Cavities and waveguide-cavity coupling in a three-dimensional layer-by-layer photonic crystal

Preeti Kohli

Iowa State University

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Cavities and waveguide-cavity coupling in a three-dimensional layer-by-layer photonic crystal

by

Preeti Kohli

A thesis submitted to the graduate faculty
in partial fulfillment of the requirements for the degree of

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Gary Tuttle, Major Professor
   Kai-Ming Ho
   Rana Biswas
   Vikram Dalal

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This is to certify that the master's thesis of

Preeti Kohli

has met the thesis requirements of Iowa State University

Signatures have been redacted for privacy
# TABLE OF CONTENTS

ABSTRACT

CHAPTER 1. INTRODUCTION 1

- Photonic Bandgap Crystals 1
- Maxwell’s equations for photonic crystals 2
- Properties of photonic band gap structure 5
- Photonic Crystal cavities 6
- Organization of thesis 12
- References 13

CHAPTER 2. PROPERTIES OF SINGLE-DEFECT ACCEPTOR CAVITY IN A THREE-DIMENSIONAL LAYER-BY-LAYER PHOTONIC CRYSTAL 13

- Abstract 13
- References 22

CHAPTER 3. TUNING OF RESONANT FREQUENCIES OF A SINGLE DEFECT CAVITY IN A 3-D PHOTONIC CRYSTAL 23

- Abstract 23
- References 29

CHAPTER 4. TRANSMISSION FROM AN INPUT TO EXIT GUIDE USING RESONANT EFFECT OF A NEARBY CAVITY 30

- Abstract 30
- References 37

CHAPTER 5. CONCLUSION AND FUTURE WORK 39
This thesis examines the fundamental characteristics of defect cavities in three-dimensional (3-D) layer-by-layer photonic crystals (PCs). Photonic crystals are engineered materials in which a 3-D periodic variation in the dielectric constant is used to prevent propagation of electromagnetic waves within a range of frequencies, thus creating a photonic band gap. By introducing a cavity defect within the periodic structure of the crystal, narrow transmission bands are formed within the photonic gap. This can be viewed as photonic resonant tunneling, in which photons tunnel through PC cladding layers surrounding the cavity. The transmission modes correspond to the resonant frequencies of the cavity.

Using a microwave-scale photonic crystal structure, we studied the effects of changing cladding layer thicknesses on resonant frequencies and the quality factors (Q) of measured transmission peaks. We found that, for a given cavity size, the quality factor increased with increasing cladding thickness, indicative of better photonic confinement within the cavity. Also, the peak transmission level decreased with increasing cladding thickness. Both of these results are consistent with resonant tunneling behavior. We also studied the effect of changing the cavity size, while keeping cladding thicknesses constant. We found that the resonant frequencies changed very little as the cavity sizes changed. Again, this is consistent with the theoretical picture of cavities within PCs.

We also studied coupling between waveguides and cavities, in which electromagnetic radiation would travel down a waveguide within a PC, couple to a nearby cavity, which
would, in turn, couple to a second waveguide. By studying variations in the placement of the coupling cavity with respect to the two waveguides, we were able to find conditions that would lead to almost 100% transfer from one waveguide to the other, within a narrow band of frequencies. These results suggest that waveguide-cavity coupling may be a useful component for building complex photonic waveguide networks within a photonic crystal.
CHAPTER 1
INTRODUCTION

Photonic Bandgap Crystals

Photonic crystals are periodically structured electromagnetic media, generally possessing photonic band gaps: ranges of frequency in which light cannot propagate through the structure. This periodicity, whose length scale is proportional to the wavelength of light in the band gap, is the electromagnetic analogue of a crystalline atomic lattice, where the latter acts on the electron wave function to produce the familiar band gaps, exhibited in semiconductors. The study of photonic crystals is likewise governed by the Bloch-Floquet theorem, and intentionally introduced defects in the crystal (analogous to electronic dopants) give rise to localized electromagnetic states: linear waveguides and point-like cavities. The crystal can thus form a kind of perfect optical “insulator,” which can confine light losslessly around sharp bends, in lower-index media, and within wavelength-scale cavities, among other novel possibilities for control of electromagnetic phenomena.

Photonic bandgap crystals are typically made by taking a unit cell containing dielectric material of differing indices of refraction and repeating this unit cell to construct the crystal. Crystals that contain metal are termed as metallic photonic band gap crystals. Photonic crystals are normally classified by the number of their dimensions. The two main groups are two-dimensional photonic bandgap crystals and three-dimensional photonic band gap crystals. Two-dimensional structures are easy to fabricate for shorter wavelengths using standard semiconductor processing techniques. A three-dimensional photonic band gap crystal is periodic in all directions and so exhibits a photonic band gap in all directions. Figure 1.1 shows all 3 kinds of photonic crystals, 1D, 2D and 3D.
Figure 1.1: Schematic depiction of photonic crystals periodic in one, two and three directions, where periodicity is in the material (typically dielectric) structure of the crystal. Only a 3-D periodicity, with a more complex topology than is shown here, can support an omnidirectional photonic band gap.

**Maxwell’s equations for photonic crystals**

The propagation of light in photonic crystals as a phenomenon of electromagnetism is governed by four Maxwell Equations.

\[ \nabla \cdot \mathbf{D} = 0, \quad \nabla \cdot \mathbf{B} = 0 \]

\[ \nabla \times \mathbf{E} = \frac{-\partial \mathbf{B}}{\partial t}, \quad \nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} \]

where \( \mathbf{E} \) and \( \mathbf{H} \) are the macroscopic electric and magnetic fields, \( \mathbf{D} \) is the displacement field, \( \mathbf{B} \) is the magnetic induction field, \( \rho \) and \( J \) are the free charges and currents. Assuming that the material under consideration has no free charges or currents, we set,

\[ \rho = J = 0. \]
Ignoring frequency dependence of dielectric constant we can write a constitutive relation

\[ D(x) = \varepsilon(x) E(x) \]

Assume \( B = \mu H \) where \( \mu \) is a constant in this case

With these assumptions Maxwell’s equations become:

\[
\nabla \cdot \varepsilon(x) E(x, t) = 0, \quad \nabla \cdot H(x, t) = 0
\]

\[
\nabla \times E(x, t) = -\mu \frac{\partial H(x, t)}{\partial t}, \quad \nabla \times H(x, t) = \varepsilon(x) \frac{\partial E(x, t)}{\partial t}
\]

We can separate out time dependence by expanding the fields into set of harmonic modes.

\[ E(x, t) = E(x) e^{i\omega t} \]
\[ H(x, t) = H(x) e^{i\omega t} \]

To find the equations of the mode of a given frequency, we insert the above equations in the two curl equations:

\[
\nabla \times E(x) - i\mu \omega H(x) = 0
\]
\[
\nabla \times H(x) + i\omega E(x) \varepsilon(x) = 0
\]

Decoupling the two equations we get:

\[
\nabla \times \frac{1}{\varepsilon} \nabla \times \tilde{H}(x) = -\left( \frac{\omega}{c} \right)^2 \frac{\partial^2}{\partial t^2} \tilde{H}
\]

where \( c = \frac{\mu}{\varepsilon} \)
An alternative way is to take the curl of first equation and then to use second equation to eliminate $H(x)$. Then the result is

$$\vec{\nabla} \times \left[ \vec{\nabla} \times \vec{E}(x) \right] = -\left( \frac{\omega}{c} \right)^2 \frac{\partial^2}{\partial t^2} \vec{E}(x),$$

Similarly we can get $H(x)$. The solution to these equations for electromagnetic waves in periodic media is done using plane wave expansion method. Here $\omega$ is a function of $k$. A complete photonic band gap is a range of $\omega$ in which there are no propagating (real $k$) solutions of Maxwell’s equations for any $k$, surrounded by propagating states above and below the gap. Figure 1.2 shows the gap in the band diagram of the periodic structure.

Figure 1.2: Left: Dispersion relation for $\omega$, frequency $\omega$ versus wave number $k$, of a uniform one-dimensional medium. Right: Effect on the bands of a physical periodic dielectric variation, where a gap has been opened by splitting the degeneracy at $k = \pm \frac{\pi}{a}$ Brillouin-zone boundaries.
Properties of photonic band gap structure:

1. **Scalability:** One of the unique features of PBG structures is that their characteristics are scalable from microwaves to optical frequency. This can be explained better by going back to Maxwell’s equations. This is derived from Maxwell’s equations which can be rewritten in magnetic field vector format as

\[
\vec{\nabla} \times \frac{1}{\varepsilon} \vec{\nabla} \times \vec{H} = \left( \frac{\omega}{c} \right)^2 \vec{H} \tag{1}
\]

where \( \varepsilon(r) \) is the magnetic field vector.

Let’s define a new dielectric constant

\[
\varepsilon'(r) = \varepsilon \left( \frac{r}{s} \right) = \varepsilon \left( r' \right)
\]

where \( s \) is some scalar parameter. We have compressed or expanded the dielectric by this scalar value \( s \). Now defining a new variable \( r' = s \cdot r \) and \( \vec{\nabla}' = \frac{\vec{\nabla}}{s} \) we can rewrite equation 1 as:

\[
s\vec{\nabla}' \times \frac{1}{\varepsilon} \left( \frac{r'}{s} \right) s\vec{\nabla} \times \vec{H} \left( \frac{r'}{s} \right) = \left( \frac{\omega}{c} \right)^2 \vec{H} \left( \frac{r'}{s} \right)
\]

which can also be written as

\[
\vec{\nabla}' \times \frac{1}{\varepsilon} (r') \vec{\nabla} \vec{H} \left( \frac{r'}{s} \right) = \left( \frac{\omega}{c} \right)^2 s^2 \vec{H} \left( \frac{r'}{s} \right)
\]

Here \( \varepsilon = \varepsilon' \left( r' \right) \) and this gives us back the master equation with mode profile

\[
H \left( r' \right) = H \left( \frac{r'}{s} \right) \text{ and frequency } \omega' = \frac{\omega}{s}
\]
The modes of photonic crystals can be studied at microwave frequencies with bigger dimension and because of scalability of the structure we can ensure that the electromagnetic properties will not change at optical frequencies.

2. **Defect modes:** Similar to the impurity doping in semiconductor, localized electromagnetic modes can be created in the bandgap region of PBG structures by introducing defects that disturb the periodicity of the structure. This can be achieved by adding extra material to the crystal which acts like a donor atom of a semiconductor. The defect can also be introduced by removing a part of material, thus creating states similar to semiconductor material having acceptor atoms. In photonic crystals with defects, the transmission spectrum is changed by the presence of a narrow transmission peak within the band gap.

**Photonic crystal cavities**

One of the most interesting similarities between semiconductors and photonic crystals is the emergence of localized defect modes in the gap region when a certain degree of disorder is introduced in the periodic structure. This effect was predicted by John in 1987 before the theory about photonic crystal were developed. We can define disorder as anything breaking symmetry properties of the lattice, and it can take many forms. A first divide which appears when we deal with disorder is between disorder deliberately introduced in the lattice, in this case we usually talk of 'defects' and random disorder in the lattice and unit cell parameters introduced by the fabrication process.
When a point defect is located in a photonic crystal, the defect may create an allowed mode at a frequency within the band gap. Because such a state is forbidden from propagating in the bulk crystal, it is trapped. The mode decays exponentially into the bulk.

Such a point defect, or resonant cavity, can be utilized to produce many important effects. For example, it can be coupled with a pair of waveguides to produce a very sharp filter (through resonant tunneling). Point defects are at the heart of many other photonic crystal devices, such as channel drop filters. Another application of resonant cavities is in enhancing the efficiency of lasers; taking advantage of the fact that the density of states at the resonant frequency is very high.

The Q factor or quality factor is a measure of the sharpness of a resonant system. Resonant systems respond to frequencies close to the natural frequency much more strongly than they respond to other frequencies. On a graph of response versus frequency, the bandwidth is defined by the frequencies at which the amplitude is half of the peak amplitude (FWHM). In decibels, the bandwidth corresponds to the -3dB points.

The Q factor is defined as the resonant frequency (center frequency) $f_0$ divided by the bandwidth $\Delta f$ or bandwidth:

$$Q = \frac{f_0}{\Delta f} = \frac{f_0}{f_2 - f_1}$$

Where $\Delta f = f_2 - f_1$, where $f_2$ is the upper and $f_1$ the lower cutoff frequency.
Most of the mentioned applications of photonic crystals were initially built around 2D PBG structures. However, to avoid the leakage problem in 2D structures, either one has to extend the size of the photonic crystal in the vertical direction, or use a strong index guiding mechanism in the vertical direction. Another way to eliminate the leakage problem is to use 3D photonic crystals in such applications. Even though the fabrication of 3-D photonic crystals is not easy, there are two advantages to 3-D structures. First, 3-D crystals have high rejection rates. Second, one can confine light within a very small volume of 3-D structures, and this property leads to very high quality factors. In 1991 a method for full confinement of the electromagnetic waves utilizing 3-D layer-by-layer photonic crystals was reported. A single rod is removed from an otherwise periodic crystal in order to construct waveguides, which are also called highly confined waveguides. Photons can propagate through the vacancy of the missing rod without any radiation losses for certain wavelengths. Various photonic components such as waveguides, waveguide bends, and power dividers, which are built around waveguides, have been experimentally demonstrated.

Our study is based on a three-dimensional layer-by-layer photonic crystal. In this study high purity alumina is used for the higher dielectric material. The PBG crystal used is a three-dimensional layer-by-layer structure, each layer built out of square cross-section alumina rods. The structure repeats itself every 4 layers. A schematic is shown in figure 1.3. The crystal is assembled by stacking together layers of dielectric material as basic building blocks. Each layer consists of parallel rods with a center-to-center separation of $a$. The rods are rotated by $90^\circ$ in each successive layer. Starting at any reference layer, the rods of every
second neighboring layer are parallel to the reference layer, but shifted by a distance of 0.5 a
perpendicular to the rod axes.

This results in a stacking sequence which repeats every four layers. Transmission and phase
dispersion properties of this structure were studied by Ozbay, et al. in 1994 at Iowa State
University. Theoretical calculations predict the band gap for the propagation direction to be
10.65GHz to 15.4 GHz. Figure 1.4 shows the band gap for this crystal in all directions. The
gap in the z-direction is a bigger gap than the complete band gap.

The construction of functional integrated photonic chips requires 3-D PCs in which a defect
state could be introduced into the crystal or an efficient light emitting element could be
incorporated. Noda, et al. were successful in creating 3-D PCs operating at infrared
wavelengths and also reported introduction of both defects and light emitters into the PC. In
1991, Yablonovitch and co-workers described donor and acceptor modes in 3-D photonic
structures. They examined donor and acceptor mode frequencies as a function of normalized
donor and acceptor defect volumes.
Ozbay, et al. did some useful work on defects, acceptor and donor defects on a crystal similar to ours, a layer-by-layer 3D structure made of cylindrical alumina rods but the highest Q that they could deduce from his cavities was 1000 at that time.

Figure 1.4: Band gap predicted by theory for this 3-D photonic crystal in all directions (Calculated by Ming Li)

Also the measurements on increasing cavity sizes were done but for less number of points. In 2002, Noda, et al. did analysis of donor defect cavities in a layer-by-layer crystal. They predicted a Q factor of about 10000 for a 19-layered crystal. Q as a function of stacking layers was determined.

Among various PC-based devices, ultra-compact channel drop filters based on resonant coupling between cavity modes of point defects and waveguide modes of line defects have drawn primary interest due to their substantial demand in wavelength division multiplexed...
(WDM) optical communication systems. So far, several structures of channel drop filters based on two-dimensional (2-D) PCs have been proposed\textsuperscript{11,12}. These structures may be classified into two categories: four-port system and three-port system. Schematics are shown in figure 1.5.

Figure 1.5: Schematics of three-port and four-port networks

The channel drop filters in a four-port system make use of resonant tunneling through a resonator between two parallel waveguides. In these structures, the resonator should support two degenerate modes of different symmetry in order to attain 100% drop efficiency, but enforcing degeneracy between two resonant modes of different symmetry requires a very sophisticated resonator design. The performance of a channel drop filter is determined by the transfer efficiency between the two waveguides. Perfect efficiency corresponds to 100% transfer of the selected channel into either the forward or backward direction in the drop, with no forward transmission or backward reflection into the bus. All other channels should remain unaffected by the presence of the optical resonator system.
Three-port channel drop filters also have been proposed by several groups. In the waveguide-cavity coupling experiments, Mehmet Bayindir and Ekmel, demonstrated trapping and dropping of photons through cavity modes\textsuperscript{13}. They were able to drop selected frequencies from waveguide modes. Noda and co-workers studied the properties of coupling between point defect cavities and line defect waveguides theoretically\textsuperscript{14}. A number of other researchers are trying to achieve an add-drop filter using photonic crystals.

**Organization of thesis**

Chapter 2 presents the photonic crystal cavities. This chapter is in the form of a paper to be submitted to *Applied Physics Letters*. This study includes study of resonant modes and quality Factors for changing confinement of acceptor type cavity. In this chapter a single cavity size is examined.

In Chapter 3, cavities of different sizes are studied and trends in resonant frequencies are presented. This is also in the form of a paper to be submitted to *Applied Physics Letters*.

Chapter 4 discusses coupling between waveguide and cavity. We studied different coupling structures and were able to get close to 100% coupling between two straight waveguides through a cavity between them. Different cavity positions were examined. This is similar to a narrow band-pass filter.

**References**

CHAPTER 2

PROPERTIES OF SINGLE-DEFECT ACCEPTOR CAVITY IN A THREE-DIMENSIONAL LAYER-BY-LAYER PHOTONIC CRYSTAL

A paper to be submitted to Applied Physics Letters

Preeti Kohli $^1$, Ming Li $^2$, Jacob Chatterton $^1$, J. R. Cao $^3$, Gary Tuttle $^1$, Kai-Ming Ho $^2$

1 Department of Electrical and Computer Engineering and Microelectronics Research Center, Iowa State University, Ames, Iowa 50011

2 Ames Laboratory and Department of Physics and Astronomy, Iowa State University, Ames, Iowa 50011

3 Canon Development Americas, Inc., 19575 Alton Pkwy, Irvine, California 92618, USA

Abstract

In this letter we demonstrate a single defect cavity inside a three-dimensional layer-by-layer photonic crystal. The defect is formed by removing part of one rod in a single layer of the crystal. The effects of increasing cladding layers in the z-direction are studied. We measured cavity $Q$ factors of the order of $10^4$ for a 22-layered crystal. The experimental results are in good agreement with theoretical transfer matrix method calculations. The high $Q$ value cavities in three-dimensional photonic crystal can be extended to laser wavelengths and will be valuable for making low threshold lasers.

In recent years photonic crystals (PCs) have attracted extensive interest due to their potential to engineer the properties of electromagnetic waves propagating within them$^{1,2}$. The
existence of a photonic band gap where radiation of a particular frequency range is completely forbidden makes PCs useful in a number of applications, like control of spontaneous emission, zero-threshold lasing and single mode LEDs. These applications involve introduction of defects or light emitters within the crystal. Good progress has been made in two-dimensional PCs\textsuperscript{3,4}, though there is an increasing interest in three-dimensional (3D) PCs. The waveguide\textsuperscript{5} and cavity structures\textsuperscript{6} built into the layer-by-layer 3D PCs have been demonstrated earlier where defect modes were present within the forbidden band gap.

The structure that we examine here is also a 3D layer-by-layer PC with square cross-section dielectric rods made of pure alumina. These rods (measured refractive index around 3.0) are commercially available and have a width of 0.32cm. The dimensions of the crystal are 15.24cm $\times$ 30.48cm in width and length with height determined by the number of layers of dielectric rods stacked on top of each other. See 2.1(a) for a 3D view of the crystal. We define the x-y plane as the plane of each layer in this structure. The stacking sequence repeats every four layers, corresponding to a single unit cell in the stacking direction. The center-to-center spacing between rods is 1.07 cm, giving a filling ratio of 29%. The defect was located in the layer that was almost equally clad above and below in the z direction. For example if the structure had 14 layers stacked on top of each other then the defect was made in the 8$^{th}$ layer and we call it the 7-1-6 configuration.
Figure 2.1: (a) Shows the design of three dimensional photonic crystal; (b) Illustrates the defect layer, the removed part has a width 'd' and center to center spacing between rods is 'a';(c) is the experimental setup.

We investigated the effect of increasing the cladding layers above and below the cavity on the resonant frequency, cavity Q and transmission intensity. A cavity with d/a = 1.0 was studied where 'd' is the length of rod removed and 'a' is the center-to-center spacing between adjacent rods. The defect volume normalized to a cube of half wavelength in dielectric medium is 1.712. We measured the cavity characteristics for 5-1-4, 6-1-5, 7-1-6, 8-1-7, 9-1-8, 10-1-9, 11-1-10 and 12-1-11 layered crystals where cavity was placed in the 6th, 7th, 8th, 9th, 10th and 12th layer respectively. The configuration of the cavity layer in x-y plane was same for all these and is shown in Fig.2.1(b).
Transmission properties were measured using a Hewlett Packard 8510A network analyzer with standard gain horn antennas to transmit and receive electromagnetic radiation. Figure 2.1(c) shows the experimental setup. The electric field is polarized in the x-y plane for all these measurements. Only x axis polarized electric field has the resonant feature while y axis polarized electric field does not have any resonant mode. This is consistent with the numerical calculations. The loss in the cables and the horns was normalized by calibrating all measurements up to the ends of the horn antennas. The Q factor was determined for all the
configurations using measured values of peak frequency and full width half maxima (-3dB) points, i.e. $Q = \frac{f_{\text{peak}}}{\Delta f}$.

![Graph showing resonant frequency trends](image)

Figure 2.3: Trends in resonant frequency found experimentally (solid lines with error bar) and calculated numerically using TMM (dotted lines). Resonant frequencies increase slightly and then oscillate approaching a constant as stacking layers in z-direction increase (Calculations by Ming Li)

The numerical calculation was performed on a 5-by-5 super cell structure using the plane-wave based Transfer Matrix Method (TMM) with interpolation technique. The 5-by-5 super cell structure is same as the experiment structure except that there are repeat cavities in both
x and y direction in the defect layer which is introduced by imposing a periodic boundary condition in TMM.

In our measurements it was found that the cavity Q factor increases when more cladding layers were used. The largest overall Q factor of 6190 was measured for the 22-layered crystal. The change of resonant frequencies was also measured with increase in cladding layers above and below the cavity. A sample data for 9-1-8 configuration is illustrated in Figure 2.2. The resonant frequency introduced by a cavity in the 10th layer can be seen within the band gap. It shows the details of both experiment and calculation transmission spectrum close to the resonant frequency. There is a second peak in the calculation spectrum which corresponds to a surface mode.

Figure 2.3 shows the trends of the resonant frequency for both experiment and calculation as the cladding layer thicknesses increase. The resonant frequency increases slightly when stacking layers were increased from 10 to 14 and oscillates and approaches a constant at higher number of stacking layers. The difference between the frequencies is within the experimental error of the measurement. There is a systematic lower resonant frequencies in the calculation compared with experiment which is due to the uncertainty of the refractive index of the rods. In the calculation, the refractive index is set exactly as 3.0, while the refractive index of the alumina rods is measured to be close to 3.0 with uncertainty ±0.1. A slight change in the refractive index will result in substantial change in the resonant frequency.

In Figure 2.4, the measured and calculated Q values are compared: the Q value increases almost exponentially in both cases. But the measured Q values are much lower than the calculated Q values. One main reason is the effect of loss in the lateral direction since the
cavity is confined by only 6 lattice constants in the y-direction and 12 lattice constants in the x-direction. This loss is not present in numerical calculation as it considers an infinite photonic crystal in the lateral direction with periodic defects in the super cell lattice.

Figure 2.4: Trends in Q value found experimentally (solid lines with square) and calculated numerically using TMM (dotted lines with square): Q value increases exponentially as stacking layers in z-direction increase. (Calculations by Ming Li)

A set of measurements was done to estimate the effect of the sideways loss. We found that as the cavity moves nearer to the edges of the crystal the Q factor reduces almost exponentially. Figure 2.5 shows the Q factor data as cavity moves towards the edge of the crystal for 22-
layered crystal. The black line with squares is the inverse of the measured Q value and the dotted line is the exponential increase fit.

\[
\frac{1}{Q_{\text{meas}}} = \frac{1}{Q_l} + \frac{1}{Q_0} = A + B \cdot e^{\alpha \Delta x}
\]

with \( \frac{1}{Q_l} = A - 2B \)

Figure 2.5: The measured Q factor decreases exponentially as the cavity is moved nearer to the edge of the crystal. \( \Delta X \) is the cavity's center position away from the center of the photonic crystal's XY plane's center. The dotted line is an exponential decay fitting with fitted result: \( Q_l = 12,000 \). The fitting equation used is \( y = A \cdot e^{x/t} + y_0 \). (Calculations by Ming Li)
Based on the above equations with the fitted data, we can get an estimated of $Q_L = 12,000$ which corresponds to the calculated $Q$ value from TMM. This value is still less than the calculated value of 18000 for this structure. Absorption of the dielectric rods and the misalignment in the photonic crystal structure and cavity used in experiments could be the factors responsible for the lower estimated $Q_L$ determined from experiments.

References

CHAPTER 3

TUNING OF RESONANT FREQUENCIES OF A SINGLE DEFECT CAVITY IN A

3-D PHOTONIC CRYSTAL

A paper to be submitted to Applied Physics Letters

Preeti Kohli 1, Ming Li 2, Jacob Chatterton 1, J. R. Cao 3, Gary Tuttle 1, Kai-Ming Ho 2

1 Department of Electrical and Computer Engineering and Microelectronics Research Center, Iowa State University, Ames, Iowa 50011

2 Ames Laboratory and Department of Physics and Astronomy, Iowa State University, Ames, Iowa 50011

3 Canon Development Americas, Inc., 19575 Alton Pkwy, Irvine, California 92618, USA

Abstract

We have studied the effect of introducing a cavity inside a layer-by-layer three-dimensional photonic crystal structure experimentally and verified the results by calculations. The cavity was formed by removing a portion of one rod from the center of the crystal. The size of the cavity was varied in small increments and the modes that emerged in the transmission spectrum were examined. Small increments in size of defect have given us a pattern of resonant frequencies not shown in previous works.

In recent years photonic crystals (PCs) have attracted extensive interest due to their potential to engineer the properties of electromagnetic waves within them 1, 2. The existence of a photonic bandgap where radiation of a particular frequency range is completely forbidden makes PCs useful in a number of applications, like control of spontaneous emission, zero-threshold lasing and single mode LEDs. These applications involve introduction of defects or
light emitters within the crystal. Good progress has been made in 2D PCs, though there is increasing interest in 3D PCs. The presence of a complete and robust band gap in a 3D layer-by-layer PC structure was demonstrated at Iowa State University. This microwave scale model made up of cylindrical alumina rods had a full photonic band gap at ku-band frequencies. Cavity structures built into this structure were demonstrated where defect modes were present within the forbidden gap. The structure that we examine is also a layer-by-layer structure but is built out of square cross-section alumina rods. These rods (measured refractive index 3.0) are commercially available and have a width of 3.2mm. The crystal has a rectangular cross section, 15.24cm × 30.48cm with height determined by the number of layers of dielectric stacked on top of each other. The stacking sequence repeats every four layers, corresponding to a single unit cell in the stacking direction. The center-to-center spacing between rods is 10.7mm, giving a filling ratio of 29%. The structure had 14 layers in total and the defect cavity was located in the 8th layer from the top. The experimental setup is shown below in figure 3.1.

Figure 3.1: (a) Experimental set up with two microwave horn antennas used for coupling. (b) Shows the cavity structure
The size of the defect is increased from 0.5a to 8a in steps of 0.25a where 'a' is the center-to-center separation between rods of the PC. There was no visible mode for a cavity size smaller than 0.5 a. We measured transmission properties using Hewlett Packard 8510A network analyzer with standard gain horn antennas to transmit and receive EM radiation. The electric field polarization is parallel to the rods of the defect layer for all these measurements. The loss in the cables and the horns is normalized by calibrating all measurements up to the ends of the horn antennas.

![Graph showing transmission peaks within the band gap for a cavity of length 1a](image)

Figure 3.2: A transmission peak within the band gap for a cavity of length 1a can be seen

The photonic crystal is very robust with respect to resonant frequencies. They do not change with slight tilting of the crystal or movement of the horns. The defect frequency does not change even if the crystal is tilted at angles up to 45 degrees. The band gap for this configuration lies between 10GHz and 16GHz. Theoretical calculations predict the bad gap for this propagation direction to be 10.65 to 15.4GHz. Transmission characteristics for
propagation along z axis for d/a=1 is shown in Fig 3.2. The peak transmission is 19db below the incident signal and lies at 12.424 GHz.

The peak frequencies with increasing defect volume do not follow a regular pattern. Modes split and reinforce at regular intervals as shown in fig 3.3. All points in the figure correspond to a resonant peak and points shown in squares are peaks with highest transmission for a particular defect size. It can be clearly seen that resonant frequencies increase as the defect size increases. Consider defect size increase from 2a to 5.5a, we can see increasing dominant modes up to defect size 3.5a after which the dominant peak shifts to a lower frequency and again starts increasing for size up to 5.5a. Before the shifting of the dominant mode at 3.5a, we can see the new modes arise and becomes dominant gradually. Also as the cavity size increases the rate of increase of resonant frequency reduces and becomes almost flat for defect size close to 7a. The maximum change in higher transmission peak frequencies on changing defect size from 0.5a to 8a is only 3.5 %. Lower transmission peaks (10-30 db below) vary by 8% over the entire range. The Q factors of modes in this configuration range from 500-1000.

This opposite trend of resonant frequency increasing with increase in cavity size can be attributed to the increase in k vectors as the size of the cavity increases\textsuperscript{14}. For 2d crystals where the slope of dispersion diagram is negative, the resonant frequency decreases on increasing d/a but since our crystal has a positive waveguide dispersion slope\textsuperscript{15} we see an opposite trend in the frequencies. This can be attributed to the abnormal modal reflectivity at the ends of the cavity in question.
Figure 3.3: Graph showing increasing peak frequencies with defect size. It also shows the diminishing modes and their counterparts arising at a lower frequency and then following the same increasing trend that flattens out for higher cavity sizes.

The numerical calculation was performed on a 5-by-5 super cell structure using the plane-wave based Transfer Matrix Method (TMM) with interpolation technique. The 5-by-5 super cell structure is same as the experiment structure except that there are repeat cavities in both x and y direction in the defect layer which is introduced by imposing a periodic boundary condition in TMM.
The resonant peaks have been calculated for cavity sizes up to 2.5a only. When cavities of sizes bigger than that are introduced in the super cell, cavity-cavity interactions are not negligible and interfere with the resonant peak calculation.

Figure 3.4 shows the calculation results for increasing cavity size up to 2.5a. The trend for frequency increase is same as experiment but there is a 1% shift in frequency values due to many reasons. Few of them are uncertainty in refractive index of rods and error in creating exact sized cavities during experiment.
References

CHAPTER 4
TRANSMISSION FROM AN INPUT TO EXIT GUIDE USING RESONANT EFFECT OF A NEARBY CAVITY

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Preeti Kohli 1, Caleb Christensen1, Jason Muehlmeier1, J. R. Cao3, Gary Tuttle 1, Rana Biswas2, Kai-Ming Ho 2

1 Department of Electrical and Computer Engineering and Microelectronics Research Center, Iowa State University, Ames, Iowa 50011

2 Ames Laboratory and Department of Physics and Astronomy, Iowa State University, Ames, Iowa 50011

3 Canon Development Americas, Inc., 19575 Alton Pkwy, Irvine, California 92618, USA

Abstract

It has been demonstrated by numerous papers that it is possible to manipulate photons by introducing artificial defects or light emitters in three-dimensional layer-by-layer photonic crystal. We have demonstrated experimentally and verified by calculations a band-pass filter where a particular frequency i.e. the resonant frequency of the cavity is selected from the input guide and transmitted to the output guide leaving out other radiation frequencies. Very good coupling, close to 100% has been seen between the waveguide and cavity in our setup.

There are a variety of applications of photonic crystals in optical components and circuits. A number of devices have been demonstrated already and there is continuing interest in photonic crystal structures for realization of channel drop filter1,2. Trapping and emission of photons by cavities has been demonstrated in 2-D structures experimentally3 and extensive
numerical calculations have been performed for coupling properties in 3D structures\textsuperscript{4}. Dropping of photons in a 3-D layer-by-layer was demonstrated earlier\textsuperscript{5}. This made way for the realization of channel drop filter and/or band pass filter in a photonic crystal.

We have examined enhanced coupling between photonic crystal waveguides using a cavity defect. The frequencies of the EM waves coupled between waveguides are determined by the resonant frequencies of the cavity. The effectiveness of the coupling depends on a number of factors, including cavity size, cavity-waveguide separation distance, and cavity confinement.

Our experiments were performed at microwave frequencies, using a large-scale photonic crystal structure. The structure is the familiar layer-by-layer crystal that has been studied extensively at Iowa State University. The crystal is built using square cross-section (3.2 mm x 3.2 mm) dielectric rods made of pure alumina (measured $n = 3 \pm 0.1$). The stacking sequence repeats every four layers, corresponding to a single unit cell in the stacking direction\textsuperscript{6}. The center-to-center spacing between rods is 1.07 cm, giving a filling ratio of 29%. The periodic crystal has a full three-dimensional photonic band gap covering the range 11 GHz – 16 GHz.

Figure 4.1 shows the experimental setup. Waveguides are formed in the crystal by removing single rods. This type of waveguide has been termed an X-waveguide\textsuperscript{7}. Cavities are formed in a similar fashion, by removing a portion of rod. However, the removed length is at most a few lattice constants. Transmission measurements were performed using an HP 8510 network analyzer to generate and receive microwave radiation. Microwaves were coupled into and out of the waveguides using a conventional horn antenna matched to an in-line photonic crystal horn antenna formed the crystal itself. This coupling scheme has been shown to be quite effective\textsuperscript{8}. The two waveguides lie in the same line within the crystal, but the
guides are each terminated within the crystal, with a separation distance \(d\) between the ends. The cavity of length \(L\) lies in line with the waveguides but one unit cell above the waveguide layer. The centerline of the cavity is aligned with centerline of the waveguide gap. This is referred to as the 'out of plane' cavity.

Figure 4.1: Configuration details of the crystal setup with spacer of length 'd' between the waveguides and cavity of length 'L'. This is the Z-X cross-section.

The separation distance between the ends of the waveguides \(d\) is an important parameter as it determines the interaction between the cavity and the waveguide and also the crosstalk between the waveguides when the cavity is not present.

Figure 4.2 shows transmission measurements for a straight waveguide, two waveguides with separation distance \(d = 8\) unit cells between the ends and no cavity, and then two waveguides with separation distance \(d = 8\) unit cells between the ends and with cavity with \(L = 0.75\) unit cell one unit cell above it. The straight waveguide shows a strong transmission band from about 11.8 GHz to about 12.8 GHz.
Figure 4.2: The introduction of the cavity close to the waveguides separated by 9a spacer makes a single frequency to be able to transmit to the exit guide, shown as dotted line in figure. It is very close to the straight waveguide transmission for that frequency, within 1 dB.

This behavior has been seen routinely in earlier waveguide measurements in our lab. By introducing the gap into the waveguide, the transmission is reduced by more than 20 dB over the entire frequency range. It can be seen that when the cavity is not present the transmission from one waveguide to the other is -45dB to -50dB in the frequency range of interest. On the introduction of cavity, a narrow transmission band appears at 12.22 GHz shows. The peak transmission of this band is at the nearly the same level as the straight waveguide case, suggesting very good coupling at these frequencies.
Figure 4.3: A cavity of length 5.5a supports three frequencies to transfer from input to the exit guide; these frequencies have close to 100% transmission with respect to the straight waveguide transmission.

We have a series of measurements where transmission is within 3 dB below the straight waveguide transmission for a particular frequency. The condition for 100% transmission is called critical coupling condition. We should note that 12.22 GHz corresponds to the resonant frequency of a cavity of length \( L = 0.75a \). (See Chap. 3.)

We studied the coupling effect for different cavity sizes \( (L) \) and different waveguide separation \( (d) \). Each cavity size has a different resonant frequency (See Chap. 3). Also, as the cavity size increases, the cavity can support more than one mode. The case with \( L = 0.75a \)
discussed above was a single-mode case. Figure 4.3 shows a cavity of size 5.5a with 3 cavity modes.

It has been shown that for any localized mode with a discrete resonant frequency, a phenomenon called critical coupling can occur between the localized mode and another mode with incident energy, in which all energy from the input channel transfers to the localized mode or vice-versa. If critical coupling can be achieved, there should be 100% transmission from one waveguide to the other. Assume that the localized mode can couple to several other modes, such as radiative or guided modes. For each of these available modes we can define a quality factor $Q$ such that $1/Q$ is proportional to the strength of the coupling. It has been shown that critical coupling will occur if $Q_{\text{input}}/Q_{\text{rest}}=1$, where $Q_{\text{input}}$ is associated with the input channel, and $Q_{\text{rest}}$ is associated with all other modes.

For our case, the localized mode is the cavity mode. The input channel is the incident waveguide, and all other modes include the output waveguide and any leakage from the cavity outside the crystal. Symmetry allows the same argument to be used assuming the output waveguide is the “input channel”. If $1/Q_w$ is proportional to the coupling between the cavity and the waveguide, and $1/Q_l$ is proportional to the leakage out of the crystal, we have:

\[
\frac{1}{Q_{\text{input}}} = \frac{1}{Q_w}, \text{ and } \frac{1}{Q_{\text{rest}}} = \frac{1}{Q_w} + \frac{1}{Q_l}
\]

The condition for critical coupling becomes:

\[
\frac{Q_{\text{input}}}{Q_{\text{rest}}} = \frac{(Q_w + Q_l)}{Q_l} = 1 + \frac{Q_w}{Q_l} = 1
\]

Since there will always be some amount of leakage in any finite crystal, we cannot get truly perfect coupling. However, if the ratio is close to one, there should be very good transmission. For our situation this gives the simple result that $Q_l/Q_w$ should be small for good transmission from the input guide to the cavity, and by symmetry, good transmission
from the cavity to the output waveguide. Not surprisingly, this requires good coupling between waveguides and cavities. Less obvious, however, is that even small amounts of leakage outside the crystal can drastically reduce the transmission across the cavity. The confinement of cavity and waveguide plays a very crucial role in the critical coupling condition.

Figure 4.4: The effect of reducing cavity confinement on resonant mode centered around 12.82 GHz. The number of layers above the cavity layer is decreased from 12 layers to 4 layers. The effect on resonant peak is opposite from the effect on the rest of the transmission. The rest of the transmission follows the usual pattern of increased attenuation with increase in number of layers in the crystal.
We examined the effects of cavity confinement on transmission in our waveguide–cavity arrangement. Leakage from the cavity to radiation modes outside crystal will reduce the cavity Q. (see Chap. 2). So for “leaky” cavities, we see a continuous reduction in signal transmission with decreasing confinement of cavity. The number of cladding layers above the cavity layer was decreased from 12 layers to 4 layers to see this effect. As the cavity becomes progressively leakier, the peak transmission drops by about 8 dB.

Figure 4.4 shows results for a structure with \( d = 9 \) unit cells and \( L = 5 \) unit cells which has two transmission peaks. The results here are interesting because the two peaks show opposite trends. The peak transmission of the dominant mode at 12.82 GHz does drop as the cavity becomes leakier, but the peak transmission of the secondary mode at 12.88 GHz actually increases.

References

CHAPTER 5
CONCLUSION AND FUTURE WORK

The work presented in this thesis is aimed at analyzing the behavior of cavities and cavity-waveguide coupling in a three-dimensional layer-by-layer photonic crystal. We have shown that defect cavity modes exist when the periodic symmetry of the crystal is disturbed. We measured high Q values (of the order $10^4$) and these values matched well with simulation results performed for the same structure. With increase in the confinement of the cavity, the quality factor increases exponentially. The resonant frequency increases for increase in confinement up to 16 layers in the z-direction and then tends to become constant with some oscillation. The simulations performed using TMM also determine similar results and values of resonant frequencies match well. The effect of finite confinement in the x and y-direction of the crystal is also measured. It gives a value of Q factor close to the calculation which assumes infinite confinement on the sides of the crystal.

We studied the effect of introducing a variable size cavity inside a layer-by-layer three-dimensional photonic crystal structure experimentally and verified the results by calculations. The size of the cavity was varied in small increments and the modes that emerged in the transmission spectrum were examined. Small increments in size of defect have given us a pattern of resonant frequencies not shown in previous works.

A roadmap of frequencies has been generated which provides the resonant property of a particular cavity size from 0.5a to 8a in the crystal. The results are compared with calculation for increasing cavity sizes up to 2.5a due to constraints like cavity-cavity interaction in the super cell lattice. Calculations show the same trend of increasing resonant frequencies.
The coupling between waveguide and cavities was studied. Different three-port and four-port networks were examined with cavities placed offset and in line with the waveguides. The results for the three-port pass filter have been presented in this thesis. Almost 100% transmission has been seen from the entry guide to the exit guide for various configurations of cavity placement and spacer between the waveguides. This can be utilized in the design of band-pass filters and WDM applications. The channel drop filters of four-port system make use of resonant tunneling through a resonator between two parallel waveguides. In these structures, the resonator should support two degenerate modes of different symmetry in order to attain 100% drop efficiency, but enforcing degeneracy between two resonant modes of different symmetry requires a very sophisticated resonator design.

While WDM signals (i.e. multi-frequency signals) propagate inside one waveguide (the bus), a single frequency-channel is transferred out of the bus and into the other waveguide (the drop) either in the forward or backward propagation direction, while completely prohibiting cross talk between the bus and the drop for all other frequencies. The performance of a channel drop filter is determined by the transfer efficiency between the two waveguides. Perfect efficiency corresponds to 100% transfer of the selected channel into either the forward or backward direction in the drop, with no forward transmission or backward reflection into the bus. All other channels should remain unaffected by the presence of the optical resonator system. Further investigation can be done into the behavior of four-port designs.
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