

## NONLINEAR ACOUSTIC PROPERTIES OF STRUCTURAL MATERIALS - A REVIEW

O. Buck

Ames Laboratory and  
Materials Science and Engineering Department  
Iowa State University  
Ames, IA 50011

### INTRODUCTION

One of the most obvious manifestations of the nonlinear stress-strain relation in elastic solids is the existence of thermal expansion due to a non-parabolic atomic potential. From the acoustic point of view, this nonlinearity immediately explains a variety of observations such as stress effects on the sound propagation velocities and acoustic harmonic generation, which is basically a distortion of the wave. Additional nonlinearities come about due to dislocation motion, or the initiation of plastic flow, and the nucleation of a new phase, such as in the case of a martensitic transformation, e.g. Other examples are nonlinear acoustic effects that are induced at free and internal surfaces caused for a variety of reasons. Detailed acoustic experiments on these phenomena have been made over the past forty years but the ideas have not been applied seriously in NDE. The present paper is a short review of work, some of which this author has been involved in. The objective is to show the utility of nonlinear acoustics for NDE of structural materials.

### BACKGROUND

Deviations from Hooke's law were first observed by Poynting [1,2] who noted a change in length and radius of a wire in torsion. Probably the first dynamic measurements were performed by Birch [3] who observed a change in the resonance frequency of torsional vibrations under hydrostatic pressure. A systematic nonlinear theory of elasticity was developed by Murnaghan [4] which is now used extensively to calculate static and dynamic problems. This theory shows that for an isotropic material there are three independent "third order" elastic constants (Murnaghan's constants) in addition to the two "second order" elastic constants (Lamé's constants). As may be expected, the complexity of the theory increases dramatically for anisotropic materials in that the number of second and third order elastic constants increases [4,5]. However, good agreement between the thermal expansion coefficients, calculated from the results of acoustic measurements, and those observed experimentally, has been reported [6].

As a consequence of Murnaghan's theory, two nonlinear acoustic effects can be predicted. The first is a dependence of the acoustic

sound velocity on the applied stress, the so-called acoustoelastic effect. The second effect is harmonic generation in which case energy, traveling with the frequency of the fundamental wave, is pumped into the second and higher harmonics. These effects depend on both, second and third order elastic constants. One may suspect immediately that, since alloying affects the second order elastic constants, third order elastic constants may also be affected and thus both, the acoustoelastic effect and harmonic generation will change. In addition, "diffusionless transformations" as a consequence of a temperature change may provide striking effects since the motion of atoms from one metastable lattice position to another more stable one must be highly nonlinear. Similarly, the onset of glide motion by dislocations should provide for nonlinear acoustic effects due to the strong deviation from elasticity as plasticity arises. Furthermore, one can expect strong effects from a weak interface on the propagation of an acoustic wave since the atomic potential is locally disturbed. In the following, we will briefly review some observations related to the above topics.

#### EFFECTS OF STRESS ON SOUND VELOCITY (ACUSTOELASTICITY)

Acoustoelasticity is definitely the most thoroughly studied [6,7] nonlinear acoustic effect and has found practical applications such as in bolt tensioning devices [8]. The relation between sound velocity,  $V$ , and the stress,  $\sigma$ , is of the form

$$V = V_0 + S\sigma \quad (1)$$

where  $V_0$  is the sound velocity without application of stress and  $S$  is a combination of second and third order elastic constants. Depending on the direction of  $\sigma$  with respect to the propagation and polarization directions of the wave,  $S$  has different forms [9]. A useful application is shown in Fig. 1 where a part of a bolt is under tension. Using an ultrasonic pulse-overlap technique, the stress can easily be determined, as shown for aluminum in Fig. 2.

More recently, applications to complex stress states have been studied. A very inhomogeneous stress field is produced due to the presence of a notch (or crack) in a structure as an external load is applied. Kino et al. [10] used longitudinal waves to determine such a stress field as shown in Fig. 3. The change in velocity is determined by the two principal stresses as

$$\frac{\Delta V_1}{V_1} = S' (\sigma_{11} + \sigma_{22}) \quad (2)$$

where  $S'$  is some combination of second and third order elastic constants. On the other hand, using two shear wave polarizations provides information on the differences in the principal stresses or the maximum shear stress  $\tau$ . The "acoustic birefringence" is then given by

$$\frac{V_{S1} - V_{S2}}{1/2 (V_{S1} + V_{S2})} = S'' \tau \quad (3)$$

with results [11] shown in Fig. 4. Again,  $S''$  is a combination of second and third order elastic constants. In both cases, such measurements provide information on the "stress intensity factor" which provides information on the driving force on a crack.

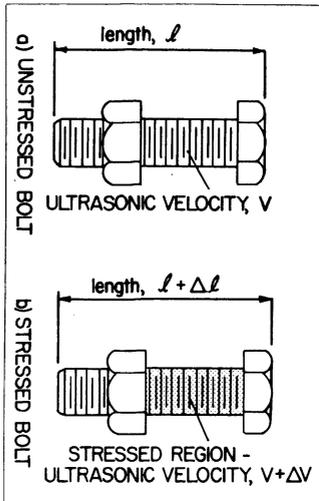


Fig. 1 Bolt tension measurement [9].

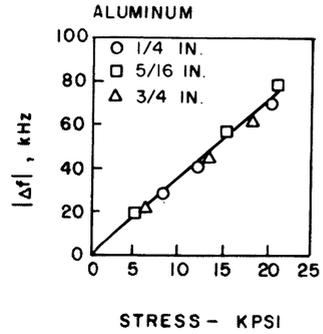


Fig. 2 Frequency shift versus applied load in a bolt tension test [9].

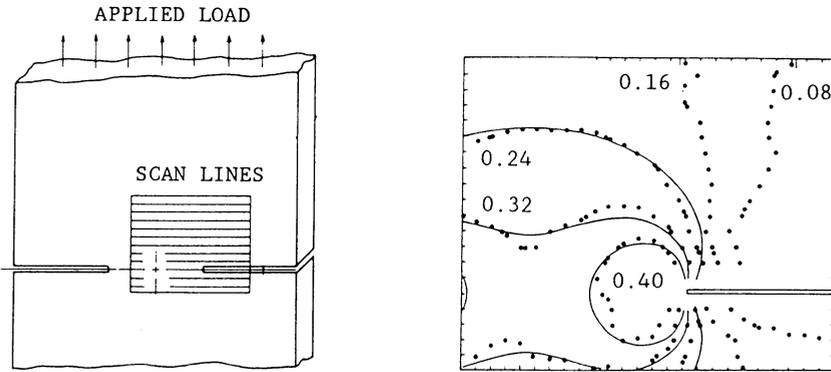


Fig. 3 Stress contours in a double edge notch panel [10].

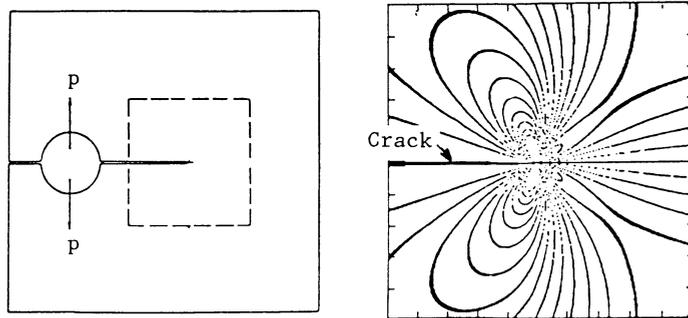


Fig. 4 Shear stress contours in a compact tension specimen [11].

#### ACOUSTIC HARMONIC GENERATION

The propagation of an acoustic wave with reasonably high amplitude  $A_1$  produces a second harmonic with amplitude

$$A_2 = S''' A_1^2 \omega^2 x \quad (4)$$

with  $S'''$  being a combination of second and third order elastic constants,  $\omega$  the input frequency and  $x$  the propagation distance of the wave [12]. In fused silica, harmonics up to the fourth one have been observed, as shown in Fig. 5, in agreement with a model by Fubini [13,14] which says that the amplitudes of the third and higher harmonics are mainly determined by the second and third order elastic constants. Effects of the boundary on second harmonic generation have to be taken into account, however. Particularly it should be noted that upon reflection of a second harmonic at a stress-free boundary,  $A_2$  decreases with propagation distance and becomes zero at the point of origin [15,16].

#### EFFECTS OF ALLOYING ADDITIONS IN NONLINEAR ACOUSTICS

Since the second order elastic constants change with alloying additions, one can immediately assume that the third order elastic constants will also change. Probably the most extensive study in this respect was done by Buck et al. [17] on the effects of hydrogen in solid solution in a niobium single crystal using the acoustoelastic effect. The changes of the third order elastic constants were found to be roughly 5-10 times larger than those of the second order elastic constants and, in both cases, can be either negative or positive. Thus one can conclude that alloying additions affect nonlinear acoustic properties more strongly than the linear acoustic properties.

The effect of carbon content on the acoustoelastic effect of four different steels was determined by Heyman et al. [18]. The so-called "acoustoelastic constant" [19] which is related to  $S$  in Eq. (1), is shown in Fig. 6 as a function of percent ferrite phase (or decreasing carbon concentration) for these steels. The carbon is located in interstitial sites as is hydrogen in niobium.

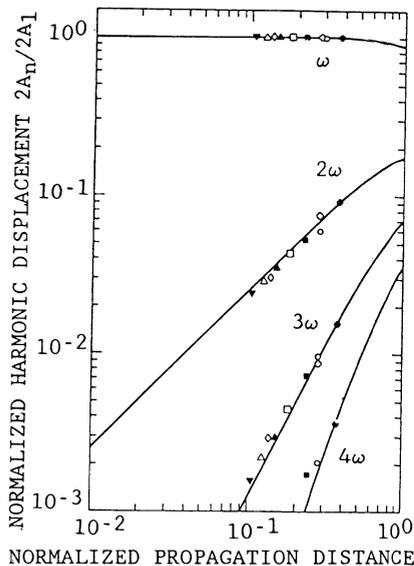


Fig. 5 Harmonic amplitudes versus propagation distance in fused silica [14].

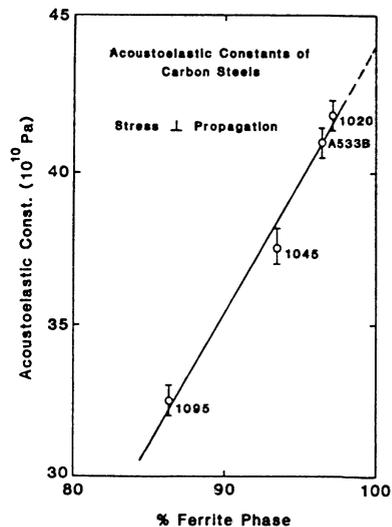


Fig. 6 Acoustoelastic constant vs. % ferrite phase present in four steels [18].

In contrast, the metallic additions in high strength aluminum alloys are on substitutional sites. By thermal aging after quenching from the solid solution phase, second phase particles precipitate and grow. As Razvi et al. [20] showed the acousto-elastic constant changes with precipitation of the second phase. As expected, the "nonlinearity parameter", determined by acoustic harmonic generation and defined in Ref. [21], similarly shows strong changes with the volume fraction of second phase present, as shown in Fig. 7.

#### DIFFUSIONLESS PHASE TRANSFORMATIONS

A large number of phase transformations occur by "diffusionless" transformations such as the  $\beta \rightarrow \omega$  transformation of titanium alloys, as shown in Fig. 8. In this case, two bcc (111) planes collapse into a hcp basal plane [22]. Connected with such transformations is a large change in the second order elastic constants (as well as a large attenuation effect) as was shown by Buck et al. [23]. Although, to the present author's knowledge, nonlinear acoustic measurements have not been performed on such systems, one would expect large effects, since the atomic potential is heavily distorted in such cases.

#### DISLOCATION CONTRIBUTIONS TO NONLINEAR ACOUSTICS

Hikata et al. [25,26] introduced the idea that the displacement due to the bowing out of dislocations,  $U_d$ , is a contributor to harmonic generation. This  $U_d$  is determined by the Koehler-Granato-Lücke vibrating string model

$$m_d(\partial^2 U_d / \partial t^2) + B(\partial U_d / \partial t) - C(\partial^2 U_d / \partial y^2 + \dots) = \sigma b \exp(i\omega t) \quad (5)$$

where  $m_d$  is the dislocation mass, B the damping coefficient, C the line tension,  $\sigma$  the stress amplitude and b the Burgers vector. This displacement  $U_d$  adds to the elastic lattice displacement,  $U_1$ , so that the equilibrium condition becomes

$$\text{div } \sigma = \rho \partial^2 (U_1 + U_d) / \partial t^2 \quad (6)$$

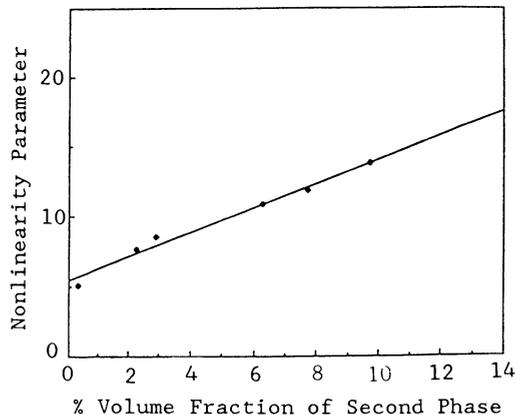


Fig. 7 Nonlinearity parameter vs. % volume fraction of second phase in high strength Al [20].

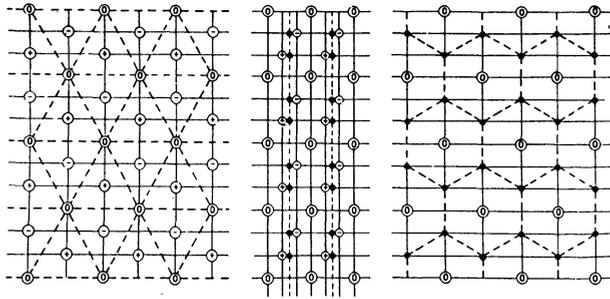


Fig. 8 Simple model of the diffusionless bcc to hcp transformation ( $\beta \leftrightarrow \omega$ ) [22].

The solution of Eq. (6) is very complex. A major conclusion, however, is that the third harmonic may be more sensitive to dislocation contributions than the second harmonic, as was indeed observed in some experiments [14,25]. The effects of fatigue on the second harmonic and the Knoop hardness of a predeformed aluminum single crystal of nominally high purity is shown in Fig. 9. The results are as expected from the Hikata et al. theory. The predeformed single crystal contains a relatively high and homogeneously distributed dislocation density and thus a short, average dislocation loop length so that  $U_d$  is almost negligible. The material fatigue softens with some of the dislocation loops becoming longer, thus contributing more strongly to second harmonic generation. Unfortunately, such dislocation effects are hard to observe in high strength alloys since the alloying additions tend to pin down the dislocations.

#### INTERFACE EFFECTS

Richardson [27] pointed out that an unbonded interface, subjected to a sufficiently intense incident acoustic wave, acts as a harmonic generator. Experiments on fatigue cracks [28,29] confirmed this prediction. Figure 10 shows the harmonic generation as small fatigue cracks develop at the surface of a high strength aluminum alloy. Generation is most efficient close to a zero stress in the surface and increases with the growth of surface cracks. If the cracks are completely closed, due to a compressive stress, or fully open, due to a tension stress, harmonic generation disappears as one would expect from this model [27].

The theory has now been expanded [30,31] so that any interface can be described by introducing a spring constant which could vary from zero (totally unbonded interface) to infinity (no interface present). Such a spring constant describes the local potential at the interface and appears to be well suited to provide a host of information using acoustic measurements.

#### SUMMARY

This short review was written with the hope that it stimulates the NDE community for further work on the nonlinear acoustic properties of solids and interfaces between solids. In many cases it appears that nonlinear acoustic effects are more sensitive than linear ones and therefore should be exploited. Furthermore, it is shown that nonlinear acoustics not only works for a homogeneous solid but also for interfaces between solids which may vary over a wide range of strengths.

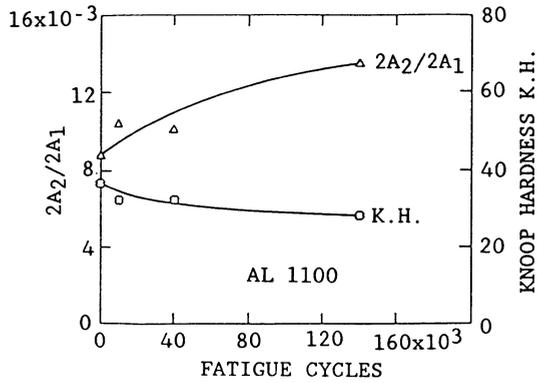


Fig. 9 Second harmonic generation and Knoop hardness as a function of fatigue [26].

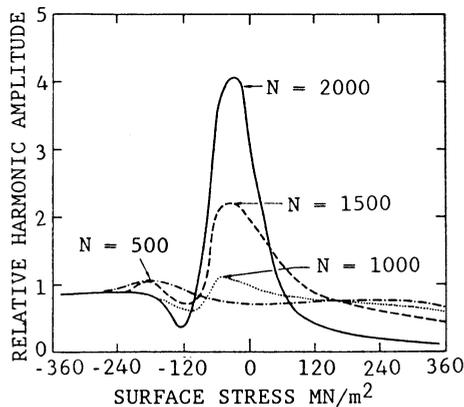


Fig. 10 Dependence of harmonic generation on surface stress and fatigue [29].

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