{\omega}(s, y)) \geq \pi' = 0 \quad \text{for some } s \in S, \text{ and } y > 0.
\]

That is, if there is a combination of output rate and QAS that makes the expected return of the value-added market to be positive, then the firm has an incentive to enter the value-added market.

Throughout the analysis, we assume for mathematical convenience that there is a continuum of stringency levels from which to choose. The convexity of the set \( S \) frees us from worrying about problems of non-existence of solutions related to discontinuities. It also allows us to use the tools of calculus to analyze the problem.

We introduce \( \omega \) to parameterize the degree to which consumers can ascertain the actual quality of the good. It measures the readiness to which quality is observed after consumption. For example, we could interpret \( \omega \in [0, 1] \) as the exogenous probability that a consumer discovers the true quality of the product. \( \omega = 1 \) implies that quality is perfectly observable after consumption, or the sought-after characteristic is an experience attribute. Credence attributes are represented by \( \omega = 0 \).

#### 3.2.1.1 Monopolist processor

We first examine the case where there is only one processor in the market. Recall that we assume processors will be trusted until proven wrong. Therefore, there are only two possible states of the world denoted by \( r = 1, 2 \). The first state denotes the periods where the processor has a good reputation, and hence faces a positive demand. In state two, the demand for the high quality product is zero. Since there is only one processor, and profits are zero in the second state of the world, the superscript of the per-period profit function will be dropped here.
Let $T$ denote the point in time where the processor loses his/her reputation (move from the first to the second state). That is, $T$ is the period in which consumers purchase a product that does not meet the standards promised, and find out. A processor that moves from state 1 to state 2 in period $T$, has profits given by

$$
\Pi(s,y) = \sum_{t=1}^{T} \beta^{t-1} \pi(y,s;a) = \pi(y,s;a) \sum_{t=1}^{T} \beta^{t-1} = \pi(y,s;a) \frac{1 - \beta^T}{1 - \beta}
$$

(3.1)

As before, $\pi(\bullet)$ represents the per-period profits of a processor that has a good reputation. Since they depend only on the state of the world, we can pull per-period profits out of the summation. In equation (3.1), $\beta$ is the relevant discount factor, $a$ again denotes the size of the market for value added products, $y$ and $s$ represent the levels of output and QAS respectively. We assume $\frac{\partial \pi}{\partial a} \geq 0$, i.e. per-period profits increase as the demand for high quality products strengthens. Profits are zero once the seller is forced out of the value added market.

However quality is random and the processor cannot exert perfect control over it. A processor can only affect the distribution of quality in the sense described above. Hence, it is not known when a processor will lose its reputation. Recognizing that, processor’s expected profits are

$$
E(\Pi(y,s)) = E\left( \pi(y,s;a) \frac{1 - \beta^T}{1 - \beta} m(s,\omega) \right) = \pi(y,s,a) \frac{1 - E(\beta^T | m(s,\omega))}{1 - \beta}
$$

where $m(s,\omega)$ denotes the probability that a processor with a QAS $s$ in place will stay in the value added market for a trait with discoverability $\omega$. In particular, note that the probability of staying in the market or keeping consumer’s goodwill for two successive periods, $m(s,\omega) = \lambda(s) + (1 - \lambda(s))(1 - \omega)$, combines the probability that resulting quality is

---

14 The rightmost equality follows by recognizing that the summation term is a partial sum of a geometric series.
high, with the probability of type II error weighted by the consumer's level of awareness. A processor will face a zero demand in the second period with probability $1 - m(s, \omega)$.

To make further progress, we need an expression for $E(\beta^T | m(s, \omega))$. Note that $T$ is just counting the number of periods until the first notorious (discovered) failure. Since the outcome in a given period is independent of the outcome of other periods, $T$ is the number of Bernoulli trials required to get the first failure. This is just the description of a geometric random variable with "success" probability $1 - m(s, \omega)$. The previous observation allows us to obtain the required expression as

$$E(\beta^T | m(s, \omega)) = \sum_{t=1}^{\infty} \beta^t \Pr(T = t) = \sum_{t=1}^{\infty} \beta^t m(s, \omega)^{t-1} (1 - m(s, \omega)) = \frac{(1 - m(s, \omega)) \beta}{1 - \beta m(s, \omega)}.$$

Plugging this back we see that the processor expected profits are

$$E(\Pi(y, s)) = \frac{\pi(y, s; a)}{1 - \beta m(s, \omega)}.$$

Suppose first that the choice of the output rate is independent of the optimal QAS. This may be the case where output and safety are nonjoint in inputs (see Chambers (1989), or when investments in QAS do not depend on the rate of output. The choice of output in this case is the standard monopolist profit maximization problem. To focus on the selection of stringency of controls, we assume that processor's only procure and sell one unit of the good (or the optimal and independently chosen $y^*$, which is fixed here). The problem then reduces to the choice of investments in QAS that maximizes profits as follow

$$\max_{s \epsilon S} E(\Pi(1, s)) = \max_{s \epsilon S} \frac{\pi(1, s; a)}{1 - \beta m(s, \omega)}.$$

A quick look at this problem reveals that the easier it is for a processor to acquire information about quality (i.e., the cheaper it is to implement a QAS that yields a given level of certainty about quality), the more likely it is that a QAS will be implemented. A rising

\[ \text{Note } m(s, \omega) = \lambda(s) + (1 - \lambda(s))(1 - \omega) = (1 - \omega) + \omega \lambda(s) \text{ is the convex combination between the true probability of having a product of high quality, and one. We see again that as } \omega \rightarrow 0, m(s, \omega) \rightarrow 1, \text{ and the processors are expected to stay in state one for a large number of periods, even if they are offering a product that does not meet the promised standards.} \]
price premium also would increase the likelihood that there exists a profitable QAS. The first order condition with respect to $s$ is

$$\frac{\partial E(\Pi(1,s))}{\partial s} = \frac{\partial \pi(1,s;a)}{\partial s} (1 - \beta m(s,\omega)) + \pi(1,s;a) \beta \frac{\partial m(s,\omega)}{\partial s} \leq 0$$

with equality if $s^* > 0$. Note that $\frac{\partial \pi(1,s,a)}{\partial s} = -\frac{\partial C(1,s)}{\partial s}$. The second order sufficient condition (S.O.S.C.) given by

$$\frac{\partial^2 E(\Pi(1,s))}{\partial s^2} = \frac{\partial^2 \pi(1,s,a)}{\partial s^2} + \pi(1,s,a) \beta \frac{\partial^2 m(s,\omega)}{\partial s^2} \frac{\partial m(s,\omega)}{\partial s} \leq 0$$

is assumed to hold.

Equation (3.2) has the usual interpretation. It states that the level of stringency should be increased until the marginal benefits of increased stringency equal the marginal costs. Marginal benefits of an increase in $s$ equals the change in the proportion of purchases that are of high quality multiplied by the probability that low quality output will be discovered\textsuperscript{16}, the per-period profit rate and a factor that takes into account the multi-period nature of the problem at hand. The marginal benefits of increased assurance rise as the quality of the good is more readily observable by the processor’s customers, and as the potential punishments for false certification become more severe. Switching to the second state of the world is a harsher punishment when per-period profits are high, and the future is important to the processor. The marginal cost of an increase in $s$ is simply the increase in costs that must be incurred to implement a more stringent QAS.

In this problem it is straightforward to show that the optimal level of investments in quality assurance is unambiguously increasing in the size of the market (or strength of the demand for value added products), and the value processor’s place in the future. These

\textsuperscript{16} Note that $\frac{\partial m(s,\omega)}{\partial s} = \omega \frac{\partial \lambda(s)}{\partial s}$
comparative statics can be respectively represented by
\[
\frac{\partial s^*}{\partial \alpha} = -\frac{\left( \frac{\partial \pi(1,s,a)}{\partial a} \frac{\partial m(s,\omega)}{\partial s} \right)}{S.O.S.C.} > 0
\]
and
\[
\frac{\partial s^*}{\partial \beta} = -\frac{\left( \frac{\partial \pi(1,s,a)}{\partial s} m(s,\omega) + \pi(1,s,a) \frac{\partial m(s,\omega)}{\partial s} \right)}{S.O.S.C.} > 0.
\]

We next show that the ability of consumers to perceive quality also increases the optimal level of stringency of the QAS. This can be seen by implicit differentiation of equation (3.2) (assuming an interior solution, and dropping the arguments of the functions to reduce notational clutter)
\[
\frac{\partial^2 \pi}{\partial s^2} (1-\beta m) + \pi \beta \frac{\partial^2 m}{\partial s^2} \frac{\partial s^*}{\partial \omega} + \pi \frac{\partial^2 m}{\partial s \partial \omega} \frac{\partial s}{\partial \omega} - \beta \frac{\partial \pi}{\partial s} \frac{\partial m}{\partial \omega} = 0.
\]

The term in square brackets is the usual second order conditions of the maximization problem, and hence negative. Therefore
\[
\text{sgn} \left( \frac{\partial s^*}{\partial \omega} \right) = \text{sgn} \left( \pi \beta \frac{\partial^2 m}{\partial s \partial \omega} - \beta \frac{\partial \pi}{\partial s} \frac{\partial m}{\partial \omega} \right).
\]

Note that \( \frac{\partial^2 m}{\partial s \partial \omega} \) and \( \frac{\partial \pi}{\partial s} \frac{\partial m}{\partial \omega} \) are both positive, and hence we cannot determine this sign directly. To show that this difference is positive add equation (3.2) to it (which is the first order condition of the problem and hence zero), multiplied by \( \frac{\beta}{\partial \omega} \frac{1}{(1-\beta m)} \) to obtain
\[
\text{sgn} \left( \frac{\partial s^*}{\partial \omega} \right) = \text{sgn} \left( \pi \beta \frac{\partial^2 m}{\partial s \partial \omega} - \beta \frac{\partial \pi}{\partial s} \frac{\partial m}{\partial \omega} + \frac{\partial \pi}{\partial s} \frac{\partial m}{\partial \omega} \frac{1}{(1-\beta m)} \right)
\]
which simplifies to
\[
\text{sgn} \left( \frac{\partial s^*}{\partial \omega} \right) = \text{sgn} \left( \frac{\pi \beta}{(1-\beta m)} \frac{\partial \lambda}{\partial s} (1-\beta) \right) \geq 0.
\]

Hence, as consumers become more able to discern quality, processors find it optimal to increase their expenditures in reputation-preserving devices such as QASs. This reveals that as the economic loss from incurring a type II error and/or being discovered increases, processors will be more careful about the product he/she certifies.
A more interesting and perhaps more realistic situation arises when the choice of the level of output is not independent of the investments in quality. Antle (2001) classifies quality control technologies for producing quality-differentiated goods as process control, inspection, testing and identity preservation. He argues that all these technologies, except testing, affect the variable costs of production. However the costs of the testing technologies are not independent of the rate of output, since it typically involves sampling a small proportion of the product. Weaver and Kim (2002) also model quantity and quality as competing goals.\footnote{They model quality as a product diversion process.}

In this case, we can write the processor’s problem, which is to choose the optimal QAS and output level to maximize expected profits as follows

\[
\max_{y>0, s \in S} E(\Pi(y,s)) = \max_{y>0, s \in S} \frac{\pi(y,s,a)}{1 - \beta m(s,\omega)}.
\]

Straightforward applications of the envelope theorem confirm the intuitive result that processors are better off when they face a larger demand in state 1 (higher \(a\)), when they are more “patient” (higher \(\beta\)), and when it is harder for consumers to determine true output quality (lower \(\omega\)). In particular, the envelope theorem immediately indicates that

\[
\frac{\partial E(\Pi(y,s))}{\partial a} = \frac{\partial \pi}{\partial a} \frac{1}{(1 - \beta m(s,\omega))} \geq 0, \quad \frac{\partial E(\Pi(y,s))}{\partial \beta} = \pi \frac{m(s,\omega)}{(1 - \beta m(s,\omega))^2} \geq 0, \quad \text{and}
\]

\[
\frac{\partial E(\Pi(y,s))}{\partial \omega} = \pi \beta \frac{\partial m(s,\omega)}{\partial \omega} \frac{1}{(1 - \beta m(s,\omega))^2} \leq 0.
\]

A direct implication of the last derivative is that as long as the certification system is imperfect \(\lambda(s) = 1 - F(q^M | s) < 1\), the profitability of participation in the market for quality-assured products is hindered by increased consumer awareness.

The first order conditions for this problem are given by

\[
\frac{\partial E(\Pi(y,s))}{\partial y} = \frac{\partial \pi(y,s;a)}{\partial y} \frac{1}{1 - \beta m(s,\omega)} \leq 0, \quad y \geq 0.
\]
\[
\frac{\partial E(\Pi(y,s))}{\partial s} = \frac{\partial \pi(y,s,a)}{\partial s} \frac{1}{1 - \beta m(s,\omega)} + \pi(y,s,a) \frac{\beta \frac{\partial m(s,\omega)}{\partial s}}{(1 - \beta m(s,\omega))} \leq 0, \quad s \geq 0,
\]
and the corresponding complementary slackness conditions.

The first order conditions can in principle be solved to obtain the optimal choices for output and stringency of controls represented by \( y^*(\omega,\beta,a) \) and \( s^*(\omega,\beta,a) \) respectively.

These conditions are equivalent to
\[
\frac{\partial E(\Pi)}{\partial y} = \frac{\partial \pi(y,s,a)}{\partial y} \leq 0, \quad y \geq 0
\]
\[
(3.3)
\]
\[
\frac{\partial E(\Pi)}{\partial s} = \frac{\partial \pi'(y,s,a)}{\partial s} + \pi'(y,s,a) \frac{\beta \frac{\partial m(s,\omega)}{\partial s}}{(1 - \beta m(s,\omega))} \leq 0, \quad s \geq 0.
\]

The Hessian for this problem is
\[
H = \begin{pmatrix}
\frac{\partial^2 \pi(y,s,a)}{\partial y^2} & \frac{\partial^2 \pi(y,s,a)}{\partial y \partial s} \\
\frac{\partial^2 \pi(y,s,a)}{\partial y \partial s} & \frac{\partial^2 \pi(y,s,a)}{\partial s^2} + \pi(y,s,a) \frac{\beta \frac{\partial^2 m(s,\omega)}{\partial s^2}}{(1 - \beta m(s,\omega))}
\end{pmatrix}
\]
which implies that the second order sufficient conditions will be satisfied when
1) \( \frac{\partial^2 \pi(y,s,a)}{\partial y^2} \leq 0 \)

2) \(|H| = \frac{\partial^2 \pi(y,s,a)}{\partial y^2} \left( \frac{\partial^2 \pi(y,s,a)}{\partial s^2} + \pi(y,s,a) \frac{\beta \frac{\partial^2 m(s,\omega)}{\partial s^2}}{(1 - \beta m(s,\omega))} \right) - \left( \frac{\partial^2 \pi(y,s,a)}{\partial y \partial s} \right)^2 \geq 0 \)

3) \( \frac{\partial^2 \pi(y,s,a)}{\partial s^2} + \pi(y,s,a) \frac{\beta \frac{\partial^2 m(s,\omega)}{\partial s^2}}{(1 - \beta m(s,\omega))} \leq 0 \), (implied by 1) and 2) together),

hold.
Natural questions to ask are what are the optimal responses in terms of output and choice of quality assurance system as the parameters change. That is, we are asking what are the signs of $\frac{\partial y^*}{\partial \omega}$, $\frac{\partial s^*}{\partial \omega}$, $\frac{\partial y^*}{\partial \beta}$, $\frac{\partial s^*}{\partial \beta}$, $\frac{\partial v^*}{\partial a}$, and $\frac{\partial s^*}{\partial a}$. Some of these questions can be answered by conducting comparative statics in the proposed model. However, the structure of the problem makes the signs of the last two derivatives inherently ambiguous. The intuition is that as $a$ increases, there is an incentive to increase the rate of output, which may in turn partially offset the gain in price. Increasing the output rate has potentially the double effect of reducing per unit price, and increasing production costs (in a case of decreasing returns to scale technologies). Also, increases in the profitability of the market for value added products provide incentives to monitor more closely product quality, and delay the transition to the second state. However, this has to be weighted against the costs incurred in doing so.

We now tackle the question of the optimal choices of output and QAS as quality becomes more readily discernible. Differentiating system (3.3) partially with respect to $\omega$, using the chain rule, we get (after some rearrangement)

$$
\begin{pmatrix}
\frac{\partial^2 \pi(y, s; a)}{\partial y^2} + \frac{\partial^2 \pi(y, s; a)}{\partial y \partial s} \\
\frac{\partial^2 \pi(y, s; a)}{\partial y \partial s} - \frac{\partial^2 \pi(y, s; a)}{\partial s^2} + \pi(y, s; a) \frac{\beta \frac{\partial^2 m(s, \omega)}{\partial s^2}}{(1 - \beta m(s, \omega))} \\
- \frac{\pi(y, s; a) \beta}{(1 - \beta m(s, \omega))^2} \left( \frac{\partial^2 m(s, \omega)}{\partial s \partial \omega} \left( (1 - \beta m(s, \omega)) \right) + \beta \frac{\partial m(s, \omega)}{\partial s} \frac{\partial m(s, \omega)}{\partial \omega} \right)
\end{pmatrix} =
\begin{pmatrix}
\frac{\partial y^*}{\partial \omega} \\
\frac{\partial s^*}{\partial \omega} \\
\frac{\partial v^*}{\partial a} \\
\frac{\partial s^*}{\partial a}
\end{pmatrix}
$$

(3.4)

Recalling that $m(s, \omega) = \lambda(s) + (1 - \lambda(s))(1 - \omega)$, and applying the same technique used in the previous problem, the second element of vector in the right hand side of the

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18 Note that the parameter $a$ enters by itself in both equations of system (3.3). See Silberberg (1990).

19 For the commonly studied case of linear demand and constant marginal cost, both $\frac{\partial y^*}{\partial a}$ and $\frac{\partial s^*}{\partial a}$ are positive.
system (3.4) can be written again as 

$$- \frac{\pi \beta}{(1 - \beta m(s, \omega))^2} \frac{\partial \gamma(s)}{\partial s} (1 - \beta)$$

which makes its negative sign clear. Samuelson's conjugate pairs theorem immediately asserts \( \frac{\partial \gamma^*}{\partial \omega} > 0 \). As consumers become more able to discern quality, processors will find it optimal to adopt more stringent controls. This result is similar in a sense to one of the findings of Darby and Kami (1973). These authors argued that it is very likely (albeit not necessarily true) that as consumers become more knowledgeable, the optimal amount of fraud is reduced. In our paper, firms would have incentives to reduce the number of mistakes they make as consumers become increasingly able to discern qualities (or become more informed).

There exists a key trade-off between the benefits and costs of information acquisition on the part of processors. Having a more precise QAS, though costly, decreases the probability that firms will lose consumers trust. Furthermore, as the expected losses derived from consumer distrust increase, the payoff from processor becoming better informed about actual quality increases.

Using Cramer's rule to solve for \( \frac{\partial y^*}{\partial \omega} \), we find that its sign is ambiguous without imposing further structure, since \( \omega \) enters by itself in the second equation of system (3.3). Moreover, system (3.4) tells us that \( \text{sgn} \left( \frac{\partial y^*}{\partial \omega} \right) = \text{sgn} \left( \frac{\partial^2 \pi}{\partial y \partial s} \right) \), which is not implied by the maximization hypothesis alone. Since it is reasonable to assume that raising the levels of controls increases the marginal costs of production, and noting that 

$$\frac{\partial^2 \pi}{\partial y \partial s} = -\frac{\partial^2 C}{\partial y \partial s},$$

we expect the optimal output rate to decrease as \( \omega \) increases.

The question of how the value that producers place on the future affects the optimal choices of QAS and output levels can be explored through a similar exercise. The derivations are as follows.

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\(^{20}\) Note that in Darby and Kami’s (1973) paper, supplying firms knew the actual quality of the product (repair services) they were offering.
\[
\begin{pmatrix}
\frac{\partial^2 \pi(y,s,a)}{\partial y^2} & \frac{\partial^2 \pi(y,s,a)}{\partial y \partial s} & \frac{\partial^2 \pi(y,s,a)}{\partial s^2} \\
\frac{\partial^2 \pi(y,s,a)}{\partial y \partial s} & \beta \frac{\partial^2 m(s,\omega)}{\partial s^2} + \pi(y,s,a) & \frac{\partial}{\partial s} \\
\frac{\partial^2 \pi(y,s,a)}{\partial s^2} & \frac{\partial}{\partial s} & 0
\end{pmatrix}
\begin{pmatrix}
\frac{\partial y^*}{\partial \beta} \\
\frac{\partial s^*}{\partial \beta}
\end{pmatrix}
= \begin{pmatrix}
\frac{\partial m(s,\omega)}{\partial s} \frac{1}{1 - \beta m(s,\omega)} \left( \frac{\beta m(s,\omega)}{1 - \beta m(s,\omega)} + \pi(y,s,a) \right) \\
\frac{\partial m(s,\omega)}{\partial s} \frac{1}{1 - \beta m(s,\omega)} \left( \frac{\beta m(s,\omega)}{1 - \beta m(s,\omega)} + \pi(y,s,a) \right) \\
0
\end{pmatrix}
\]

Solving the system by Cramer's rule we obtain

\[
\frac{\partial y^*}{\partial \beta} = \frac{\left[ \frac{\partial m(s,\omega)}{\partial s} \frac{1}{1 - \beta m(s,\omega)} \left( \frac{\beta m(s,\omega)}{1 - \beta m(s,\omega)} + \pi(y,s,a) \right) \right] \frac{\partial^2 \pi(y,s,a)}{\partial y \partial s} - \left[ \frac{\partial m(s,\omega)}{\partial s} \frac{1}{1 - \beta m(s,\omega)} \left( \frac{\beta m(s,\omega)}{1 - \beta m(s,\omega)} + \pi(y,s,a) \right) \right] \frac{\partial^2 \pi(y,s,a)}{\partial s^2}}{|H|}
\]

\[
\frac{\partial s^*}{\partial \beta} = -\frac{\left[ \frac{\partial m(s,\omega)}{\partial s} \frac{1}{1 - \beta m(s,\omega)} \left( \frac{\beta m(s,\omega)}{1 - \beta m(s,\omega)} + \pi(y,s,a) \right) \right] \frac{\partial^2 \pi(y,s,a)}{\partial y \partial s} - \left[ \frac{\partial m(s,\omega)}{\partial s} \frac{1}{1 - \beta m(s,\omega)} \left( \frac{\beta m(s,\omega)}{1 - \beta m(s,\omega)} + \pi(y,s,a) \right) \right] \frac{\partial^2 \pi(y,s,a)}{\partial s^2}}{|H|}
\]

thus we know

\[
\text{sgn} \left( \frac{\partial y^*}{\partial \beta} \right) = \text{sgn} \left( \frac{\partial^2 \pi(y,s,a)}{\partial y \partial s} \right)
\]

\[
\frac{\partial s^*}{\partial \beta} \geq 0
\]

As the future becomes more important, it is more valuable for processors to invest in quality assurance systems that give them a longer expected presence in the market. The sign of \(\frac{\partial y^*}{\partial \beta}\) is ambiguous as before (and due to the exact same reasons). However, the discussion above suggests it is negative. Increasing the expenses incurred to "learn" about the actual quality of the good increases variable costs of production, and hence it is optimal to cut back on the output rate.

To summarize, we have modeled the decision of a monopolistic processor that chooses her per period output rate and the stringency of the QAS to implement in a potentially infinitely lived market. The duration of the market is unknown to the processor.
However, she can influence it (stochastically) through her choices. In this setting, processors or firms implementing more stringent QAS are expected to keep their reputation for a larger number of time periods. Our main findings thus far are that a) the stringency of the QAS will be higher for more easily discoverable traits, more patient firms, and more attractive markets (only when the output rate is fixed); b) firms are more likely to implement a QAS when the future is important, the quality trait is harder to observe, and of course when the demand for the differentiated product is stronger; and c) the effect of both the discoverability of the quality trait and the value firms place on the future on the per-period output rate is in general ambiguous, however we argued that an inverse relationship between both variables and the output rate is more likely.

The current analysis ignores the potential existence of other firms in the market. Strategic interactions among firms are important, especially in markets susceptible to free riding and Akerlof's lemons problem. To study strategic interactions between firms, we need to have at least two participants in the market. The next section develops a duopoly model to achieve that end.

### 3.2.1.2 Strategic interactions between firms

#### 3.2.1.2.1 Reputation is a public good

We consider first the situation where the entire industry can lose consumers trust as a result of the actions of one participant. In this situation, consumer trust can be viewed as a public good, and as we will see, in some situations firms may have incentives to free ride on the rival’s investments in safety or quality. An industry that has recently experienced such loss of reputation is the beef chain, as a consequence of the appearance of “mad-cow” disease (bovine spongiform encephalopathy, or BSE) in Europe (Northen, 2001), Canada and the United States.

Henessy, Roosen and Miranowski (2001) model a related issue, and conclude that even when firms in the food industry may profit by increased investments in safety (assuming a leadership role), they may obtain higher levels of benefits by free riding on the efforts of other chain participants.

---

21 Note that the current problem is different than the one considered by Akerlof, since processors are only imperfectly informed about the actual quality of a given unit of product they offer for sale.
Suppose consumers buy a product as long as no irregularity or hazard is detected. In this framework, we still have two possible states of the world. In the first state, all processors have a good reputation, whereas in the second all sellers are punished by consumers. This is likely to occur when it is hard to differentiate the product of different producers, or when consumers believe that all participants in the industry follow similar procedures. This case can be contrasted to the "notorious firm" situation to be described next.

We model this situation as a potentially infinitely lived game, where the termination time is random. The uncertainty comes again from the fact that processors cannot exert absolute control on the quality of the good they procure and/or produce. Rob and Sekiguchi (2001) model a similar situation. However, in their model firms compete in price in the second period, and only one firm is able to make sales until it loses its reputation and the other occupies its place. When the second firm loses the market, consumers switch back to the first firm and so on.

As in the previous sections, we assume that the industry will be trusted until proven wrong. In the first stage, the two processors once and for all decide independently and in a non-cooperative fashion, what QAS to implement. After observing each others choice of QAS, processors compete a la Cournot. Therefore, production and QAS technologies are known by both firms. Consumers buy the product at the end of the first period and update their beliefs about the quality of the industry's output. In the periods to follow, firms keep competing in output, and consumers keep updating their beliefs, until a failure occurs and is detected by consumers. In that period confidence is lost by the entire industry forever. In terms of a stochastic process, the second state of the world is an absorbing one. The monopolist problems analyzed in the previous sections can be thought of as following the same structure; first choose QAS, and then rate of output. However, since there is only one decision maker in the monopoly model, the result is clearly the same as under the simultaneous choice modeled.

As is standard for this sort of dynamic programming problems, we work backwards. We first find the equilibrium level of output after technologies have been chosen, and then

\[22\] Klein and Leffler (1980) refer to the "notorious firm" situation, as one in which every consumer learns about a firm that has cheated.
solve the first stage, keeping the second period equilibrium rules in mind. The second stage problem is a standard Cournot game. Following the same argument used to construct the monopolist’s objective function of the previous section we find that firm $i$’s expected profits are

$$E \Pi^{i,l}(y, s) = \frac{\pi^{i,l}(y, y_{-i}, s_i)}{1 - \beta m_i(s_i) m_{-i}(s_{-i})},$$

where $\pi^{i,l}$ denotes per-period profits of player $i$ in state 1, and $y = (y, y_{-i})$ represents a vector of output. Because per-period profits are zero if the firm has lost its reputation, we drop the superscript that indexes the state of the world. Since both $s_i$ and $s_{-i}$ are predetermined, the problem is just to choose the level of output that maximizes per-period profits in the standard way to find the Nash equilibrium levels $y^*_i(s_i, s_{-i}), i = 1, 2$. We can now write the first stage problem for firm $i$ as

$$\max_{s_i \in S} \left( \frac{\pi^i(y^*_i(s_i, s_{-i}), y^*_{-i}(s_i, s_{-i}), s_i)}{1 - \beta m_i(s_i) m_{-i}(s_{-i})} \right) = \max_{s_i \in S} \left( \frac{\pi^i(s_i, s_{-i})}{1 - \beta m_i(s_i) m_{-i}(s_{-i})} \right)$$

which has first order conditions given by

$$\frac{\partial \pi^i(s_i, s_{-i})}{\partial s_i} \left( 1 - \beta m_i(s_i) m_{-i}(s_{-i}) \right) + \pi^i(s_i, s_{-i}) \beta m_{-i}(s_{-i}) \frac{\partial m_i(s_i)}{\partial s_i} \leq 0 \quad s_i \geq 0 \quad i = 1, 2$$

$$\Leftrightarrow$$

$$\frac{\partial \pi^i(s_i, s_{-i})}{\partial s_i} + \frac{\pi^i(s_i, s_{-i}) \beta m_{-i}(s_{-i})}{\left( 1 - \beta m_i(s_i) m_{-i}(s_{-i}) \right)} \frac{\partial m_i(s_i)}{\partial s_i} \leq 0 \quad s_i \geq 0 \quad i = 1, 2 \quad (3.5)$$

and the corresponding complementary slackness conditions. Note that if it is difficult to detect quality deviations (i.e. when $\omega \rightarrow 0$), the model predicts that processors will find it optimal not to invest in quality assurance (the solution will tend to the corner
The same holds of course if processors do not value future periods, or when the probability that your rival is caught is close to one.

The second order sufficient conditions are

\[
A = \begin{bmatrix} a_{i,1} & a_{i,2} \\ a_{2,1} & a_{2,2} \end{bmatrix} \frac{1}{(1 - \beta m_i(s_i) m_{i-1}(s_{i-1}))^2} = \begin{bmatrix} \frac{\partial^2 E \Pi^{i}}{\partial s_i^2} & \frac{\partial^2 E \Pi^{i}}{\partial s_i \partial s_{i-1}} \\ \frac{\partial^2 E \Pi^{i}}{\partial s_{i-1} \partial s_i} & \frac{\partial^2 E \Pi^{i}}{\partial s_{i-1}^2} \end{bmatrix}
\]

\[
a_{i,j} = \frac{\partial^2 \pi^i(s_i, s_{i-1})}{\partial s_i^2}(1 - \beta m_i(s_i) m_{i-1}(s_{i-1})) + \pi^i(s_i, s_{i-1}) \frac{\partial^2 m_i(s_i)}{\partial s_i^2} - \beta m_i(s_i) \leq 0
\]

\[
a_{i,j} = \frac{\partial^2 \pi^i(s_i, s_{i-1})}{\partial s_i \partial s_{i-1}}(1 - \beta m_i(s_i) m_{i-1}(s_{i-1})) + \frac{\partial \pi^i(s_i, s_{i-1})}{\partial s_i} \frac{\partial m_i(s_i)}{\partial s_i} + \frac{\pi^i(s_i, s_{i-1}) \beta}{\partial s_i} \frac{\partial m_i(s_i)}{\partial s_i} + \frac{\pi^i(s_i, s_{i-1})}{(1 - \beta m_i(s_i) m_{i-1}(s_{i-1}))}
\]

and \( \Omega = a_{i,i} - a_{i,i} a_{i-1,i} > 0 \), for \( i = 1, 2 \).

The system of equations (3.5), implicitly define the processor’s best response functions \( b_i(s_{i-1}) \), \( i = 1, 2 \). A vector of investments in safety \( (s^*_1, s^*_2) \) is a Nash equilibrium for this model if and only if \( s^*_i = b(s^*_{i-1}) \) for \( i = 1, 2 \). If an equilibrium for this stage exists, we can then compute the output quantities and prices using the equilibrium rules for the Cournot game that will be played at the second stage.

We now investigate the nature of the competition outlined studying a key property, the slope, of the best response functions. To that end we substitute the best response function of firm \( i \) into its first order condition, and differentiate it (at an interior solution) with respect to the choice of its rival. After rearranging (and omitting arguments for brevity) we get

\[
\frac{\partial b(s_{i-1})}{\partial s_{i-1}} = -\frac{\partial^2 \pi^i}{\partial s_i \partial s_{i-1}}(1 - \beta m_i m_{i-1}) + \frac{\partial \pi^i}{\partial s_i} \frac{\partial m_i}{\partial s_i} \beta m_{i-1} + \frac{\pi^i \beta}{\partial s_i} \frac{\partial m_i}{\partial s_i} + \frac{\pi^i}{(1 - \beta m_i m_{i-1})}
\]

\[
\frac{\partial^2 \pi^i}{\partial s_i^2}(1 - \beta m_i m_{i-1}) + \pi^i \frac{\partial^2 m_i}{\partial s_i^2} \beta m_{i-1}
\]

---

23 To see this, recall that \( m(s_i) = 1 - \omega + \omega \lambda(s_i) \), and \( \lim_{\omega \to 0} \frac{\partial m_i(s_i)}{\partial s_i} = \lim_{\omega \to 0} \omega \frac{\partial \lambda(s_i)}{\partial s_i} = 0. \)
The denominator of this expression is negative by the sufficient conditions for a maximum. Then, the sign of the slope of the best response function is the same as the sign of the numerator (see Dixit, 1986). The first term is negative, since as one firm increases investments in safety, it reduces the benefits derived from increases in their rival’s expenditures in QASs. By a similar argument, and the assumption that more stringent systems yield better products, the last two terms are positive. Overall the sign is ambiguous without further structure.

However, some features can be read from the form of the previous equation: (a) if the probability of success (stay in the good state of the world) for both processors is high, and the future is valued, the best response function will slope upwards (b) if the cross-partial terms are close to zero, for example if the products are not close substitutes, the sign will also be positive. But neither condition is necessary to obtain upward sloping reaction functions. Downward sloping reaction functions will arise only when the cross effects are strong compared to the profitability of the industry. Of course, nothing says that the reaction curves will be monotonic.

Hence, the structure of the problem makes several types of interaction plausible. Free riding would be represented by downward sloping best response functions. In the term introduced by Bulow, Geanakoplos, and Klemperer (1985), quality assurance systems are “strategic substitutes”. If one firm invests heavily in QAS, it will be a weak competitor in the second stage. It then may be worthwhile for the other processor to free ride on the periods warranted by the other firm’s investments, and enjoy a high level of per-period profits for as long as the market lasts. Though mathematically plausible, it is hard to rationalize a situation where a processor would choose stringent levels of assurance, believing that its rival will put a lax system in place. If the processor’s rival invests little in quality, he/she will face a tough Cournot competitor for the few periods the market is expected to last, which act as a double incentive to implement less stringent QASs.

24 By increasing its first period investments in QASs, a firm is raising its own costs for the second stage competition, which benefits its rivals. However, these benefits are lower if the rivals also increase their costs.

25 Unless of course when $\frac{\partial m_i(s)}{\partial s_i} \frac{\partial m_{-i}(s_{-i})}{\partial s_{-i}} \to 0$, $i = 1, 2$, faster than $\beta m_i(s_i)m_{-i}(s_{-i}) \to 1$. 
A more plausible and intuitively appealing scenario is that of QAS being, in the language of Bulow, Geanakoplos, and Klemperer (1985), “strategic complements”. That is, reactions functions have an upward slope\(^{26}\). When a processor’s rival puts a lax QAS in place, he/she will find, as discussed in the previous paragraph, little incentive to invest in quality assurance. As the processor’s rival increases the stringency of the QAS, he/she will have two types of incentives to raise his/her investments. First, the market will last for a longer number of periods. Second, he/she will be facing a milder competitor, which increases per period margins and makes more stringent systems worthwhile.

The direction of the response in the levels of assurance when the capacity of consumers to perceive quality increases becomes ambiguous. Following Dixit (1986), we totally differentiate the system of first order conditions to get

\[
\begin{bmatrix}
    a_{1,1} & a_{1,2} \\
    a_{2,1} & a_{2,2}
\end{bmatrix}
\begin{bmatrix}
    1 \\
    1 - \beta m_1(s_1) m_2(s_2)
\end{bmatrix}
\begin{bmatrix}
    ds_1 \\
    ds_2
\end{bmatrix}
= \begin{bmatrix}
    \frac{\partial^3 E \Pi^1}{\partial s_1^2} & \frac{\partial^3 E \Pi^1}{\partial s_1 \partial s_2} \\
    \frac{\partial^3 E \Pi^2}{\partial s_1 \partial s_2} & \frac{\partial^3 E \Pi^2}{\partial s_2^2}
\end{bmatrix}
\begin{bmatrix}
    ds_1 \\
    ds_2
\end{bmatrix}
= \begin{bmatrix}
    \frac{\partial^3 E \Pi^1}{\partial s_1 \partial s_2 \partial \omega} \\
    \frac{\partial^3 E \Pi^2}{\partial s_2 \partial \omega}
\end{bmatrix}
\begin{bmatrix}
    \frac{\partial^3 E \Pi^1}{\partial \omega} \\
    \frac{\partial^3 E \Pi^2}{\partial \omega}
\end{bmatrix}
\]

where \(a_{i,i}\) and \(a_{i-1,i}\), for \(i = 1, 2\) are as above. The system can in principle be solved to obtain

\[
\begin{align*}
\frac{ds_1^*}{d\omega} &= \frac{1}{\Omega} \begin{bmatrix}
    -a_{2,2} & a_{1,2} \\
    a_{2,1} & -a_{1,1}
\end{bmatrix} \begin{bmatrix}
    \frac{\partial^3 E \Pi^1}{\partial s_1 \partial \omega} \\
    \frac{\partial^3 E \Pi^2}{\partial s_2 \partial \omega}
\end{bmatrix} \\
\frac{ds_2^*}{d\omega} &= \frac{1}{\Omega} \begin{bmatrix}
    -a_{2,2} & a_{1,2} \\
    a_{2,1} & -a_{1,1}
\end{bmatrix} \begin{bmatrix}
    \frac{\partial^3 E \Pi^1}{\partial s_2 \partial \omega} \\
    \frac{\partial^3 E \Pi^2}{\partial \omega}
\end{bmatrix}
\end{align*}
\]

where again \(\Omega = \left(a_{1,1}a_{2,2} - a_{1,2}a_{2,1}\right) > 0\), and \(a_{i,i} < 0\), for \(i = 1, 2\) are the stability conditions (see Dixit, 1986).

The decisions of both processors are equal, so that it suffices to analyze the responses of processor 1. From equation (3.6), the change in the optimal stringency of assurance for firm 1 is given by

\[
\frac{ds_1^*}{d\omega} = \frac{1}{\Omega} \begin{bmatrix}
    -a_{2,2} & \frac{\partial^3 E \Pi^1}{\partial s_1 \partial \omega} + a_{1,2} \frac{\partial^3 E \Pi^2}{\partial s_2 \partial \omega}
\end{bmatrix}.
\]

\(^{26}\) If the parameterization of the problem were such that the game is supermodular, upward slopping best response functions would be implied, and Nash equilibrium in pure strategies exist (See Milgrom and Roberts (1990). However we do not impose that structure a priori.
A first look at this expression suffices to observe that its sign is ambiguous in general. However, we can analyze some cases. From the stability conditions, we know that $\Omega$ is positive, and $a_{2,2}$ is negative. Nothing more can be said without imposing further structure. If symmetry is assumed (as we do) the cross partial terms are equal and hence we only need to sign one of them.

We study the case where quality assurance systems are strategic complements. From the previous analysis this case arises when $a_{1,2} > 0$. Using the stability conditions we immediately see that \(\text{sgn}\left(\frac{ds_1^*}{d\omega}\right) = \text{sgn}\left(\frac{\partial^2 E \Pi^t}{\partial s_1 \partial \omega}\right)\). Cross-partial differentiation of the objective function, and some rather tedious rearranging yields

\[
\frac{\partial^2 E \Pi^t}{\partial s_1 \partial \omega} = \frac{\pi^t(s_1, s_2)}{(1 - \beta m_1(s_1)m_2(s_2))^3} \frac{\partial \lambda(s_1)}{\partial s_1}(m_2(s_2)(2 - \beta m_2(s_2)) - 1),
\]

which implies \(\text{sgn}\left(\frac{ds_1^*}{d\omega}\right) = \text{sgn}\left(m_2(s_2)(2 - \beta m_2(s_2)) - 1\right)\). The expression within parenthesis is a quadratic equation with negative coefficient $-\beta \in (-1, 0)$. It is straightforward to check that the expression is positive if and only if $m_2 \in [m^0, 1]$, where $m^0$ is the only root (in the unit interval) of the quadratic equation. This shows that if assurance systems are complementary, processors will increase their investments in quality only if their rival's probability of success is above a certain threshold given by $m^0$. Note also that this condition is conducive to the property of strategic complementarity. For small values of $m_2$, processor 1 will find it optimal to reduce investments in quality as consumers become more knowledgeable.

In summary, we have extended the monopoly model presented above as a way to capture and explore plausible strategic behavior among firms in an environment where

\[\text{For strategic substitutes, the optimal adjustment will depend not only on the sign of the cross partial studied below, but also on the relative magnitude of } a_{1,1} \text{ and } a_{1,2}. \text{To see this, impose symmetry and rewrite}\]

\[
\frac{ds_1^*}{d\omega} = \frac{1}{\Omega^2} (-a_{2,2} + a_{1,2}) \frac{\partial^2 E \Pi^t}{\partial s_1 \partial \omega}.
\]
reputations are not firm specific. In that sense a discovered substandard performance by a firm will also have negative effects on other firms. The scenario was modeled as a potentially infinitely lived game, with two stages. In the first stage firms choose simultaneous and independently their QAS (once and for all), and output rates are chosen in the second stage, after the QAS of both firms is observed. In this scenario, the nature of the competition is ambiguous, in the sense that QASs can potentially be strategic complements or substitutes. However, we argued that the former is more plausible. For this case, we showed that firms will implement more stringent QASs as the ability of consumers to perceive quality increases (or the absence of the trait is more easily discoverable) only if their rival’s probability of success is sufficiently high (or equivalently if they have a QAS of enough stringency in place).

3.2.1.2.2 Reputation is a private good

This section formalizes the situation where reputation is a private good. The structure of the problem is similar in the sense that firms have to choose the optimal level of investments in quality assurance systems, which affect quality stochastically, and then compete in quantities for the random number of periods in which they have consumer’s approval. The key difference with the previous section is that in this scenario when a processor fails, the other is benefited because he/she increases his/her market share. We still maintain the assumption that firms will be trusted until proven wrong.

The sequence of the model is as follows. In the current period (e.g. period zero), processors, independently and non-cooperatively, choose the stringency of the quality assurance they wish to implement. After observing the choice of their competitor, they compete a la Cournot, until one processor loses his/her reputation. Unlike the situation where reputation is a public good, now the other processor has an "expanded" demand, and behaves as a monopolist until his/her product fails and is discovered by consumers.

To accommodate this sequencing we need to expand the number of states of the world from 2 to 4. In state 1, both processors are trusted. State 2 arises if processor 2 loses its reputation. In state 3, processor 2 acts as a monopolist. The market disappears when state 4 is reached (both processors lose consumer’s goodwill). Reputations can then be modeled as following a stochastic process, where the probabilities of reaching the different states of the
world are affected by the choices made by the market participants. Since only the immediate past determines the state of the world in the following period, the stochastic process exhibits the Markov property. Therefore, it is natural to use the concept of a Markov process to model the dynamics of this market.

A Markov process is defined by the possible states of the system, the transition matrix, and a vector that records the initial state (Ljungqvist and Sargent (2000)). The states of the world are described in the previous paragraph, and the assumption that processors will be trusted until proven wrong is equivalent to assuming that the system starts at the first state with probability one (and hence the remaining three entries of the initial state vector are assigned zero probability). The transition matrix $M$ is as follows:

$$
M = \begin{pmatrix}
m_1 m_2 & m_1 (1-m_2) & m_2 (1-m_1) & (1-m_1)(1-m_2) \\
0 & m_1 & 0 & 1-m_1 \\
0 & 0 & m_2 & 1-m_2 \\
0 & 0 & 0 & 1
\end{pmatrix}.
$$

The probabilities of success $m_1(s_1)$ and $m_2(s_2)$ are defined as before. Then, entry $M_{i,j}$ denotes the probability that the system will be in state $j$ in the next period, given that the current state is $i$. Note that any state can follow state 1, but neither state 1 nor 3 can be reached after state 2. In the jargon of Markov chains, state 4 is absorbing, i.e. once it is reached the system stays in it forever, or alternatively, it will go to other states with probability zero. Note that as required in a Markov matrix, all the entries are nonnegative and $\sum_{j=1}^{4} M_{i,j} = 1$ for $i=1,\ldots,4$.\textsuperscript{28}

With this in place and noting that processor one makes zero profits in states 3 and 4, we can write his/her first stage problem as

$$
\max_{s_i \in S} E \Pi_i^l = \max_{s_i \in S} \left\{ \left( \pi_1^l \left( s_1, s_2 \right) \sum_{t=0}^{\infty} \beta^t e_1 M^t e_1^T \right) + \left( \pi_1^l \left( s_1, s_2 \right) \sum_{t=0}^{\infty} \beta^t e_2 M^t e_2^T \right) \right\}. \quad (3.7)
$$

\textsuperscript{28} Note that the case where reputations is a public good, is also a Markov process, but with only two possible states (that would correspond to the first and fourth states of the current setting).
For \( k = 1,2 \), \( \pi_k \), denotes per period profits for processor one when there are \( k \) market participants, \( e_k \) is a four by one vector that has zeros everywhere except for a 1 in the \( k \)th position, and \( T \) is the transpose operator.

Equation (3.7) specifies that processors maximize the profits in the duopoly and monopoly situation, weighted by the likelihood of the different scenarios, which they can affect by the stringency of the QAS they choose. The first order condition for this problem (omitting arguments for brevity) is given by

\[
\frac{\partial \pi_k}{\partial s_i} \sum_{i=0}^{\infty} \beta^i e_i M^i e_i^T + \pi_k \sum_{i=0}^{\infty} \beta^i e_i M^i e_i^T + \frac{\partial \pi_k}{\partial s_i} \sum_{i=1}^{\infty} \beta^i e_i M^i e_i^T + \pi_k \sum_{i=1}^{\infty} \beta^i e_i M^i e_i^T \leq 0, \tag{3.8}
\]

with equality if \( s_i > 0 \). Of course, a symmetric equation exists for the other processor.

Equation (3.8) has the usual interpretation. Processors will weight marginal benefits (given by the second and fourth terms in (3.8)) against marginal costs (given by the first and third terms in (3.8)) in their choice of QAS. Now there are new terms in the marginal benefits and costs side of the equation. These new terms are associated with the introduction of the possibility of being a monopolist if firm 2 loses its reputation first. An equilibrium for this model is again a pair of QASs \((s_1, s_2)\) such that \( s^*_i = b_i(s^*\cdot_i) \), \( i = 1,2 \), where \( b_i(\cdot) \) represents the best response function of player \( i \).

The nature of the competition, some comparative statics, and a comparison to the previous industry and reputation structures (monopoly and reputations as a public good) for this model is studied in the next section, under the special but frequently encountered case of linear demands and constant marginal costs functions.

### 3.3 Numerical simulations

Assume that consumer's valuation of the homogenous final product in each period can be represented by a utility function of the form

\[
U(y_1, y_2) = a(y_1I_1 + y_2I_2) - \frac{b}{2}(y_1^2 + y_2^2 + 2y_1y_2), \tag{3.9}
\]

where \( a \) and \( b \) are parameters, and \((I_1, I_2)\) are indicator functions denoting the state of the world. The specific form of the indicators will depend on the structure of the market, and the
nature of the reputations. Consumers choose quantities to maximize $U(y_1, y_2) - p_1 y_1 - p_2 y_2$. Consumers discount the future at the same rate as producers.

Processors convert inputs (e.g. farm output) into a good that can be for final consumption (this is consistent with the terminology we have been using so far), or used as an input by downstream processors or retailers (i.e. within a supply chain). In the latter case, equation (3.9) would be interpreted as the profit function of downstream processors or retailers. In any case, suppose that processor's transform the input into its product in a Leontief fashion with constant marginal costs given by their investments in QASs. Similar utility function and technologies (though they normalize marginal costs to zero) are used by Hueth and Marcoul (2002).

The link between stringency of the QAS and the probability of obtaining a product of good quality is given by the monotonic and concave function $\lambda(s) = \frac{s}{s+1}$. Therefore, we assume that if investments in quality are zero, processors will be obtaining a good product with probability zero. This sort of link is more likely to occur when quality is given by cost increasing practices. When the variability is natural, and processors just need to select what input to buy, it may be more reasonable to employ a link that assigns equal probability to obtaining a good product and incurring in statistical type II error.

### 3.3.1 Monopolist processor

The monopoly situation can be obtained by setting $I_2 = 0$. That is, the good produced by processor two is not valued. $I_1 = 1$ if the processor is trusted and zero otherwise. This yields an inverse demand function for good 1 given by $p_1(y_1) = a I_1 - b y_1$. Plugging this back in the monopolist problem we obtain

$$\max_{s, y_1, y_2 \geq 0} \frac{(a I_1 - b y_1) y_1 - s_1 y_1}{(1 - \beta (1 - \omega + \omega \lambda(s_1)))}$$

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20 An example would be $\lambda(s) = \frac{e^s}{e^s + 1}$

30 Note that this does not mean that consumers can distinguish between the two goods. It is just to be able to exclude processor 2 to study the monopolist problem as a special case of the general framework.
3.3.2 Duopoly situation when reputation is a public good

This case is obtained letting $I_1$ and $I_2$ equal one if no failure has been detected and zero otherwise. The inverse demand for processor $i$ is given by $p_i(y_1, y_2) = aI_i - b(y_1 + y_2)$. Then, equilibrium quantities and for the second stage and per period profits are easily checked to be $y_i^* = \frac{(a - 2s_i + s_{-i})}{3b}$, and $\pi_2^*(s_i, s_{-i}) = \frac{(a - 2s_i + s_{-i})^2}{9b}$, $i = 1, 2$. Plugging this in the first stage problem we get that processor’s 1 objective is given by

$$\max_{s_1 \in S} \frac{(a - 2s_1 + s_2)^2}{9b \left(1 - \beta \left(1 - \omega + \omega \lambda (s_1) \right) \left(1 - \omega + \omega \lambda (s_2) \right) \right)}$$

3.3.3 Duopoly situation when reputation is a private good

For this case, let $I_1$ equal one in states one and two, and zero otherwise. Also, define $I_2$ to be one in states one and three, and zero otherwise. As long as the stochastic process stays in the initial state (both processor’s have good reputation), per period profits are as in the duopoly situation presented above. When the system reaches states 2 or 3, the monopolist per period profit (also presented above) becomes relevant. Here the problem of processor 1 is

$$\max_{s_1 \in S} \left\{ \frac{(a - 2s_1 + s_2)^2}{9b} \sum_{j=0}^{\infty} \beta^j e_j M^j e^T_j + \frac{(a - s_1)^2}{4b} \sum_{j=1}^{\infty} \beta^j e_j M^j e^T_j \right\}$$

3.3.4 Results and discussion

In this section, we examine the solutions to the problems posed, and study how changes in the economic environment affect the equilibrium levels of stringency of the QAS implemented, associated expected profits for firms, and utility for consumers. Specifically, we study how the structure of the industry, nature of reputations, and ability of consumers to detect quality deviations affect the equilibrium outcomes.

Figure 3.1 presents the best response functions for both duopoly scenarios. Under the functional specifications and reputation being a public good, reaction curves slope upwards (this is the most likely scenario identified before). Processors find few incentives to invest in QAS, when the market is not going to last long (i.e. when their rivals invest little in quality), and they expect to face a tough (low cost) Cournot competitor in the following
periods. However, as discussed above, processors find it worthwhile to put more stringent systems in place if they anticipate their rivals will do the same thing. For the market to last more than a few periods, and compensate the investments in quality assurance, both players need to collaborate in this scenario.

On the other hand, when reputations are private goods, reaction curves have a negative slope. That is, when reputations are private goods, as processors anticipate their rivals will put a lax system in place, it is worthwhile to invest in a more stringent QAS. The driving force is that processors find it beneficial to give up some of the duopoly profits, while increasing the likelihood of outlasting the rival and capturing the entire market. But when a processor anticipates its rival will invest heavily in quality assurance (and becomes harder to outlast), the best he/she can do is to reduce his/her own expenses, and try to capture a higher per-period duopoly profit. However, note that a processor profit maximizing level of QAS is quite insensitive to its rival’s choice under the current parameterization.

Comparing the stringency of assurance across the different types of reputation, Figure 3.1 reveals that, as expected, the equilibrium investments in QAS are lower when reputation is a public good. This has the classic textbook interpretation. As firms do not capture the full returns of their QASs, they will under-invest in quality, and try to free ride on other firm’s

Figure 3.1. Best response functions of firms \((a = 50, b = 1, \omega = 0.5, \beta = 0.9)\)
investments. Branding for example, could be seen as an effort to convert reputations into a private good, and hence capture a higher share of the returns to investments in QAS.\textsuperscript{31} Economic theory predicts that if branding allows processors to capture the returns to their investments, it will result in higher levels of quality assurance. Also, though costlier QAS result in lower per period outputs, the expected total production (and hence consumption) is higher when reputation is a private good. This is driven by the fact that more stringent controls result in a longer duopoly stage, in addition to the monopoly periods afforded by being able to identify the firm that has provided a product that does not reach the standards promised (see Figure 3.2).

![Figure 3.2. Equilibrium responses of expected output to changes in quality observability (\( a = 50, b = 1, \beta = 0.9 \))](image)

When there is only one firm in the market, expected output will be higher than under the duopoly and public good nature of reputations (see Figure 3.2). However, expected output appears to be slightly higher in the duopoly with private reputations, for most levels of \( \omega \).

We now study the effects of consumers' ability to discern quality on the optimal levels of investments. As discussed before, when quality is a credence attribute, firms will

\textsuperscript{31} However, as Hennessy, Roosen, and Jensen (2002) note, branding also provides a target to consumers in the marketplace when a problem occurs (more on this below).
find it optimal to not invest in QAS. Firms do not have incentives to avoid punishments that have zero probability of occurrence. Figure 3.3 shows that as consumers can detect quality deviations more easily, the equilibrium levels of stringency increase, for all market structures and natures of reputation considered.\(^3\)

![Figure 3.3. Equilibrium responses of the stringency of controls to changes in quality observability (\(a = 50, b = 1, \beta = 0.9\))](image)

The level of stringency is always greater when reputation is a private good. Furthermore, it reveals that the monopolist will provide the highest quality (in expectations). Monopolists are the ones that would lose more if a quality deviation occurs and is detected. They have the entire market for themselves from the beginning. In the case of a duopoly, processors can at best hope to capture the market in the future, and of course future earnings are less valuable than current profits. However, it is not possible to state which situation yields higher welfare in general. Processors would prefer, as expected, to be the only market participants and reputations to be private rather than public goods (see Figure 3.4). It is also clear from Figure 3.5 that consumers prefer the private reputations situation to the collective reputations and monopoly scenario. However, consumer’s choice between monopoly and duopoly with public reputations depend on the level of quality observability (\(\omega\)). In short,

\(^3\) Though not presented, as the demand for high quality products, (parameterized by \(a\)) rises, the stringency of the QAS under all scenarios increases.
we can only say that the duopoly situation with collective reputations is Pareto dominated by
the other duopoly scenario considered.\footnote{Note if we consider that consumers value more the future than producers, or beta is close to 1 (higher than 0.98), the expected utility from the monopoly scenario exceeds that of the duopoly with public reputations for all $\omega$. In those cases, the monopoly scenario would also Pareto dominate the duopoly with public reputations.}

Figure 3.4 may explain the efforts of many firms to be able to distinguish and trace their products along the supply chain. For example, Niman Ranch has a trace-back system that allows its loins to be traced back to each producing farmer. Also, many food retailers sell their own-brand labeled products as high quality (Noelke and Caswell (2000)). The drawback of this strategy is that firms identify themselves when a problem occurs, and are clear targets in the marketplace (Hennessy, Roosen, and Jensen, (2002). If they can differentiate their products enough, firms could create a market for themselves (closer to a monopoly situation), or create a reputation that is independent from the actions of other market participants. That is to say, many of the investments in QAS can be seen as efforts to move away from a market where it is hard to identify who produced what, and hence reputations are collective, to a scenario where consumers can identify the firms that deliver goods of substandard quality.

![Figure 3.4. Equilibrium responses of expected profits to changes in quality observability ($a = 50, b = 1, \beta = 0.9$)](image)

Figure 3.4. Equilibrium responses of expected profits to changes in quality observability ($a = 50, b = 1, \beta = 0.9$)
The above discussion provides support for some of the policies proposed by Hennessy, Roosen and Jensen (2002), to overcome some causes of systemic risk in the food industry. The authors advocate for improving traceability, testing, mandate labeling, interpreting policies on mergers more leniently for the food industry, etc. Our results suggest that market concentration and/or identification of firms producing a given unit of output may result in higher overall welfare.

Figure 3.5. Equilibrium expected utility for the three scenarios considered for different levels of quality observability \((a = 50, b = 1, \beta = 0.9)\)

### 3.4 Conclusions

The examination of existing QASs reveals a great disparity in the stringency (and associated costs) of the systems employed. We provide a rationale for those differences based on the market structure, nature of the reputation mechanisms, and size of the markets or strength in demand for value-added products, when the sought after attributes are credence, experience, or a mixture of both. We argue that QAS can be seen as efforts made by firms by position themselves strategically in the market place. Firms may be trying to move away from a market where is hard to identify who produced what to an environment where consumers can identify the firm that deliver goods of substandard quality.

Three models are developed, to accommodate the different scenarios (monopoly, duopoly with collective reputations, and duopoly with private reputations) and predictions are obtained through comparative statics and numerical simulations. The later are performed
under the widely used assumptions of linear demands and constant marginal costs, given by
the levels of stringency chosen by processors.

Our results suggest that monopolists would invest more heavily in quality assurance
than duopolists. In addition, being able to capitalize the full returns (no free riding or
externalities) of investments in quality provides further incentives to employ more stringent
QASs. That is under the collective reputations scenario, duopolists will reduce their expected
quality. Also, as the ability of consumers to detect the actual quality of the good increases, so
does the stringency of the quality controls\textsuperscript{34}, for all scenarios. Perhaps not surprisingly, our
numerical simulations show that the size of the market (and hence the potential premiums)
has a positive impact on the level of investments in quality.

In terms of welfare, we can only say that the duopoly with private reputations Pareto
dominate the collective reputation scenario. However, less can be said about how the
monopoly situation compares to the other scenarios. Processors of course prefer the
monopoly over any of the duopoly situations. Consumers prefer the duopoly with private
reputations over the other two scenarios considered. However, under some conditions (when
the level of quality observability is high, or if consumers value the future highly), consumers
will prefer the monopoly over the duopoly with public reputations. Hence, our model
suggests that it is not clear that market concentration hurts consumers, when there is a
tradeoff between quantity and quality, and the later is imperfectly observable. In the same
line, if there is no way that the producer of a given unit of output can be identified among
several firms (it is not possible to develop private reputations), overall welfare may be
increased by promoting the existence of a monopoly (under the conditions just noted).

The model could be expanded in several directions. For example, the ability to
observe quality may be endogeneized, by giving the processor's customers (namely
consumers or downstream firms) the chance to expend resources and test or inspect the
processes employed, or introduce groups of consumers that do just that as in Feddersen and
Gillgan (2001). Alternatively, one could investigate other forms of punishment to producers
(in addition or in place of seclusion), such as liabilities under the relevant tort law.

\textsuperscript{34}This is shown to be true analytically for the monopolist case. For the duopoly with collective reputation, the
conditions for this to occur are identified. Only numerical predictions were performed for the duopoly with
private reputations scenario.
CHAPTER 4. TOWARDS THE DESIGN OF A
"GUARANTEED TENDER BEEF" CERTIFICATION
SYSTEM: TRADE-OFFS IN IMPERFECT CERTIFICATION
TECHNOLOGIES

4.1 Introduction

In most situations a participant in a supply chain has at best imperfect information about some quality attributes of a particular input she/he is buying, handling and/or processing, and selling to its downstream customers. In many cases, the quality of the final product, the destination, and/or appropriate processing of the input is contingent on these unobservable quality attributes. Assessing the quality of an input is particularly important in value-added agriculture, where new producer alliances are creating niche markets by differentiating their products by some experience or credence attributes. The success or failure of these ventures is often dependent on whether they are seen as dependable and trustworthy by their customers.

Participants in a supply chain can choose to implement some sort of quality assurance systems (QAS) as a way to attempt to learn some relevant aspect about those attributes, and provide quality certification to buyers (see Caswell, Bredahl, and Hooker, 1998). Assurance can potentially come from a system run by the alliances, from reliance on certification by a private or public third party, or both. For some examples from the meats sector see Lawrence (2002), or Carriquiry, Babcock, and Carbone (2003). The QASs presented in Chapter 3 vary in their precision and effectiveness in identifying products likely to satisfy consumers according to certain criteria. We identified several economic factors affecting the precision that any decision maker will choose to use in their supply chains. Important factors include the market structure, the discoverability of product quality by downstream chain participants,

1 For example, tough meats or non-juicy oranges have very different potential markets, uses, or require different preparation than their opposites.
2 The terms search, experience were introduced by Nelson (1970). The term credence attributes was coined by Darby and Karni (1973). For search and experience attributes, consumers can gain information about product quality before or after consumption respectively. Quality can not be learned by consumers (even after consumption) for credence attributes.
3 A partial list is organic, natural, tender beef, and free range. For a list of existing alliances in the beef sector and a rationale for that form of organization see Schroeder and Kovanda (2003).
and the nature of reputations. In Chapter 3, however, the quality level considered acceptable was fixed and known, but the firm was only able to affect product quality stochastically.

This chapter focuses on the use of a QAS to learn about the quality of a particular input used by a firm in a supply chain, in a framework similar to the one developed in the previous paper. However, it differs from the previous work in four ways: 1) we introduce variability in the level of quality that will be considered acceptable; 2) we are more explicit with regard to the sorting nature of the problem and the distribution of quality of both certified and rejected product; 3) we keep track separately of the costs of certification and those associated with not being able to observe quality and/or tastes and match customers accordingly; and 4) we focus on the choice of thresholds within a given testing or quality assessment protocol. This work also differs in scope, in the sense that it intends to be more applied by providing an easy to use framework that is flexible enough to accommodate different objectives and quality measurements (provided the relevant information is available).

The aim of the QAS considered here is to control a physical attribute of the input, which is discoverable by the firm’s customers only after purchase and use of the product. Thus, an experience attribute will be studied. For concreteness this article focuses on beef tenderness. However, the economic forces at work apply broadly to any supply chain that contains processed inputs with some imperfectly observable quality attribute and consumer heterogeneity. A need for certification from a trusted party comes into play due to the fact that tenderness is an experience attribute, and hence buyers can only make inferences about the true level of tenderness of a steak. Producers can use certification systems to provide information to consumers about beef tenderness (and associated eating experience). Consumers can use these cues to better select the product to buy.

Meat and animal scientists have dedicated much effort at finding objective measurements to infer the palatability and acceptability of beef by consumers to be used on a commercial scale to certify quality (or to provide cues to consumers willing to pay premiums for better beef). Considerable work has also been devoted to identify thresholds for those

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4 Tenderness has been recognized as the prime determinant of consumer satisfaction by both the industry and the meat and animal science community (more on this below).
measurements that can be used to sort beef into guaranteed tender and non-certifiable beef. The relevant literature is reviewed below. Identification of those thresholds, however, has proved elusive. The main problem identified by a review of the literature is a lack of clear objectives in defining appropriate thresholds. Optimal thresholds are contingent on the objective the producer wants to achieve and/or the environment. What is best for a given objective (e.g. minimization of errors in classification) may, as we show below miss the target grossly under another objective (e.g. profit maximization). The objective of this paper is to develop a method that can be used to determine optimal threshold levels in a tenderness certification scheme for niche beef marketing, paying special consideration to the goals to achieve by the classification procedure, and to the potential costs and benefits of each decision.

4.2 Literature review

4.2.1 Importance of perceptions

Perception of foods in general, and meats in particular, depend both on their intrinsic properties and on how they interact with external factors, such as price, information, and previous experience (Dransfield, Zamora, and Bayle, 1998). It is generally accepted that consumers’ expectations about product quality are based on perceptions of one or more quality cues (Steenkamp and Van Trijp, 1996). These authors refer to quality cues as any “informational stimuli that can be ascertained through the senses prior to consumption, and, according to the consumer, have predictive validity for the product’s quality performance upon consumption” (p.197). Quality cues can be divided into intrinsic and extrinsic. The former refer to attributes that are part of the physical product and cannot be changed without altering the product itself (e.g. color and marbling for beef). Extrinsic quality cues (e.g. price, brands, etc) are not part of the product, but rather the result of marketing efforts.

Providing consumers with information has a potentially significant impact on quality perception and preferences (Dransfield, Zamora, and Bayle, 1998). For example, samples of beef labeled “75% lean” received better quality ratings than identical samples labeled “25% fat” (Levin and Gaeth, 1988). Shackelford et al., (2001) indicated that 80% of consumers

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5 The most commonly used measure in experiments are Warner-Bratzler and slice shear force, the force needed to shear cooked beef (see for example, Shackelford et al. 1991; Huffman et al. 1996; Miller et al. 2001).
believed that steaks carrying a label “Tender Select” (and a statement guaranteeing tenderness) were more tender than other fresh beef cuts. Explicit information (in terms of guaranteed tenderness) significantly altered consumers’ perceptions and revealed preferences about steaks in a study conducted by Lusk et al. (2001). In that study, consumers were asked to test two samples coming from steaks that differed widely in terms of tenderness (measured by slice shear force). Information about the tenderness differences of the steaks increased the percentage of consumers preferring the tender steak from 69.16 to 83.72%, and the percentage of consumers willing to pay a premium from 36.12 to 51.16%. In another study, Boleman et al. (1997) found that informing people about the tenderness classification increased the proportion of steaks purchased by families (after tasting) from the tenderest category from 55.3% to 94.6%. The current discussion suggests that this provides credit to producers, in the sense that consumers’ acceptability (or perceptions) of products after consumption is positively affected by expected quality.

A study conducted by Dransfield, Zamora, and Bayle (1998) indicated that for some consumers quality is more important than price in determining purchasing decisions. According to the authors, there is a wide range of qualities in the market that do not command price differences because the eating experience is not known to the consumer prior to purchase. A careful sorting of carcasses and appropriate labeling could in principle be used by beef alliances to cash in on consumers’ willingness to pay for high quality products, in particular for “guaranteed tender beef”.

4.2.2 Guaranteed tender beef

Guaranteeing a good eating experience is a top priority for the meat industry in general, and the beef sector in particular. In this context, segregating carcasses by tenderness is increasingly becoming an important topic, especially in the beef sector which has seen a decline in demand every year from 1979 to 1998 (Purcell, 1998). The problem of inadequate beef tenderness is not new. It has appeared within the top quality concerns identified in the National Beef Quality Audits of 1991, 1995, and 2000 (conducted by Smith et al., 1991;

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6 In this study the information was provided in terms of the very suggestive names of “guaranteed tender” and “probably tough” (after an objective measurement of tenderness). “Guaranteed tender steaks had a slice shear force lower than 15 kg, whereas “probably tough” steaks had slice shear force higher than 35 kg.

7 Morgan (1995), reported losses of $216,976,000 for tenderness related problems.
Moreover, the National Cattlemen's Beef Association (NCBA) created a working group of industry professionals to address this challenge (NCBA, 2001). The concern of the beef industry is warranted. Beef tenderness has been repeatedly reported as the most important quality attribute of meat (Huffman et al., 1996; Miller et al., 2001; CCA, 2002). The NCBA called tenderness “the one attribute that consumers most associate with eating quality” (NCBA, 2001).

Recent research has shown that consumers are able to distinguish differences in tenderness of beef loin steaks that have been classified based on Warner-Bratzler shear (WBS) force (e.g. Miller et al., 2001; Boleman et al., 1997; Shackelford et al., 2001; Lusk et al., 2001). The same authors report that consumers are willing to pay premiums for these tender steaks. For fresh meats, however, flavor and tenderness are experience attributes, and consumers have to rely on other cues to make inferences about how the product will perform (Acebrón and Dopico, 2000; Dransfield, Zamora, and Bayle, 1998; Steenkamp and Van Trijp, 1996; Bredahl, Grunert, and Fertin, 1998). Studies differ on the assessment of the accuracy of those inferences. Steenkamp and Van Trijp found a slightly significant (at a one sided p-value of 0.1) relationship between quality expectations and quality experienced. The other two studies cited found moderate positive relations between the two. This indicates that consumers are often disappointed because their eating experience falls short of their expectations.9

An interesting approach aimed at improving consumer satisfaction was undertaken by Meat and Livestock Australia (MLA). MLA has developed a grading system called Meat Standards Australia (MSA), with the focus of providing consumers a guaranteed eating experience (Polkinghorne et al., 1999). In contrast to assessment of carcass traits at the chiller, MSA has taken a total system approach to grading meat, based on the principles of Palatability Assurance at Critical Control Points, a concept borrowed from the food safety sector (Polkinghorne et al., 1999). The aim is to control the critical points that impact meat quality from production, processing and value-adding links of the beef chain. Briefly, MLA has conducted large-scale, carefully-planned experiments using consumer tasting panels,

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8 Morgan et al. (1991) also reported that beef tenderness variability was of primary concern to the US industry.
9 Lockhart (2000) quotes Brad Morgan, assistant professor of Animal Science at Oklahoma State University saying “one out of five beef eating experiences will be less than desirable.”
with a focus on factors identified in the literature as affecting beef palatability. Consumers were asked to rate samples of beef for tenderness, juiciness, flavor, and overall satisfaction. These ratings were used to construct an index of quality, with the highest weight (40%) placed on tenderness. Consumers were also asked to classify the sample as either "unsatisfactory" (no grade), "good everyday" (3 star), "better than everyday" (4 star), or "premium quality" (5 star). The quality index was then used to calculate optimal boundaries for the grades assessed by consumers using linear discriminant analysis. The boundary between unsatisfactory and 3 star (representing the pass/fail criteria), was then adjusted up for further protection.

The MSA example indicates that participants in the beef supply chain can transform the experience attribute tenderness into a search attribute (see footnote 2) by testing, sorting, and carefully labeling. This creates an opportunity for producer alliances who wish to develop a niche marketing venture. It seems that there are premiums that can be fetched by "guaranteed tender" beef. Some producer groups and food corporations have already tapped into this market. One of the enterprises, Beefmaster Cattlemen LP (located in Texas), has been allowed by the USDA to label its products as "all natural tender aged beef", and they command significant premiums for their products. The procedure used by this venture to assure tenderness is selection of USDA Select carcasses, with yield grades lower than 3, having a certain weight and ribeye area, and being devoid of visible defects. Selection of carcasses is accomplished with the BeefCam technology (more on this technology below). Eligible carcasses are electrically stimulated and aged at least 14 day before they are shipped to retail warehouses.

However, the technology used to identify and certify guaranteed tender must be accurate enough to result in beef products that are recognized by consumers as superior in tenderness (Ward et al., 2004). Most often, beef products are sorted for tenderness by type of

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10 The issues in transforming credence and experience attributes into search attributes in food products are discussed by Caswell and Mojuszka (1996).
11 For the industry as a whole classifying cuts by tenderness is a more delicate issue. If classification becomes the norm, there will be a reduction in value for all the non-qualifying (probably tough) cuts, and the overall balance is not clear. This is a concern for the beef industry (Schroeder et al. 1997; Ward et al. 2004), but the efforts devoted by the NCBA to develop better (tenderness) sorting technologies seem to indicate the industry perceives benefits of doing so.
cut and USDA quality grades (NCBA, 2002). Although USDA quality grades are correlated with consumer’s ratings of beef palatability, using this classification criterion will fail to convey precise information to consumers with regard to the tenderness of the product (Savell et al., 1987). USDA quality grades have been recognized (Wulf et al., 1997) as not appropriately differentiating longissimus tenderness of USDA Select or Low Choice fed-beef carcasses. Further, Shackelford et al. (2001) argued that consumers can detect differences in tenderness within Select strip loins after 14 days of postmortem aging. This is significant, since Choice and Select account for over 90% (Boleman et al., 1998) of the graded carcasses. More broadly, Wheeler Cundiff, and Koch (1994) found that marbling explained at most just 5% of the variation in beef palatability. Comparing the USDA grading system, with MSA, and the Japanese Meat Grading Association System, Strong (2001) concluded that the wide variation of eating quality within each USDA quality grade is not surprising, since the system does not consider many factors proven to affect quality.

It is well established that to increase the probability of obtaining satisfactory tenderness, the best genetics should be used, and appropriate management practices should be followed during growth, slaughter, and processing of carcasses (Koohmaraie et al., 1996). However, the same authors cautioned, that the relation between breed and tenderness is not strong, since variation of tenderness within breeds is larger than variation across breeds. Hence, as Schroeder et al. (1997, p.10) conclude “…producer alliances with the goal of targeting beef to specific markets demanding particular quality attributes will likely find success elusive if they rely predominantly on current beef quality grades, cattle breeds, and genetics to ensure tenderness and consistency of their products. Producers may also need to employ some type of tenderness testing.” This claim is significant, since only one of the 40

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12 These grades are based on the relationship between marbling of the 12th rib cross section of the longissimus and cooked beef palatability (USDA, 1997).
13 Miller et al. (2001) were able to produce (strip loin) steaks within the Select grade that were rated from extremely tough to extremely tender by a panel of consumers, by varying the aging period of the different steaks. The animals were all steers of the Simbrah breed, fed in the same feedlot, subject to similar implant and feeding strategies. Before being slaughtered at a commercial processing facility, all animals were fed 180 to 210 days. Steaks were cooked by trained research teams to the same degree of doneness, at the same time, and on the same day, to avoid other sources of variability.
14 Hilton et al. (2004) found no differences in WBS force and sensory panel tenderness ratings among six different phenotypes (all including less than ¼ Brahman). The authors then concluded that any cattle phenotypes with ¼ or less phenotypic expression of Brahman can be used in a guaranteed tender program.
certified beef programs registered with the USDA rely on such measurements.\textsuperscript{15} Broadly speaking, all these programs require is some distinctive genotypic and/or phenotypic characteristics combined with eligible USDA quality and yield grades (with variable stringency), and absence of visible defects such as hemorrhages or dark cuts.\textsuperscript{16} All these requirements are conducive to more tender meat, but there are still significant amounts of unexplained variation in consumers' perceptions. Scientists have concluded that "the beef industry must identify more precise methods [than USDA quality grades] of distinguishing palatable from unpalatable beef" (Wulf and Page, 2000, p.2595), and in the same line "a direct measure of meat tenderness is needed to supplement quality grade" (Wheeler, Cundiff, and Koch, 1994, p.3150).

\textbf{4.2.3 Prediction of tenderness}

For commercial utilization, real-time measurements are needed to classify carcasses according to tenderness. Despite a tremendous amount of effort by animal and meat scientists, accurate prediction of consumer perceptions of beef tenderness has proved elusive. Further, Lorenzen et al. (2003) concluded that predicting consumers' ratings of beef based on laboratory procedures (trained panels, Warner-Bratzler shear force) is inherently difficult, because cooking method, degree of doneness, seasoning and individual heterogeneity in preferences all affect the way consumers perceive the quality of the product. Shackelford, Wheeler, and Koohmaraie (1997, p.2421) argued that the ideal method to measure or predict meat tenderness would be through "an accurate, rapid, automated, tamper-proof, noninvasive machine." Several technologies have been investigated for the purpose of replacing the time consuming and destructive measurement of Warner-Bratzler shear force, a widely used device for measuring the tenderness of cooked meat. Some examples include near-infrared reflectance spectra (Park et al., 2001); image texture analysis (Li, Tan, and Shatadal, 2001); neural network modeling (Hill et al., 2000); measurements of muscle color, pH and electrical impedance (Wulf and Page, 2000); and BeefCam (Vote et al., 2003). Wheeler et al. (2002) showed that slice shear force was more accurate (than the later two) at identifying beef cuts

\textsuperscript{15} The only program that attempts to measure tenderness is Nolan Ryan Tender-Aged Beef\textsuperscript{TM}. However, research conducted by Gheno et al. (2001) concluded that the resulting tenderness in this program as measured by WBSF or trained panel evaluations is no better than USDA Choice.

\textsuperscript{16} The most salient of these alliances, Certified Angus Beef accounted for 5.7% of all the fed cattle slaughtered in 2001 (Schroeder and Kovanda, 2003).
that can be guaranteed tender. Further, the authors concluded that commercial BeefCam provided added assurance of acceptable tenderness over USDA quality grade, but refinements seem necessary to enhance the ability of BeefCam to identify carcasses that will yield acceptably tender meat. In another work, Wyle et al. (2003) reached a very similar conclusion that BeefCam might ultimately be useful in identifying carcasses for inclusion in branded beef programs, but further development and testing of the technology is warranted.

In view of the difficulty in predicting consumers’ ratings accurately, researchers often try to segregate carcasses into classes (e.g. either tender or tough) according to some criteria. The most commonly used procedure is to classify carcasses according to the Warner-Bratzler or slice shear force, measured in the longissimus.\textsuperscript{17} Since sufficiently accurate and less invasive (costly) methods are not available (yet), effort is being placed on determining thresholds for Warner-Bratzler or slice shear force values to be used in the classification process. In particular, this is one of the objectives of the Instrumentation Working Group of the National Beef Instrument Assessment Plan II (NCBA, 2002). The first thresholds for WBS force were proposed by Shackelford et al. (1991), who proposed thresholds values of 4.6 kg and 3.9 kg as approximate cutoffs to obtain 50% (corresponding to the mean) and 68% (mean minus standard deviation)\textsuperscript{18} chances of a steak being rated slightly tender or better respectively, recognizing that the confidence of having more stringent standards may be offset by higher production costs because of lower acceptance rates. Though not mentioned in the study, the trade-offs between type I and type II statistical errors are nicely illustrated. For example, 6.6% and 16.9% of the predicted tough were actually tender (rated at or higher than slightly tender by a trained panel), for the 50% and 68% confidence limits respectively. On the other hand when comparing the 50% and 68% confidence limits, the percentage of predicted tender that were actually tough (rated lower than slightly tender) decreased from 6.7% to 1.6%. Another example can be found in Wheeler, Shackelford, and Koohmari (1997). These authors adapted the data from Huffman et al. (1996) to show that steaks rated as acceptable had WBS forces as high as 5.7 kg, whereas values as low as 3 kg were rated

\textsuperscript{17} Of course, this only predicts the tenderness of this muscle. The correlation between shear force of the longissimus and other muscles is generally low (Shackelford, Wheeler, and Koohmari, 1995).

\textsuperscript{18} To conduct the analysis the authors assumed shear force in the population of cattle to be normally distributed.
unacceptable, indicating a significant overlap. Both types of error will arise for the proposed threshold of 4.1 kg, which led to 98 percent consumer overall satisfaction with the steaks.

The problem just discussed is not to be seen as exclusive to the prediction of tenderness. Thompson et al. (1999) argued that the MSA system worked well in the sense that it reduced the probability that a consumer will receive an unsatisfactory steak (11% for striploins). However, there were a significant proportion of carcasses (71%) that failed to meet the 3 star specifications but were deemed acceptable by consumers. The developers of MSA recognize this as a problem with the methods or pathways used to determine acceptability, but argue that a “minimal risk approach was necessary in the interests of guaranteeing consumer satisfaction” (Thompson et al., 1999, p.2).

The number of tenderness classes needed to maximize carcass value and consumer satisfaction is an unsettled question (Wheeler, Shackelford, and Koohmaraie, 1997; Shackelford, Wheeler, and Koohmaraie, 1999). It is also unclear where the lines separating the tenderness classes should be drawn. In this paper we focus on where to draw the line for two classes. Future research needs to address the optimal number of tenderness classes needed. In previous research, the best system to classify carcasses is the one that matches most closely the perceptions of either trained panels or consumers. Arbitrary values of WBS (e.g. 5 kg in Shackelford, Wheeler, and Koohmaraie, 1999) force at 14 days of aging were used as benchmarks. However, none of the studies provide an economic assessment of the implications of different benchmarks. For example, setting the standards higher may confirm the prediction more frequently, but it will also lead to a large number of false rejections (or statistical type I error). Setting the standards lower, will likely lead to fewer false rejection, but a higher proportion of the certified tender beef will be perceived as not complying by consumers (type II statistical error). The correct standard from an economic standpoint must balance these two types of errors with the potential premiums and cost of segregation by tenderness in an economically meaningful fashion.

Testing and/or classifying a product (even with nondestructive methods) entail costs. Producers will be willing to implement a certification system if they perceive there are price premiums that can be fetched through better sorting. Hence, before any effort is incurred in designing a sorting and certification system, one question should be answered: are there
premiums that can be fetched by guaranteed tender steaks? More importantly, are those premiums higher than the costs of implementing the certification system?

4.2.4 Benefits and costs of a tenderness-certified program

Costs are involved in any tenderness certification system. These costs can be broadly classified into tenderness assessment (or testing) and sorting costs. The cost of classification using an automated system using slice shear force to measure tenderness (once developed) has been "crudely estimated" at $4.35 per carcass (Wheeler, Shackelford, and Koohmaraie, 1999). The same author estimated the costs of manual classification at $8.50 per carcass. Included in those cost estimates are labor (at $25/hr), equipment, and the sample that needs to be destroyed for testing (a 1-inch ribeye steak valued at $4). However, as Lusk et al. (1999) observed, actual costs will likely be larger since the previous estimate ignores additional sorting costs and the cost of capital financing. As we will see below, we would add the costs associated with errors in the certification process. Other available technologies, such as BeefCam may result in lower costs.

Consumer willingness to pay premiums for tender beef has been documented by recent studies. Lusk et al. (2001) reported average premiums of $1.84 per pound for guaranteed tender steaks and 20% of consumers were willing to pay a premium of $2.67 per pound or more. In another study conducted by Shackelford et al. (2001), half of the consumers indicated that they would definitely or probably be willing to pay 50¢ more per pound for the guaranteed tender Select steak. Boleman et al. (1997) reported that most of the families who purchased steaks in an experiment (94.6%) chose revealed tender, over both intermediate and tough steaks (segregated by shear force), even though a $1.10/kg difference was placed between each category. In a somewhat surprising response, half of the consumers (surveyed by Shackelford et al. (2001) in Denver, Colorado) indicated they do not let price govern their food-purchasing decisions.

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19 This study does not give us a good sense of consumer's willingness to pay for tender steak, since it only asked "How willing would you be to pay 50¢ per pound...more to purchase that steak?" Hence the only information we can extract is that half of the consumers are willing to pay at least 50¢ per pound of premium for the guaranteed tender Select steak.

20 The sample used was not representative of the general population, since 58.1% of the respondent households reported incomes above $60,000. However, this could be a good sample for research of acceptance of value added products.
Benefits or premiums accrue only on carcasses that are certified as tender, whereas costs are incurred for all the carcasses tested. In an error free world, where cuts can be perfectly classified (based on the result of the measurements) into acceptable and unacceptable, the profitability of a certification system depends only on the price premiums for high quality, costs of testing and sorting, and the fraction of the tested population that can be certified (assuming that packers cannot strategically misrepresent the results of the tests). Ward et al. (2004) argue that the proportion of qualifying carcasses plays a key role in determining whether a system to guarantee tenderness is feasible. Letting $\lambda$ be the fraction of tested carcasses that can be sold in the high quality market, $p$ be the price premium for that quality, and $C$ the incurred costs per tested carcass, a system will be implemented whenever, $\lambda p - C > 0$. Rearranging the inequality as $p > C/\lambda$ makes clear that a classification system will be feasible only if the price premiums exceed the costs per certified carcass. The inequality could also be rearranged as $\lambda > C/p$, the larger the costs of certification relative to the premiums, the higher is the fraction of high quality carcasses needed for the system to be profitable (more on this below).

In the real world, the issue is not that simple. Packers face uncertainty about both the real quality of the carcasses (even after grading and/or performing tests), and about differences among consumers. A steak that is perceived as tender by a consumer in a given circumstance may not be rated in the same way by another buyer or situation. Additionally, the proportion of carcasses that are certified is under the alliances’ control whose objectives may conflict with minimization of the number of classification errors. In what follows we present a model that allows us to address these issues formally.

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21 This could be the case for example when packers reveal the result of the measurements, and consumers would buy only those cuts they know will meet their expectations. In reality consumers will not be able to perfectly “self-select”, due to a lack of perfect links between quality measurements and actual perceptions. We discuss this more thoroughly when we present the model.

22 Suppose that only the rib and loins area can be certified (usually the area tested; more on this below). According to Miller et al., (2001) those cuts would represent 137 lbs of scalable steaks per carcass. For the manual system costs reported by Wheeler, Shackelford, and Koohmaraie, (1999) and the conservative premiums (lower bound) reported by Shackelford et al., (2001) a proportion $\gamma \geq (\$8.5/137/lbs)/\$0.5 = .12$ is enough to warrant the implementation of a certified tender system, in an error free world.
4.3 The model

Suppose an alliance is considering launching a line of guaranteed tender steaks to the market. Tenderness of any given steak is unknown to the alliance. However, it can attempt to measure it (at some cost) before guaranteeing it as tender. Suppose that measurements of quality can be condensed in an index $w$. In this setting, the quality index and tenderness are used interchangeably. As discussed above, the method of shear force has been proven to be the most accurate that can be used at a commercial level to infer tenderness, hence we base our discussion on terms of that testing technology assuming it summarizes all the information available. Since tougher steaks require higher forces to shear them, lower values of the index will be representing more tender meat. However, the analysis is presented here in a way that can be easily adapted when new (nondestructive) technologies such as BeefCam, for example, are more precisely calibrated, or to a process certification system such as the MSA. Since testing and certification entails costs, high value cuts (e.g. loins) are natural candidates for the analysis. When they are ready to market, producers in the alliance ship their cattle to the packing plant, where they are tested with the selected technology. At this stage the decision maker has to decide whether to certify the product or not. If certification occurs, the steaks from that loin are labeled as “guaranteed tender”. Beef that fails to meet the certification standards are sold as commodity under the usual USDA quality grades. The commodity market is assumed to be competitive, implying zero premiums for non-qualifying animals.

Consumers are willing to pay a premium of $p$ per unit over the commodity price of beef for certified tender steak, and each consumer has his or her own (potentially different) idea of what is the sensory perception delivered by a tender steak. However, if after buying and cooking the steak a consumer disagrees with the statement of the seller, that is the steaks falls short from the sensory perception expected, he/she can punish the alliance. The punishment an individual customer can impose to the alliance equals $\delta$ per unit certified.

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$^{23}$ Meat scientists warn us against using the results of a cut to predict tenderness of a different cut. Round cuts for example are less likely to be worthy of certification due their high within carcass variability (Shackelford, Wheeler, and Koohmaraie, 1997). The same authors argue that measuring a minor muscle (as a way to avoid the need to destruct high value product) would provide little information regarding longissimus tenderness due to their poor correlation. Meat Livestock Australia conducted experiments at the cut level, and MSA assigns “stars” to different cuts.
Plausible interpretations of this punishment are given below for the different scenarios modeled. We also assume that bad news does not spread throughout the market (as in Klein and Leffler; 1981, and Shapiro, 1983). Only consumers whose expectations were not met punish the alliance certifying the guaranteed tender beef.

If the true quality (tenderness) measured by \( w \) can be assessed by testing the product, and the acceptable perceptions for tenderness of consumers are equal and known by the alliance, then with repeated purchases, as we assume here (or when firms can be punished with enough severity for non-compliance), there is an equilibrium where high quality is provided to the market at a price above marginal costs of production (see for example Klein and Leffler, 1981; Shapiro, 1983; Grossman, 1981).\(^{24}\) One could argue, that if the alliance is able to assess \( w \) with certainty, then in theory, it can just reveal the results of the test, set a premium for each steak \( p(w) \), and allow buyers to self-select. This would imply that the alliance has information about consumer willingness to pay for each available quality, and that consumers can determine the value of the index \( w \) that satisfies them. The two assumptions are highly problematic from a practical standpoint. As noted before, researchers are only starting to uncover consumers' willingness to pay for tenderness. Moreover, the studies only indicate willingness to pay for guaranteed tender beef, and not for different levels of tenderness (an exception is Miller et al., 2001). Additionally, consumers have some confusion with regards to the information provided by USDA beef quality grades (Cox, McMullen, and Garrod, 1990), even when these grades have been in place (with changes) for more than 70 years.\(^{25}\) Hence, it is at best questionable to assume that consumers are able to infer the acceptability of the steaks, at least at the beginning, based on the value of the index. A less stark assumption would be to assign steaks into a number of classes based on the result of the index, and price equally all the steaks in the same class. This is exactly the approach taken in MSA.\(^{26}\) The optimal number of classes is, as argued above, an open

\(^{24}\) Relaxing this, Antle (2001) argued that when preferences are tightly concentrated around a central value of a distribution, and producing a variety of products is costly, firms will concentrate in qualities around that central value. Rob and Sekiguchi (2001) provide a model where producers can only affect the distribution of quality, but the acceptable quality level is known.

\(^{25}\) For a brief history of quality grades see Schroeder et al (1997).

\(^{26}\) Also, classification by ranges of \( w \) makes sense when there is error in measurement.
question. Here we focus on the threshold for only two classes. Steaks are either certified as tender or they are not.

In any given steak, however, the true $w$ is not known. After testing, the packer only observes a noisy signal $w_0 | w$, where $w_0$ is the result from the test.\(^{27}\) Additionally, a given $w$ can be rated as either satisfactory or not satisfactory for different consumers and/or circumstances. Myriad factors can affect the sensory experience that is rated as acceptable both within and across consumers (e.g. the degree of doneness and cooking method, geographic location, etc\(^{28}\)). This implies that there may be two forms of heterogeneity in perceptions that will be considered acceptable. First, acceptable perceptions may be consumer specific in the sense that what would be rated as satisfactory by a buyer may not make the grade for another consumer. In this interpretation we have consumer heterogeneity. Another form of heterogeneity arises if the ex-ante identical perceptions that would be considered acceptable change each time the consumer purchases a steak.\(^{29}\) The two forms of heterogeneity have different implications in terms of modeling and outcomes. We model the two forms of heterogeneity explicitly, paying close attention to their effects on the optimal choices of the alliance. The next section analyzes the situation where the alliance can learn the true value of the index.

4.3.1 True tenderness can be learned

The general structure of the model is presented in this section assuming that $w$ can be learned with certainty through some testing technology, and its distribution in the population of cattle available is known to be $f(w)$. The following subsections consider the implications of the two types of heterogeneity introduced. If consumers who are willing to pay a premium are homogeneous with respect to the quality that meets their expectations, and that value is known, the problem of the alliance is very simple. They can just certify steaks known to be satisfactory, and compare the revenues (given by the premiums and proportion of pounds

\(^{27}\) When Warner Bratzer or slice shear force is used, this is usually the average of the shear force needed to cut 5 or 6 cores taken parallel to the fiber muscles of a steak.

\(^{28}\) See McKenna et al. (2004). Also, a given consumer can find a steak acceptable under some conditions but not under other (see footnote 33).

\(^{29}\) This is "as if" each consumer receives a random shock from a common distribution before each purchase.
certified) against certification costs, to see whether a niche marketing venture is warranted.\(^{30}\) We delay a formal proof of this claim until the relevant notation has been introduced. In general, however, consumers in the target population differ for example in the quality (or sensorial perception) that meets their expectations. This heterogeneity is captured by \(y_j\), which represents consumer's \(j\) tastes, tenderness acceptability levels, or "type". Consumer \(i\) will be less demanding in terms of tenderness than consumer \(j\) whenever \(y_j < y_i\). The alliance does not know the type of any particular consumer, hence it cannot match her with steaks that will be acceptable. It only knows that types can be represented by the random variable \(y\) which has distribution \(g(y)\).\(^{31}\) From the alliance's standpoint, any consumer that buys a particular certified steak is seen as a random draw from that probability distribution. A certified steak that measures \(w\) will meet the expectations of a random consumer approaching the meat counter if \(w \leq y\). If on the other hand the consumer who buys the steak has expectations \(y > w\), he/she will be disappointed and will not repeat the purchase. For any particular certified steak with tenderness measured by \(w\), the probability that it satisfies a random customer that approaches the meat counter is given by \(\Pr_y(y \geq w) = 1 - G(w)\), where \(G(\cdot)\) is the cumulative distribution function of the consumer tenderness acceptability level. Defining \(\theta(w) = 1 - G(w)\), we have that \(1 - \theta(w) = G(w)\) is the probability that a consumer is disappointed.

The problem of the alliance is to choose a threshold for certification \(\hat{w}\). This can be seen as a classification problem. The alliance is using the results of the measurement to separate steaks from two populations; tender or tough steaks, as rated by consumers. Letting higher values of \(w\) be associated with lower probability of acceptability by consumers (e.g. more force is needed to cut a tougher steak), every steak with \(w \leq \hat{w}\) will be certified as tender. This leads to a distribution of tenderness of certified steaks given by \(f(w|w \leq \hat{w})\). As in any decision under imperfect information, "errors" will be made. Some consumers will be

\(^{30}\) This is true if the loss from cheating is higher than the premium obtained in a single period (more on this below).

\(^{31}\) A similar description of heterogeneity, but in a food safety setting where households make decision under the belief that health risks follow a given distribution, is presented in Antle (2001).
dissatisfied with the purchased steak. Strictly speaking, the problem may be one of mismatch, and there may have been no disappointments had the alliance been able to observe the “type” or expectations of each consumer buying the product (and ordered the steaks accordingly). Recall that the alliance knows the distribution of consumer expectations, but not the expectations of a particular consumer that shows up at the store. The alliance can control the proportion of dissatisfied consumers through the choice of the threshold \( \hat{w} \). Hence, dissatisfaction may also occur because the alliance has chosen to certify non-tender steaks. Figure 4.1 illustrates. The vertical line in Figure 4.1 represents a hypothetical threshold, and region \( E \) represents the product that will be certified as tender. Whereas consumers whose tastes belong to region \( B \) will be assured 100% satisfaction, it is likely that some consumers from region \( A \) will be disappointed. Figure 4.1 also makes clear that some of the non-certified steaks (region \( D \)) would have been found acceptable by some consumers with tastes falling in region \( B \).

Figure 4.1. Implications of an arbitrary threshold for hypothetical distributions of \( y \) and \( w \)
Certified steaks that fail to meet the expectations of a consumer, lead to a punishment of $\delta$ if purchased by that consumer. This punishment could represent the net present value (NPV) of the consumer's future purchases or it could represent restitutions or replacements to the consumer such as would occur with a money back guarantee. It could also represent the cost of replacing a disappointed consumer.\(^{32}\) Strictly speaking, under the net present value interpretation of consumer value to the alliance, the punishment each customer can impose would be contingent on the quality distribution of certified steaks, which depends on the threshold chosen. We delay the full treatment of how the size of the punishment interacts with the distribution of quality until the next section. Some of the rejected steaks in region $B$ of Figure 4.1 could have been certified, and fetched the premium $p$ with some customers, which of course, is an opportunity cost. However, potential punishments are also avoided by not certifying them. We will ignore the potential rewards or costs associated with that region and only include realized costs and revenues in the profit function. Denial of certification entails only testing costs, whereas correct certification (in the sense that the consumer is satisfied with the purchased steak) brings a net benefit $p - C$. When the alliance observes a given $w \leq \hat{w}$ (and hence certifies the steak), it obtains expected profits per pound given by

$$E(\pi|w) = E_r(p-C|y \geq w)Pr_r(y \geq w) + E_r(p-C-\delta|y < w)Pr_r(y < w).$$

$E_r$ is the mathematical expectations operator with respect to the variable $y$. Using the notation introduced above, the previous expression can be rewritten as

$$E(\pi|w) = (p-C)\theta(w) + (1-\theta(w))(p-C-E_r(\delta|y < w)). \tag{4.1}$$

This is the average of the profits obtained from sales of certified steak measuring $w$ to an ex-post satisfied or dissatisfied consumer respectively, weighted by the probability of each event. Recall that quality is an experience attribute, and consumers will pay for the product before discovering the actual quality. Averaging over the acceptance region, expected profits are

$$E(\pi|w \leq \hat{w}) = E_w \left(p - (1-\theta(w))E_r(\delta|y < w) - C|w \leq \hat{w}\right).$$

\(^{32}\) Caswell and Henson (1997) argued that the losses associated with customer dissatisfaction (e.g. market share, bad reputation) are costlier for firms than those resulting from legal action under the current tort law.
The expression \( p - E_{\hat{w}} \left( (1 - \theta(w)) E_\gamma \left( \delta \mid y < w \right) \mid w \leq \hat{w} \right) \) can be seen as a long run equilibrium premium. In long run equilibrium consumers will have near perfect information about product quality for some experience attribute (Segerson, 1999).

For a steak yielding \( w > \hat{w} \), certification is denied and expected profits (losses) per unit over the denial of certification region are \( E(\pi \mid w > \hat{w}) = -C \), the costs of conducting the test. This equation assumes that the commodity market price just covers the alliance's production and processing costs (i.e. zero profits from commodity production).

Now we have all the elements needed to write the per unit profit function of an alliance that chooses to implement a guaranteed tenderness certification system with threshold \( \hat{w} \) as

\[
E(\pi \mid \hat{w}) = E(\pi \mid w \leq \hat{w}) \Pr_{\tau}(w \leq \hat{w}) + E(\pi \mid w > \hat{w}) \Pr_{\tau}(w > \hat{w})
\]

\[
= E_{\hat{w}} \left( p - (1 - \theta(w)) E_\gamma \left( \delta \mid y < w \right) \mid w \leq \hat{w} \right) \Pr_{\tau}(w \leq \hat{w}) - C \quad (4.2)
\]

Equation (4.2) shows that, in general, the optimal threshold will depend on the objective of the alliance, the price premium and punishments, the distribution of types in the target population of consumers, and the distribution of tenderness of the population of cattle owned and/or marketed by the alliance. The costs of testing and certification play no role in determining the optimal threshold, but determine whether the alliance is profitable. Equation (4.2) together with Figure 4.1, also make clear the tradeoffs involved. More stringent certification levels lead to higher profits from each certified steak due to a reduction in expected punishments. This can be seen by noting the reduction in area \( A \) in Figure 4.1 or the increase in the \( E_{\hat{w}}(\cdot \mid w \leq \hat{w}) \) term in equation (4.2). But the proportion of steaks certified given by \( \Pr_{\tau}(w \leq \hat{w}) \) or area \( E \) is reduced, and the number of steaks that were not certified but would have satisfied some customers is increased. The later can be seen as an increase of areas \( B \) and \( D \) in Figure 4.1.

**4.3.1.1 Value of a customer is not “taste specific” with fixed \( \delta \)**

Suppose first that consumers are homogeneous in tastes, but the way in which they perceive tenderness when they eat a steak interacts with other factors such as the dining environment, side dishes, or the entrée served. This leads to heterogeneity in acceptability
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levels (or acceptable perceptions) among consumers. Distinct y's are the result of different shocks that affect the level of tenderness that different consumers will rate acceptable in any time period. Shocks are independent within and across time periods. The alliance knows that random shocks result in the acceptability levels for tenderness of consumers to follow the distribution g(y). In this situation, each customer is valued the same in expectations by the alliance, since someone very demanding in the current periods may be easy to please the next period (more on this below). When the cost of losing any consumer is the same regardless of his/her type equation (4.1) simplifies to

$$E(\pi|w) = \theta(w)(p-C) + (1-\theta(w))(p-\delta-C) = p - (1-\theta(w))\delta-C.$$ 

As argued above, the optimal threshold depends on the objective of the alliance, on whether it acknowledges or not the presence of the opportunity costs associated with different choices, and of course on the relative sizes of premiums and punishments. Suppose first that the alliance is striving to maximize aggregate profits of its members. In this case, the optimal threshold for certification is defined as the level that maximizes expected profit $E(\pi|\hat{w})$ from testing and certification, which is given by

$$E(\pi|\hat{w}) = E(p - (1-\theta(w))\delta|w \leq \hat{w}) Pr_{w}(w \leq \hat{w}) - C$$

and can be rewritten in integral form as

$$E(\pi|\hat{w}) = \int_{-\infty}^{\hat{w}} (p - (1-\theta(w))\delta) dF(w) - C$$

or

$$E(\pi|\hat{w}) = pF(\hat{w}) - \delta \int_{-\infty}^{\hat{w}} (1-\theta(w)) dF(w) - C$$ (4.3)

In this expression $F(\bullet)$ is the cumulative distribution function of $w$, and the integral represents the costs associated with not being able to match the results of the test with consumers expectations. Those mismatches could be used to quantify the amount the alliance

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33 The fact that people's perceptions interact with many different factors is widely recognized among meat and animal scientists. For example, Miller et al. (2001) conducted their consumer evaluations at the same time and on the same day to reduce variation. Huffman et al (1996) served wine to their subjects in a restaurant setting, but instructed them not to drink until they had completed rating the steaks so that it would not affect the results. Other researches provide crackers between samples to avoid confounding, or use red lights to minimize the impacts of visual perceptions on tenderness evaluations.
would be willing to invest in a more accurate technology, or in targeting the customers more effectively. Notice that whereas costs are paid for all the pounds tested, premiums are only obtained over certified pounds. This has relevance, since as recognized by Shackelford et al. (2001) and Ward et al. (2004) among others, a certification program would be feasible only if testing costs can be spread over a relatively large proportion of animals that can be certified. It may be important to combine certification programs with some form of presorting to only test animals that are likely to meet the criteria for classification (especially when premiums are not large compared to testing costs). Many producer groups and alliances already have programs in place that use cattle with characteristics known to be conducive to tender meat. For example presorting could be done to exclude breeds likely to present tenderness problems. Certified Angus Beef is one case; the program relies basically on certain breeds (assessed phenotypically mainly by color and hump height), USDA quality grades, and absence of visible defects (USDA). This would concentrate testing costs on carcasses that have a high chance of meeting the certification criteria.

We have now all the elements needed to address the problem of the choice of the optimal threshold. In the previous section we claimed that if consumer's acceptability perceptions were fixed and known \( \bar{y} \), the optimal threshold entails choosing a value such that no customer will be disappointed (i.e. \( \bar{w} = \bar{y} \)). The next result states this formally for both profit maximization and minimization of expected punishment objectives.

\textbf{Result 1}: If \( \bar{y} = \bar{y} \) is fixed, and \( \delta > p \), then setting the threshold at \( \bar{w} = \bar{y} \) is optimal both under a profit maximization and expected punishment minimization objectives. Only steaks known to be satisfactory will be certified.

\footnote{The example provided is that if only 1/5 of the tested carcasses can be certified, and testing costs are $4 per animal each certified carcass has to obtain a $20 premium for the system to break even. On the other hand, if 4/5 of the carcasses can be certified the premium needed is reduced to $5 per carcass.}

\footnote{Tatum et al. (1999), proposed placing a constraint of 3/8 Bos indicus inheritance, as a way to reduce the incidence of tenderness problems. Proven genetics is also mentioned by the authors as a way to improve tenderness.}

\footnote{Since acceptable tenderness is known, the problem of minimizing errors in classification is trivial.}

\footnote{A similar result was found by Shapiro (1983) and Klein and Leffler (1981). However, these authors modeled a situation where product quality was homogeneous. Producers just needed to choose its level.}
Proof: From the definition of \( \theta(w) \) we have that \( \theta(w) = 1 \) for all \( w \leq \bar{y} \) and hence \( \theta(w) = 0 \) for all \( w > \bar{y} \). When \( \hat{w} \leq \bar{y} \) the per unit profit function can be written as
\[
E(\pi|\hat{w}) = \int_{w}^{\hat{w}} p - (1 - \theta(w)) \delta dF(w) - C = pF(\hat{w}) - C
\]
which is clearly maximized by setting \( \hat{w} = \bar{y} \). Expected punishments are zero for all \( \hat{w} \leq \bar{y} \). However, there will be opportunity costs for any \( \hat{w} < \bar{y} \) since steaks known to be acceptable will not be certified. For our purposes, \( \hat{w} = \bar{y} \) minimizes expected punishments (as any other \( \hat{w} \leq \bar{y} \)). When \( \hat{w} \geq \bar{y} \) the profit function is given by
\[
E(\pi|\hat{w}) = \int_{\hat{w}}^{\bar{y}} p - (1 - \theta(w)) \delta dF(w) + \int_{\bar{y}}^{\infty} p - (1 - \theta(w)) \delta dF(w)
\]
simplifies to
\[
E(\pi|\hat{w}) = pF(\bar{y}) + (p - \delta)(F(\hat{w}) - F(\bar{y}))
\]
The second term in the previous equation represents the expected punishments. Since \( \delta > p \), profits are maximized when costs of misclassification are minimized at \( \hat{w} = \bar{y} \).

When there is heterogeneity in acceptable perceptions, the optimal threshold is contingent, as expected, on the distribution of perceptions and the objective pursued. In this case, the necessary condition for maximizing profits is
\[
\frac{\partial E(\pi|\hat{w})}{\partial \hat{w}} = pf(\hat{w}) - \delta(1 - \theta(\hat{w}))f(\hat{w}) = 0.
\]
Equation (4.4) indicates that the profit maximizing threshold is implicitly defined by
\[
\theta(\hat{w}^*) = \frac{\delta - p}{\delta}.
\]
Hence, if any effort is to be put in sorting, the costs associated with losing the business of a customer would be at least as big as the premium in a given purchase. When

\[38\] Sufficient conditions are given by \( \theta'(\hat{w}^*) \leq 0 \), which is satisfied by the definition of a cdf.
larger punishments are imposed, the alliance finds it optimal to be more stringent in the certification threshold, and hence increase the likelihood that certified steaks meet consumers' expectations. Since the probability of acceptance by consumers is assumed to be a decreasing function of $w$, this translates into lower levels of $\hat{w}^*$. The formula also implies that it will be optimal for the alliance to admit some dissatisfaction unless the punishment goes to infinity. Since $\theta(\hat{w}^*) = 1 - G(\hat{w}^*)$, we obtain $G(\hat{w}^*) = \frac{D}{\delta}$, which represents the probability that a consumer is disappointed when eating a steak of tenderness $\hat{w}^*$. Note that in this setting the threshold depends solely on the premium, the punishment, and the distribution of shocks. In particular, the optimal threshold does not depend on the distribution of quality of the cattle population employed. The latter distribution only has a role in determining whether the venture is profitable.

It is worth noting that the profit maximizing threshold will in general not coincide with the one that minimizes classification costs or mismatches as obtained in the absence of uncertainty in acceptable perceptions. Equation (4.3) shows that expected cash punishments are increasing with the threshold, which implies that to minimize this quantity only steak that will be found acceptable by every consumer should be certified, which would only be profit maximizing for very large punishments relative to the premium. More generally, there are two types of misclassification costs associated with the two decisions certify and do not certify. For the certified steaks, we already argued that the costs of misclassification arise due to consumer dissatisfaction. For the non-certified product, the expected (opportunity) cost arises because the steaks could have been sold for a premium to some consumers (recall region $D$ in Figure 4.1).\(^{39}\) The expected costs of misclassification ($ECM$) for threshold $\hat{w}$ are

$$ECM = E_w \left( \delta \left( 1 - \theta(w) \right) | w \leq \hat{w} \right) Pr(w \leq \hat{w}) + E_w \left( p\theta(w) | w > \hat{w} \right) Pr(w > \hat{w}).$$  \quad (4.5)

It is straightforward to show that the threshold ($\hat{w}^*_{c}$) that minimizes equation (4.5) is implicitly defined by the rule $\theta(\hat{w}^*_{c}) = \frac{\delta}{\delta + p}$, thereby will be generally different than the threshold obtained through profit maximization. Defining thresholds as to minimize the expected costs of misclassification is a more rigorous approach than the profit maximization framework developed in Section 4. However, note that by not certifying a steak the alliance is also obtaining the "potential benefit" of not being punished, which would oppose the opportunity costs just mentioned.

\(^{39}\) Note however, that by not certifying a steak the alliance is also obtaining the "potential benefit" of not being punished, which would oppose the opportunity costs just mentioned.
number of mismatches (as in Platter et al., 2003a), is not likely to lead to maximum profits for the alliance either. Error in matching minimization in this context is equivalent to assume that both mistakes are equally costly, and \( \theta(\hat{w}^E) = \frac{1}{2} \) obtains.

The probabilities of acceptance for the first two thresholds can be ranked for arbitrary combinations of \( \delta \) and \( p \) such that \( 0 < p \leq \delta \) as follows \( \theta(\hat{w}^*) < \theta(\hat{w}^C) < 1 \). This would translate into \( \hat{w}^* > \hat{w}^C \), implying that a more stringent system will result if the objective is to minimize the expected costs of misclassifications. The place in the ranking for threshold that minimizes the number of mismatches can not be determined in general. We can only say that \( \theta(\hat{w}^E) \leq \theta(\hat{w}^C) \), and hence that \( \hat{w}^E \geq \hat{w}^C \). However, for sufficiently high punishments (relative to the premium) for noncompliance (\( \delta/p \geq 2 \)), the three thresholds can be ranked as \( \hat{w}^E \geq \hat{w}^* > \hat{w}^C \), otherwise \( \hat{w}^* > \hat{w}^E \geq \hat{w}^C \) holds. Profit maximizing decision makers will adopt more stringent thresholds than the ones that minimize classification errors, whenever the costs of a mismatch in a certified steak are severe compared to the premium. In these situations, alliances will prefer to take extra precautions, or are willing to incur in some opportunity costs, in order to reduce the amount of certified products that do not meet customer expectations. For fixed premiums, when punishments for consumer dissatisfaction decrease, profit maximizing alliances will be more willing to certify carcasses of doubtful tenderness. This result already arose in the previous Chapter. There we showed that for all the market structures and nature of reputations considered, when consumers were more able to discern quality (and hence the expected punishment for any low quality item certified is higher), processors chose more stringent quality assurance systems.

Interestingly, when heterogeneity in acceptable perceptions is introduced, neither of the possible objectives considered lead to satisfaction of all consumers (under finite punishments). In fact, as we state and prove in the next result if the consumer that receives the median shock is going to be always satisfied with his/her purchase, \( \delta/p \geq 2 \) is needed under the profit maximization objective.

**Result 2**: Let \( f(w) > 0 \) for all \( w \) in the support. The consumer receiving the median shock will be satisfied with his/her experience with probability one (if the median shock can
be satisfied with the base cattle population) for $\delta/p \geq 2$ under profit maximization, (i.e. for sufficiently large punishments relative to the premium) and $\delta/p \geq 1$ for cost of misclassification minimization, for any distribution of shocks.

Proof: Let $\bar{y}_M$ be the median shock. The median shock will be satisfied with his/her experience with probability $\Pr_w\left(w \leq \bar{y}_M \mid w \leq \hat{w}\right)$. Since $\Pr_w\left(w \leq \bar{y}_M \mid w \leq \hat{w}\right) = 1$ for all $\hat{w} \leq \bar{y}_M$ and $\Pr_w\left(w \leq \bar{y}_M \mid w \leq \hat{w}\right) < 1$ for all $\hat{w} > \bar{y}_M$, to guarantee that the median shock will be satisfied with the purchased product $\hat{w} \leq \bar{y}_M$ is needed. This threshold implies $G(\hat{w}) \leq G(\bar{y}_M) = 1 - G(\bar{y}_M) = 0.5$. Hence, at the profit maximizing threshold we need $G(\tilde{w}^*) \leq G(\tilde{w}) \leq 0.5$, or equivalently $\frac{\delta}{p} \leq 0.5$, which implies $\delta/p \geq 2$. At the cost minimizing threshold we need $G(\tilde{w}^c) \leq 0.5$, or equivalently $\frac{\delta}{p} + \frac{p}{\delta} \leq 0.5$, which implies $\delta/p \geq 1$. \(\Box\)

The current discussion would imply that the threshold used by Shackelford, Wheeler, and Koochmaraie (1999) to classify a steak into the tender class using a (3 days) slice shear force of 23 kg would be too stringent from all the perspectives analyzed here. In their study, the threshold resulted in 100% of the steaks classified as tender being rated slightly tender or higher by a panel (which was considered a success in classification). However, 91% of the carcasses classified as intermediate (48% of the carcasses in the experiment), and 28% of the carcasses classified as tough (5% of the carcasses in the experiment) were rated slightly tender or higher by the panel, and hence were suitable for certification. Note that we do not explicitly recognize that more stringent thresholds have the “potential benefit” discussed in footnote 41. Maybe this component was highly weighted by Shackelford, Wheeler, and Koochmaraie 1999. In any case, the criterion used by proponents of thresholds is rarely articulated explicitly.\(^{40}\)

\(^{40}\) The developers of MSA are an exception. They justify the large number of rejections of acceptable steaks on the grounds that it is necessary to guarantee consumer satisfaction (Thompson et al., 1999). Shackelford, Wheeler, and Koochmaraie (1999) are not explicit about the criterion used to choose their shear force threshold for the tender class, but it seems to be in a way such that there are not disappointed consumers. Other authors proposed threshold to assure a given percentage of consumer satisfaction, but they do not explain why that percentage is the correct target (e.g. Shackelford et al., 1991; Huffman et al., 1996; Miller et al., 2001).
For estimation of the probability of consumer satisfaction, or of acceptance, consumer $j$'s perceptions upon tasting the steak are assumed to be summarized by the dichotomous variable $\lambda_j$, where:

$$\lambda_j = \begin{cases} 1 & \text{if } y_j \geq w \\ 0 & \text{if } y_j < w \end{cases}$$

Therefore, for certified tender steaks, values of $\lambda_j = 0$, reflect disappointment by buyer $j$. Conditional on $w$, $\lambda$ can be viewed from the modeler's perspective to follow a Bernoulli distribution $\theta(w)$.\(^{41}\) We know that the probability of acceptance will vary systematically with quality $w$ and one would expect the probability of acceptance for any given Bernoulli trial to vary systematically with it. For estimation purposes, the systematic component is assumed linear in the parameters ($\alpha$ and $\beta$), and we use the logit as the link function connecting $w$ and $E(\lambda|w) = \theta(w)$ which yields the logistic regression model

$$\text{logit} (\theta_j) = \alpha + \beta w_j.\(^{42}\)$$

For the logit model, the thresholds can be obtained in closed form. For example, under the profit maximization criteria

$$\hat{w} = \frac{1}{\beta} \left( \ln \left( \frac{\delta - p}{p} \right) - \alpha \right).$$

### 4.3.1.2 Value of a customer is consumer specific

Consumer heterogeneity from differences in tastes not shocks complicates the analysis because the value assigned to any particular consumer and hence the punishment that that consumer can impose depends on how easy or difficult he or she is to please. In this section, we interpret the punishment a consumer can impose on the alliance as the loss of the expected net present value of that customer's future purchases. More demanding consumers will be less valued (for any fixed premium) because they are easier to lose. In other words, it is more likely that a consumer who has high standards will be disappointed in any given purchase. The alliance expects him/her not to repeat the purchase with high probability. Hence, expected punishments are consumer specific. Suppose that in any given period, the

\(^{41}\) Recall that $\theta(w)$ denotes the probability that a random consumer that shows up at the meat counter to buy steak with measured tenderness $w$ is satisfied by the product.

\(^{42}\) The logistic model has been used to model the relationship between shear force, and consumer's acceptability of steaks (Platter et al., 2003a).
alliance knows that a consumer will be disappointed $T$ periods down the road. Hence, the punishment this consumer can impose on the alliance if he/she is disappointed in that period is given by 

$$\delta = \sum_{t=1}^{T} p\gamma^{t-1} - p = \frac{p\gamma(1-\gamma^{T-1})}{1-\gamma},$$

where $\gamma \in (0,1)$ represents the appropriate discount factor. The equation represents the punishments the consumer can impose after paying for the current period steak. When there will be no repeated purchases, i.e. when $T=1$, the alliance cannot be punished for noncompliance in any given period. In these situations, it has been shown that markets for experience attributes will fail to develop (Klein and Leffler, 1981; Grossman, 1981; Antle, 2001; Marette, Bureau, and Gozlan, 2000). Note that consumers from whom the alliance expects more purchases over time have the capability to impose stiffer penalties in expectations. However, the time period in which customer $y$ will be disappointed is random, and depends on the interaction between his/her preferences (unknown to the alliance) and the quality distribution of steaks certified. Hence the alliance can only evaluate the punishment in an expectations sense $E_{\gamma} \delta = \frac{p\gamma(1-E_{\gamma}(\gamma^{T-1}))}{1-\gamma}$. As before, a consumer with taste $y$ will be satisfied whenever, by purchasing a certified steak, he/she finds that $w \leq y$. In any given period this scenario occurs with probability 

$$m(y, \hat{w}) = \Pr_{w \leq \hat{w}}(w \leq y) = \frac{F(y)}{F(\hat{w})} = F_{w \leq \hat{w}}(y),$$

for any consumer with $y \leq \hat{w}$, where $F_{w \leq \hat{w}}(\cdot)$ represents the cumulative distribution of tenderness in the certified population. By the definition of a cdf, $\frac{\partial m(y, \hat{w})}{\partial y} \geq 0$, which just reflects the fact that less demanding consumers will be satisfied with higher probabilities for any distribution of qualities. The same consumer is disappointed in any given period with probability $1 - m(y, \hat{w})$. With this in place, $T$ can be given the interpretation of a geometric random variable, with “success” probability $1 - m(y, \hat{w})$, which is counting the number of periods until customer $y$ is disappointed.

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43 For customers with $y > \hat{w}$, $m(y, \hat{w}) = 1$. Even the less tender of the certified steaks will satisfy these consumers, implying that mismatches do not occur for this segment of the customer base.
disappointed. Then \( E_r \delta = \frac{p y m(y, \hat{w})}{1 - \gamma m(y, \hat{w})} \) and \( \frac{\delta E_r \delta}{\delta y} \geq 0 \). The previous derivative implies that it is costlier to mismatch a less demanding customer. Note also that the value placed upon each customer, given by his/her punishment capability is endogenous. As \( \hat{w} \) increases (and the distribution of certified quality spreads towards less tender beef), the probability that any customer with \( y < \hat{w} \) is satisfied in any given period decreases, and so does the expected net present value of that consumer. More formally \( \frac{\delta E_r \delta}{\delta \hat{w}} = \frac{p y}{(1 - \gamma m(y, \hat{w}))} \leq 0 \), since by the definition of \( m(y, \hat{w}) \), \( \frac{\partial m(y, \hat{w})}{\partial \hat{w}} = -\frac{F(y) f(\hat{w})}{F(\hat{w})^2} \leq 0 \).

Note that this does not imply that the alliance can reduce total punishments and increase profits just by increasing the threshold. There is a balancing effect. As the alliance increases the threshold, a higher proportion of consumers are likely to be disappointed. Therefore, the net effect of increasing the threshold on expected total punishments is in general ambiguous. If the expected total punishment is decreasing over the relevant range of \( w \) we would have a corner solution, with the alliance certifying all the steaks. The trade-off just discussed is captured in the necessary conditions for the alliances' problem. We will return to it after we present the problem formally.

Stepping back for a moment, in parallel to our exposition in the previous section, suppose that every consumer has the same known perception of what is an acceptable steak. In this situation, all consumers will be satisfied only if the common expectations are not "too high" relative to the population of cattle available to the alliance. The next result states and proves this claim formally:

**Result 3:** For any given and known level of expectations \( y = \bar{y} \) and distribution of quality in the base cattle population \( F(\cdot) \) such that \( f(\bar{y}) > 0 \), all customers will be satisfied with probability one only if \( F(\bar{y}) > 1/2 \), that is, if the median quality will be found acceptable by all consumers. Otherwise, no effort will be incurred in sorting (recall that higher expectations in terms of quality translate into lower \( y \)).
Proof: For fixed \( y = \tilde{y} \), all customers will be satisfied with probability one if \( \hat{w} \leq \tilde{y} \).

We first show that \( \hat{w} < \tilde{y} \) does not maximize profits. For any arbitrary \( \hat{w} < \tilde{y} \),
\[
E(\pi | \hat{w}) = \int_{-\infty}^{\hat{w}} \left( p - (1 - \theta(w)) E_{T, \delta} \right) f(w) dw,
\]
which simplifies to \( E(\pi | \hat{w}) = pF(\hat{w}) \) by the definition of \( \theta(*) \). By the properties of a cdf and the assumption that \( f(\tilde{y}) > 0 \), the previous function does not attain a maximum in the region \( \hat{w} < \tilde{y} \). Therefore, only \( \hat{w} \geq \tilde{y} \) are candidate solutions for the profit maximization problem, and only \( \hat{w}^* = \tilde{y} \) result in the satisfaction of all customers with probability one. For any particular \( \hat{w} \geq \tilde{y} \), profits are
\[
E(\pi | \hat{w}) = \int_{-\infty}^{\tilde{y}} \left( p - (1 - \theta(w)) E_{T, \delta} \right) f(w) dw + \int_{\tilde{y}}^{\hat{w}} p - (1 - \theta(w)) E_{T, \delta} f(w) dw,
\]
which can be rewritten as \( E(\pi | \hat{w}) = pF(\tilde{y}) + (p - E_{T, \delta})(F(\hat{w}) - F(\tilde{y})) \) by use of the definition of \( \theta(*) \).
Therefore, \( \hat{w}^* = \tilde{y} \) will result only when \( (p - E_{T, \delta}) \leq 0 \) for all \( \hat{w} \geq \tilde{y} \). Since \( (p - E_{T, \delta}) \) is monotonically increasing in \( \hat{w} \), it is sufficient to identify the conditions needed for \( \hat{w} \to \infty \).
\[
\lim_{\hat{w} \to \infty} (p - E_{T, \delta}) = \lim_{\hat{w} \to \infty} \left( p - \frac{pF(\tilde{y})}{F(\hat{w}) - F(\tilde{y})} \right) \leq 0 \text{ occurs when } F(\tilde{y}) \geq 1/2,
\]
which implies \( F(\tilde{y}) > 1/2 \). Otherwise, there exist some \( \hat{w}' \) such that \( (p - E_{T, \delta}) > 0 \) and no effort will be incurred in sorting, since the profit function is monotonically increasing for all \( \hat{w} \geq \hat{w}' \).

Ex ante profits given by equation (4.2) can be written as
\[
E(\pi | \hat{w}) = E_{w} \left( p - (1 - \theta(w)) E_{T, \delta} | y < w \right) w \leq \hat{w} \right) Pr_{w} \left( w \leq \hat{w} \right) - C.
\]
which can be written in integral form as
\[
E(\pi | \hat{w}) = \int_{-\infty}^{\hat{w}} \left( p - \int_{-\infty}^{\gamma m(y, \hat{w})} dG(y) \right) dF(w) - C
\]
or
\[
E(\pi | \hat{w}) = \int_{-\infty}^{\hat{w}} \left( dG(y) \int_{-\infty}^{\gamma m(y, \hat{w})} dF(w) \right) - C.
\]

The double integral in equation (4.6) represents the expected losses due to matching or classification errors. Again, the losses arise because disappointed customers will not repeat purchases. The necessary condition for a maximum for this problem is given by
with equality only if \( \hat{w} > 0 \). Equation (4.7) captures formally the trade-off between the size of the punishment and the proportion of disappointed consumers. The third term in equation (4.7) denotes the change in the expected size of the punishment. This term combined with the preceding negative sign are conducive to higher thresholds. The second term of equation (4.7) represents the punishments coming from an increased proportion of disappointed consumers. Note that this is the only term in the equation (combined with the preceding sign) providing incentives to the alliance to not certify all the steaks. Using the definition of \( m(\cdot) \) and \( G(\cdot) \) the first order condition can be written as

\[
\frac{\partial E(\pi|\hat{w})}{\partial \hat{w}} = p \left\{ \frac{f(\hat{w}) - \int_{-\infty}^{\hat{w}} \frac{\gamma m(y, \hat{w})}{1 - \gamma m(y, \hat{w})} g(y) dyf(\hat{w}) - \int_{-\infty}^{\infty} \frac{\gamma m(y, \hat{w})}{(1 - \gamma m(y, \hat{w}))^2} g(y) dyf(w) dw}{-\int_{-\infty}^{\hat{w}} \frac{f(\hat{w}) - \gamma F(y)}{F(\hat{w}) - \gamma F(y)} g(y) dyf(\hat{w})} \right\} \leq 0 \tag{4.7}
\]

The optimal threshold depends on the distribution of consumer’s tastes, and in contrast with the fixed punishment case (presented in the previous section), on the distribution of measured tenderness in the cattle population. This equation has the usual interpretation. The alliance should increase the threshold until the costs outweigh the benefits of doing so. Benefits of increasing the threshold accrue due to larger proportions of certifiable product, and reductions in the punishment each disappointed customer can inflict. On the costs side, punishments due to mismatches are more likely to occur. For the alliance to find it worthwhile to put some effort in sorting, the term in parenthesis in equation (4.8) has to be positive for some \( \hat{w} \). This will only be satisfied if the decline in net present value of customers is strictly smaller than the punishments due to the increased proportion of

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\(^{44}\) Since the premium enters multiplicatively on the punishment function, it washes away from the first order condition. However, whether the alliance is successful (profitable) or not will be ultimately determined by the premium.
mismatches. Figure 4.4 presents the expected punishment function obtained from the data analysis in section 4.4.

The sufficient condition for this problem is given by

$$\frac{\partial^2 E(\pi|\hat{w})}{\partial \hat{w}^2} = -\gamma g(\hat{w}) + 2\gamma f(\hat{w}) \left( \int_{\gamma F(\hat{w})}^{1} \frac{F(y)}{1-\gamma} g(y) dy - \int_{\gamma F(\hat{w})}^{1} \frac{F(y)}{F(\hat{w})-\gamma F(y)} g(y) dy f(w) dw \right) \leq 0.$$  

A potential extension to the work presented here would be to study the effects of the distribution of tastes in the population on the optimal threshold, and alliance profitability. In contrast to the situation where all consumers have identical tastes, the effects of different expectations distributions on the optimal threshold can not be explored analytically. However, the effects of consumer heterogeneity on the threshold (and alliance profitability) can be assessed, in principle, numerically, using for example mean preserving spreads on the distribution of tastes, or solving the problem for alternative plausible distributions. McKenna et al. (2003) reported variations in consumers’ ratings of beef that can be attributed to cooking methods, degree of doneness, and interactions with other factors such as city. Research has shown it is common for cities to have grade differences (see McKenna et al., 2003 and cites therein). This implies that appropriate thresholds may differ for different regions. The alliance could attempt to improve the overall levels of acceptability, by targeting certain market segments (or geographical locations) known to rate beef tenderness better. A pragmatic implication is that different thresholds could be tailored to different populations leading to higher proportions of certified product (and profitability). In short, the alliance would be engaging in more sorting than just an absolute threshold.\(^{45}\)

\(^{45}\) This would also have practical implications if the alliance can exert efforts to reduce consumer heterogeneity in perceptions, for example by educating customers on cooking methods that will result in consistent (or improved) tenderness. For example, if the alliance can convince a customer to cook the steaks to a lesser degree, the perception for that consumer will be better (and likely to meet the expectations of marginal consumers). CCA (2002) found that 88% of customers not satisfied with their steaks blamed it on the product, whereas only 6% and 5% cooked correctly their boneless cross ribs and inside round respectively. When instructions on correct cooking methods were provided, those figures rose to 52% and 29% respectively. However, this would be another line of research requiring an alternative interpretation of \(W\), as a noisy signal of the true quality. Consumers cannot learn the true tenderness of a steak (even after consumption), only a signal of it affected by for example by the abilities of the cook.
4.3.2 True tenderness cannot be learned

There is no testing technology currently available allowing us to determine true tenderness index $w$. This index has to be inferred from other measurements. Shackelford, Wheeler, and Koohmaraie (1999) point out that the repeatability of slice shear force measurements is 0.89, indicating that there is still uncertainty of the true “tenderness” of a steak even after the test has been performed. In this situation, the alliance has to rely on the noisy signal $w_0 | w$ to make decisions. The threshold has to be identified in terms of $w_0$.

Associated with any value of $w_0$ there is a posterior probability distribution for the true tenderness $q(w | w_0)$. Formally, $q(w | w_0) \propto f(w) h(w_0 | w)$, where $h(w_0 | w)$ is the sampling distribution of the tests results, and $f(w)$ represents the prior information the alliance has about the cattle available. Tenderness of any given steak is now a random variable, and the results of the test give us some information about it. The acceptance region is the interval of feasible $w_0$ such that $w_0 \leq \hat{w}_0$. In general, for any $w_0$ in the acceptance region, the expected profits are given by $E_{w | w_0} \left( p - (1 - \theta(w)) E_{\delta | y < w} \delta y < w - C | w_0 \right)$, and averaged over all the acceptance region are $E_{w_0 \leq \hat{w}_0} E_{w | w_0} \left( p - (1 - \theta(w)) E_{\delta | y < w} \delta y < w - C | w_0 \leq \hat{w}_0 \right)$. We only present here the case where all consumers are valued identically and $\delta$ is fixed ($E_{\delta | y < w} = \delta$).

Over the rejection region expected profits are $E_{w_0 > \hat{w}_0} E_{w | w_0} (-C | w_0 > \hat{w}_0) = -C$. Hence, ex-ante expected profits for an alliance that has chosen the threshold $\hat{w}_0$ are given by

$$E(\pi | \hat{w}_0) = \int_{-\infty}^{\hat{w}_0} \int_{-\infty}^{\infty} \left( p - (1 - \theta(w)) \delta \right) q(w | w_0) h(w_0) dw dw_0 - C,$$

(4.9)

where $h(w_0)$ is the marginal distribution of the results of the test,

$$h(w_0) = \int h(w_0 | w) f(w) dw.$$

Equation (4.9) can be rearranged to yield

$$E(\pi | \hat{w}_0) = p H(\hat{w}_0) - C - \delta \int_{-\infty}^{\hat{w}_0} \int_{-\infty}^{\infty} \left( (1 - \theta(w)) \right) q(w | w_0) h(w_0) dw dw_0 ,$$

(4.10)
where, \( H(\cdot) \) is the cumulative distribution of the test results. The term involving the double integral in equation (4.10) represents the costs of not being able to sort the product perfectly to match consumers' expectations. Equation (4.10) can be rewritten to obtain

\[
E(\pi | \hat{w}_0) = (p - \delta) H(\hat{w}_0) - C + \delta \int_{\hat{w}_0}^{\infty} E \theta(w|w_0) h(w_0) dw_0.
\]

The first order condition for an interior solution for this problem is given by

\[
\frac{\partial E(\pi | \hat{w}_0)}{\partial \hat{w}_0} = (p - \delta) h(\hat{w}_0) + \delta E \theta(w|\hat{w}_0) h(\hat{w}_0) = 0,
\]

which yields \( \theta(w|\hat{w}_0) = \frac{\delta - p}{\delta} \). The expected probability of acceptance at the threshold is the same as when the true \( w \) was observable. The problem is to find the value of \( \hat{w}_0 \) such that the previous equation holds. Now the optimal threshold is not independent of the distribution of tenderness in the population of cattle available to the alliance. In particular, the distribution of tenderness on the cattle population affects the threshold through the posterior distribution. When \( w \) is observable, the distribution of tenderness only affects the feasibility of the system, that is, whether or not the system will be implemented. When \( w \) is not observable, the distribution of quality of the population plays a role in both, the optimal threshold of the signal, and whether the system is feasible.

Note that the equation determining the threshold can be rewritten in a way that makes clearer the factors involved as

\[
\theta(w|\hat{w}_0) = \int \theta(w)dQ(w|\hat{w}_0) = \int (1 - G(w))dQ(w|\hat{w}_0) = \frac{\delta - p}{\delta},
\]

which is equivalent to \( \int G(w)dQ(w|\hat{w}_0) = \frac{p}{\delta} \). \( G(\cdot) \) is again the cumulative distribution function of perceptions that will be considered acceptable. If we assume that different \( w_0 \)'s lead to first order stochastically dominated shifts of the distribution of \( w \), the left hand side of the equality is increasing in \( \hat{w}_0 \), implying that if a solution exists, it is unique.
4.4 Data and estimation results

The organization of the discussion and the approach used in this section follows closely that in Babcock, Carriquiry and Stern (1996). To illustrate the approach proposed, data was obtained and estimation of the relevant distribution functions was conducted. The data obtained are measurements of Warner-Bratzler shear force in two experiments (described below) conducted at Iowa State University. Some summary information of the data used here is presented in the first two rows of Table 1. The means and standard deviations of shear force obtained in both experiments seem to be smaller than those reported in other studies (Table 1), hence care is warranted when interpreting our results. As argued before, a “guaranteed tender” certification system is most likely to be successful when combined with some type of pre-sorting of cattle (e.g. as a complement to some of the existing ventures that sort by breed, number of days on high energy diets, etc), because this allows the concentration of efforts (and testing costs) on animals that have higher chances of being certified. The cattle population used in this study could be seen as being the result of selection from the general cattle population, by some criteria conducive to obtain tender beef. Our intention here is to illustrate the usage of the framework proposed, which can be easily adjusted to the cattle population available to the alliance and relevant economic environment.

The functions needed to solve the preceding model are

a) the sampling distribution of shear force measurement represented by \( h(w_0 | w) \)

b) the prior density function of shear force in the population of cattle of interest \( f(w) \)

c) the distribution of perceptions that will be considered acceptable for the target population \( g(y) \)

---

46 Hilton et al (2004) also obtained low averages of shear force. In that study, selection was conducted by trained evaluators. Animals were chosen from over 2000 feedlot steers and heifers. Additional sorting was conducted after slaughter, retaining only A-maturity carcasses with marbling scores between Slight and Modest.
Table 4.1. Summary statistic from seven selected studies from the animal and meat science literature

<table>
<thead>
<tr>
<th>Study</th>
<th>n</th>
<th>Aging (days)</th>
<th>Mean (kg)</th>
<th>SD (kg)</th>
<th>Grades^a</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gonzalez (2003)</td>
<td>10</td>
<td>1 &amp; 7</td>
<td>2.34</td>
<td>0.80</td>
<td>C</td>
</tr>
<tr>
<td>Iowa Beef Tenderness Project</td>
<td>376</td>
<td>N/A</td>
<td>2.53</td>
<td>0.45</td>
<td>P,C,S</td>
</tr>
<tr>
<td>Wheeler et al. (2002)</td>
<td>306</td>
<td>14</td>
<td>4.0</td>
<td>0.90</td>
<td>C,S</td>
</tr>
<tr>
<td></td>
<td>400</td>
<td>14</td>
<td>3.82</td>
<td>0.86</td>
<td>C,S</td>
</tr>
<tr>
<td>Hilton et al. (2004)</td>
<td>303</td>
<td>14</td>
<td>2.41-2.57</td>
<td>N/A</td>
<td>C,S</td>
</tr>
<tr>
<td>Shackelford et al. (1997)</td>
<td>400</td>
<td>14</td>
<td>5.3</td>
<td>1.6</td>
<td>N/A</td>
</tr>
<tr>
<td></td>
<td>594</td>
<td>14</td>
<td>4.1</td>
<td>1.1</td>
<td>N/A</td>
</tr>
<tr>
<td>Platter et al. (2003b)</td>
<td>550</td>
<td>14 &amp; 21</td>
<td>3.54-4.46</td>
<td>N/A</td>
<td>P,C,S</td>
</tr>
<tr>
<td>Vote et al. (2003)</td>
<td>399</td>
<td>14</td>
<td>4.0</td>
<td>0.9</td>
<td>C,S</td>
</tr>
<tr>
<td></td>
<td>195</td>
<td>14</td>
<td>4.5</td>
<td>1.1</td>
<td>C,S</td>
</tr>
<tr>
<td></td>
<td>304</td>
<td>14</td>
<td>4.3</td>
<td>0.9</td>
<td>N/A</td>
</tr>
<tr>
<td></td>
<td>184</td>
<td>14</td>
<td>4.7</td>
<td>1.0</td>
<td>S</td>
</tr>
</tbody>
</table>

^a USDA quality grade: P=prime, C=choice, S=select. ^b Six different phenotypes were considered. ^c For different hormonal implanting strategies. ^d Only standard errors for the means are reported in the study, and the sample size after post-harvest sorting was not reported (see footnote 1).

Data for estimation of a) and b) are available, whereas the parameters of the function \( g(y) \) are obtained from the animal science literature. Specifically, Platter et al. (2003a) used the logistic regression model presented before to predict probability of acceptance of steaks by consumers, as a function of shear force. Here we interpret \( y \) as a given consumer’s perception. Steak with shear force \( w > y' \) will be considered unacceptable by all consumers with acceptability perceptions \( y \leq y' \). In this context, \( G(w) \) represents the probability that a steak with shear force \( w \) will be deemed unacceptable. In Platter et al. (2003a), the predicted probability of acceptance was given by \( \theta(w) = \frac{e^{4.7331-1.0701w}}{1+e^{4.7331-1.0701w}} \), hence, the predicted probability of rejection would be \( G(w) = 1 - \theta(w) = \frac{1}{1+e^{4.7331-1.0701w}} \). We take the parameters of the distribution as given ignoring uncertainty in the estimates.

4.4.1 Prior density and sampling distribution

The prior distribution captures the alliance’s prior information about the actual tenderness as captured by shear force of the cattle available to them, that is, their beliefs about shear forces, before obtaining any new information through testing. A three parameter
gamma distribution, an informative prior, will be used in this illustration. This distribution was chosen based on an analysis of shear force measurements. Data for this purpose was collected from an experiment designed to assess the effects of different power ultrasound treatments on properties of the longissimus muscle (Gonzalez, 2003). The longissimus of 10 animals were used in this study. Each muscle (20 in total), was cut into 10 steaks, and each steak was divided into four samples. Three treatments were applied, and the remaining sample was left as a control. In total 800 samples were obtained (and 1600 cores were sheared). The data represents Warner-Bratzler test results rather than actual tenderness, but we ignore measurement errors in constructing the prior distribution. The idea here is to try to get a sense of the distribution of tenderness that will obtain from the cattle available. The treated samples from the previous data set may not be appropriate for this purpose, since the objective of the treatment was to alter the distribution of tenderness of the meat. Instead, only the samples from the control group (in their natural state) were used.

Several tests for normality (using the procedure CAPABILITIES of SAS, on WBS force measurements of the 200 untreated samples in Gonzalez (2003)) were conducted on the data set described below, and the null hypothesis that the data comes from a normal distribution is rejected. Graphical analysis of the distribution of the tests results from the untreated samples indicated that a positively skewed distribution might be appropriate. Shackelford et al. (1991, p. 293) conducted the same tests for normality concluding “WBS values were normally distributed”. However, discrepancies between their results and those expected if WBS were to follow a theoretically normal distribution are explained by the lack of “perfectly normal” data. In their discussion they note that the data was positively skewed since “the majority of the carcasses had WBS values less than the mean while the upper tail…extended beyond 10 kg (4.9 standard deviations above the mean)”. Hence, we chose a three parameter gamma distribution to represent the prior belief of the alliance about the tenderness $w$ in the cattle population available. The density of the proposed distribution is

$$f(w) = \frac{(w-\kappa)^{r-1} \exp\left(-\eta^{-1}(w-\kappa)\right)}{\eta^r \Gamma(r)}, \quad \tau, \eta, \kappa > 0; \ w > \kappa.$$  

$^{47}$ The tests conducted were: Shapiro-Wilk, Kolmogorov-Smirnov, Cramer-von Mises, and Anderson-Darling. All returned p-values lower than 0.01. Applying the same analysis to the other data set available confirmed these results.
The parameters $\eta$, $\kappa$, and $\tau$ were estimated (through maximum likelihood) to be $\hat{\eta} = 0.4338$, $\hat{\kappa} = 1.1821$ and $\hat{\tau} = 2.8142$, implying that the expected value and standard deviation of $w$ are 2.40 and 0.73 kg respectively, which are equal to the same sample statistics obtained for the control group (the first row of Table 1 reports the same statistics for all the treatments).

To explore the shape and estimate the parameters of the sampling distribution $h(w_0|w)$ a different data set was used. Data for this purpose was obtained from the Iowa Beef Tenderness Project, a three year project (2000-2002) that sought (among other objectives) to identify sires that produce superior offspring in terms of tenderness. The longissimus dorsi of 376 steers were used in this study. The animals used in this project were mostly from the Angus breed, and finished on high energy diets on a central testing site. This may explain the low average shear forces measured and the homogeneity (low standard deviation) in tenderness of the group. A plot of the mean (of six cores) and standard deviation of shear force for each sample is presented in Figure 4.2.

Standard deviations appear to be increasing with the mean shear force, and conditional on the mean, the data do not appear skewed. A normal distribution with mean $w$ and variance $(\psi w)^2$ is hypothesized for the sampling distribution of the individual cores. The sampling distribution for the result of the test is just the average of independent (if the testing
is done correctly) measurements obtained from \( n \) cores from a steak, and hence follows a normal distribution with mean \( \psi \) and variance \( \psi^2 / n \). \( \psi^2 \) needs to be estimated from the data. As in Babcock, Carriquiry, and Stern (1996), the parameter \( \psi \) was estimated using the ratio estimator

\[
\hat{\psi}^2 = \frac{\sum_{i=1}^{k} S_i^2}{\left( \sum_{i=1}^{k} \frac{X_i^2}{r_i} - \frac{1}{r} \right)},
\]

where \( k \) is the number of samples available, \( r_i \) is the number of cores sheared in the \( i^{th} \) sample and \( \bar{X}_i^2 \) and \( S_i^2 \) are the mean and variance of the measurements obtained from the \( i^{th} \) sample. For the data set available, \( \sum_{i=1}^{k} r_i = 376 \times 6 = 2256 \), and the parameter was estimated to be \( \hat{\psi}^2 = 0.0402 \). Hence, ignoring the uncertainty in this estimate, the sampling distribution of test results is normal with mean \( \psi \) and variance \( \psi^2 / n \). Research has shown that to achieve high repeatability, six cores \( (n = 6) \) are recommended (Wheeler et al., 1997).

The posterior distribution of \( \psi \) (again ignoring uncertainty in the estimate of \( \hat{\psi}^2 \)) can be written as

\[
q(w|w_0) = \frac{\sqrt{n} \exp\left\{-0.5\left(\frac{(w_0 - w)^2}{\psi^2} - \eta^{-1}(w - \kappa)\right)\right\}(w - \kappa)^{r-1}}{h(w_0)\sqrt{2\pi\psi^2\eta^r}\Gamma(r)}, \quad \kappa \leq w \leq \infty,
\]

where the normalizing constant, represented by the derived marginal distribution of \( w_0 \) is given by

\[
h(w_0) = \int_0^\infty \frac{\sqrt{n} \exp\left\{-0.5\left(\frac{(w_0 - w)^2}{\psi^2} - \eta^{-1}(w - \kappa)\right)\right\}(w - \kappa)^{r-1}}{\sqrt{2\pi\psi^2\eta^r}\Gamma(r)} dw.
\]

Figures 4.3a and 4.3b present the derived marginal distribution for Warner-Bratzler shear force measurements, and the posterior distribution conditional on different test results.

Figure 4.3b highlights the fact that significant uncertainty with regards to tenderness persists, even after completion of the best test currently available to measure it. Also, the distributions are not centered on the result of the test, especially for values that differ from
the a priori expected result. This reflects the effect of the informative prior distribution used. This combined with the observation that tougher steaks are associated with larger sampling variability (as reflected by the sampling distribution proposed), lead to posteriors with higher variances for larger tests results.

Figure 4.3. (a) Derived marginal distribution for test results; (b) Posterior distributions for three different test results

4.4.2 Determining the profit maximizing thresholds

To find the optimal thresholds, information is needed about the distribution of consumer tastes or types in the target population. Since data to gain that information is not available, we rely here on the results obtained by Platter et al. (2003a) who fitted a logistic regression to determine the probability of acceptance as determined by WBSF. We first obtain the threshold for the case where tenderness can be observed by the alliance through testing. The scenario when testing is imperfect is solved next.

4.4.2.1 True tenderness can be learned through testing

We assume in this section that the prior distribution obtained is an accurate reflection of the quality of the cattle available to the alliance. For the scenario where the value of a consumer is not taste specific and \( \delta \) is fixed, we showed in section 4.3.1.1 that for the logit model the optimal threshold can be obtained in closed form as 

\[
\hat{w}^* = \frac{1}{\beta} \left( \ln \left( \frac{\delta - p}{p} \right) - \alpha \right).
\]

Under the expected net present value interpretation of the punishment, if the alliance’s
planning horizon is long enough, \( \delta = \frac{py}{1-\gamma} \) obtains, and the profit maximizing threshold can be rewritten as \( \hat{w}^* = \frac{1}{\beta} \left( \ln \left( \frac{2\gamma - 1}{1 - \gamma} \right) - \alpha \right) \). In the logistic regression setting, \( \beta \) has the interpretation of the change in the log-odds of success (that the steak is found acceptable) that corresponds to a one unit increase in the independent variable \( w \). Therefore, it is expected to be negative (tougher steaks have lower probabilities of success). For the distribution of tastes implied by Platter et al. (2003a), \( \beta = -1.0701 \), and \( \alpha = 4.7331 \). As the punishment that can be imposed upon the alliance parameterized by the discount factor increases, the optimal threshold decreases, provided that the alliance puts sufficient weight on future earnings.\(^48\) For a discount factor of 0.9, the optimal threshold is \( \hat{w}^* = 2.48 \) kg, and for the distribution of tenderness of the cattle available, 61.9% of the loins tested will be certified.

When the value of a consumer is taste specific, the threshold needs to be obtained numerically, either by maximizing equation (4.6) (or equivalently the parenthesized portion of the equation), or solving equation (4.7). We follow the first approach. In this case, the optimal threshold depends on both the distribution of tastes, and the distribution of quality. Figure 4.4 explores the shape of the expected profits equation for different thresholds, by plotting the parenthesized term of equation (4.6). We explore this portion of the equation rather than expected profits because the shapes are the same and it allows us to avoid assuming a premium and testing costs. Expected profits are obtained just by rescaling that term by the appropriate premium and subtracting testing costs.

The two marked curves in Figure 4.4 represent the individual terms in the parenthesized portion of equation (4.6). The curve labeled “CDF” is the cumulative distribution function of the population of cattle available. The “E. Punishment” represents the expected total punishments by disappointed costumers respectively. Figure 4.4 depicts a situation where expected total punishments are increasing in the threshold, affording an

\(48\) To see this \( \frac{\partial \hat{w}^*}{\partial \gamma} = \frac{(1-\gamma)^{-1}}{\beta(2\gamma - 1)} < 0 \), provided \( \gamma > 1/2 \), or equivalently that the relevant interest rate is less than 100\%. 
interior solution. The dashed line in Figure 4.4 represents the term in parenthesis in equation (4.6). Clearly, profits will be highest when this term attains a maximum. That happens at $w^* = 3.98$ kg. The distribution function of tenderness indicates that at the profit maximizing threshold 96% of the cattle tested will be certified under the current scenario.

![Cumulative distribution function of tenderness, expected punishments from dissatisfied customers, and difference between the fraction certified under alternative thresholds and expected punishments. $\gamma = 0.9$](image)

At the profit maximizing threshold, the term in parenthesis is 0.85, hence the venture is profitable only if $C/p < 0.85$, or if the costs of testing do not surpass 85% of the premium. Notice the trade-off between the expected punishments by dissatisfied customers and opportunity costs. For very low thresholds, all consumers will be satisfied, but only a small fraction of the product is certified.

The optimal threshold obtained when tastes are consumer specific is noticeable higher than the one arising for non-consumer specific tastes and a long planning horizon. The difference between the two is due to the way in which the alliance perceives the punishments. In a sense, by increasing the threshold, the alliance is diminishing the punishment capabilities of each consumer. However, Figure 4.4 indicates that expected punishments increase as the threshold is increased. Figure 4.4 also shows that the benefits associated with certifying a higher proportion of steaks are only offset by the punishments at higher values of the threshold.
A high proportion of steaks certified were expected for this environment. First, the distribution of quality used is such that there are a large proportion of animals with low shear forces (see Table 4.1). The population of cattle used could be seen as coming from a successful presorting. Additionally, the first order conditions of the problem indicate that as the threshold is relaxed, the alliance knows it is putting a lower quality product in the market (in an expectations sense) and lessening the punishment potential of consumers (this was already argued above). Lastly subjects used by Platter et al. (2003a), whose taste distribution we borrow, may have been “too lenient” in their answers, since they were only asked if the steak was acceptable, and not if it would have been acceptable in a tenderness certified program. For example, increasing marginally (in absolute value) the parameter $\beta$, which would be representing a more demanding population of consumers led to more a stringent threshold (simulations not shown).

4.4.2.2 True tenderness cannot be learned through testing

We turn now to the situation where the alliance can only observe the noisy signal of tenderness provided by WBS force measurement. In parallel to the presentation of the conceptual model, only the scenario where tastes are not consumers specific and $\delta$ is fixed will be considered. Figure 4.5 illustrates the determination of the optimal threshold for the test. From the analysis in section 4.3.2, the optimal threshold is implicitly defined by $E\theta(w|\tilde{w}_0^*) = \frac{\delta-P}{\delta}$. Returning to the interpretation of the punishment as the expected NPV of a consumer for a time horizon that is long enough, i.e. $\delta = py/(y-1)$, (or letting $\delta$ be proportional to $p$ ) we obtain $E\theta(w|\tilde{w}_0^*) = (2y-1)/y$, the threshold depends only on the discount factor (or proportionality constant). The downward sloping curve in Figure 4.5 represents the expected probability of acceptance for alternative test results ($E\theta(w|w_0)$). The profit maximizing threshold is determined by finding the test result at which the expected probability of acceptance curve intersects the relevant horizontal line, representing alternative values of $\frac{\delta-P}{\delta}$, which for the current interpretation of the punishments translate into alternative discount factors.
For the same discount factor used in the previous section, that is \( \gamma = 0.9 \), the required expected probability of acceptance is \( \frac{2\gamma - 1}{\gamma} = 0.89 \). The test result associated with an expected probability of acceptance of 0.89 and therefore profit maximizing is \( \tilde{w}_\theta = 2.47 \text{ kg} \) (see Figure 4.5). To provide a measure of the uncertainty involved in the probability if acceptance, we report that for the proposed threshold the posterior interval is given by (0.84, 0.92). Also, from the derived marginal distribution of test results (Figure 4.3a), 61.3% of tested steaks will be certified for this discount factor.

![Figure 4.5. Determination of the profit maximizing threshold for two different premium-punishment combinations](image)

Note that this threshold is significantly lower than the one obtained for the case of customer specific tastes. This is due in part to the discussed fact that in the later environment punishments were endogenous (and affected by the alliance).

The threshold proposed by Shackelford et al. (1991) to obtain 68% of steaks rated slightly tender or better was 3.9 kg. Figure 4.5 shows that the proposed threshold, correspond to an expected probability of acceptance of 66%, which would be profit maximizing under a discount rate of 0.75. The percentage of steaks certified under this threshold (again obtained from the marginal distribution of test results, Figure 4.3a) would be 96.1%. For the prior and sampling distributions obtained, a test result of 3.9 kg, result in an expected posterior shear
force of 3.79 kg, and an expected probability of acceptance comparable (slightly lower) to that obtained by Shackelford et al. (1991). The acceptability level proposed by these authors (68%) is included in the 95% posterior interval (0.51, 0.77) for the probability of acceptance at $w_0 = 3.9$.\textsuperscript{49} When quality can be measured perfectly an expected probability of acceptance of 0.66 can be obtained by $w = 3.8$ kg, which is as just argued the expected posterior shear force of a test result of 3.9 kg. The informative prior distribution is indicating that such a high test result may be overestimating the true tenderness and corrects accordingly. In the same line, if the required shear force to attain the optimum probability of acceptance falls below the prior mean, the alliance needs to allow for some “margin of error” when an imperfect test is used.\textsuperscript{50}

Direct comparisons are hard to make, because of the way in which questions were posed to consumers. In Shackelford et al. (1991), acceptability was not recorded directly (tenderness ratings higher than “slightly tender” were considered acceptable), whereas it was recorded in the work of Platter et al. (2003a). Huffman et al. (1996) concluded from their study that steaks with less than 4.1 kg of WBS force will ensure satisfaction of 98% of customers. Miller et al. (2001) obtained 100% tenderness acceptability with steaks of measured WBS force lower than 3, and 93% tenderness acceptability for values of WBS force between 3 and 4.6 kg. The tenderness acceptability levels reported in the later two studies seem to be higher than those obtained in the present work and by Shackelford et al. (1991). The discrepancy with our work may be partially explained by the fact that the distribution of tastes borrowed from Platter et al. (2003a) referred to overall and not just tenderness acceptability.

4.5 Conclusions

One of the main problems identified by U.S. beef industry participants is the lack of a uniform and consistently tender product. This problem is important because there is widespread agreement that tenderness is the single most important factor affecting consumer satisfaction with beef. The lack of adequate product tenderness has been blamed, at least

\textsuperscript{49} The posterior interval associated with a test result of 3.9 kg is wider than the one for 2.44 kg. This takes into account the larger uncertainty associated with higher test results.

\textsuperscript{50} For example, if the discount factor is $\gamma = 0.95$ the optimal threshold when quality can be assessed without error is 1.67 kg, whereas the same threshold is 1.60 if there are testing errors.
partially, for the decline in beef market share to other sources of protein. The beef industry has been slow in implementing systems to sort beef better into tenderness classes. This has been fueled by the feeling of packers and feedyards that no one would want to purchase steaks in the least tender classes (Schroeder et al. 1997). Agricultural economists, along with meat and animal scientists, documented that consumers are willing to pay premiums exceeding the costs of testing and sorting for meat that is guaranteed tender (Boleman et al., 1997; Lusk et al., 2001; Miller et al., 2001; Shackelford et al., 2001). Hence, there is an opportunity for certain groups of producers to create a niche market and add value to their cattle by categorizing meat according to tenderness. This marketing strategy is beginning to be used in the U.S.\textsuperscript{51}

Some of the difficulties of implementing a tenderness certification system for red meats (with particular emphasis on beef), and the main economic factors to consider are identified in this paper. Specifically, we have shown that the optimal threshold depends in general on the distribution of quality of the livestock available, the precision available to measure it, the objective pursued, the distribution of consumer tastes, and how the venture perceives the likelihood that a consumer will be disappointed with any given quality provided. We develop a flexible framework that can be used to determine optimal certification thresholds for a wide variety of economic environments, including perfect and imperfect testing technologies, different objectives, and different consumer valuations. The commonality to all scenarios considered is that sorting consumers out perfectly is not possible. That is, only partial sorting is allowed.

The paper sheds some light on why determining thresholds for certification of tenderness in red meats has been so elusive. A key factor is that the objective function needs to be clearly identified in order to obtain a meaningful optimal threshold. Thresholds designed under a profit maximizing, cost minimizing, or error minimizing (with different weight for different types of errors) objectives will generally differ.

If the objective is to maximize profits, the optimal threshold when true quality cannot be learned will be different than that in the perfect testing scenario. This reflects that when

\textsuperscript{51} However, it has been in use for a few years in Australia, using a process based classification system (see the literature review).
there are testing errors, the alliance needs to take some "margin of error" in the certification scheme, which in turn results in a different proportion of certified steaks. For all the scenarios analyzed, the least stringent thresholds were obtained when customer valuation was taste specific and for a perfect testing technology. This follows from the fact that the value of each customer for the alliance depends on the distribution of product quality in the market and on how hard is he/she to please. Customers that are very demanding are more likely to be disappointed sooner, hence, less repeated purchases or business is expected from them. This in turn undermines their punishment capabilities. In this model, by increasing the threshold, the alliance knows that is providing a product to the market that is of lower expected quality. The alliance will expect fewer repeated purchases from all consumers. By loosening the threshold, the alliance decreases the punishment each customer can impose (the market is less valuable when you do not expect it to last). The balancing factor is that a looser threshold will also yield a higher proportion of consumers that are disappointed.

The analysis presented considers only classification of tenderness into two categories. The optimal number of tenderness classes is an open question, not addressed in this dissertation. Fruitful exploration of that question will require more information regarding consumer's expectations and willingness to pay for different tenderness levels.

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52 This is consistent with the Australian experience, where the boundary for the pass/fail criteria was adjusted in order to ensure a high level of confidence (Polkinghorne et al., 1999).
CHAPTER 5. CONCLUSIONS

This dissertation analyzed some key economic issues in supply chain management for value added products that arise when qualitative or quantitative uncertainties are present. Both types of uncertainties entail different problems and plausible solutions. A commonality is that they both demand more information gathering and/or tighter coordination among chain members than that required for commodity production and afforded by open markets. When investments in processing capacity are needed, or the sole existence of the market is contingent on the presence of a sought after quality attribute that is difficult (if not impossible) to verify even after consumption, some chain participants need to take actions to obtain more information or alter the quantity or quality of the product that will be available.

The actions analyzed in this dissertation are: a) contracting (in Chapter 2) as a way to manage the amount of an input a processor will obtain from a group of farmers; b) requiring input suppliers to adopt a particular QAS system to affect (stochastically) the quality of the output a firm is going to be able to deliver (Chapter 3); and c) the decisions by a group of producers with regards to the design of a certification system, focusing on finding thresholds for certification on an index of quality measurements (Chapter 4).

Chapter 2 studied the merit and feasibility of co-existence of spot and contract markets for a value added (specialty grain) product. For that purpose, a stylized theoretical model was constructed. The situation was modeled as a two stage game, where oligopsonistic processors contracted upstream with price taking farmers. In our model, co-existence was obtained as Nash equilibrium under a wide range of distinct parameters. This would indicate that the introduction of contracting into an agricultural industry does not necessarily mean that open markets for those products will disappear. There may be a balance to be obtained for the relative size of the two markets. The fundamental economic factors influencing the prevalence of each market were identified. It was shown that in order to obtain co-existence, both yield uncertainty and a vigorous competition (Bertrand style) whenever a spot market arises are needed. From a policy perspective, if the goal is to maintain viable spot markets where independent producers can sell their output, it may be necessary to monitor the behavior of the buying side whenever an open market arises. The simulations also revealed that processors would prefer the disappearance of spot markets if marginal costs of
contracting are affected by spot acreage, whereas farmers profits are higher under coexistence. All our results apply to the particular form of the contract observed. An interesting line of research would be to verify whether the same results obtain under alternative contractual arrangements, and explore the shape of the optimal contract.

Chapter 3 analyzed the optimal choice of a QAS a processing firm should require from its suppliers, or in other words, the choice of a quasi-voluntary system of assurance. This case is becoming pervasive in the food industry. The model presented has great generality. It allows us to analyze process control systems, that is verification that certain production methods were observed and followed (e.g. free range), as well as control over physical attributes of the input (e.g. lean beef). Also, any type of quality attribute, from credence to experience can be accommodated within the proposed framework. Our findings indicate that when the sought after quality attribute is harder to discover, the firm finds it optimal to reduce the stringency of QAS. This effect is magnified when firms cannot obtain the full return on their investments in quality; when reputations are not firm-specific firms will try to free ride on other firms’ investments. Our analysis indicates that increasing the number of firms (from a monopoly to a duopoly) is welfare improving only if firms that deliver substandard goods can be identified. Firms prefer the firm specific to the industry wide reputation. In short, we provide a rationale for both the wide variety of QAS observed in the real world, and the branding efforts of many firms to differentiate themselves in the marketplace. The chapter also has policy implications. Chief among them is that welfare can be enhanced by improving traceability, facilitating branding efforts, and/or mandating labeling.

The last issue addressed (Chapter 4) points to some of the fundamental economic factors and challenges that should be considered when designing a quality assurance system. This chapter intends to be pragmatic. It proposes a flexible framework that can be used empirically by a group of producers to sort their product into quality classes, based on the result of some (potentially many and imperfect) measures of quality. In particular, the models presented are designed to obtain thresholds for certification. The analysis in conducted in terms of beef tenderness, but the framework is clearly more general.
The search for thresholds for beef tenderness certification, based on Warner-Bratzler shear force measurements (pursued mainly by meat and animal scientists) has been ongoing for a long time now. Our models provide insights on why finding those thresholds has been such an elusive task. Optimal thresholds depend in a complex way on the particular objectives of the designer, the quality of the cattle population available, the distribution of consumer tastes (in particular of tenderness acceptability), the accuracy of the testing technology, and the way in which the designer believes tenderness acceptability is determined (customer versus non-customer specific). The last item in turn determines the value the designer will attach to different customers. An illustration of how to put our framework to work is provided for different combinations of the mentioned factors.

The optimal number of quality classes is an open question. We limit the analysis to two classes. Although the theoretical models presented can be expanded to address this issue, meaningful answers for that question will require carefully designed experiments to elicit consumers’ expectations and willingness to pay for different tenderness classes, as well as to assess their ability to detect subtler differences in tenderness that will result as the number of classes increase.

Chapters 3 and 4 highlight the importance of the economic environment in determining the incentives that shape the quality provision efforts of firms. When customers have the means or the ability to punish firms that deliver substandard products, firms will put more stringent systems in place or choose higher thresholds, even if that implies higher procurement costs, or having a lower proportion of the product to sell for a premium. The analysis points to environments that will result in higher expected quality delivered to the market and markets where low quality firms can be singled out easily. Those are the types of markets where reputational mechanisms for quality provision will work best. When firm reputational mechanisms do not work, government intervention may be required to prevent customer deception, either by providing certification services, requiring quality claims to be backed by a trusted independent third party, or facilitating the linkage between firms and products.
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