REFLECTION OF ULTRASONIC WAVES BY FLAWS IN THICK COMPOSITES

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INTRODUCTION

A layer of a fiber reinforced composite consists of many plies. For wavelengths of the same order of magnitude as the ply thicknesses, but still much larger than the fiber diameters, the ply interfaces will act as surfaces of reflection and transmission. A thick layer of a fiber-reinforced composite may then be expected to display many of the features of a solid with periodic structuring. For through-the-thickness propagation of time-harmonic mechanical disturbances, these features include passing and stopping bands.

The first passing band is at low frequencies. At such frequencies, the specific effects of reflections at closely spaced interfaces are difficult to distinguish. It is then also difficult to distinguish bad from good interfaces, particularly if their reflective properties would not be too different. For that reason it is desirable to probe the composite at higher frequencies.

In this paper we consider a composite as consisting of a stack of identical layers with reflecting interfaces. First the reflection and transmission coefficients of a single interface are examined. The coefficients are analyzed as functions of the frequency for a broad class of interfaces. In the next section the propagation of harmonic waves through an infinite solid with equally spaced interfaces is investigated. Results for a typical example show the appearance of passing and stopping bands in the frequency spectrum. Next propagation through a finite sequence of interfaces is investigated by the use of the propagator matrix formalism. The total reflection coefficient for a sequence of interfaces is calculated for the case that all interfaces are good, and for the case that one of the interfaces is bad. Finally, a layer containing bad and good interfaces is considered. For transfer of ultrasonic wave motion, either buffer rods are thought to be connected at opposite positions on the faces of the layer or the layer is immersed in a water bath. For a single bad interface placed in a sequence of good interfaces, the solution for an inverse problem yields the reflection coefficient of the bad interface as a function of the frequency

REFLECTION AND TRANSMISSION

A plane, conveniently defined by $x = 0$, is assumed to act as a plane of homogeneous reflection and transmission for normal incidence of an ultrasonic wave. The wave fields are

$$u(x) = e^{ikx} + Re^{-ikx} \quad , \quad x \leq 0 \tag{1}$$

$$u(x) = Te^{ikx} \quad , \quad x \geq 0 \tag{2}$$

Here $k = \omega/c$ is the wavenumber, where ω and c are the circular frequency and the wave speed, respectively. Also, R is the reflection coefficient and T is the transmission coefficient. Equations (1) and (2) certainly apply for any kind of homogeneous contact across an interface, but they also are applicable for locally inhomogeneous conditions, such as apply for equally spaced cracks, cavities and inclusions. In the latter case the expressions (1) and (2) are valid at some distance from $x = 0$, and at not too high frequencies. For example, for a distribution of periodically spaced spherical cavities, the reflection and transmission coefficients have been calculated by Kitahara and Achenbach [1], for frequencies below a certain cut-off frequency. Above that cut-off frequency an incident plane wave gives rise to higher-order modes. In a similar manner reflection coefficients for periodically spaced cracks [2] and periodically spaced inclusions [3] have been presented.

By virtue of conservation of rate of energy, the reflection and transmission coefficients are related by

$$|R|^2 + |T|^2 = 1 \tag{3}$$

A single example of a homogeneous plane of reflection and transmission is provided by a spring connection between two half spaces of equal material properties. Such a spring connection can be considered as an approximation to an extremely thin layer of material properties different from those of the adjoining half-spaces. For this case the interface conditions are:

$$\left.\frac{\partial u}{\partial x}\right|_{x=0^+} = \left.\frac{\partial u}{\partial x}\right|_{x=0^-} = \left.\frac{\partial u}{\partial x}\right|_{x=0} \tag{4}$$

$$C\,\frac{\partial u}{\partial x} = K\left(u\Big|_{x=0^+} - u\Big|_{x=0^-}\right) \tag{5}$$

In Eq.(5) C is an elastic constant, namely $(\lambda+2\mu)$ for longitudinal waves and μ for transverse waves. Equation (4) represents continuity of traction, while (5) relates the traction to the displacement discontinuity across the spring connection. Substitution of Eqs.(1) and (2) into (4)-(5) yields

$$R = -\frac{i\,\alpha kh}{1-i\alpha kh} \quad , \quad T = \frac{1}{1-i\alpha kh} \tag{6a,b}$$

where

$$\alpha = C/2Kh \tag{7}$$

It is noted that

$$R + T = 1 \tag{8}$$

It can be shown that Equation (8) has validity for a wider class of interfaces. Then, if we leave the specifics of the interface undefined, except that Eq.(8) is assumed to hold, it follows from (3), which is always valid, and Eq.(8) that R and T may be expressed in terms of a single parameter, χ, as

$$R = \sin\chi \, \exp[i(\chi - \tfrac{1}{2}\pi) \quad , \quad T = \cos\chi\exp(i\chi) \tag{9a,b}$$

Figure 1 shows $|R|$ and $|T|$ as functions of a dimensionless frequency.

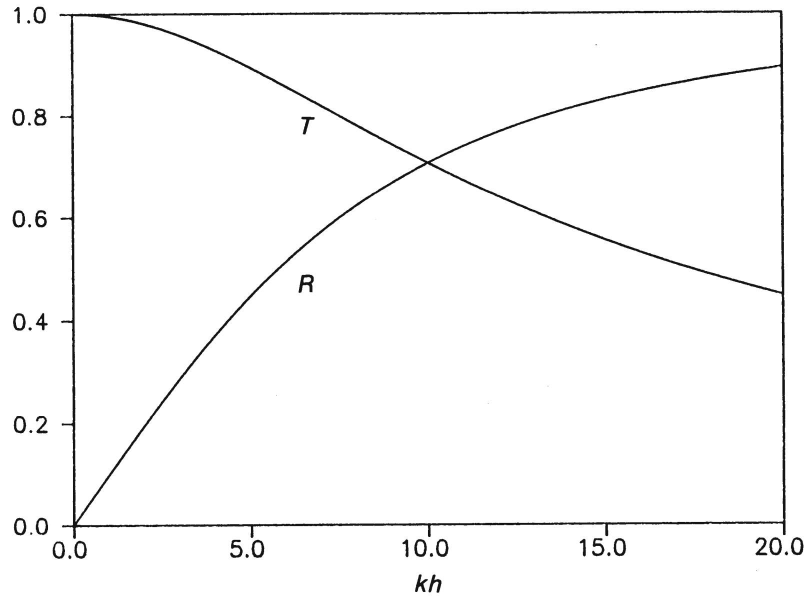

Fig. 1 Reflection and transmission coefficients versus kh according to Eq.(6a,b), for ah = 0.1.

PROPAGATION THROUGH AN INFINITE SOLID WITH EQUALLY SPACED INTERFACES

An infinite solid containing equally spaced interfaces (spacing distance h), with reflection and transmission coefficients R and T, is considered in this Section. A solid of this kind is periodic, and propagation of harmonic waves normal to the interfaces may be expected to display passing and stopping bands. Indeed, if in an unbounded medium the wave motion is considered of the form

$$u(x,t) = u(x)e^{-i\omega t} \tag{10}$$

where

$$u(x) = U(x)e^{iqx} \tag{11}$$

it is not difficult to show, [4], that q and k, where $k = \omega/c$, c being the phase velocity for waves in the bulk material, are related by the following relatively simple dispersion equation

$$\cos(qh) = \frac{1}{2T} [(T^2 - R^2 - 1)e^{ikh} + 2\cos(kh)] \tag{12}$$

The curves for q(kh,R,T), as functions of kh, do indeed show passing bands (regions of real-valued qh) and stopping bands (complex-valued qh). A typical example, using the reflection and transmission coefficients shown in Fig. 1, is displayed in Fig. 2.

PROPAGATION THROUGH A FINITE SEQUENCE OF INTERFACES

Figure 3a shows two interfaces separated by a distance h in an unbounded solid. First we consider the interface 1. Amplitudes of waves to the left of this interface are denoted by A_o' and B_o', while amplitudes to the right are denoted by A_1 and B_1. Let $A_o'\exp(ikx)$ and $B_1\exp(-ikx)$ be considered as incident waves. Then if R and T are the reflection and transmission coefficient, respectively, we have

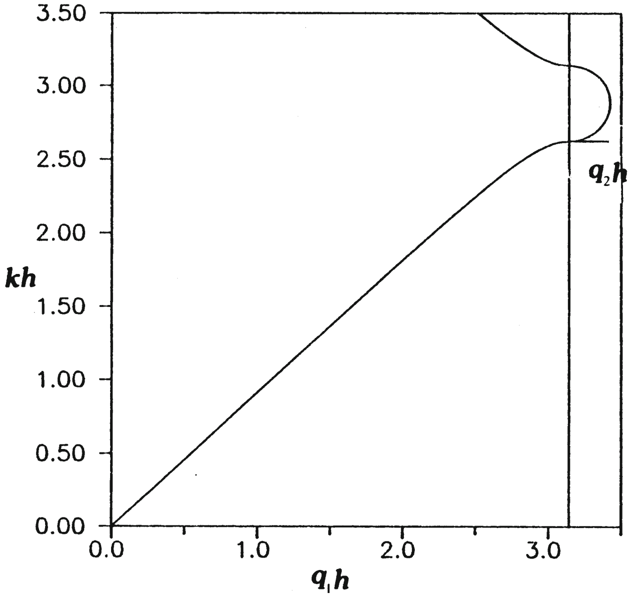

Fig. 2 One passing and one stopping band for a solid with periodically spaced interfaces with reflection coefficients according to Fig. 1.

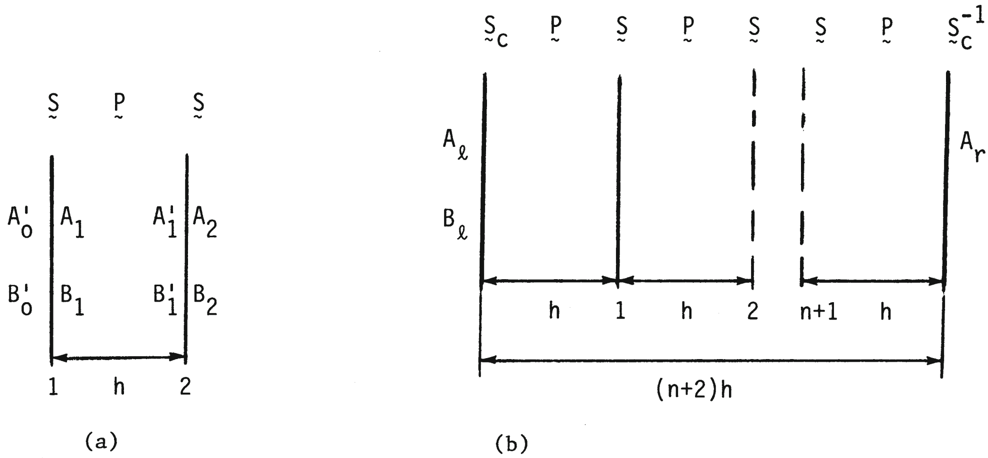

Fig. 3 (a) two interfaces in an unbounded solid; (b) layer containing n+1 interfaces, coupled to adjoining media.

$$B_o' = R\,A_o' + T\,B_1\ , \qquad (13)$$

$$A_1 = T\,A_o' + R\,B_1\ , \qquad (14)$$

This system of equations may be expressed in the form

$$\begin{bmatrix} A_o' \\ B_o' \end{bmatrix} = \underset{\sim}{S} \begin{bmatrix} A_1 \\ B_1 \end{bmatrix}\ , \qquad (15)$$

where $\underset{\sim}{S}$ is the scattering matrix

$$\underset{\sim}{S} = \frac{1}{T} \begin{bmatrix} 1 & -R \\ R & T^2 - R^2 \end{bmatrix} \qquad (16)$$

Next we consider interface 2 in Fig. 3a. The amplitudes to the left and right of this interface are denoted by A_1', B_1', and A_2, B_2, respectively. It is not difficult to show that the following relation holds

$$\begin{pmatrix} A_1 \\ B_1 \end{pmatrix} = \underset{\sim}{P} \begin{pmatrix} A_1' \\ B_1' \end{pmatrix} , \tag{17}$$

where the matrix $\underset{\sim}{P}$ is of the form

$$\underset{\sim}{P} = \begin{pmatrix} e^{-ikh} & 0 \\ 0 & e^{ikh} \end{pmatrix} \tag{18}$$

Analogously to Eq.(17) we have

$$\begin{pmatrix} A_1' \\ B_1' \end{pmatrix} = \underset{\sim}{S} \begin{pmatrix} A_2 \\ B_2 \end{pmatrix} , \tag{19}$$

By combining Eqs.(18) and (19) we can subsequently write

$$\begin{pmatrix} A_o' \\ B_o' \end{pmatrix} = \underset{\sim}{S}\, \underset{\sim}{M} \begin{pmatrix} A_2 \\ B_2 \end{pmatrix} , \tag{20}$$

where the propagator matrix $\underset{\sim}{M}$ is defined as

$$\underset{\sim}{M} = \underset{\sim}{P}\, \underset{\sim}{S} = \frac{1}{T} \begin{pmatrix} e^{-ikh} & -Re^{-ikh} \\ Re^{ikh} & (T^2 - R^2)e^{ikh} \end{pmatrix} \tag{21}$$

A straightforward extension of Eq.(20) to (n+1) interfaces, with wave incidence in the positive x-direction (from the left) only, immediately yields the result

$$\begin{pmatrix} A_o' \\ B_o' \end{pmatrix} = \underset{\sim}{S}\, (\underset{\sim}{M})^n \begin{pmatrix} A_{n+1} \\ 0 \end{pmatrix} , \tag{22}$$

Now suppose that one interface, the (m+1)th interface is a bad interface with reflection coefficient R_b and transmission coefficient T_b. The corresponding propagator matrix is $\underset{\sim}{M}_b$, where $\underset{\sim}{M}_b$ follows from Eq.(21) by replacing R by R_b and T by T_b. Instead of (22) we now have

$$\begin{pmatrix} A_o' \\ B_o' \end{pmatrix} = \underset{\sim}{S}\, (\underset{\sim}{M})^{m-1}\, \underset{\sim}{M}_b (\underset{\sim}{M})^{n-m} \begin{pmatrix} A_{n+1} \\ 0 \end{pmatrix} , \tag{23}$$

Next we consider the case that a layer of the composite containing n + 1 interfaces is coupled to another medium on both sides, see Fig. 3b. Now two additional scattering matrices enter, namely for the coupler-solid interface ($\underset{\sim}{S}_c$) and the solid-coupler interface ($\underset{\sim}{S}_c^{-1}$).

The amplitudes on the insonified and shadow sides are now related by

$$\begin{pmatrix} A_\ell \\ B_\ell \end{pmatrix} = \underset{\sim}{S}_c\, \underset{\sim}{P}\, \underset{\sim}{S}\, (\underset{\sim}{M})^{m-1}\, \bar{\underset{\sim}{M}}_b\, (\underset{\sim}{M})^{n-m}\, \underset{\sim}{P}\, \underset{\sim}{S}_c^{-1} \begin{pmatrix} A_r \\ 0 \end{pmatrix}$$

$$= \underset{\sim}{S}_c\, (\underset{\sim}{M})^{m}\, \bar{\underset{\sim}{M}}_b\, (\underset{\sim}{M})^{n-m}\, \underset{\sim}{P}\, \underset{\sim}{S}_c^{-1} \begin{pmatrix} A_r \\ 0 \end{pmatrix} \tag{24}$$

Equation (24) can be solved for A_r and B_ℓ in terms of A_ℓ (the amplitude of the incident wave). For a very small number of interfaces explicit expressions can be obtained. For a larger number of interfaces the matrix multiplications become unwieldy. In that case the Cayley-Hamilton theorem, [5], can, however, conveniently be employed to obtain higher order powers of a matrix. The first described calculations lead directly to the definition and the

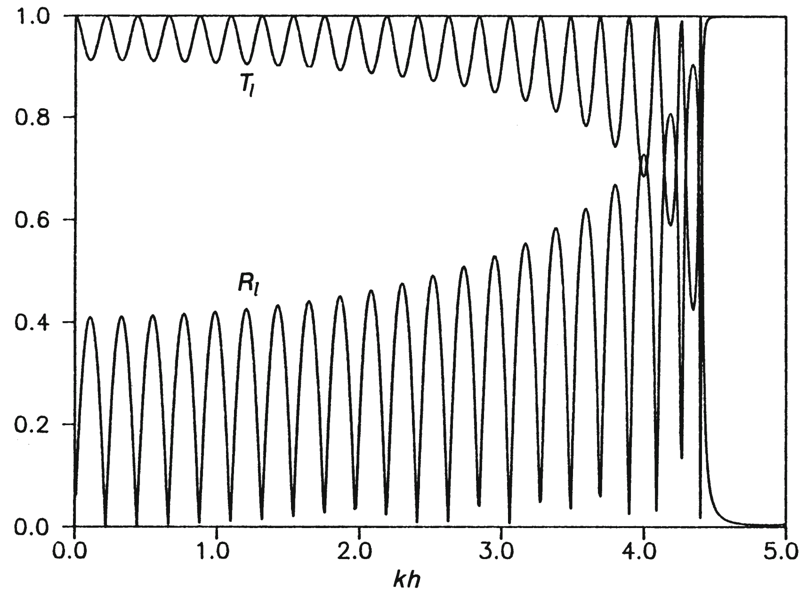

Fig. 4 Reflection and transmission coefficients for a layer containing a sequence of 21 identical interfaces, with R and T according to Fig. 1.

solution of appropriate reflection and transmission coefficients for the composite layer. They are

$$R_\ell = B_\ell / A_\ell , \quad T_\ell = A_r / A_\ell \tag{25a,b}$$

The direct reflection problem is solved by Eqs.(6)-(7). For the case that all interfaces are the same, and of the form shown in Fig. 1, results are displayed in Fig. 4. For the case that one interface is different (R_b and T_b), the curves for $|R_\ell|$ and $|T_\ell|$ are shown in Fig. 5.

INVERSE PROBLEM

Of practical interest is a thick layer of a composite material. For transfer of ultrasonic wave motion, either buffer rods are connected at opposite positions on the faces of the layer for the use of contact transducers, or the layer is immersed in a water bath. A test of this configuration would have the objective of detecting a bad interface among many good interfaces. For example, through the thickness there might be m good ply interfaces, then 1 bad interface, and then again n-m good interfaces, for a total of n+1 interfaces. The reflection and transmission coefficients of the good ply interfaces, which may be frequency dependent, are denoted by $R(\omega)$ and $T(\omega)$. The bad interface, which may contain voids, microcracks, reduced bond strengths, etc., has corresponding coefficients $R_b(\omega)$ and$T_b(\omega)$. The objective of a test would be to detect the bad interface, and to obtain $R_b(\omega)$ and $T_b(\omega)$ from measured data for $R_\ell(\omega)$ and/or $T_\ell(\omega)$. Knowledge of the coefficients $R_b(\omega)$ and $T_b(\omega)$ subsequently provides information on the strength of the bad interface.

The calculation of $R_b(\omega)$ and/or $T_b(\omega)$ from measured reflection and/or transmission data is clearly an inverse problem. A potentially successful attempt to the solution of the inverse problem should be preceded by a solution to the direct problem. The direct problem of calculating the reflection coefficient for an immersed layer with n good interfaces and one bad interface can be solved by the use of the propagator matrix method, as shown in the preceding Section.

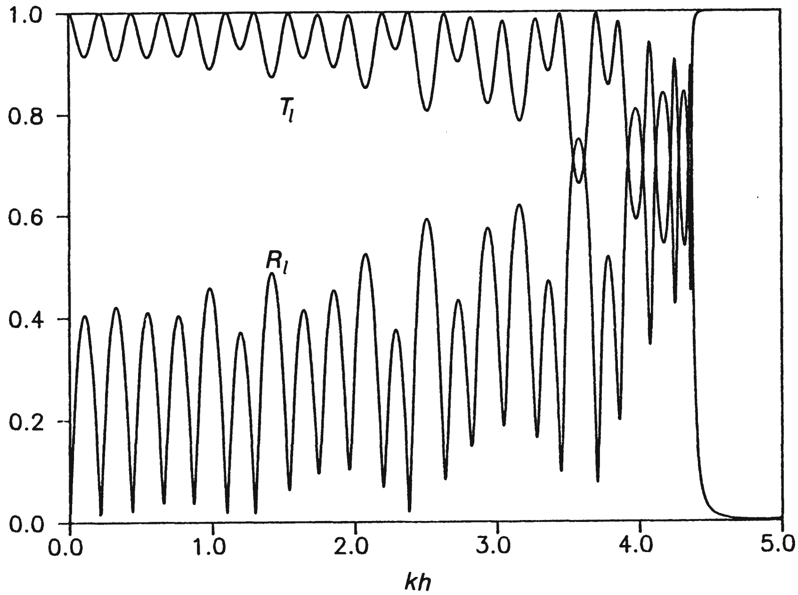

Fig. 5 Reflection and transmission coefficients for a layer containing a sequence of 21 interfaces, where the ninth interface is a bad interface with coefficients R_b and T_b according to Fig. 6.

To solve the inverse problem we use a method which avoids the use of actual absolute values of the reflection coefficient, $R_\ell(\omega)$. Absolute values of reflection coefficients are affected by attenuation and experimental error, and they will therefore not completely agree with theoretical results under the best of circumstances. As a function of frequency a reflection coefficient does, however, display a number of local maximums whose positions are generally not greatly affected by deviations between experimental conditions and theoretical assumptions. The positions of the maximums generally agree quite well with theoretical results, and hence they will be used as input for the inverse problem. The inverse method will proceed as follows. Suppose R_ℓ has maximums at $kh = (kh)_i$. Near each $(kh)_i$ two neighboring points are determined by the relations

$$(kh)_{i+} = (kh)_i + [(kh)_{i+1} - (kh)_i]\gamma \qquad (26a)$$

$$(kh)_{i-} = (kh)_i - [(kh)_i - (kh)_{i-1}]\gamma \quad , \qquad (26b)$$

where $0 < \gamma < 0.5$. Next a trial and error method is initiated, which takes advantage of the fact that m(the number of the bad interface) can only be one of a finite number of integers (n+1, the total number of interfaces), while α_b is a real-valued quantity in a bounded interval. An integer value is selected for m (m=1, m=2, --- m = n/2 for n even and m = (n+1)/2 for n odd), and values of α_b are considered with small increments in the region of interest. For each value of α_b, the reflection coefficient is computed at the three points $(kh)_i$, $(kh)_{i+}$ and $(kh)_{i-}$. For those values of α_b at which the R_ℓ is larger at $(kh)_i$ than at the neighboring points, a maximum is possible at $(kh)_i$, and this value of α_b is registered. For each m there will be a list of α_b values computed at the $(kh)_i$ that is being considered. Next the procedure is repeated for $(kh)_{i+1}$, etc. The actual value of m and the actual value of α_b are the ones that the lists for different $(kh)_i$ have in common. This procedure has been tested by the use of synthetic data provided by Fig. 5. The dots in Fig. 6, which are the results of the inverse problem show that correct result has been obtained.

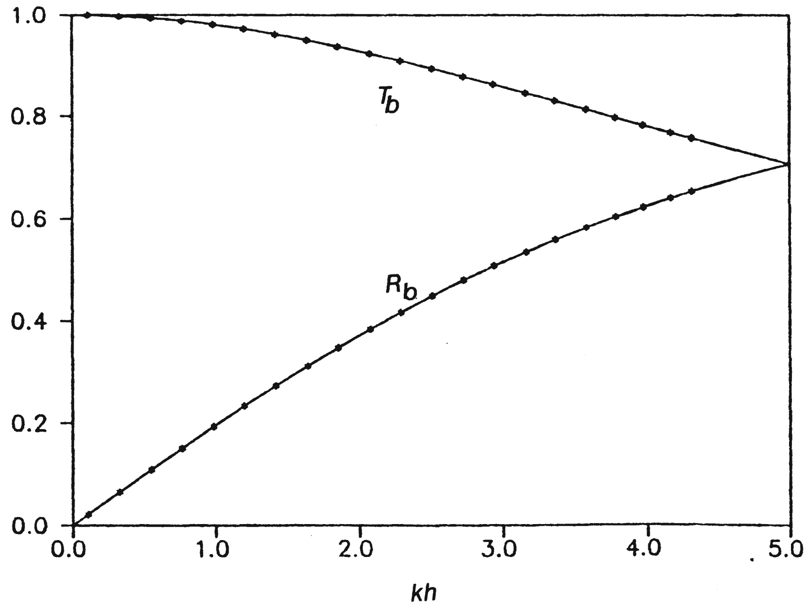

Fig. 6 Results (dots) for the inverse problem of determining R_b and T_b from the position of the maximum values of R_ℓ and T_ℓ shown in Fig. 5.

The inversion procedure discussed above would simplify considerably if all the good interfaces would be essentially perfect, i.e., $R \sim 0$ and $T \sim 1$. Then only the bad interface would reflect, and the problem of characterizing $R_b(\omega)$ will be easily solvable. This case essentially corresponds to the use of the effective modulus theory for the undamaged composite material. When the thick composite contains several bad interfaces, this may be the only practical way to proceed.

In an actual testing procedure, the method of characterizing bad interfaces quantitatively, might be combined with an imaging procedure. The imaging procedure would provide a visual indication of damage, and the method discussed above could then be used to provide quantitative information on the extent of the damage and the degree to which cohesive properties across an interface have been reduced.

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