

ULTRASONIC TRANSDUCER CALIBRATION FOR
 RECIPROCITY-BASED MEASUREMENT MODELS

David D. Bennink and Anna L. Pate

Center for NDE
 Iowa State University
 Ames, IA 50011

INTRODUCTION

A general formulation regarding the relationship between scatterer properties and experimental signals was presented in a companion paper^[1]. This formulation was developed for the case of acoustic wave scattering in an immersion measurement. In order to perform the theoretical calculations, the formulation requires knowledge of the radiation and reception characteristics of the probes involved and the combined properties of the pulser and the receiver (system efficiency). Thus, the objective of this paper is to discuss methods for obtaining the characteristics of measurement systems as required in the above-mentioned formulation. As an example, the case of acoustic wave scattering from a sphere of finite size is examined. Experimental verifications of results are included.

In principle one could calculate properties of measurement systems from the physical construction of the transducers and equipment; however, it is currently more practical to determine them experimentally. Our method is based on Eq. (7b) of^[1], repeated here for convenience:

$$\Delta S_{ij} = \frac{1}{2} \oint_s (\hat{p}'_i \underline{v}'_j - p'_j \hat{v}'_i) \cdot \underline{n} dS \quad (1)$$

Specifically, it is applied to the commonly used on-axis profiling technique in which a scatterer is moved along the acoustic axis of a probe operated in the pulse-echo mode^[2]. Both exact and approximate methods of evaluating Eq. (1) are considered for this technique, and the results are applied to the determination of the system efficiency.

ON-AXIS PROFILING

The measurement arrangement to be considered is depicted in Fig. 1. An object is placed on the acoustic axis of a transducer operating in the pulse-echo mode at a distance of z from the probe's front surface, and the signal resulting from scattering is measured. Assuming a reciprocal probe with identical reception and radiation characteristics gives $(\hat{p}', \underline{v}') = (p', \underline{v}')$. With the radiated field given by the Rayleigh integral, then

$$p'(\underline{r}) = -\rho c \left(\frac{ik}{2\pi} \right) \int_{s_0} V_n(\underline{x}) \frac{e^{ikR}}{R} dS(\underline{x})$$

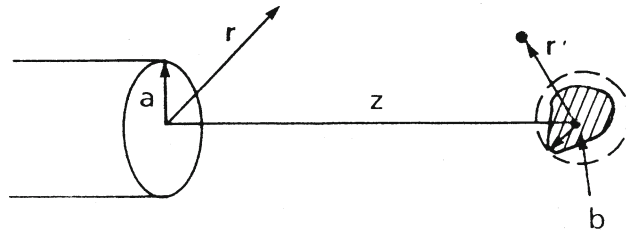


Fig. 1. Measurement arrangement for on-axis profiling.

where S_0 is the surface of the probe, here assumed to be a circle of radius a , and V_n is the normal velocity on that surface. By expanding $R^{-1}e^{ikR}$ in spherical waves, the pressure can be written in the form

$$p^i(\underline{r}) = \sum_{n,m} S_{10}(n,m) h_n^{(1)}(kr) Y_{nm}(\Theta, \Phi), \quad r > a \quad (2)$$

where the coefficients $S_{10}(n,m)$ describe the probe's radiation characteristics and are given by

$$S_{10}(n,m) = \begin{cases} (-1)^l \sqrt{4\pi(4l+1)} \frac{(2l-1)!!}{(2l)!!} I_l(ka), & n=2l \text{ \& \ } m=0 \\ 0, & \text{otherwise} \end{cases} \quad (3a)$$

with

$$I_l(y) = y^2 \int_0^1 f(x) j_{2l}(xy) x dx \quad (3b)$$

To arrive at Eq. (3a), the normal velocity is assumed to be symmetric about the acoustic axis and to have a maximum value of V_0 with $\rho c V_0$ taken to be unity in order to correspond to the case $a_0=1$. The function $f(x)$ in Eq. (3b) represents the normalized radial distribution of the normal velocity and takes on the value $f(x)=1$ for a rigid piston source.

To find the incident field in the vicinity of the scatterer, the translational addition theorem for spherical waves along the z-axis^[3] is applied to Eq. (2) with the result

$$p^i(\underline{r}') = \sum_{n,m} \alpha(n,m) j_n(kr') Y_{n,m}(\Theta', \Phi') \quad (4a)$$

where

$$\alpha(n,m) = \sum_{\nu} C_{n\nu}(kz) S_{10}(\nu, m) \quad (4b)$$

and r', Θ', Φ' are the spherical coordinates of the field point \underline{r}' with respect to an origin located at z . The coefficients $C_{n\nu}(kz)$ account for the displacement of the coordinate system along the z-axis. Using the T-matrix method^[4] to solve the scattering problem gives the scattered field as

$$p^s(\underline{r}') = \sum_{n,m} b(n,m) h_n^{(1)}(kr') Y_{nm}(\Theta', \Phi') \quad (5a)$$

where

$$b(n,m) = \sum_{\nu, \mu} T(n, m | \nu, \mu) \alpha(\nu, \mu) \quad (5b)$$

and the T-matrix $T(n, m | \nu, \mu)$ contains all the information about the scatterer. When this scattered field and the incident field from Eq. (4) are placed in the scattering coefficient formula of Eq. (1) and the reciprocity of the transducer is applied, the result is

$$\Delta S_{11} = \frac{1}{2\rho ck^2} \sum_{n, m} \sum_{\nu, \mu} T(n, m | \nu, \mu) a(n, -m) a(\nu, \mu) \quad (6)$$

Eq. (6), together with Eqs. (3a), (3b), and (4b), represents the exact solution for the case of acoustic wave scattering from an object placed on the acoustic axis of a reciprocal probe with an axially symmetric normal velocity distribution and operated in the pulse-echo mode. Note that the assumptions made, or restrictions placed upon the transducer, are not necessary for the analysis but were used to keep the equations from becoming overly complicated. In Eq. (6), each of the measurement aspects can be clearly recognized: the transition matrix $T(n, m | \nu, \mu)$ represents the scattering process, and the amplitudes $a(n, m)$ include the transducer characteristics through the coefficients $S_{10}(n, m)$ and the wave propagation through the coefficients $C_{n\nu}(kz)$.

Before examining the predictions of Eq. (6), it is useful to consider the limiting case of planewave incident fields. In this case, it is assumed that the incident fields in the vicinity of the scatterer can be approximated by

$$p'(r') = C(z) e^{ikz} e^{i\mathbf{k} \cdot \mathbf{r}'} \quad (7)$$

where \mathbf{k} is a vector directed along the z-axis of magnitude k and $C(z)$ represents the normalized pressure (recall $\rho c V_0 = 1$) at the location z divided by the planewave phase factor e^{ikz} . Expanding $e^{i\mathbf{k} \cdot \mathbf{r}'}$ in Eq. (7) in spherical waves gives the incident pressure in the form of Eq. (4a) with

$$a(n, m) = 4\pi i^n C(z) e^{ikz} Y_{nm}^*(0, \phi) \quad (8)$$

When Eq. (8) is substituted into Eq. (6), the resulting double sum is recognized, yielding

$$\Delta S_{11} = \frac{2\pi i}{\rho ck} C^2(z) e^{2ikz} A(-\mathbf{k} | \mathbf{k}) \quad (9)$$

where $A(-\mathbf{k} | \mathbf{k})$ is the farfield back-scattering amplitude when a plane wave is incident in the direction of \mathbf{k} . For the purpose of comparing the results of this limiting case with the exact solution of Eq. (6), it is convenient to define the normalized on-axis profile by

$$C(z) \equiv \left\{ \frac{\rho ck \Delta S_{11}}{2\pi i A(-\mathbf{k} | \mathbf{k})} \right\}^{1/2} e^{-ikz} \quad (10)$$

If the incident fields can be approximated by plane waves, then $C(z)$, as given by Eq. (10) with the exact results of Eq. (6) used for ΔS_{11} , will correspond to the normalized on-axis pressure divided by the planewave phase factor e^{ikz} .

A comparison between the planewave approximation and the exact solution appears in Fig. 2 for the case of scattering from a rigid sphere when a piston source is used as the transducer. In the figure, the magnitude of the normalized profile $C(z)$ is plotted against the on-axis location given in terms of the S parameter, $S = z\lambda/a^2$, with z measured from the front of the rigid sphere and a the radius of the piston. The radius of the sphere is b , and the curve with b/a of 0 is

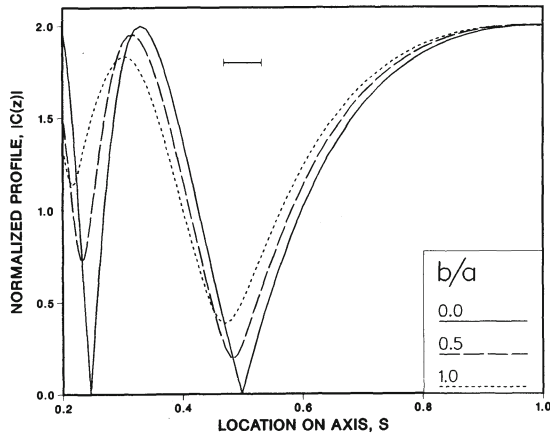


Fig. 2. Normalized on-axis profile for scattering from a rigid sphere for a reciprocal piston source with $ka = 100$.

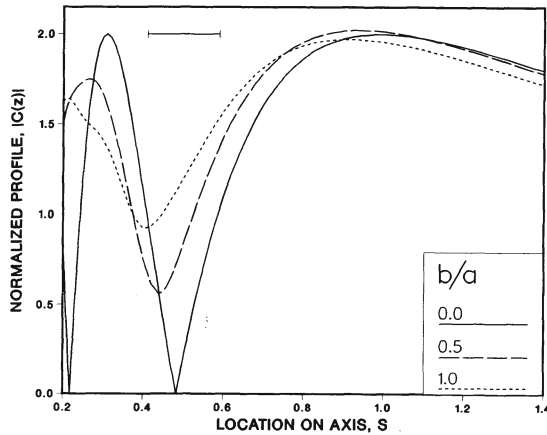


Fig. 3. Normalized on-axis profile for scattering from a rigid sphere for a reciprocal piston source with $ka = 35$.

the familiar on-axis pressure profile of a rigid piston. The calculations were carried out for a ka value of 100, and to put a scale on the profile variations the probe radius in terms of S is represented by the horizontal bar in the figure. A similar graph for a ka value of 35 appears in Fig. 3.

Several interesting features can be observed in these figures. As expected, the finite size of the scatterer causes the on-axis zeroes to become only minima, which become shallower for larger spheres, and the maxima to decrease in amplitude, except for the last one which is relatively unaffected in height. Of interest is the fact that the locations of these maxima and minima move as the size of the sphere changes, but without any direct correlation to the size. Of course these effects are more pronounced for the lower frequencies for which the scatterer size becomes larger with respect to the scale of the field variations. Also as expected, the exact curves for nonzero values of

b/a approach the curves of the normalized on-axis pressure as S increases. This property may be used to determine when the planewave approximation is valid.

SYSTEM EFFICIENCY

To characterize a measurement process completely, more than the determination of the scattering coefficient is required. As discussed in^[1], the component of the received signal due to scattering is given by

$$V_{sc} = \beta \Delta S_{ij} \quad (11)$$

where β is referred to as the system efficiency and contains information about the properties of the pulser and the receiver. Since the system efficiency does not depend on the scattering process, it may be determined from a reference or calibration experiment as

$$\beta = V_{ref} / \Delta S_{ref} \quad (12)$$

Once β is determined, it can be used to convert the scattering coefficients into predicted received voltages through Eq. (11). It is important to note that the system efficiency determined from Eq. (12) is valid only for the equipment settings used in the reference experiment and that these settings must be maintained for Eq. (11) to predict the results of other scattering experiments. For this reason, it is desirable to have a variety of possible reference experiments involving different response levels or scattering strengths.

In our work, Eq. (6) was used to calculate ΔS_{ref} for substitution in Eq. (12). Results of measuring the system efficiency in this manner for a commercial 10 MHz, 1/4" probe (Panametrics model V312) and for three separate reference experiments are presented in Fig. 4. Each experiment consisted of scattering from a steel sphere placed 5 cm away from the active face of the probe, which corresponds to an S value of 0.84 at the nominal frequency of 10 MHz and puts the sphere still somewhat in the nearfield (the steel spheres were mounted on the end of steel rods of smaller diameter so the scatterer did not correspond exactly to the idealization of a free but stationary rigid sphere). What varied among the three experiments was the radius of the sphere, ranging from 7/64" down to 3/64". Reasonable agreement was achieved among the three experiments, although they did not give exactly identical results. Errors in the measurement, as well as errors associated with the idealizations used in the theoretical calculations, could easily account for the discrepancies observed.

To test these β values, we used them in Eq. (11) to predict what the received signal would be when the scatterer was moved to a different on-axis location. The results of Fig. 5 were obtained when the spheres were moved to a location 1.5 cm from the active face of the probe, which corresponds to an S value of 0.25 at 10 MHz and is well into the nearfield. Two spheres were considered with radii of 7/64" and 3/64". For being far into the nearfield, the agreement between the predicted signal spectrum and that actually measured is quite reasonable. As the sphere was moved farther out and closer to the location at which β was measured, the agreement between the predicted and the measured signals improved as can be observed from Fig. 6. In Fig. 6, results are shown for a sphere location of 3.4 cm from the face of the probe ($S = 0.57$ at 10 MHz). Of course, a more severe test of these β values would be to use them to predict the signals received from scatterers other than a steel sphere; however, the results presented here are sufficient to demonstrate the validity of the approach.

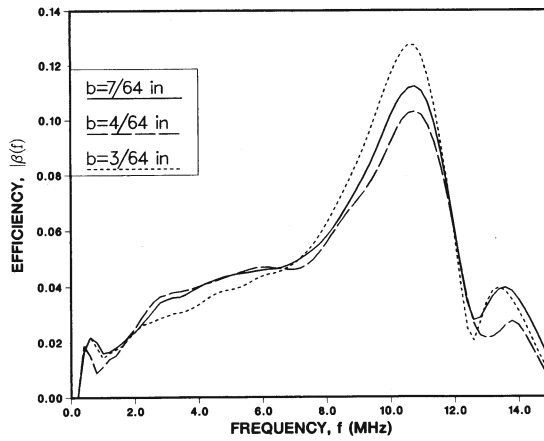


Fig. 4. System efficiency as determined from the scattering from a steel sphere at $z = 5$ cm.

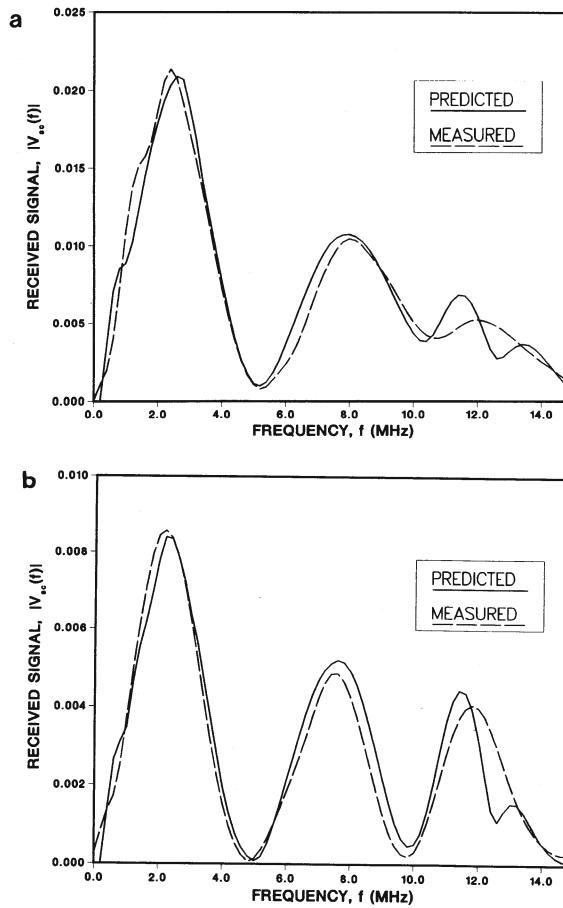


Fig. 5. Received signal for scattering from a steel sphere at $z = 1.5$ cm: (a) sphere radius of $7/64$ " and (b) sphere radius of $3/64$ ".

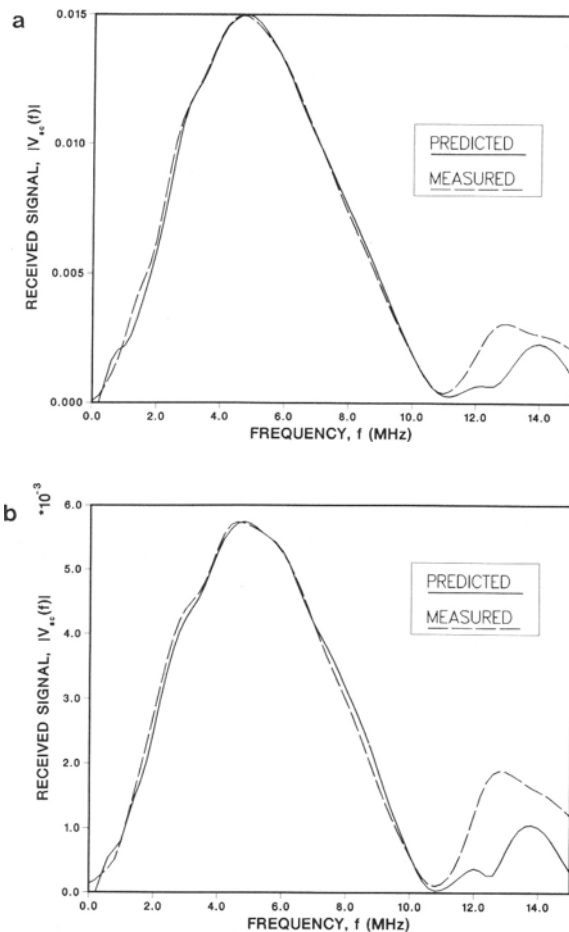


Fig. 6. Received signal for scattering from a steel sphere at $z = 3.4$ cm: (a) sphere radius of $7/64$ " and (b) sphere radius of $3/64$ ".

DISCUSSION

An application of the formulation developed in^[1] has been presented for the case of acoustic wave scattering from an object placed on the acoustic axis of a reciprocal probe operated in the pulse-echo mode. This particular measurement arrangement is commonly used to examine the radiation properties of probes, and the current work allows one to investigate the effects resulting from the finite size of the scatterer. Both exact and approximate methods of evaluating the scattering formula, Eq. (1), were considered. The application of the results to the determination of the system efficiency factor was also examined. It was shown that the system efficiency could be determined from a reference experiment and subsequently used to predict the signals received in other scattering measurements. As mentioned in^[1], this ability to predict the signals received in scattering experiments has many applications in quantitative NDE, including the computation of the probability of detecting flaws and the development of flaw characterization techniques.

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