THE ULTRASONIC FIELD OF FOCUSED TRANSUDCERS THROUGH A LIQUID-SOLID INTERFACE

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INTRODUCTION

This paper presents theoretical and experimental results on the ultrasonic field of focused immersion transducers. The French Atomic Energy Commission (C.E.A.) has developed a software which calculates the ultrasonic field produced by a focused (or unfocused) transducer through a liquid-solid interface at normal or oblique incidence. The radiation of the transducer is formulated by the method of the Rayleigh integral, extended to take into account the liquid-solid interface. Firstly we describe this model, then we present measurements of the ultrasonic field produced by focused transducers in steel blocks. Experiments have been made using, at low frequencies, an electrodynamic probe, and, at high frequencies, an optical probe.

MODEL

According to the scalar theory of diffraction, the radiation in an infinite isotropic medium due to the motion of a planar baffled piston can be expressed by the Rayleigh integral. In its temporal form the Rayleigh integral is written:

\[ \Phi(M,t) = \int_{\text{transducer}} \frac{v_0(S,t - r/c)}{2\pi r} ds \]  \hspace{1cm} (1)

where \( \Phi(M,t) \) is the velocity potential at a point \( M \) and time \( t \), \( v_0(S,t) \) is the normal velocity at a source point \( S \), \( r \) is the distance separating \( S \) and \( M \) and \( c \) is the sound velocity. In the following we will only consider uniform transducers: \( v_0(S,t) = v_0(t) \).

The field emitted by a focused transducer can also be described by the Rayleigh integral [1]. The multiple diffusions on the curved surface are then neglected, which is a
reasonable approximation for usual emitting surfaces [2]. Lastly, an acoustic lens is modeled as a pure phase changer.

Equation (1) amounts to apply the Huyghens principle, that is to postulate that each surface element radiates a hemispherical wave. The approximation of geometrical optics can then be applied to express the refraction of such a hemispherical wave incident on a planar interface [3,4]. In this approximation, the hemispherical wave is refracted with the same transmission coefficient as the plane wave. It can be shown that this approximation is applicable as soon as the field point is sufficiently far from the interface in comparison with the wavelength [4]. Let us consider an interface between a fluid (medium 1) and an isotropic solid (medium 2). The elastic field in the solid can't be described by a scalar quantity. Nevertheless, we can apply the Rayleigh integral method and the geometrical approximation to calculate separately the two components, longitudinal and transverse, of the field. Thus, if we are interested by the calculation of the displacement field, we can write

\[ u^{(S)}(M,t) = C_{\alpha}^{\Phi,u}(\theta(S,M),t) \ast \frac{v_0(t - T(S,M))}{2\pi R(S,M)} \]  

(2)

If \( \alpha = L \) (resp. \( \alpha = T \), \( u^{(S)}_{\alpha}(M,t) \, ds \) is the amplitude of the longitudinal (resp. transverse) component of the displacement field at point \( M \) and time \( t \) resulting from the motion of the surface element \( ds \) around \( S \). The ray joining \( S \) and \( M \) according to the Snell law defines the point of impact \( I \) on the interface and the angle of incidence \( \theta_i \). The flying time \( T \) of the wave issued from \( S \) is determined by the ultrasonic path \( SI \) and the geometrical factor \( R \) takes into account the divergence of the rays. \( R \) is obtained from the condition of energy flux conservation inside the ray tube. Therefore, \( T \) and \( R \) are given by the following expressions:

\[ T(S,M) = \frac{R_1 + R_2}{c_1} \]  

(3)

and

\[ R(S,M) = \sqrt{\left( \frac{R_1 + R_2}{c_1} \right) \left( \frac{R_1 + R_2}{c_1} \right) \left( \frac{R_2}{c_2} \cos^2 \theta_i \right)} \]  

(4)

c_1 and \( c_2 \) are respectively the sound velocity in the fluid medium and the velocity of the elastic waves (longitudinal or transverse) in the solid. \( R_1, R_2 \) are the paths in the two media (\( R_1 = SI \) and \( R_2 = IM \), see figure 1), \( \theta_i \) (resp. \( \theta \)) is the angle of incidence (resp. refraction). In the case of an acoustic lens, \( T \) contains an additional term \( w / c_l \) where \( w \) is the lens thickness at point \( S \) and \( c_l \) is the velocity of the wave. \( C_{\alpha}^{\Phi,u}(\theta_i,t) \) is the temporal form of the transmission coefficient which relates the velocity potential in medium 1 and the amplitude of longitudinal or transverse displacement in medium 2. It can be shown that the general form of \( C_{\alpha}^{\Phi,u}(\theta_i,t) \) is [5]:

\[ C_{\alpha}^{\Phi,u}(\theta_i,t) = A_{\alpha}^{\Phi,u}(\theta_i) \left( \cos(\varphi_{\alpha}^{\Phi,u}(\theta_i)) \delta(t) + \frac{\sin(\varphi_{\alpha}^{\Phi,u}(\theta_i))}{\pi t} \right) \]  

(5)

where \( A_{\alpha}^{\Phi,u} \) and \( \varphi_{\alpha}^{\Phi,u} \) are the amplitude and the phase of the transmission coefficient calculated in harmonic regime.
In the case of the longitudinal wave, the vector displacement produced by the motion of a point source $S$ is collinear to the ray $IM$, while, in the case of the transverse wave, it is normal to $IM$. Therefore, it is possible from equation (2) to determine the three components $u^{(S)}_{\alpha i}$ of the displacement vector $\vec{u}^{(S)}_{\alpha}$. Lastly, we can apply the Rayleigh integral representation and calculate the three components of the displacement field $\vec{u}_\alpha$ resulting from the motion of the whole transducer by integration on the emitting surface:

$$u_{\alpha i}(M,t) = \int_{\text{transducer}} u^{(S)}_{\alpha i}(M,t) ds$$ (6)

**EXPERIMENTAL**

A computer program based on the calculation of the equations 2-6 has been implemented in the new system, named CIVA, developed by the French Atomic Energy Commission [6]. This system enables the direct comparison of calculated results and experimental data. In order to estimate the reliability of the model, we have carried out measurements of the ultrasonic field produced by focused transducers in steel blocks immersed in water.

The field calculation requires the knowledge of the excitation signal $v_0(t)$. Actually $v_0(t)$ is deduced from a measurement of the ultrasonic displacement field of the transducer in the focal area. It can be shown that, for the longitudinal polarization or for the transverse polarization when the angle of incidence is below the first critical angle, the displacement field in the focal area has the same pulse shape than $v_0(t)$. For the transverse polarization and an angle of incidence greater than the first critical angle it is necessary to take into account the distortion induced by the transmission coefficient (Eq. 5).

The principle of these experiments is to measure the motion of the block surface opposite to the transducer, assuming that this motion is identical to the incident displacement field which would exist in the absence of a boundary. The position of the
probe is fixed while the transducer is moved along one axis (x-axis) parallel to the block surface. At each position of the transducer the signal recorded by the probe corresponds to the elastic field of the transducer at a different point \( M \). The x-axis, the axis of the transducer and the center of the probe are in the same plane. In other words, all the field points \( M \) are in the plane of incidence of the transducer. The referential \((O,x,z)\) adopted in the following is shown on figure 2. In order to obtain results for a large range of frequencies we have used two different technics of detection. The field of low frequency transducers (1 MHz) have been measured in thick steel blocks with an electrodynamic probe, while the field of higher frequency transducers (15 MHz) have been measured with an optical probe. At a high frequency, the spatial resolution of the electrodynamic probe becomes too large compared to the wavelength and at a low frequency the dimensions of the steel blocks are too important for the optical probe installation.

**Measurements with the Electrodynamic Probe**

The experimental apparatus is described on figure 2. The coupling liquid is water \((c_1 \simeq 1480 \text{ ms}^{-1})\) and the solid is ferritic steel \((c_2 \simeq 5950 \text{ ms}^{-1}\) for longitudinal waves and \(c_2 \simeq 3230 \text{ ms}^{-1}\) for transverse waves). The measurements have been made on blocks of different thickness: 20 mm, 50 mm, 75 mm and 120 mm. The electrodynamic probe (Sofranel MML 83010) measures the normal component of the solid-air boundary displacement (see figure 2).

We present on figures 3 and 4 results concerning a 1 MHz transducer used at a normal incidence (L0 examination). The piezoelectric crystal is shaped according to the Fermat surface in order to focus in steel at \( z = 75 \text{ mm} \), the water height being 150 mm. The diameter of the transducer is 100 mm. Figure 3 shows echo dynamic curves (amplitude of the displacement versus the position of the transducer) recorded on three blocks of different thickness: 20 mm, 75 mm and 120 mm. As expected, at \( z = 20 \text{ mm} \) we are in the near field of the transducer, at \( z = 75 \text{ mm} \) the field is concentrated in a narrow beam, and at \( z = 120 \text{ mm} \) the beam is spread out. Figure 4 shows three ascans (displacement versus time) recorded on the 20 mm block at three positions of the transducer: \( x = 0 \text{ mm}, x = 15 \text{ mm} \) and \( x = 30 \text{ mm} \). The calculated results are superimposed on the curves presented on figures 3 and 4. We can see that the model predicts quite well the relative amplitude in the beam (figure 3) and the recorded waveforms.

![Figure 2](image-url)
Figure 3. Ultrasonic field of the 1 MHz-L0 focused transducer: calculated (continuous lines) and experimental (dashed lines) echodynamic curves at three different depths: a) $z = 20$ mm; b) $z = 75$ mm; c) $z = 120$ mm.

Figure 4. Ultrasonic field of the 1MHz-L0 focused transducer: calculated (continuous lines) and experimental (dashed lines) Ascans at three different points: a) (0, 20); b) (15, 20); c) (30, 20).
Figures 5 and 6 show results concerning a 1 MHz transducer used to produce a transverse beam refracted at 45° (T45 examination). The focusing power of this transducer comes from an acoustic lens having two different curvature radii (bifocal transducer). The curvature radii have been calculated for the transducer to focus in steel at \( z = 70 \) mm, the water height being 160 mm. The diameter of the transducer is 60 mm. On figure 5 the echodynamic curve calculated at \( z = 50 \) mm and measured on a 50 mm thick steel block are compared. Figure 6 concerns three ascans recorded on the same block for three positions of the transducer: a) \( x = 40 \) mm; b) \( x = 50 \) mm; c) \( x = 60 \) mm. We can see on figures 5 and 6 that again, the calculated and experimental results again correspond quite well.

**Measurements with the Optical Probe**

At a higher frequency the measurements have been performed on a 1.25 mm thick steel plate (\( c_2 \equiv 5950 \text{ ms}^{-1} \) for longitudinal waves and \( c_2 \equiv 3230 \text{ ms}^{-1} \) for transverse waves) with an optical probe. As shown in figure 7, the steel plate is immersed in a water tank in front of an optical probe. The optical probe is a Mach-Zehnder heterodyne interferometer [7]. The probe beam, the frequency of which is shifted by a Bragg cell, is focused into a diameter less than 50 \( \mu \text{m} \) on the plate. The motion of the plate surface modulates the phase

![Figure 5. Ultrasonic field of the 1 MHz-T45 focused transducer: calculated (continuous line) and experimental (dashed line) echodynamic curves at \( z = 50 \) mm.](image)

![Figure 6. Ultrasonic field of the 1 MHz-T45 focused transducer: calculated (continuous lines) and experimental (dashed lines) Ascans at three different points: a) (40, 50); b) (50, 50); c) (60, 50).](image)
of the reflected optical wave. The beating of the reflected beam with a reference beam on a photodiode and a coherent demodulation of the photocurrent with broadband electronics (30 kHz-30 MHz) provides an electrical signal proportional to the displacement of the surface plate.

We present on figures 8-10 some results concerning a 15 MHz transducer. The piezoelectric crystal is spherically shaped in order to focus, at normal incidence, at 1.25 mm in steel, the water height being 10 mm. The diameter of the transducer is 4.5 mm. This transducer has been used both at a normal incidence (L0 examination) and at a 19° oblique incidence (T45 examination). In figure 8 we present recorded and calculated L0 ascans corresponding to three different field points: a) $x = 0$ mm, $z = 1.25$ mm; b) $x = 0.4$ mm, $z = 1.25$ mm; c) $x = 0.6$ mm, $z = 1.25$ mm. We can see on this example that again, the waveforms are well predicted at a high frequency. Lastly, figures 9 and 10 show a good agreement between the calculated and measured echodynamic curves at a normal incidence (figure 9) and at an oblique incidence (figure 10).

![Figure 7](image)

**Figure 7.** Measurements with the optical probe: Experimental apparatus.

![Figure 8](image)

**Figure 8.** Ultrasonic field of the 15 MHz-L0 focused transducer: Calculated (continuous lines) and experimental (dashed lines) Ascans at three different points: a) (0, 1.25); b) (0.4, 1.25); c) (0.6, 1.25).
CONCLUSION

We have presented a method enabling the calculation of the ultrasonic field produced by focused transducers through a liquid-solid interface. The model is based on the Rayleigh integral representation of the acoustic field emitted in the coupling liquid and applies the geometrical optics approximation to take into account the liquid-solid boundary. We have carried out measurements of the elastic field produced by different transducers. The measurements have been made with an electrodynamic probe for 1 MHz transducers and with an optical probe for 15 MHz transducers. The experiments and the calculations correspond quite well, showing the reliability of the model.

REFERENCES

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