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Relationship of Mean Stress, Volumetric Strain and Dynamic Loads in Soil

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Abstract
The changes in soil consolidation resulting from externally applied forces and the effect of these changes on the physical properties of the soil have been studied by many individuals. Unfortunately their results have not produced an adequate agricultural soil mechanics. The development of soil stress-strain relationships which will permit the prediction of the changes in the state of compaction caused by various implements and power units will be a major contribution toward controlling soil compaction.

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Relationship of Mean Stress, Volumetric Strain and Dynamic Loads in Soil

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THE changes in soil consolidation resulting from externally applied forces and the effect of these changes on the physical properties of the soil have been studied by many individuals. Unfortunately their results have not produced an adequate agricultural soil mechanics. The development of soil stress-strain relationships which will permit the prediction of the changes in the state of compaction caused by various implements and power units will be a major contribution toward controlling soil compaction.

An investigation by VandenBerg (5)* revealed that the concept of continuum mechanics could be used as a mathematical model for studying the soil-compaction problem. With this model the forces acting on a volume element may be described by a set of quantities in the form of a stress tensor. He found that the volumetric strain, which is the change in compaction, can be expressed by the change in bulk density or the change in percentage of total pore space. To define the state of stress at a point requires the determination of six independent values. The hypothesis that volume strain is governed by the mean normal stress acting on the element was proposed by VandenBerg (5).

The purpose of the investigation reported in this paper was to use continuum mechanics in the study of various soil stress-strain relationships. The hypothesis that changes in the mean normal stress control changes in volumetric strain was tested by measuring the components of the stress tensor and changes in bulk density while the soil was subjected to dynamic loads of various magnitudes.

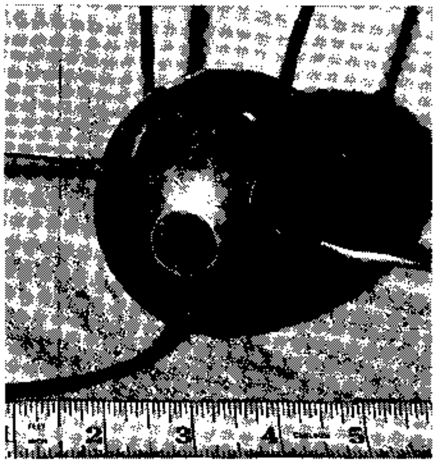

FIG. 1 Six-directional stress transducer used to measure components of stress tensor.

A series of 27 laboratory tests of five replications composed of three depths below the loading surface, three moisture contents, and three rates of loading were conducted using a Brookston sandy loam. A description of the tests is given in Table 1.

Electrical strain-gage transducers, Type A, developed by Cooper (1) and a six-directional stress transducer (6DST) developed by Harris (2) Fig. 1, were used to measure and record the normal stresses necessary to calculate the components of the stress tensor. A strain-gage force transducer was used to measure the total vertical force applied to the loading plate. Recording volumetric transducers similar to the one developed by Hovanesian (3) were used to measure changes in bulk density.

Procedure

The controlled variables in this investigation were moisture content, rate of loading and state of stress. Each series of tests was conducted by filling a 55-gal drum (soil tank) to the desired level below the loading surface. A circle 12 in. in diameter was located in the center of the tank. The stress transducers and balloons for measuring changes in bulk density were placed on the periphery of the circle as shown in Fig. 2. The tank was then filled to the operating level and the surface leveled. The loading plate was properly positioned and the recording instruments activated. The surface load was then applied hydraulically.

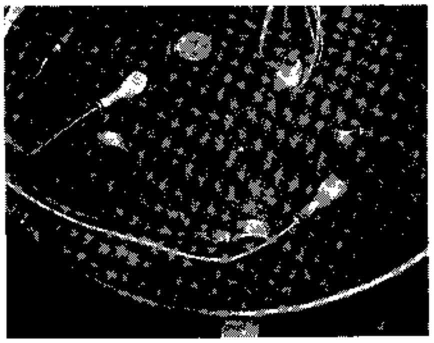

FIG. 2 Stress transducers and balloons used to obtain data.

TABLE I. DESCRIPTION OF THE LABORATORY TESTS

Test no.	Depth, in.	Rate of loading, in./sec	Moisture content, percent	Initial bulk density, gm/cc
1	10	0.62	8.63	1.08
2	5	0.62	9.89	1.08
3	15	0.62	11.59	1.06
4	10	0.38	11.56	1.06
5	5	0.38	8.89	1.07
6	15	0.38	7.97	1.08
7	5	0.38	11.35	1.08
8	15	0.38	11.39	1.09
9	10	0.38	11.38	1.06
10	15	0.62	12.63	1.06
11	10	0.62	12.43	1.08
12	5	0.62	12.46	1.09
13	10	1.00	17.41	0.94
14	5	1.00	17.41	0.94
15	15	1.00	14.85	1.01
16	10	1.00	14.85	1.02
17	5	1.00	14.34	1.03
18	15	1.00	12.31	1.08
19	10	1.00	10.95	1.06
20	5	1.00	10.79	1.06
21	15	0.62	17.93	0.93
22	10	0.62	17.52	0.91
23	5	0.62	17.67	0.95
24	15	0.38	16.16	1.00
25	10	0.38	15.78	0.98
26	5	0.38	16.05	0.98
27	15	1.00	16.54	0.96

Upon completion of a test the soil and instruments were removed from the tank. The soil was passed through a ¾ × 2-in. screen to remove large blocks of soil formed during the compaction process.

Results and Discussion

In order to verify the hypothesis that the changes in soil compaction developed under dynamic conditions are controlled by the changes in mean normal stress, two things must be demonstrated:

(a) That mean normal stress does correlate with changes in bulk density

(b) That the deviator stress tensor does not correlate with changes in bulk density.

The only measure of the spherical stress tensor is mean normal stress. Many expressions can be used as a measure of the deviator tensor. Since earlier investigations had indicated a

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* Numbers in parentheses refer to the appended references.

relationship between maximum shear stress (an invariant of the deviator tensor) or maximum normal stress (which depends on the deviator tensor as well as on the mean stress) and bulk density, these relationships were investigated.

The values of mean normal stress (σm), the maximum shear stress, the maximum normal stress and the second invariant of the stress deviator tensor were computed from four measured normal stress values obtained with Type A cells using the appropriate formulas as reported by VandenBerg (5). The values for the 6DST were computed from six measured normal stresses using the formulas reported by Harris (2). Mistic, an electronic digital computer at Michigan State University, was used to make the lengthy calculations involved in evaluating the equations and the statistical analysis of the data.

The sum of least squares method was used to determine the best predicting relationship for the data plotted on semilogarithmic paper. The regression equations, estimates of standard error (S_{xy}) and confidence limits for both the Type A and 6DST data are given in Table 2. The Type A data is designated by an *A* following the test number and the 6DST data by only the test number. The calculated values of *t* were compared with the distribution of *t* using the degrees of freedom (DF) shown. All calculated values were highly significant, which means that the regression coefficients or slopes are different than zero. The true regression coefficient is within the limits presented for each relationship. Assuming a normal distribution of error, one standard error (S_{xy}) would include 68.3 percent of the values used to determine the regression equation. The data obtained with the six directional transducer are consistently more varied than the data

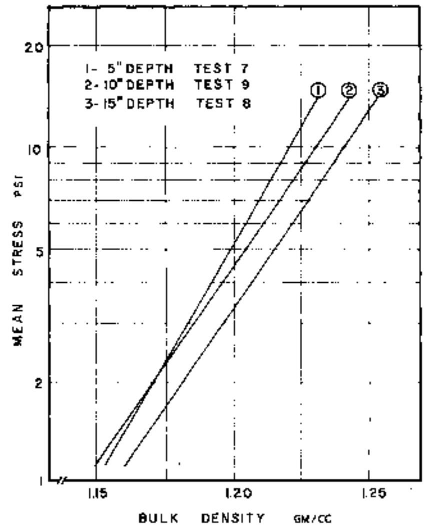

FIG. 3 Mean stress vs bulk density relationship.

TABLE 2. STATISTICAL ANALYSIS FOR MEAN NORMAL STRESS VERSUS BULK DENSITY

Test no.	Regression equation	Syx	DF	t	Confidence limits
1	$\ln_{\sigma}m = -36.82 + 30.50_{\delta}$	0.13	26	38.46	28.63-32.27
1A	$\ln_{\sigma}m = -26.11 + 22.10_{\delta}$	0.12		25.68	20.33-23.87
2	$\ln_{\sigma}m = -37.41 + 31.28_{\delta}$	0.20	33	27.93	24.60-29.04
2A	$\ln_{\sigma}m = -25.41 + 21.65_{\delta}$	0.19		16.31	18.94-24.36
3	$\ln_{\sigma}m = -23.08 + 20.26_{\delta}$	0.27	38	15.35	18.03-22.49
3A	$\ln_{\sigma}m = -21.53 + 19.07_{\delta}$	0.20		18.93	17.37-20.77
4	$\ln_{\sigma}m = -36.29 + 30.99_{\delta}$	0.24	18	12.01	25.57-36.41
4A	$\ln_{\sigma}m = -29.90 + 25.72_{\delta}$	0.20		12.28	21.31-30.13
5	$\ln_{\sigma}m = -34.47 + 28.76_{\delta}$	0.15	28	26.39	26.53-30.99
5A	$\ln_{\sigma}m = -25.14 + 21.27_{\delta}$	0.12		24.00	19.45-23.09
6	$\ln_{\sigma}m = -27.59 + 23.96_{\delta}$	0.19	38	21.59	22.09-25.83
6A	$\ln_{\sigma}m = -25.98 + 22.44_{\delta}$	0.19		20.07	20.55-24.33
7	$\ln_{\sigma}m = -34.21 + 29.83_{\delta}$	0.23	33	21.31	26.98-32.68
7A	$\ln_{\sigma}m = -26.68 + 23.58_{\delta}$	0.18		21.60	21.36-25.88
8	$\ln_{\sigma}m = -30.10 + 26.30_{\delta}$	0.50	38	10.08	21.90-30.70
8A	$\ln_{\sigma}m = -23.97 + 21.14_{\delta}$	0.42		9.65	17.45-24.83
9	$\ln_{\sigma}m = -26.84 + 24.41_{\delta}$	0.45	33	13.16	22.81-31.15
9A	$\ln_{\sigma}m = -24.17 + 21.23_{\delta}$	0.25		14.26	18.20-24.26
10	$\ln_{\sigma}m = -26.84 + 24.41_{\delta}$	0.45	38	11.05	20.68-28.14
10A	$\ln_{\sigma}m = -20.19 + 18.77_{\delta}$	0.31		12.37	16.21-21.33
11	$\ln_{\sigma}m = -34.02 + 29.84_{\delta}$	0.32	33	15.07	25.81-33.87
11A	$\ln_{\sigma}m = -24.66 + 22.12_{\delta}$	0.21		17.16	19.50-24.74
12	$\ln_{\sigma}m = -23.49 + 22.14_{\delta}$	0.26	33	16.77	19.46-24.82
12A	$\ln_{\sigma}m = -20.02 + 19.06_{\delta}$	0.21		17.71	16.86-21.26
13	$\ln_{\sigma}m = -38.41 + 37.96_{\delta}$	0.25	22	15.43	32.86-43.06
13A	$\ln_{\sigma}m = -31.44 + 31.40_{\delta}$	0.25		12.95	26.38-36.42
14	$\ln_{\sigma}m = -53.62 + 51.83_{\delta}$	0.25	18	14.44	44.29-59.37
14A	$\ln_{\sigma}m = -41.76 + 40.72_{\delta}$	0.18		15.46	35.19-46.25
15	$\ln_{\sigma}m = -51.75 + 49.72_{\delta}$	0.35	33	14.54	42.76-56.68
15A	$\ln_{\sigma}m = -38.15 + 37.13_{\delta}$	0.27		14.00	31.74-42.52
16	$\ln_{\sigma}m = -34.82 + 34.68_{\delta}$	0.17	33	26.05	32.11-37.53
16A	$\ln_{\sigma}m = -31.73 + 31.75_{\delta}$	0.17		22.19	28.84-34.66
17	$\ln_{\sigma}m = -32.79 + 32.83_{\delta}$	0.23	33	19.43	20.39-36.27
17A	$\ln_{\sigma}m = -26.36 + 26.65_{\delta}$	0.26		13.74	22.70-30.60
18	$\ln_{\sigma}m = -37.77 + 32.16_{\delta}$	0.18	38	31.84	30.46-33.86
18A	$\ln_{\sigma}m = -29.06 + 25.19_{\delta}$	0.17		26.41	23.59-26.79
19	$\ln_{\sigma}m = -43.71 + 37.50_{\delta}$	0.22	38	14.88	34.77-40.23
19A	$\ln_{\sigma}m = -33.44 + 29.05_{\delta}$	0.17		22.63	26.89-31.21
20	$\ln_{\sigma}m = -35.36 + 30.42_{\delta}$	0.22	38	22.53	28.14-32.70
20A	$\ln_{\sigma}m = -27.75 + 24.18_{\delta}$	0.21		18.31	21.95-26.41
21	$\ln_{\sigma}m = -40.48 + 39.81_{\delta}$	0.16	18	18.10	35.19-44.43
21A	$\ln_{\sigma}m = -33.23 + 32.88_{\delta}$	0.15		16.34	28.66-37.10
22	$\ln_{\sigma}m = -36.92 + 36.75_{\delta}$	0.22	13	9.10	28.02-45.48
22A	$\ln_{\sigma}m = -33.06 + 32.76_{\delta}$	0.14		12.10	26.91-38.61
23	$\ln_{\sigma}m = -27.91 + 27.87_{\delta}$	0.32	13	6.26	18.26-37.48
23A	$\ln_{\sigma}m = -21.16 + 21.37_{\delta}$	0.20		7.50	15.21-27.53
24	$\ln_{\sigma}m = -29.03 + 28.54_{\delta}$	0.29	23	13.27	24.09-32.99
24A	$\ln_{\sigma}m = -22.30 + 22.20_{\delta}$	0.20		14.68	19.08-25.32
25	$\ln_{\sigma}m = -23.63 + 23.73_{\delta}$	0.21	23	16.37	20.73-26.73
25A	$\ln_{\sigma}m = -19.24 + 19.56_{\delta}$	0.20		14.02	16.68-22.44
26	$\ln_{\sigma}m = -30.92 + 30.84_{\delta}$	0.29	23	11.34	25.21-36.47
26A	$\ln_{\sigma}m = -23.56 + 23.95_{\delta}$	0.24		10.51	19.23-28.67
27	$\ln_{\sigma}m = -26.19 + 25.23_{\delta}$	0.17	18	21.56	22.77-27.69
27A	$\ln_{\sigma}m = -19.32 + 18.70_{\delta}$	0.14		18.59	16.58-20.82

obtained with the Type A cells. The reduced size of the diaphragm pressure cells (¾-in. diameter) used in the 6DST compared with the 2-in. diameter Type A cells could have caused the average normal stress measured to be more varied. The same size particles probably were not acting on the pressure cells during each replication.

If mean normal stress is related to bulk density in a general manner, the regression lines for each different stress state should not be significantly different for a given soil condition. The lines should be parallel or the difference between slopes should not be significant. The *t* test was used to test for differences among the lines for different stress states for each method used.

Approximately fifty percent of the comparisons were significant. At the high moisture contents and high rates of loading, significant differences appeared between the 10 and 15-in. depths. At the lower moisture content and lower rates of loading, significant differences appeared between the 5 and 10-in. depths and 5 and 15-in. depths. Based on the data obtained, the hypothesis that changes in bulk density are controlled by mean normal stress cannot be accepted or rejected.

A typical set of data showing the relationship between mean normal stress and bulk density is shown in Fig. 3.

To determine the effect of moisture content on the relationship between mean normal stress and bulk density the *t* test for the regression coefficients was used. The coefficients from relationships determined at the same depth and rate of loading but with different moisture contents were compared. In general, the moisture content does affect the relationship at the deeper depths and higher rates of loading. For a given value of mean stress developed, the higher the moisture content the greater the changes in bulk density.

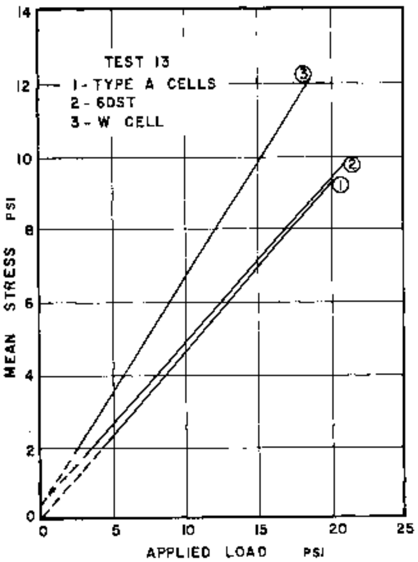

FIG. 4 Comparison of three methods used to determine mean stress.

The *t* test was used to determine the effect of the rate of loading on the relationship between mean normal stress and bulk density. The results indicated that the mean stress-bulk density relationship was not affected by the rate of loading within the range of rates studied. However, there was some variation in moisture content within a group of the tests used. This may have had some effect on the analysis.

A strain-gage transducer (*W* cell) capable of measuring mean stress directly was developed and the values of mean stress calculated from the type A and 6DST data were compared (Fig. 4). The sum of the least squares method of obtaining the best predicting straight line for the points permitted the use of a statistical method for comparing the three methods.

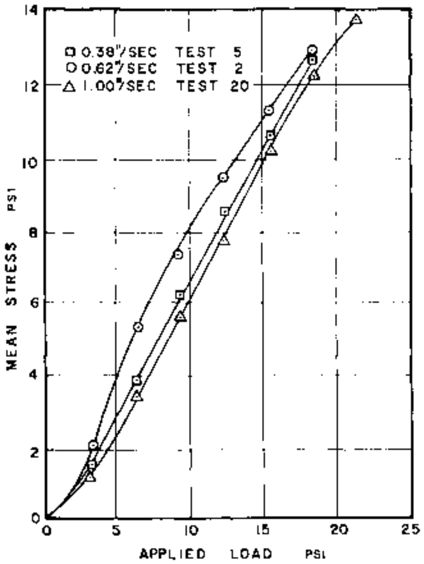

FIG. 5 Effect of rate of loading on mean stress-applied load relationship.

The *t* test was applied to the regression coefficients to check for significant differences and the results are presented in Table 3. The most consistent results were obtained between the type A cells and the W cell. The differences between the regression coefficients of Type A and 6DST data were the most varied.

To determine the effect of the rate of loading on the relationship between mean normal stress and applied surface load, the average values of mean stress for the five replications were plotted versus applied load. Fig. 5 shows a typical set of curves obtained.

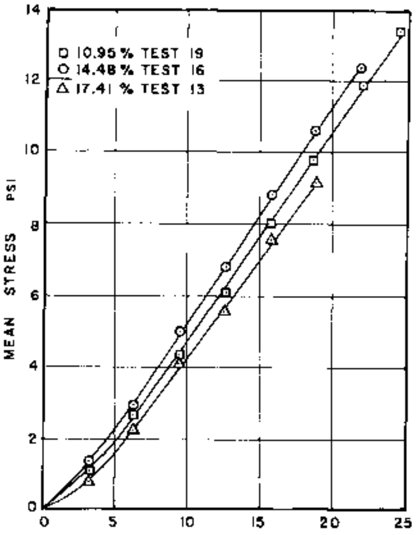

FIG. 6 Effect of moisture content on mean stress-applied load relationship.

Based on the data obtained, the conclusion is that for a given applied load the lowest values of mean stress will be obtained for the 1.00-in. per second rate of loading.

The same analysis was used to determine the effect of moisture content on the relationship between mean stress and applied load. Fig. 6 shows a typical set of curves. The conclusion was that moisture content within the range used for these tests had little or no effect on the relationship between mean stress and applied load.

Regressions equations, estimates of standard error, confidence limits and a comparison of regression coefficients were determined for the following:

1 Relationship between second invariant and bulk density

2 Relationship between maximum normal stress and bulk density

3 Relationship between maximum shear stress and bulk density

4 Relationship between mean stress and bulk density.

The data of the above relationships were fitted by statistical procedures to a straight line on semilogarithmic paper. Of the four invariants of the stress tensor investigated, the maximum shear stress was found to be best related to changes in bulk density.

Conclusions

In the loose soil used for the experimental tests, data indicated the following:

1 The data obtained with the six directional stress transducer were more

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TABLE 3. STATISTICAL ANALYSIS OF MEAN STRESS-APPLIED LOAD RELATIONSHIP FOR 1.00 INCH PER SECOND RATE OF LOADING

Test no.	Depth, in.	M.C., %	t, type A 6DST	t, type A W cell	t, 6DST W cell	DF
14	5	17.41	4.07**	1.22	0.33	36
20	5	10.79	3.57**	1.36	1.26	76
13	10	17.41	0.06	0.65	2.75**	44
19	10	10.95	2.72**	2.82**	0.81	76
18	15	12.31	0.44	1.32	3.90**	76

RELATIONSHIP OF MEAN STRESS, VOLUMETRIC STRAIN AND DYNAMIC LOADS IN SOIL

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varied than the data obtained with the Type A cells.

2 The hypothesis that changes in bulk density are controlled by mean normal stress cannot be accepted or rejected.

3 The maximum shear stress was best related to changes in bulk density.

4 The relationships between the invariants and bulk density were exponential.

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