IMPEDANCE VARIATIONS DUE TO A SINGLE CYLINDRICAL HOLE IN A RUBBER SHEET

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INTRODUCTION

The absorption of acoustic energy by a rubber sheet containing a single cylindrical hole was reported in reference 1. In that case, the hole was sited so that the top of the hole was located in the middle of the rubber sheet and the bottom of the hole was against an infinitely hard surface upon which the sheet was mounted. There were two resonant systems, the first being the walls of the holes vibrating radially as the thickness of the rubber varied and the second being the motion of the top of the hole normal to the hard surface, similar to a drum-head. We have extended this work to the case where the hole entends through the rubber sheet and is capped with a steel plate. The additional mass of the steel plate causes the resonant frequency of the rubber-steel-hole system to be at a much lower frequency than the rubber alone and also eliminates the drum-head resonance. We have analyzed this system using a dedicated, time dependent finite element program and have also verified the results of the finite element program by use of a small water-filled guided wave tube. The results show that the impedance is a useful parameter which is both sensitive to the geometry of the entire system and is a good descriptor of the resonant system.

THEORETICAL STUDIES

The model which was analyzed by the finite element computer code consisted of a 5.0 cm diameter rubber disk approximately 1.0 cm in height perforated by a centrally located hole. The bottom of the rubber disk was rigidly attached to a hard surface and the top of the disk was covered by a 6.0 mm thick steel disk. Roller type fixities were applied at the disk edges so that motion at the edges in the vertical direction was permissable while horizontal motion was not. A sinusoidal force was applied normal to the free surface of the steel disk and the complex displacement of the surface of the disk was used to calculate the impedance. See figure 1.

The finite element program can be visualized as a computer code in which the internal stress-strain energy is equated to the external forces multiplied by their displacements at the points at which they are applied. This is an application of the principle of virtual work and the task of the computer is to solve this equation for the displacements. In this study, the finite element technique was applied to a second order differential equation with a forcing function proportional to that of the applied acoustic field. As a viscoelastic material has both a real and storage modulus, it was described by a complex modulus and the

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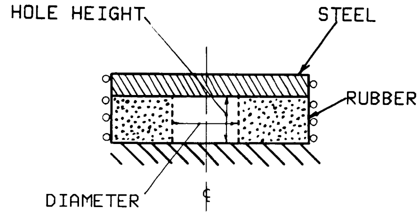

Figure 1. The axisymmetric absorber. The steel disk was 6.0 mm thick.

differential equation split into real and imaginary parts, with the damping and stiffness matrices also having two components. The solution of these coupled equations yielded a complex number representation of the displacements showing their phase relationships with that of the driving force. A single differentiation yielded the complex velocity from which the impedance, which is equal to the driving pressure divided by the complex velocity, was computed. Eight noded isoparametric elements were used in an axisymmetric arrangement and each element had nodes only in the corners and in the center of each side.

For a viscoelastic material, the shear modulus and loss factor are functions of the frequency at which the material is used: thus values of these parameters corresponding to each frequency employed in the finite element calculation must be chosen. Although there are analytic represntations for these functions, and the Kramers-Kronig relationship specifying their interdependence, we have found that a computer look-up table gave us the most accurate results.

EXPERIMENTAL

One means of measuring the acoustic absorption of a viscoelastic material is through the use of a guided wave tube as shown in figure 2. Such a tube consists of a pipe, filled with water, and of sufficient

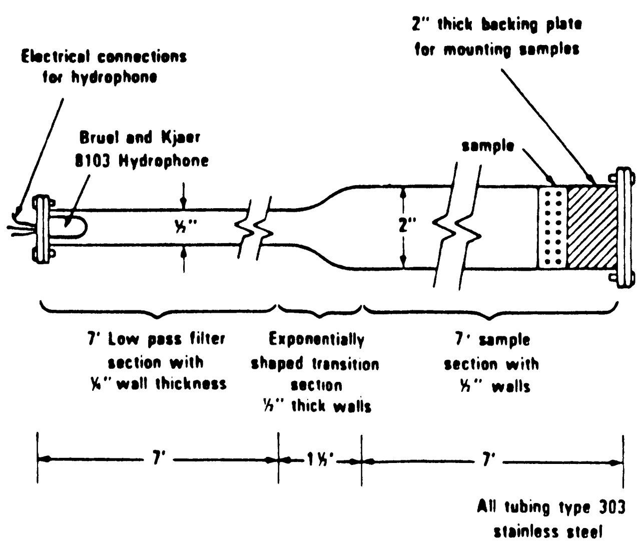

Figure 2. The NSWC guided wave tube. The small diameter section permits operation to 60 kHz.

length to allow several wavelengths of the sound wave to propagate at the same time in the water. A diode switching network which is located at one end of the tube permitted a single miniature transducer to serve as both a sound projector and a receiving hydrophone. The sample was placed at the other end. Sound waves were emitted from the projector and traveled through the water in the tube to the sample. Here they were reflected back torward the transducer where they were received and their relative amplitudes measured. Dividing the receiving amplitude by the transmitted amplitude of the sound waves yielded the absorption. Taking the log of the absorption and multiplying by -20.0 converted the absorption coefficient into a logarithmic value.

VERIFICATION OF FINITE ELEMENT PROGRAM

The guided wave tubes at NSWC have been in operation for a period of almost ten years. Data obtained from these tubes has been shown to compare favorably with that obtained from other measurement systems, such as panel tests. As a means of verifying the finite element results, a 5.0 cm diameter cylindrical rubber sample with a 6.0 mm cylindrical void penetrating the cylinder and capped by a 3.2 mm steel plate was constructed and its absorption spectrum obtained by use of a guided wave tube. Modeled in the finite element program by a similar perforated rubber washer, capped by a steel disk and having fixities only along the bottom edge, a comparison of the results is shown in figure 3 and good qualitative agreement is observed. The discrepancies in absorption are possibly due to the problem that the guided wave tube must be pressurized to at least 200 psi while the finite element program calculates the absorption for an external steady state pressure of 0 psi. An external pressure would dampen the response of the absorber to an acoustic wave and this may be what is happening here.

IMPEDANCE

The impedance is a measure of the dynamic resistance of a material to an acoustical wave. On a an impedance graph, the real part of the impedance is plotted along the abscissa and is proportional to the absorption while the imaginary part of the impedance appears on the ordinate and indicate whether the response of the sample is lagging or leading the applied stimulus. See figure 4. As the frequency is varied, the impedance traces out a trajectory in the impedance plane, with each frequency possessing a different impedance. Resonance is said to occur when the path of the impedance crosses the abscissa and the sample response is in phase with the driving force. In this study, the impedance is normalized to the impedance of water and thus the point of maximum absorption occurs when the distance between the impedance trajectory and the point (1,0) is a minimum. The graph on the right in figure 4 shows the absorption as a function of frequency for the same system. Note how

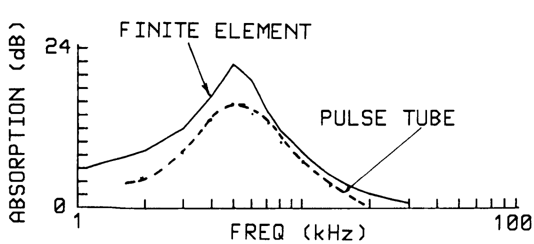

Figure 3. A comparison between panel data and guided wave tube data.

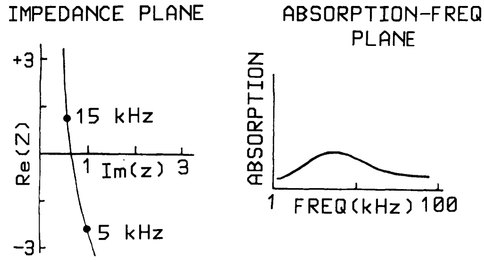

Figure 4. The relationship between the impedance plane the the absorption-frequency plane. The curve in the impedance plane crosses the abscissa at the resonant frequency while the abosrption-frequency plane shows a maximum in absorption at that frequency.

the absorption reaches a maximum at the point where the trajectory crosses the abscissa in the impedance plane.

RESULTS AND CONCLUSION

In the rubber-hole-steel disk system studied, there are two major factors determining the resonant frequency, the hole size and the hole height, referred to in figure 1. In general, the resonant frequency is proportional to the square root of the viscoelastic shear modulus so that a hole in the rubber material would tend to decrease this modulus, and hence the resonant frequency. On the other hand, the resonant frequency is inversely proportional to the thickness of the rubber so that increasing the hole height tends to lower the frequency.

In figure 5, we present a graph of absorption as a function of frequency for a constant rubber height of 6.0 mm. One notes the occurence of an absorption maximum at a hole radius of 1.5 cm when the vibrations on the sides of the holes are directly in phase with the vertical vibrations of the steel disk. In figure 6, we see the same system plotted on the impedance plane. This figure is for systems with a rubber height of 6 mm and hole radii varying from .008 mm to .015 mm. Referrring to the curve at the extreme left of the figure, we note that this curve starts at the bottom of the graph and moves to the top for a radius of .015 meters. The extreme bottom point represents a frequency of 2 kHz and each subsequent square represents an increase of frequency of 2 kHz. The large black square is a frequency marker of 4 kHz. At about 3.8 kHz, the curve crosses the abscissa and this is represented as a peak in the frequency-absorption curve in figure 5. That trajectory which passes closest to the point (1,0) produces the greatest absorption. It appears that the impedance plane and the absorption-frequency graph can be used as a guide to determine that set of geometry which produces the greatest absorption.

In figure 7, we have plotted the absorption as a function of frequency for a constant radius of 2.54 cm and varying rubber heights. Again, there is evidence of a maximum absorption for a particular selected geometry. Figure 8 shows the impedance plane for the same system. Note that a difference of only 1 mm in the height of the rubber disk produces a significant change in the trajectory, so that the impedance plane could serve as a sensitive indicator of the absorber thickness.

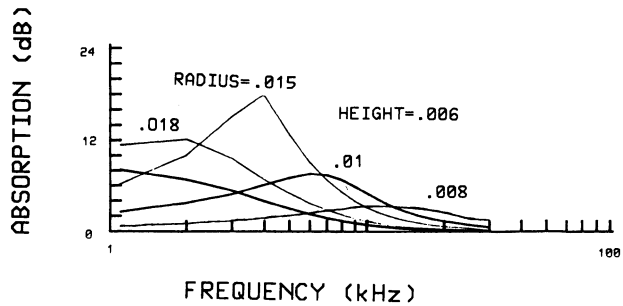

Figure 5. Absorption vs frequency for a rubber disc height of 6.0 mm. for holes of varying radii. Note the maximum in absorption for a hole of .015 meters and a hole height of .006 meters. All dimension are in meters.

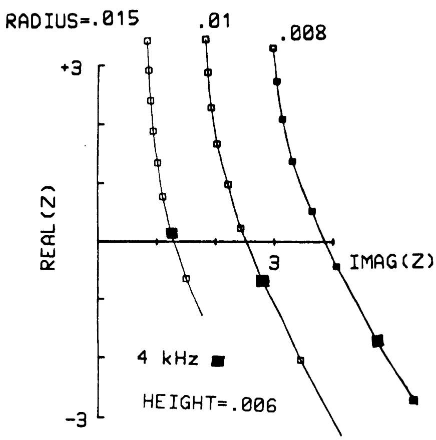

Figure 6. The impedance plane for an absorber with different radii and a rubber disk height of 6.0 mm. The large black square is a frequency marker at 4 kHz. Spacing between square markers is 2 kHz and all dimensions are in meters.

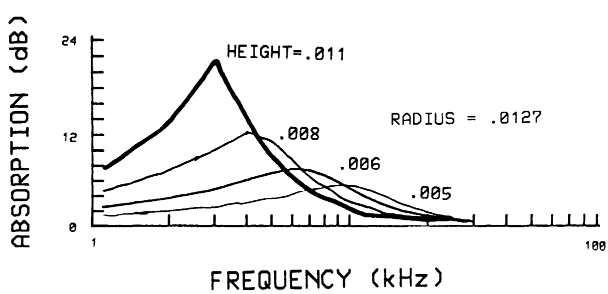

Figure 7. Absortion vs frequency for absorbers with a hole diameter of 2.54 cm and different hole heights. All dimensions are in meters.

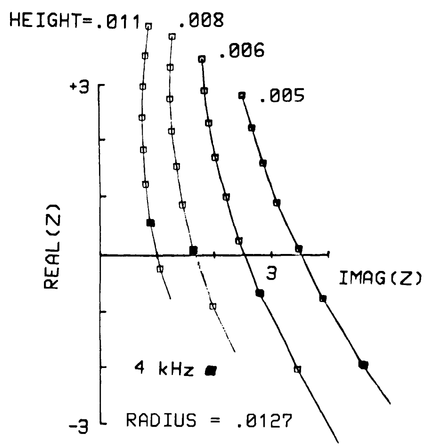

Figure 8. The impedance plane for absorbers with a hole diameters of 2.54 cm and different hole heights. All dimensions are in meters.

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