NONLINEAR EFFECTS IN THE REFLECTION FROM ADHESIVE BONDS

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INTRODUCTION

The nondestructive testing of adhesive joints is difficult, because three basic types of defects must be detected and characterized. These are complete disbonds and voids, poor cohesion (i.e., a weak adhesive layer), and poor adhesion (i.e., a weak bond between the adhesive layer and one or both adherends). For recent reviews of the non-destructive testing of adhesively bonded structures, we refer to Refs.[1] and [2].

The real challenge to nondestructive ultrasonic methods for adhesive bond testing is to obtain actual information on the strength of the bond. Most tests give a stiffness parameter which must be correlated in an ad-hoc way with strength.

In this paper it is assumed that failure of an adhesive bond is preceded by deviations from linear behavior, usually in thin layers (~ 10μm thickness) at the interfaces of the adherends and the adhesive layer. It is postulated that the corresponding overall nonlinear behavior of the adhesive layer can be represented by a simple nonlinear elastic model, up to the point of failure where further deformation would not require a further increase of the load. It is subsequently shown that the reflection of two ultrasonic signals of different amplitudes, where one should deform the adhesive linearly, but the other should pull the adhesive into the nonlinear range, will provide sufficient information to extrapolate to the failure point of the adhesive layer. Hence information on the strength of the adhesive bond is directly obtained from ultrasonic reflection data.

NONLINEAR BEHAVIOR

A typical thickness of an adhesive layer is 100μm. A dominant wavelength of a pulse of ultrasonic wave motion may well be substantially longer, which implies that the strain in the layer may be considered as homogeneous, and as a further approximation the inertia of the adhesive layer may be ignored. The mechanical behavior of the adhesive is then effectively modeled by a one-dimensional element (a spring) which relates the tractions on the faces of the adherents to the displacement discontinuity across the thickness of the adhesive. This model has the great advantage that nonlinear mechanical behavior of the adhesive can be taken into account in a relatively simple manner.

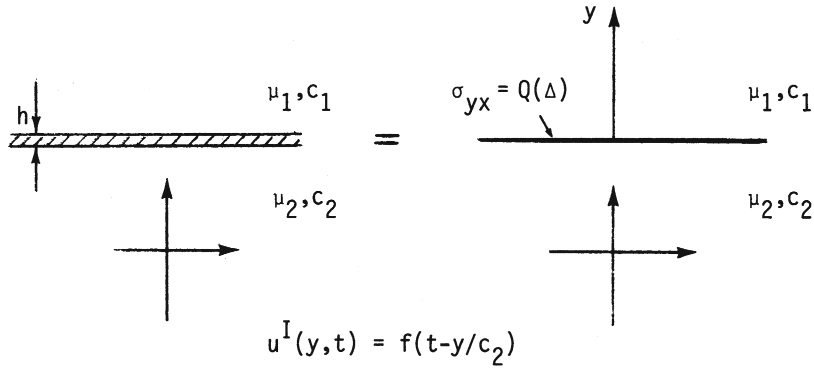

Fig. 1. Pulse of transverse wave motion incident on an adhesive bond.

Let us consider the one-dimensional configuration shown in Fig. 1. An ultrasonic pulse of transverse wave motion with a displacement directed in the x-direction, but propagating in the y-direction, is transmitted and reflected by the adhesive layer. The pulse places the adhesive layer in a state of shear. For the spring model that is being considered here, the thickness of the layer is shrunk to a surface at y = 0, across which the following conditions apply

$$\sigma_{yx}\Big|_{y=0^+} = \sigma_{yx}\Big|_{y=0^-} = \sigma_{yx}(0) \tag{1}$$

$$\sigma_{yx}(0) = Q(\Delta) \tag{2}$$

$$\Delta = u\Big|_{y=0^+} - u\Big|_{y=0^-} \quad , \tag{3}$$

where $Q(\Delta)$ may be a nonlinear function of Δ. As an example we will consider

$$Q(\Delta) = \alpha\Delta - \beta\Delta^3 \tag{4}$$

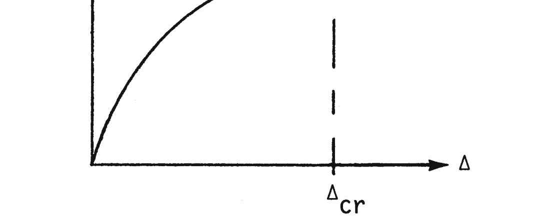

Fig. 2. Nonlinear relation between $\sigma_{yx}(0)$ and Δ

Equation (2) with $Q(\Delta)$ defined by Eq.(4), is shown in Fig. 2. Clearly there is a critical value of Δ beyond which the required stress decreases with increasing Δ. The values $(\sigma_{yx})_{cr}$ and Δ_{cr} define the failure point of the adhesive. We easily obtain

$$\Delta_{cr} = \left(\frac{\alpha}{3\beta}\right)^{\frac{1}{2}} \quad , \quad \left(\sigma_{yx}\right)_{cr} = \frac{2}{3}\left(\frac{\alpha^3}{3\beta}\right)^{\frac{1}{2}} \tag{5a,b}$$

It would be of interest if these critical values of the adhesive bond could be obtained from ultrasonic wave reflection and transmission data.

REFLECTION AND TRANSMISSION

Let the adhesive bond be located in between two semi-infinite elastic solids of different shear moduli and mass densities, see Fig. 1. The solids defined by $y \leq 0$ and $y \geq 0$ are defined by μ_2, c_2 and μ_1, c_1, respectively. Here μ defines the shear modulus and c is the velocity of transverse waves. A transverse wave pulse is incident from the underlying solid. It is of the general form

$$u^I(y,t) = f(t-y/c_2) \tag{6}$$

Here $f(t)$, where $f(t) \equiv 0$ for $t \leq 0$, defines the waveform. The reflected and transmitted displacement pulses are

$$u^R(y,t) = g(t+y/c_2) \tag{7}$$

$$u^T(y,t) = h(t-y/c_1) \tag{8}$$

In terms of $f(t)$, $g(t)$ and $h(t)$, the displacement jump defined by Eq.(3) may be written as

$$\Delta = h(t) - f(t) - g(t) \tag{9}$$

Continuity of shear stress across the bond yields

$$\mu_1 \left.\frac{\partial u^T}{\partial y}\right|_{y=0} = \mu_2 \left.\frac{\partial u^I}{\partial y}\right|_{y=0} + \mu_2 \left.\frac{\partial u^R}{\partial y}\right|_{y=0} \tag{10}$$

Substitution of Eqs.(6)-(8) into Eq.(10) yields

$$\dot{h}(t) = \frac{c_1}{c_2}\frac{\mu_2}{\mu_1}[\dot{f}(t) - \dot{g}(t)] \quad , \tag{11}$$

where the dot denotes differentiation. Elimination of $g(t)$ by the use of Eq.(9), reduces Eq.(11) to

$$\dot{h}(t) = m[2\dot{f}(t) + \dot{\Delta}] \; , \tag{12}$$

where

$$m = \frac{c_1\mu_2}{c_2\mu_1} \Big/ \left[1 + \frac{c_1\mu_2}{c_2\mu_1}\right] \tag{13}$$

Next we introduce the nonlinear relation given by Eq.(4). Since $\sigma_{yx} = -(\mu_1/c_1)\dot{h}(t)$, we find by the use of Eqs.(2) and (12)

$$\dot{\Delta} + \frac{c_1}{\mu_1 m} Q(\Delta u) = -2\dot{f}(t) \tag{14}$$

This nonlinear ordinary differential equation governs the displacement discontinuity as a function of time. The solution must satisfy the initial condition

$$\Delta(0) = 0 \tag{15}$$

When Δ has been computed, the transmitted waveform follows from Eq.(12) as

$$h(t) = m[2f(t) + \Delta] \tag{16}$$

The reflected waveform follows from Eqs.(9) and (16) as

$$g(t) = \left\{\left[\frac{\mu_2 c_1}{\mu_1 c_2} - 1\right] f(t) - \Delta\right\} / \left[\frac{\mu_2 c_1}{\mu_1 c_2} + 1\right] \tag{17}$$

We now give details for the nonlinear relation given by Eq.(4). For this case Eq.(14) becomes

$$\dot{\Delta} - \frac{\beta}{m}\frac{c_1}{\mu_1}\Delta^3 + \frac{\alpha}{m}\frac{c_1}{\mu_1}\Delta = -2\dot{f}(t), \tag{18}$$

where $\Delta(t)$ must satisfy the initial condition given by (15). For the linear case ($\beta \equiv 0$), Eq.(18) can be solved analytically. For the nonlinear case ($\beta \neq 0$), Eq. (18) must be solved numerically. This can be done by using the Runge-Kutta-Verner fifth order and sixth order methods.

The incident pulse, $f(t)$, is characterized by its duration t_o. For $t \leq 0$ and $t \geq t_o$ we have $f(t) \equiv 0$. It may be verified that the solution for $t \geq t_o$ may be obtained from the relations

$$\frac{\Delta^2}{|\Delta^2 - \alpha/\beta|} = \exp[D-2(\alpha/m)(c_1/\mu_1)t] \quad \text{for } \beta \neq 0, \tag{19}$$

and

$$\Delta = E\exp[-(\alpha/m)(c_1/\mu_1)t] \quad \text{for } \beta = 0, \tag{20}$$

where the constants D and E must be obtained from the value of Δ at $t = t_o$.

For the numerical solution of Eq.(18) it is convenient to introduce the following dimensionless variables

$$\bar{\Delta} = \Delta/h \ , \quad \bar{A} = A/h \ , \quad \bar{t} = t/(h/c_1) \ , \tag{21a,b,c}$$

where h is the actual thickness of the adhesive layer. The adhesive is characterized by

$$\bar{\alpha} = \alpha h/\mu_1 \ , \quad \bar{\beta} = \beta h^3/\mu_1 \tag{22a,b}$$

By the use Eqs.(21a,b,c) and (22a,b), Eq.(18) reduces to

$$\frac{d\bar{\Delta}}{d\bar{t}} - \frac{\bar{\beta}}{m}\bar{\Delta}^3 + \frac{\bar{\alpha}}{m}\bar{\Delta} = -2\frac{d\bar{f}}{d\bar{t}} \tag{23}$$

where

$$\bar{f} = f/h \tag{24}$$

It should be noted that $\bar{\alpha} = 1$ corresponds to an adhesive whose linear behavior is comparable to that of the solid for $y \geq 0$.

As an example we consider an incident wave pulse of the form

$$f(t) = \frac{1}{2}A\ [1 - \cos(2\pi t/t_o)]\ , \quad 0 \leq t \leq t_o\ , \tag{25}$$

where A is the amplitude. Figure 3 shows $\bar{\Delta}$ versus the dimensionless time $\bar{t}$, for $m = 0.5$, that is, for a nonlinear adhesive in between solids of identical mechanical properties: The incident pulse is characterized by the dimensionless constants

$$A/h = 0.5\ , \quad t_o/(h/c_1) = 2.0 \tag{26a,b}$$

The curves in Fig. 3 have been calculated for $\bar{\alpha} = 1.0$, $\bar{\beta} = 0$ (solid line) and $\bar{\alpha} = 1.0$, $\bar{\beta} = 2\bar{\alpha}$ (dotted line). It should be noted that for $\bar{\beta} = 2\bar{\alpha}$, the maximum value of $\bar{\Delta}$ is $(1/6)^{\frac{1}{2}} = 0.408$, in order that the critical value of Δ, given by Eq.(5a) will not be exceeded. Figure 3, shows that $\bar{\Delta}$ remains smaller than 0.408. For $0 \leq \bar{\beta} \leq 2\bar{\alpha}$, the curves for $\bar{\Delta}$ remain in between the curves for $\bar{\beta} = 0$ and $\bar{\beta} = 2\bar{\alpha}$. Figure 4 shows the maximum value of $\bar{\Delta}$ as a function of $\bar{\beta}/\bar{\alpha}$. Once Δ is known the transmitted pulse readily follows from Eq.(17) as $g(t) = -\Delta(t)/2$, since $\mu_2 c_1 = \mu_1 c_2$ for this case.

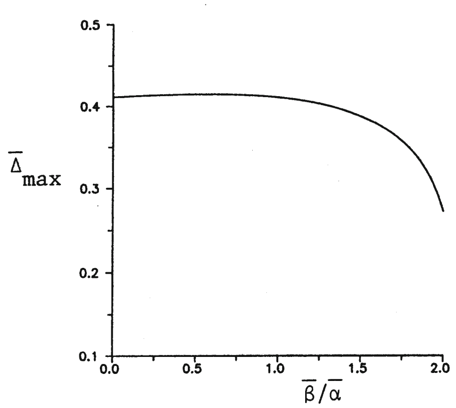

Fig. 3. $\bar{\Delta}$ versus $\bar{t}$ for identical solids.

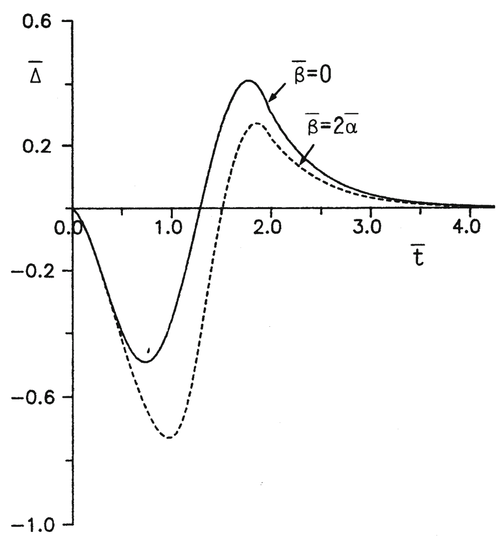

Fig. 4. Maximum value of $\bar{\Delta}$ as a function of $\bar{\beta}/\bar{\alpha}$ for m=0.5.

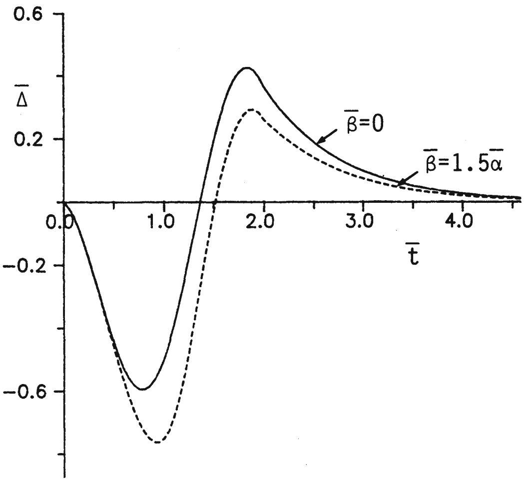

Fig. 5. $\bar{\Delta}$ versus $\bar{t}$ for solids of different mechanical properties.

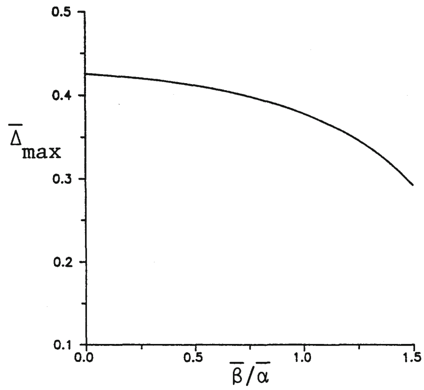

Fig. 6. Maximum value of $\bar{\Delta}$ as a function of $\bar{\beta}/\bar{\alpha}$ for m=0.75.

Figure 5 displays the effect of different mechanical properties of the adhered elastic solids, by displaying $\bar{\Delta}$ versus the dimensionless time $\bar{t}$, for m = 0.75. As can be seen from Eq.(13), the case m = 0.75 corresponds to $(c_1/c_2)(\mu_2/\mu_1) = 3$, which would mean $\mu_2/\mu_1 = 9$ for materials of equal mass densities. The calculations were again carried out for $\bar{\alpha} = 1.0$, $\bar{\beta} = 0$ (solid line) and $\bar{\alpha} = 1.0$, $\bar{\beta} = 1.5\bar{\alpha}$ (dotted line). For $\bar{\beta} = 1.5\bar{\alpha}$, the maximum allowable value of $\bar{\Delta}$ is 0.471. Figure 5 shows that $\bar{\Delta}$ remains smaller than this value. Figure 6 shows the maximum value of $\bar{\Delta}$ as a function of $\bar{\beta}/\bar{\alpha}$. Comparing Figs. 4 and 6, it is seen that the nonlinear effects should be easier to measure for the case of dissimilar adherents.

For adhered solids of the the same material properties (m = 0.5), the effect of the amplitude of the incident wave is displayed in Fig. 7. Figure 7 shows the complete curves for $\bar{f}(t)$, $\bar{h}(t)$ and $\bar{g}(t)$, for $\bar{A} = 0.5$, $\bar{\alpha} = 1$ and for both $\bar{\beta} = 0$ and $\bar{\beta} = 2\bar{\alpha}$. The curves for $\bar{\beta} = 2\bar{\alpha}$ are substantially different from those for $\bar{\beta} = 0$. The corresponding curves for $\bar{A} = 0.1$ (not shown here), do not show appreciable differences for the cases $\bar{\beta} = 0$ and $\bar{\beta} = 2\bar{\alpha}$. Now the amplitude of the incident wave is not large enough to pull the adhesive in the nonlinear range. Figure 8 shows the maximum value of $\bar{\Delta}$ as a function of $\bar{A}$ for both $\bar{\beta} = 0$, i.e., the linear case, and $\bar{\beta} = 2\bar{\alpha}$, i.e., the nonlinear case. It is seen that the nonlinear effects become prominent at higher amplitudes.

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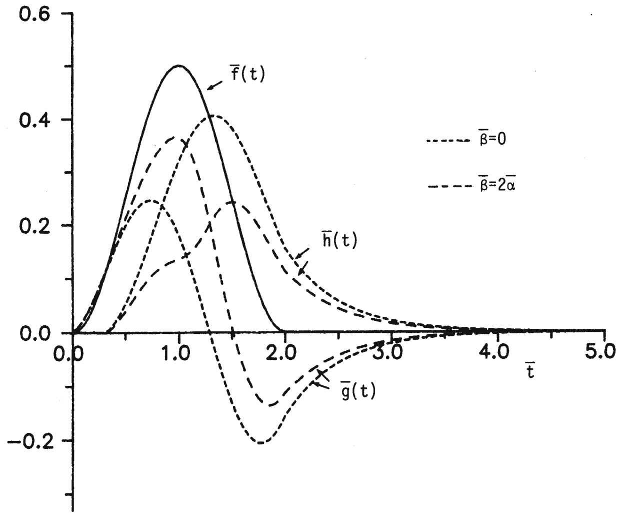

Fig. 7. The incident, reflected and transmitted pulses for $\bar{A}$=0.5.

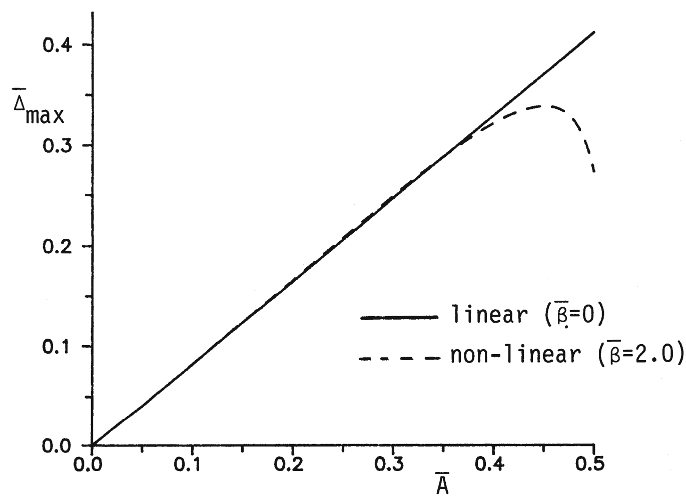

Fig. 8. Maximum value of $\bar{\Delta}$ as a function of the amplitude.

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