

## ULTRASONIC EVALUATION OF IN-PLANE AND OUT-OF-PLANE ELASTIC PROPERTIES OF COMPOSITE MATERIALS

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### INTRODUCTION

Evaluation of elastic properties of composite materials using ultrasound is important for the generation of output data for the design of composites. It is also extremely important as a nondestructive tool for quality evaluation of the composites after manufacturing. The problem was addressed in the seventies [1-3] when Markham [1] suggested using the time-delay through transmission technique with obliquely incident ultrasonic waves from water onto a composite plate. The full set of elastic constants was measured later by Kriz and Stinchcomb [4] on samples cut out in different directions from a composite plate. Recently several works have appeared where the set of elastic constants was measured by using Markham's technique [5-7].

In this work, in-plane and out-of-plane elastic properties of composite materials are measured nondestructively. The out-of-plane properties are measured by a modified through-transmission technique with rotation of the experimental sample. The double transmission method used in this work has advantages in precision over Markham's method since it removes the necessity of shifting the receiving transducer to the central axis of the refracted beam with change of incident angle. These measurements give a reconstruction of the out-of-plane slowness surfaces of the quasi-longitudinal and the two quasi-transverse waves. The in-plane elastic properties are measured by a critical angle technique using a novel type of single transducer reflection goniometer. All three critical angles can easily be measured with such a technique.

Finally, the elastic constants are reconstructed by a least-square optimization technique from the measured slowness surfaces. Previous investigators [5,6] used reconstruction in planes of symmetry while our method is valid for an arbitrary plane in a composite material.

### EXPERIMENTAL APPARATUS

The experimental apparatus which implements a novel type of goniometer is shown in Fig. 1. It is distinguished by its ability to make simultaneous measurements of both doubly-transmitted (through the sample) and doubly-reflected (from the sample) ultrasonic signals with the use of

only one ultrasonic transducer. A plane sample is mounted in the center of an aluminum cylinder. The sample is rotated around a cylinder axis which lies in the frontal plane of the sample.

Incident and reflected ultrasonic beams propagate along cylinder radii. After being reflected from the cylinder wall the ultrasonic signal is reflected a second time from the sample surface and returns to the receiving transducer. The transmitted ultrasonic signal is reflected and returned to the transducer after the second through-transmission. The amplitude values of the received signal after analog peak detection are digitized, averaged and collected by computer as a function of the angle of rotation.

When necessary, ultrasonic RF signals are digitized and analyzed in the frequency domain by a LeCroy 9400 125MHz digital oscilloscope. Both signal and spectrum are fed to the computer through an IEEE interface.

For sample rotation by angle  $\theta$  around the cylinder axis a computer controlled DC motor is used. Angle resolution and repeatability of  $0.01^\circ$  are achieved in our setup. In addition, the sample may be rotated in its plane by an angle  $\Omega$ , so different orientations of the incident plane relative to fiber orientation may be selected. The water tank is temperature stabilized within  $0.1^\circ\text{C}$ .

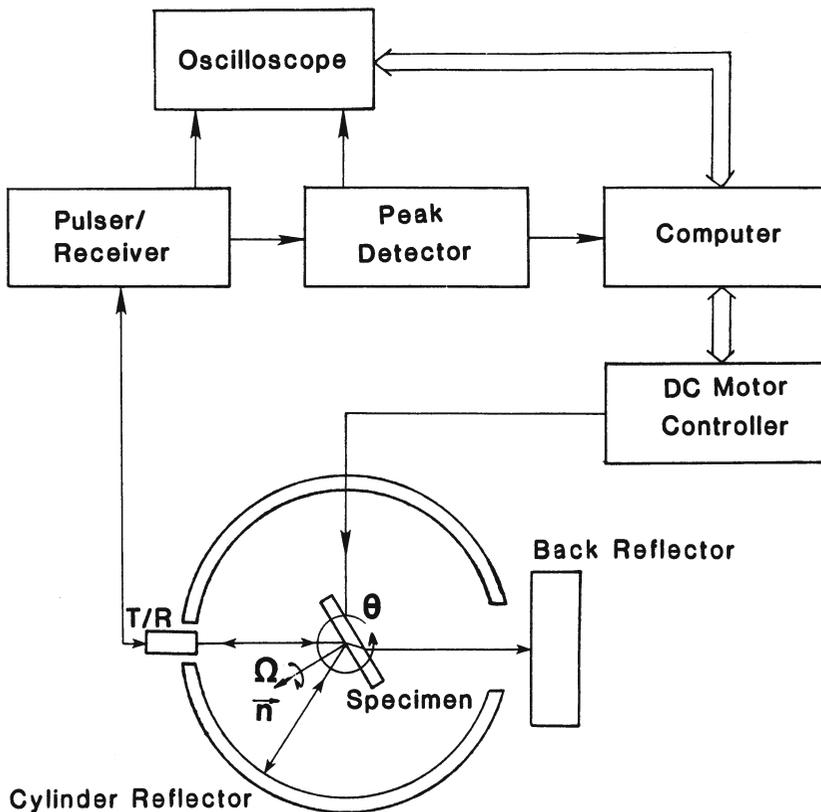


Fig. 1 Schematic diagram of the experimental system.

## CRITICAL ANGLE MEASUREMENTS (IN-PLANE PROPERTIES)

Our recent theoretical work [8] has shown that a strong, sharp maximum in the reflection coefficient appears at the critical angles for quasi-longitudinal and fast quasi-transverse waves. The third critical angle, which corresponds to the slow quasi-transverse wave, also manifests itself strongly in the theoretical dependence of the reflection coefficient on the incidence angle. When a composite sample is rotated around its surface normal (which lies in the plane of incidence), the value of the critical angles change in correspondence with the in-plane slowness surface.

The last statement requires special consideration. The critical angle may be defined as the angle of incidence for which the direction of energy flow for the refracted wave is parallel to the sample surface. Since in anisotropic material the energy flow (group velocity) direction and the wave vector direction do not coincide, it led Henneke and Jones [9] to the conclusion that critical angle measurements may not be used in general for phase velocity measurements in anisotropic materials. They supported their conclusion by theoretical calculation for quartz. For a general direction for propagation in two-directional composite materials the angle of deviation between phase and group velocities has both in-plane and out-of-plane components [8] (in-plane means here in the plane of the composite plate). But for the in-plane direction of propagation the out-of-plane component of the deviation angle will equal zero. This is clear from symmetry considerations since the in-plane elastic properties of a composite plate are invariant with change of the direction of the normal to that plane. This means that at the critical angle the wave vector (phase velocity vector) will be parallel to the plate surface and will lie in the plane of incidence. The group velocity vector will also be parallel to the plate surface but may not lie in the plane of incidence and will deviate from this plane, remaining in the plate plane. The practical conclusion from this is that Snell's law may be used at the critical angle for the calculation of the in-plane phase velocity.

Typical traces of the amplitudes of a doubly-reflected ultrasonic signal for a unidirectional composite plate is shown in Fig. 2 as a function of incident angle  $\theta_i$ . The data are shown for only one angle  $\Omega = 20^\circ$  of deviation of the fiber direction from the plane of incidence. In Fig. 1 the plane of the figure is the plane of incidence, in which lie the incident, reflected and transmitted ultrasonic rays and the normal to the plane of the sample. The axis of sample rotation "0" which lies in the sample plane is perpendicular to the plane of incidence. By rotating the sample along the normal " $\vec{n}$ " to the sample surface one can determine the critical angle and phase velocity as functions of the angle  $\Omega$ , where the angle between the "0" axis and the fiber direction will be  $90^\circ - \Omega$ . The corresponding results will be summarized on the last section of this paper.

## DOUBLE-THROUGH-TRANSMISSION MEASUREMENTS (Out-Of-Plane Properties)

The double-through-transmission method is illustrated schematically in Fig. 3. As in Markham's method an ultrasonic wave is obliquely incident on the composite sample. The angle of refraction and therefore the time of flight inside the material depends on the velocity in the direction of the refraction angle. At the second boundary the wave is transmitted into the liquid and propagates at the same angle as the incident. It reaches the back plate reflector let us say at point A (Fig. 3). If the angle of incidence is changed, the transmitted beam will shift; therefore, in the through-transmission method [1-3,5-7] the transducer should be shifted to the optimum receiving position. This is difficult to do precisely and

therefore an error is introduced into time-of-flight measurements. In our system we use a plane reflector which reflects all transmitted waves and after the second transmission returns them to the transmitter-receiver. This offers a significant advantage in simplicity of measurement and in method sensitivity since double paths in the material are used. A back-reflected-plate method has often been used in the past for NDE measurements and inspection with relation to other applications.

The actual situation is complicated by the deviation in angle between phase and group velocities, discussed in the previous section. This is illustrated schematically in Fig. 3, where the direction of the wave vector is indicated by dashed lines and the direction of the group velocity (direction of acoustic energy propagation) by solid lines. This problem was recognized by Gieske and Allred [3] who point out that the time of flight in a composite material measured by this technique is that which corresponds to the group velocity while the equations for calculations used the phase velocity. Therefore one may suspect that the results of measurement are somewhat incorrect. This question has not been clarified previously.

We have demonstrated by direct analytical calculations that this method gives absolutely correct results for arbitrary orientation of the composite material relative to the incident beam. This is illustrated in Fig. 4, where the wave vector  $\vec{k}$  and the group velocity (ray) are shown. The calculation based on Snell's law accounts for time delay as the signal arrives with certain phase velocity at point A, while in fact the signal arrived with the corresponding group velocity at point B. The time of flight to point A is equal to the time of flight to the actual point of the signal arrival C. From point C to point B there is an additional time delay  $t$ :

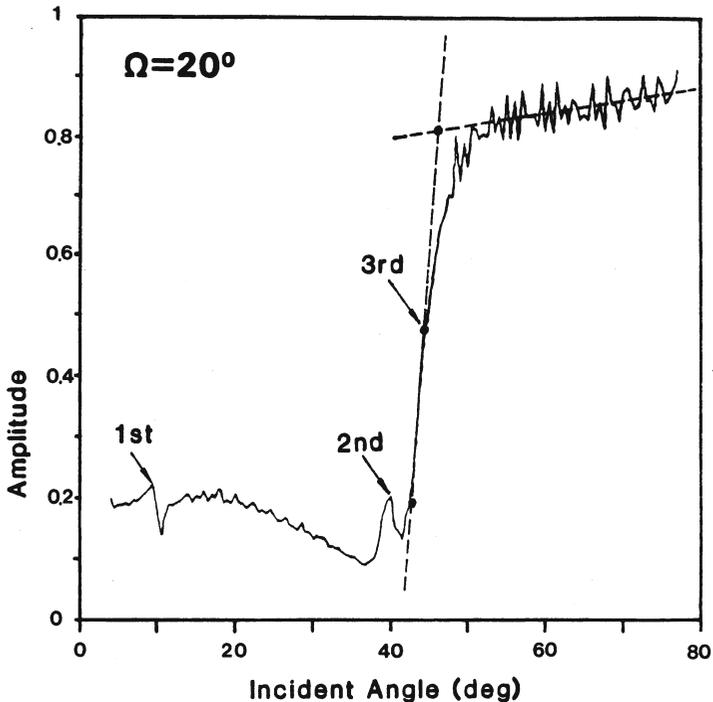


Fig. 2 Typical trace of the amplitude of a doubly-reflected ultrasonic signal from a unidirectional composite material. Three critical angles are indicated by arrows.  $\Omega$  is angle of fibers deviation from the incident plane.



$$t = (h/V_o) \sin \theta_i \sin \alpha / (\cos(\theta_r + \alpha) \cos \theta_r) \quad (1)$$

where  $V_o$  is velocity in a fluid, and the meaning of the other variables are clear from Fig. 4. It may be shown by direct calculations that exactly the same time is required for propagation in water from point A to point D. Therefore, the (imaginary) time of arrival to the receiving transducer for the signal propagating at a velocity equal to the phase velocity will be exactly the same as the actual time of flight of the signal propagating at the group velocity.

The measurements were performed by the method discussed here for different angles of orientation of the fibers relative to the incident plane. When the incident plane does not coincide with planes of symmetry, quasi-longitudinal and slow and fast quasi-transverse waves have been observed and their velocities measured. As an example the results for planes  $\Omega = 0^\circ$  and  $\Omega = 45^\circ$  are shown in Fig. 5.

#### RECONSTRUCTION OF ELASTIC CONSTANTS FROM VELOCITY DATA

The value of the elastic constants may be calculated from phase velocity measurements in several specific directions of the anisotropic material. While this approach is possible, it is somewhat sensitive to errors of measurement for reconstruction of the full matrix of elastic constants. We used a least square optimization technique for best data fitting at different angles. This is in line with the work of Hosten et al. [6], which method was developed for planes of symmetry in hexagonal materials only. Our algorithm is valid for materials of arbitrary anisotropy and for arbitrary directions.

To do optimization it is best to have an analytical function which is related to the phase velocity and the elastic constants. Since the Christoffel equation may be written for any anisotropy as a third-order polynomial in the square of the velocity, it is useful to use Cardan's solution of the cubic equation which may be written in the form:

$$\rho V_k^2 = Z_k - a/3; \\ Z_k = 2 \sqrt{p/3} \cos\left(-\frac{\psi}{3} - \frac{2k\pi}{3}\right), \quad k = 1, 2, 3$$

where  $\psi = \arccos(-q/[2(p/3)^{3/2}]);$

$$p = a^2/3 - b; \quad q = c - ba/3 + 2(a/3)^3; \quad a = -G_{ii}$$

$$b = -(G_{12}^2 + G_{13}^2 + G_{23}^2 - G_{11}G_{22} - G_{11}G_{33} - G_{22}G_{33});$$

$$c = -(G_{11}G_{22}G_{33} + 2G_{12}G_{13}G_{23} - G_{11}G_{23}^2 - G_{22}G_{13}^2 - G_{33}G_{12}^2);$$

$$G_{im} = C_{ijkl}n_jn_l; \quad C_{ijkl} \text{ are the elastic constants in tensorial form.}$$

The best fit to the experimental velocity data obtained from critical angle measurements is shown in Fig. 6. Elastic constants reconstructed for this fit are given in Table 1. Taking this set of elastic constants the velocity data were calculated for out-of-plane measurements (solid lines in Fig. 5). It is seen that comparison of the calculated and the experimental data are satisfactory, which indicates that this particular material may be considered to be transversely isotropic for planes perpendicular to the fiber direction.

TABLE 1 Reconstructed constants in GPa and density in g/cm<sup>3</sup>

$C_{11} = C_{22}$	$C_{33}$	$C_{12}$	$C_{13} = C_{23}$	$C_{44} = C_{55}$	$C_{66}$	$\rho$
17.0	162	8.2	11.8	8.0	4.4	1.61

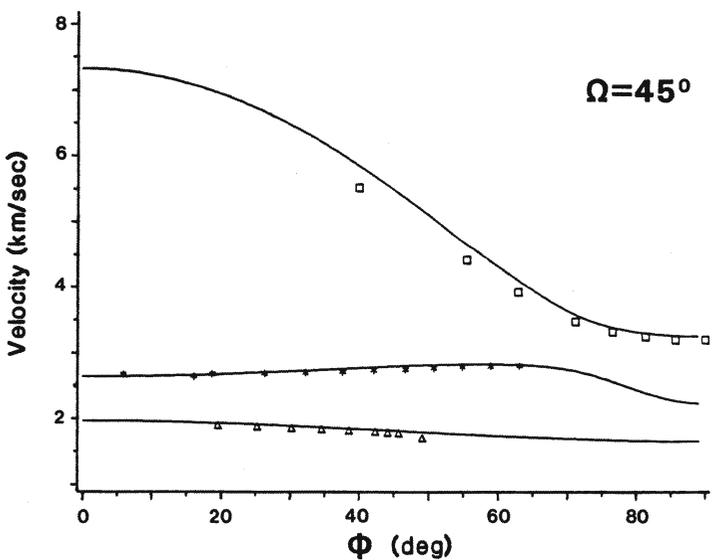
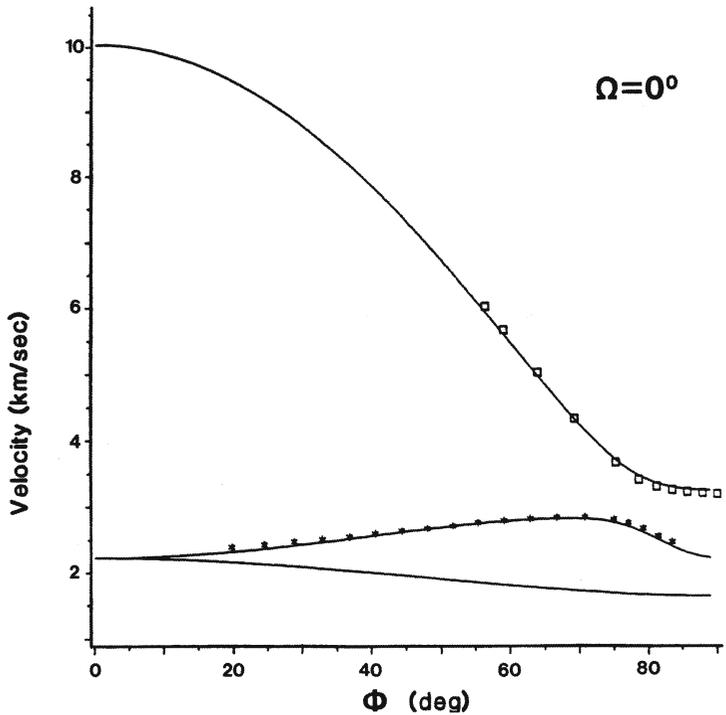


Fig. 5 Points are phase velocity calculated from through-transmission data. Solid lines are calculated velocity based on elastic constants reconstructed from critical angle measurements. Data for two angles between incident plane and fibers direction (a)  $\Omega = 0^\circ$  and (b)  $\Omega = 45^\circ$ . Angle  $\phi$  is angle between refracted wave vector and interfaces  $\phi = 90^\circ - \theta_r$ , where  $\theta_r$  is angle of refraction.

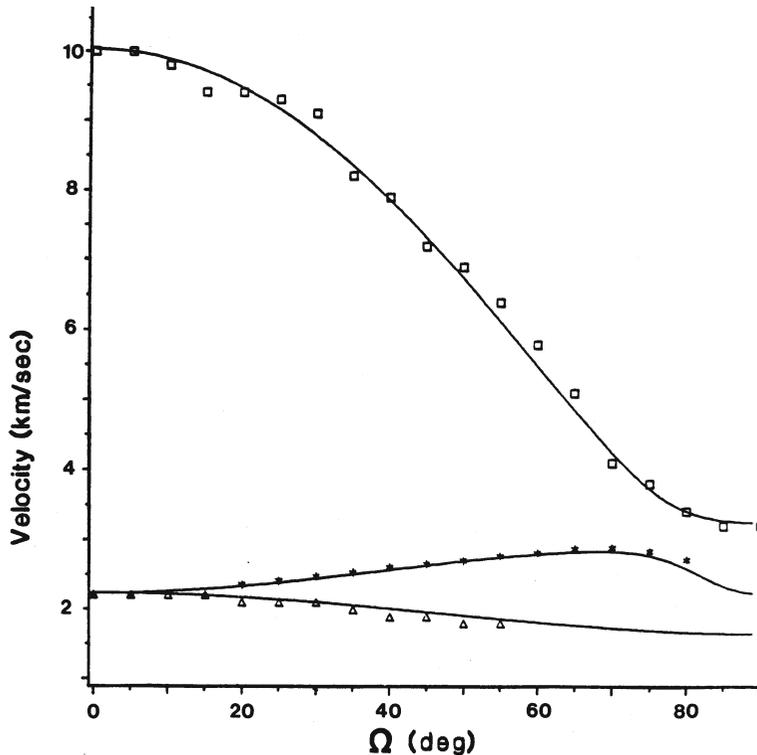


Fig. 6 Experimental velocity data points are obtained from critical angle measurements. Solid lines are best fit to experimental data obtained by nonlinear square optimization technique. The elastic constants reconstructed optimization are listed in Table 1.

#### ACKNOWLEDGEMENT

This work was in part sponsored by the Center for Advanced NDE, operated by the Ames Laboratory, USDOE, for the Air Force Wright Aeronautical Laboratories/Materials Laboratory under contract #SC-88-148B. The assistance of Mr. L. Wang with calculations is also appreciated.

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