LIQUID-MEMBRANE COUPLING RESPONSE AND MEASUREMENT OF ABSOLUTE DISPLACEMENT AMPLITUDES WITH A SUBMERSIBLE ELECTROSTATIC ACOUSTIC TRANSDUCER

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INTRODUCTION

In order for a transducer to give useful information to the investigator, it should have well known and well understood properties. Its frequency response, amplitude sensitivity, and phase shift, are important characteristics of its output. If it is to be used as a device for measuring finite amplitude and non-linear effects in a medium, it must be capable of an absolute calibration. The Electrostatic Acoustic Transducer (ESAT)[1] is such a transducer. It can also be used for calibration of other transducers.

This work establishes the linearity of this device by comparing its output with the Raman-Nath parameter (RNP) for tone bursts of various amplitudes. We also present a model to explain the effects of the membrane on the measurement of particle displacement amplitudes of ultrasonic waves in liquids A technique for their measurement is also presented.

ESAT CONSTRUCTION

Figure 1 is a cross section of the ESAT. An optically flat central electrode of 1/2 inch diameter is axially mounted with a spacing of approximately 10 μm from a thin conducting membrane, which is stretched across the housing, and held in place by a retainer ring. Alignment and spacing are assured by the fact that the central electrode is coaxially mounted in a sliding assembly whose optically flat annular front surface is pushed against the membrane. This front surface is parallel to the recessed central electrode to better than 12 arc-seconds.

The system is sealed to pneumatically adjust and control gap spacing between the membrane and central electrode. The air pressure inside the ESAT is adjusted by a bellows, and monitored by a pressure gauge with a sensitivity of 10^{-4} psig.

An ultrasonic wave incident on the ESAT membrane varies the gap spacing at the frequency of the ultrasonic wave. With a DC bias on the central electrode, an alternating electrical signal is developed. We have shown[1] that the ESAT output is given by

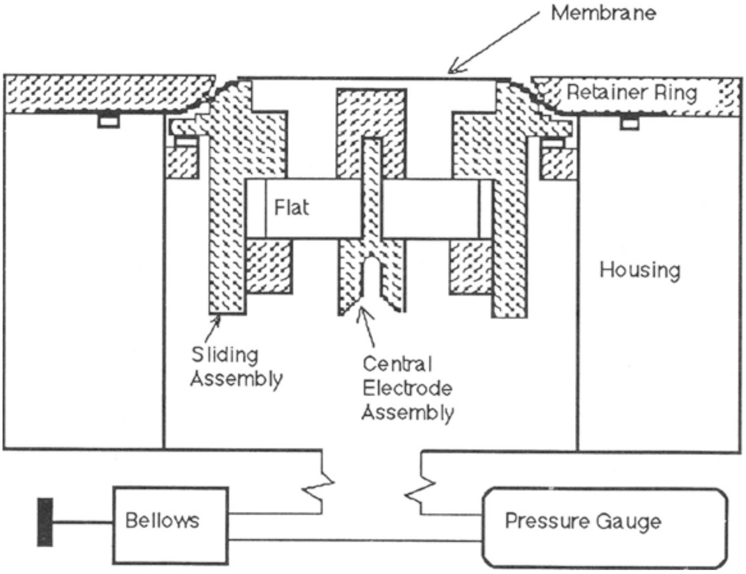

Fig. 1. Mechanical Features of the ESAT.

$$v = V_b\left(\frac{\eta}{S_o}\right), \tag{1}$$

where v is the ac voltage generated by the ESAT, V_b is the bias voltage, S_o is the gap spacing, and η is the membrane displacement from equilibrium.

THEORY

A. Raman-Nath Diffraction of Light

A light wave passing through an ultrasonic sound field in a liquid experiences a spatially periodic variation across its phase front, caused by the sinusoidal variation in the index of refraction of the liquid. These variations cause the light to be diffracted into various orders[2,3]. The normalized intensity of light diffracted into the m^th^ order, I_m, is given by

$$\mathrm{I}_m = J_m^2(v), \tag{2}$$

where

$$v = \frac{2\pi K \iota P_o}{\lambda}, \tag{3}$$

the Raman-Nath parameter, and J_m = the mth order Bessel Function, ι = light propagation distance through the ultrasonic beam, λ = wavelength of light, K = piezooptic coefficient, P_o = excess pressure amplitude.

The relationship between the excess pressure and displacement amplitude[3] is given by

$$P = -\rho c^2 \frac{\partial \xi}{\partial z}, \tag{4}$$

where ρ = mass density of the liquid, ξ = particle displacement, c = ultrasonic phase velocity in the liquid.

Consider an ultrasonic wave travelling in the z-direction. Letting

$$\xi = \xi_o e^{j(\omega t - kz)}, \tag{5}$$

where k = propagation constant, ξ_o = particle displacement amplitude, and $j = (-1)^{1/2}$,

Eqs. (3), (4), and (5) combine to give

$$v = j\left(\frac{2\pi\iota K\rho c}{\lambda}\right)\omega\xi_o. \tag{6}$$

We note that, by holding frequency, ω, constant, the Raman-Nath parameter, v, is proportional to the particle displacement amplitude, ξ_o.

MEMBRANE COUPLING RESPONSE

Referring again to Figure 1, we examine the elements involved in the determination of the liquid-membrane coupling response. At the top of the membrane is the liquid medium, while at the bottom is air. We use the fact that the acoustic impedance of air is much lower than that of the membrane to make the assumption of free surface conditions at the membrane-air interface.

Consider the interface between the liquid and the membrane. The transmission coefficient across the interface is

$$T = \frac{2Z}{Z + Z_1}, \tag{7}$$

where

$$Z_1 = j\rho_2 c_2 \tan(k_2 l), \tag{8}$$

ρ_2 = mass density of the membrane, k_2 = propagation constant in the membrane ($2\pi/_\lambda$), c_2 = compressional phase velocity in the membrane, l = membrane thickness.

Combining Eqs. (7) and (8) gives

$$T = \frac{2}{1 + j\frac{\rho_2 c_2}{\rho c}\tan(k_2 l)}, \tag{9}$$

When $\lambda \gg l$

$$\tan(k_2 l) \approx k_2 l. \tag{10}$$

Combining Eqs. (9) and (10) we have

$$T = \frac{2}{1 + j\frac{\omega\sigma}{\rho c}}. \tag{11}$$

where

$$\sigma = \rho_1 l, \tag{12}$$

with σ being the areal mass density of the membrane.

Consider a wave traveling in the z-direction, described by

$$\xi = \xi_o e^{j(\omega t - kz)}. \tag{5}$$

The transmitted portion crossing the liquid-membrane interface is

$$\xi_T = \left(\frac{2\xi_o}{1 + j\frac{\omega\sigma}{\rho c}}\right)e^{j(\omega t - kz)}. \tag{13}$$

At the membrane-air interface

$$\eta = \frac{2}{1 + j\frac{\omega\sigma}{\rho c}}\xi_o e^{j(\omega t - kz - \delta)}, \tag{14}$$

where

$$\delta = \frac{\omega l}{c_2}, \tag{15}$$

and η is the membrane displacement. Writing the denominator in polar form gives

$$\eta = \frac{2\xi_o}{\sqrt{1 + \left(\frac{\omega\sigma}{\rho c}\right)^2}} e^{j(\omega t - kz - [\delta + \theta])}, \tag{16}$$

where

$$\Theta = \mathrm{Arctan}\left(\frac{\omega\sigma}{\rho c}\right).$$

Solving for the particle displacement amplitude of the incident wave gives

$$\xi_0 = \sqrt{\frac{1+\left(\frac{\omega\sigma}{\rho c}\right)^2}{2}}\ \eta_0, \tag{17}$$

where η_0 is the membrane displacement amplitude.

Therefore, by measuring the membrane displacement amplitude, one can calculate the particle displacement amplitude impinging on the ESAT membrane.

EXPERIMENTAL CONSIDERATIONS

The linearity of the electrical output of the ESAT is established by comparing the particle displacement amplitude measured with the ESAT to the RNP[2,3]. The experimental arrangement for measurement of the particle displacement amplitudes is shown in Figure 2.

The gap spacing between the membrane and the central electrode is controlled pneumatically. A pressure gauge is connected to the pneumatic control as a monitor. Adjustments are made throughout the measurement procedure to assure that the pressure, which controls the gap spacing, is held fixed. The gap spacing is determined from a measurement of the capacitance between the central electrode and the membrane.

Consider the diagram in Figure 2. The DC bias voltage (V_b), measured by the DC Voltmeter, is applied through circuitry in the line driver to the central electrode of the ESAT. A tone burst (frequency of 4.70 MHz) is amplified by the linear amplifier and applied across the piezoelectric transducer (a 1/2 inch diameter Lithium Niobate 36° Y-cut 5 MHz plate) in the water tank. The generated ultrasonic wave traverses the water path, impinging upon the membrane of the ESAT. The high impedance AC signal (v) developed by the ESAT is sent through the line driver to a preamplifier (gain of 40 dB), whose output is marked on an oscilloscope. The line driver is disconnected from the ESAT and connected to the signal substitutional box (SSB), whose internal circuit is set to be electrically equivalent to the ESAT.

A signal generator, operating at the same frequency as the tone burst, is connected to the input of the SSB. The output of the signal generator is adjusted until the amplitude of the displayed waveform is identical to that caused by the ESAT output. The voltage input to the SSB is measured with an RF Voltmeter. Under these conditions, the RF voltmeter reading is equal to v.

The essentials of the experimental arrangement to measure the RNP is shown elsewhere[1]. The light source, a helium-neon laser, is aligned so that the beam almost grazes the surface of the piezoelectric transducer, and is focused onto the surface of a photodiode, positioned to measure the intensity of the zeroth diffraction order. Its output is fed into a current-to-voltage converter, whose output is displayed and measured on the oscilloscope.

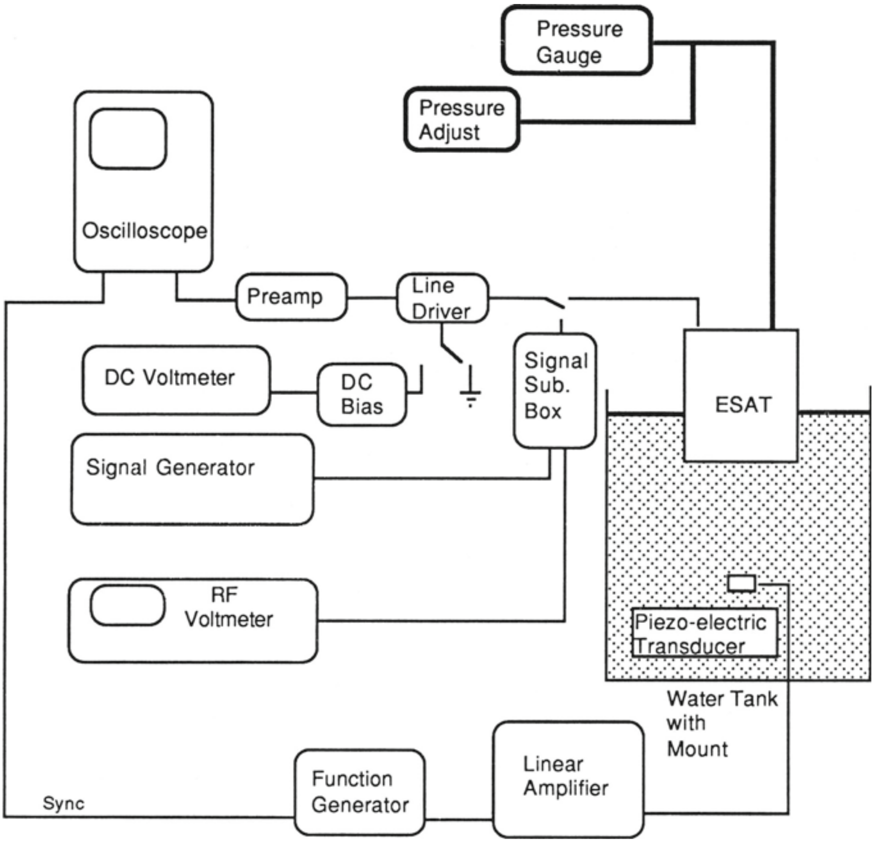

Fig. 2. Experimental Setup.

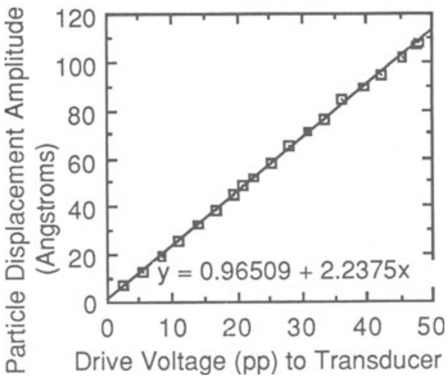

Fig. 3. Particle displacement. Amplitude vs. transducer drive voltage (peak to peak).

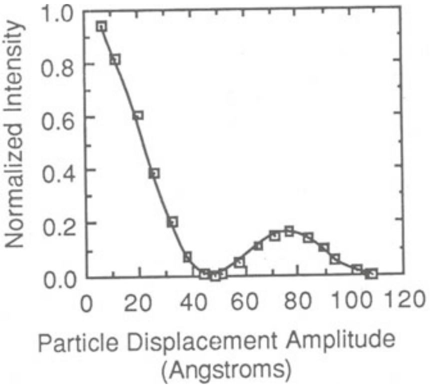

Fig. 4. Normalized intensity vs. particle displacement amplitude.

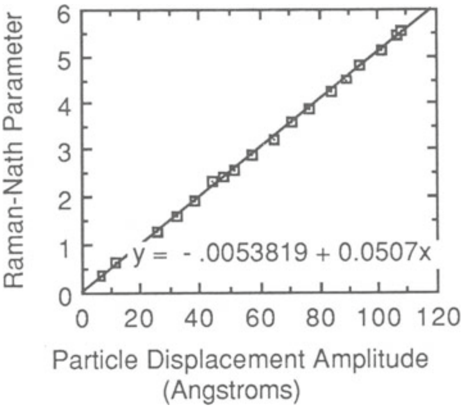

Fig. 5. Raman-Nath parameter vs. particle displacement amplitude.

DATA AND RESULTS

A plot of particle displacement amplitude vs. drive voltage (peak to peak) applied to the piezoelectric transducer is shown in Figure 3. The line is a least squares plot whose equation is given in the lower portion of the graph. We see from this plot that the particle displacement amplitude is linear in drive voltage to the transducer. A plot of normalized intensity vs. particle displacement amplitude is shown in Figure 4. The solid line in the graph is an interpolated plot between the data points.

Using a best fit algorithm to the normalized light intensity against ξ_o, we determined a best fit between the RNP and ξ_o, which is shown in Figure 5. The solid line is a least squares fit, whose equation is shown in the lower part of the graph.

CONCLUSIONS

We have shown that particle displacement amplitudes and excess pressure amplitudes, given by Raman-Nath diffraction are linearly related, as predicted by the theory. This work has also shown that the liquid-membrane coupling response analysis and measurement of the output voltage of the ESAT makes it possible to make absolute measurements of particle displacement amplitudes of ultrasonic waves. These measurements can be taken for any frequency in a range of better than 2 decades.

We anticipate that the ESAT will become a useful tool in the measurement of liquid-state anharmonicity and other nonlinear effects. With its well understood characteristics, and its extremely broadband frequency response[4], we also expect that it will find use in studies where frequency dependence is important. It will also be a useful instrument in transducer calibration.

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