RECIPROCITY-BASED MEASUREMENT MODELS
FOR ULTRASONIC NDE

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INTRODUCTION

Quantitative NDE objectives such as the evaluation of flaw signals and the assessment of inspection capabilities generate the need for models of ultrasonic scattering measurements capable of including all aspects of the measurement process: radiation and reception characteristics of probes, propagation of finite beams, scattering of nonplane-wave fields, and the like. Once such models are developed and validated, that is, proven able to predict the received voltages obtained in actual ultrasonic experiments, they can be used to gain a thorough understanding of the NDE inspection process. Specifically, these models will be useful in investigations of the factors that significantly affect ultrasonic inspections, and they will form the basis for the proper interpretation of flaw signals[1].

In order to model the measurement process, we formulate the distributed field aspects of radiation, propagation, scattering, and reception in terms of a single function of frequency, called the scattering coefficient[2]. This function can be convolved with the frequency response of the electronics, specifically the pulser and receiver, to arrive at the received voltage. Formulas for these scattering coefficients have been developed in the past on the basis of reciprocity relations for the fields involved[3,4]. In this paper, the formulas are arrived at by another method that explicitly uses a description of the transducer characteristics. This approach extends previous formulations to the case of nonreciprocal probes. The calculation of these scattering coefficients is discussed herein and then illustrated through examples in a companion paper[5].

DEVELOPMENT OF SCATTERING FORMULAS

The case to be considered is illustrated in Fig. 1. It represents a generic pitch-catch, immersion measurement with scattering from a single submerged object. From this arrangement, it is also possible to

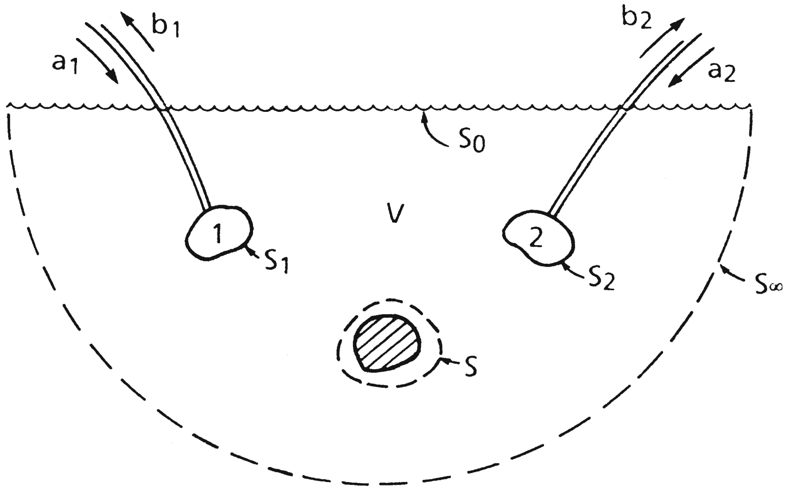

Fig. 1. Generic pitch-catch, immersion measurement.

consider a pulse-echo measurement for which the same probe acts as both the transmitter and the receiver. The probes, labeled 1 and 2 in Fig. 1, are connected to the electronics by cables that are assumed to support only one propagating mode; all other modes are attenuated as they propagate. On each cable, this propagating mode has two components: one moving toward the probe with amplitude a_1 or a_2, depending on with which probe the cable is connected, and one moving away from the probe with amplitude b_1 or b_2. For linear operation, the outgoing mode amplitudes b_1 and b_2 are related to the incident mode amplitudes a_1 and a_2 through the matrix equation [2]

$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = [S] \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \tag{1}$$

The entire radiation, scattering, and reception process is represented by the scattering coefficients S_{ij}. The first subscript on the scattering coefficient refers to the probe used as the receiver, and the second subscript to the probe used as the transmitter, so that all possible transmitter and receiver combinations are considered. The form of Eq. (1) represents single frequency excitation with the assumed time dependence of $e^{-i\omega t}$ suppressed; this will be done throughout the paper.

The first step toward the development of formulas for the scattering coefficients is to choose a general method for describing the properties of the transducers. One suitable choice is a spherical wave scattering matrix (SWSM) description. This description is exactly analogous to the plane wave scattering matrix (PWSM) approach[6,7], which would also be a suitable choice. For the SWSM, one starts with the spherical wave expansion of the pressure field outside the smallest spherical surface that completely encloses the probe (see Fig. 2):

$$p(\underline{r}) = \sum_{n,m} [b(n,m)h_n^{(1)}(kr) + a(n,m)j_n(kr)]Y_{n,m}(\Theta,\phi), \qquad r > a \tag{2}$$

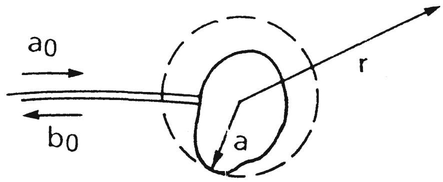

Fig. 2. General immersion transducer with feeding cable.

In Eq. (2), $b(n,m)$ is the mode amplitude of the outgoing spherical wave $h_n^{(1)}(kr)Y_{nm}(\Theta,\phi)$, where $h_n^{(1)}(kr)$ is the outgoing spherical Hankel function of order n and $Y_{n,m}(\Theta,\phi)$ is the spherical harmonic of degree n and order m, and $a(n,m)$ is the mode amplitude of the incoming spherical wave $j_n(kr)Y_{n,m}(\Theta,\phi)$, where $j_n(kr)$ is the spherical Bessel function of order n and r, Θ, and ϕ are the spherical polar coordinates of the field point $\underline{r}$. The transducer characteristics are then defined by the equations

$$b_o = S_{oo}a_o + \sum_{n,m} S_{01}(n,m)a(n,m) \tag{3a}$$

$$b(n,m) = S_{10}(n,m)a_o + \sum_{n,m} S_{11}(n,m\,|\,\nu,\mu)a(\nu,\mu) \tag{3b}$$

In Eq. (3), a_o and b_o are the amplitudes of the propagating mode in the cable connected to the general transducer under consideration, just as a_1 and b_1 are the amplitudes for the cable connected to probe 1. The scattering matrix that describes the behavior of the transducer has been partitioned into the four submatrices S_{10}, S_{01}, S_{11}, and S_{00}, representing the probe's radiation, reception, acoustic scattering, and electromagnetic reflection properties, respectively.

If we consider probe i to act as the receiver for the measurement, then it is clear from Eq. (3a) that the outgoing mode amplitude b_i on the cable due to the acoustic field incident upon the probe can be found from

$$b_i = \sum_{n,m} S_{01}^i(n,m)a_i(n,m)$$

where S_{01}^i represents the reception properties of the probe and the a_i's are the mode amplitudes for the incoming spherical wave expansion of the incident field. The value of b_i can be made numerically equivalent to S_{ij} by equating the field incident upon probe i to the field resulting when probe j is excited with an incident mode amplitude on the cable of unity, that is, taking $a_j = 1$. Furthermore, only the scattering from the submerged object is of direct interest. Thus, the component of the scattering coefficient due to scattering alone, ΔS_{ij}, and not due to any interaction between the two transducers, will be separated out as

$$\Delta S_{ij} = \sum_{n,m} S_{01}^i(n,m)a_{ij}^s(n,m) \tag{4}$$

where the a_{ij}^s's are the mode amplitudes for the incoming spherical wave expansion of the field incident upon probe i that results from scattering off the submerged object (with probe j acting as the transmitter).

The amplitudes a_{ij}^s can be determined from the integral representation of the scattered field. Consider an arbitrary closed surface S enclosing the scatterer with $p_j(\underline{r}')$ and $\underline{v}_j(\underline{r}')$ being the pressure and velocity, respectively, on that surface when probe j is used as the transmitter. Then the field scattered by the submerged object is given by

$$p_j^s(\underline{r}) = \oint_S [p_j(\underline{r}')\underline{\nabla}' G(\underline{r}',\underline{r}) - i\omega\rho \underline{v}_j(\underline{r}')G(\underline{r}',\underline{r})] \cdot \underline{n}(\underline{r}') dS(\underline{r}') \tag{5}$$

where $G(\underline{r}',\underline{r})$ is the free-space Green's function, with $\underline{r}'$ denoting the location on S; $\underline{r}$ is the field point outside S; $\underline{n}(\underline{r}')$ is the outward normal to the surface; and $\underline{\nabla}'$ is the gradient with respect to $\underline{r}'$. To determine the amplitudes a_{ij}^S, the spherical wave expansion of the free-space Green's function about the origin for probe i (see Fig. 3),

$$G(\underline{r}',\underline{r}) = ik \sum_{n,m} [h_n^{(1)}(kr')Y_{nm}^*(\Theta',\phi')] j_n(kr) Y_{nm}(\Theta,\phi)$$

is inserted into Eq. (5) and the result compared to Eq. (2). This yields the values of the amplitudes a_{ij}^S, which can then be substituted into Eq. (4) to give

$$\Delta S_{ij} = \frac{1}{2}\oint_S \{\underline{v}_j(\underline{r}')\Big[\sum_{n,m} 2\rho c k^2 S_{01}^i(n,m) h_n^{(1)}(kr') Y_{n,m}^*(\Theta',\phi')\Big]$$

$$-p_j(\underline{r}')(i\omega\rho)^{-1}\underline{\nabla}'\Big[\sum_{n,m} 2\rho c k^2 S_{01}^i(n,m) h_n^{(1)}(kr') Y_{n,m}^*(\Theta',\phi')\Big]\} \cdot \underline{n} dS \tag{6}$$

From Eqs. (2) and (3b), the factor within the square brackets is recognized as the field that would be radiated by probe i if it were excited by an incident amplitude a_i of unity and were considered to have the radiation characteristics

$$\hat{S}_{10}^i(n,m) = 2\rho c k^2 S_{01}^i(n,-m)$$

It can be shown that these radiation characteristics are equivalent to those of the (hypothetical) transducer adjoint to probe i [7] so that Eq. (6) can be written in a general form as

$$\Delta S_{ij} = \frac{1}{2}\oint_s \left(\hat{p}_i^I \underline{v}_j - p_j \hat{\underline{v}}_i^I\right) \cdot \underline{n} dS \tag{7a}$$

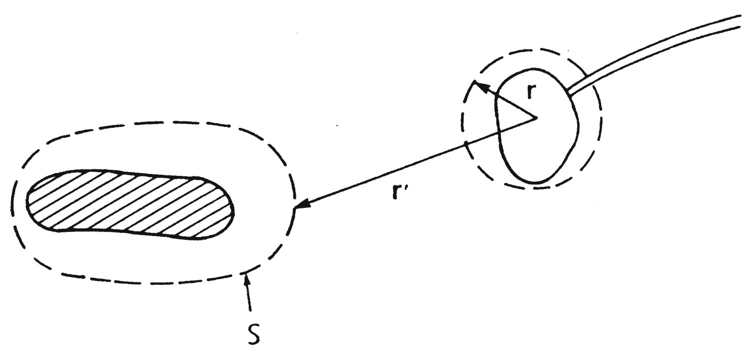

Fig. 3. Reception of scattered field by receiving probe.

where $(p_j, \underline{v}_j)$ are the total fields when probe j is used as the transmitter and $(\hat{p}_i^I, \hat{\underline{v}}_i^I)$ are the incident fields (upon the scatterer) when probe i is used as the transmitter with the radiation characteristics of its adjoint. In essence, the right-hand-side of Eq. (7a) has been normalized to a power level of $\frac{1}{2}$, since the power incident upon probe j carried by the mode in the cable is $\frac{1}{2}|a_j|^2$ and $a_j = 1$. Eq. (7a) is therefore dimensionless, although it does not at first appear to be so. The total fields $(p_j, \underline{v}_j)$ can be written as the sum of incident and scattered fields

$$p_j = p_j^I + p_j^S \quad \text{and} \quad \underline{v}_j = \underline{v}_j^I + \underline{v}_j^S$$

and the incident fields do not contribute to the scattering component ΔS_{ij}. Thus, Eq.(7a) can also be written as

$$\Delta S_{ij} = \frac{1}{2}\oint_S \left(\hat{p}_i^I \underline{v}_j^S - p_j^S \hat{\underline{v}}_i^I\right) \cdot \underline{n} \quad dS \tag{7b}$$

Eqs. (7a) and (7b) are the desired general formulas for the component of the scattering coefficient solely because of scattering from an object enclosed within the surface S.

CALCULATION OF SCATTERING COEFFICIENTS

To demonstrate how the various aspects of the measurement are incorporated into the formulas of Eqs. (7a) and (7b), the steps involved in a calculation are discussed herein. First, it is necessary to determine the radiation and reception characteristics of the transducers to be used. This will in general be done experimentally on the basis of a description of the transducers like that of the SWSM. Second, once the probe characteristics are known, the transducer fields are calculated. The fields incident upon the scatterer $(p_j^I, \underline{v}_j^I)$ and $(\hat{p}_i^I, \hat{\underline{v}}_i^I)$ contain the radiation and reception characteristics of the probes. Third, the scattered fields $(p_j^S, \underline{v}_j^S)$ must be solved for, which brings the properties of the scatterer into the calculation. Finally, the integral of Eq. (7a) or (7b) is evaluated with the substitution of the previously calculated fields. Thus all the distributed field aspects of the measurement are combined into a single, frequency-dependent function, ΔS_{ij}. The first three steps of the calculation represent three generic problems in NDE that have been, and still are being, investigated extensively, although usually separately. The final step will in most cases simply replace the surface or volume integral required for the commonly used farfield scattering amplitude. It should also be noted that the integrals called for in Eqs. (7a) and (7b) are extremely flexible in terms of the solution methods that can be used to determine the scattered fields. Numerical techniques like the T-matrix and boundary element methods are particularly well suited for use in the formulae, as are approximate techniques like the Born and Kirchhoff approximations. Eqs. (7a) and (7b) are also useful starting points for the development of approximate linear system models in which each measurement aspect is treated as a separate factor [8].

DISCUSSION

General formulae have been derived that predict the change in the scattering coefficient of an immersion measurement due to acoustic wave scattering from a submerged object. These formulae are valid for both reciprocal and nonreciprocal probes. The extension to the case of elastic wave scattering from a defect internal to the submerged object can be made by applying elastic wave reciprocity to the appropriate fields within the object. The scattering coefficient represents the outgoing mode amplitude in the cable of the receiver normalized by the incident mode amplitude in the cable of the transmitter. As such, it incorporates all of the distributed field aspects of the measurement; however, it does not include the properties of the pulser and the receiver, which are required to convert the scattering coefficient into a predicted received voltage. This conversion can be achieved by using the relation

$$V_{sc} = \beta \Delta S_{ij} \tag{8}$$

where V_{sc} is the component of the received voltage due to scattering and β, referred to as the system efficiency factor, contains the characteristics of the pulser and the receiver. The factor β also depends on the transducers involved since the loading on the pulser will depend on the probe, but it does not depend on the scattering process. The need for transducer and equipment characterization is thus apparent, and the application of Eqs. (7a) and (7b) in this regard is considered in a companion paper[5]. Other applications for these formulae, as well as their extension to the elastic wave case, include the computation of the probability of detecting flaws and the development of flaw characterization techniques.

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