THEORETICAL STUDY OF HIGH FREQUENCY ULTRASONIC WAVE ATTENUATION IN POLYCRYSTALLINE MATERIALS

S. I. Rokhlin, T. K. Bolland, and L. Adler
Department of Welding Engineering
The Ohio State University
190 West 19th Avenue
Columbus, OH 43210

INTRODUCTION AND PROBLEM STATEMENT

Three different regimes for scattering of ultrasonic waves in polycrystalline materials exist, depending on the ratio of the mean grain size to the wavelength: (i) the low frequency (Rayleigh) region with scattering-induced attenuation proportional to the fourth power of the frequency and to the cube of the mean grain diameter, (ii) the medium frequency (stochastic) region with scattering proportional to the square of the frequency and to the mean grain diameter, and (iii) the high-frequency (geometric) region with scattering independent of frequency.

Ultrasonic wave scattering in the Rayleigh and stochastic regions has been studied intensively [1-5] in the past. Major contributions to the theory have been made by Lifshits and Parkhomovski [2] whose approach was used in later studies. More recently Stanke and Kino [6] generalized the results of Lifshits and Parkhomovski using the perturbation theory of Karal and Keller [7] developed for analysis of stochastic wave propagation in random media.

Previous theories [2,4] are not valid in the high-frequency range and ultrasonic wave propagation in the geometric region is much less understood. However, it was predicted by Mason and McSkimin [8] and verified experimentally by Merkulov [9] that in the geometric region scattering is independent of frequency and inversely proportional to the grain size, since the scattering is proportional to the number of grain boundaries along the acoustic path.

While the method of Stanke and Kino [6] is formally valid for high frequency, it has limited application in the geometric scattering region since the plane-wave condition is not satisfied in the region and, therefore, the perturbation method cannot be used. This is explained by the non-collinearity of elastic rays in different grains due to the random orientation of the grains. The wave propagation may be considered to be in a particular direction only in an average sense. Stanke and Kino's theory confirms the independence on frequency and the inverse proportionality to grain size of attenuation due to scattering in the geometric region. But the coefficient of this proportionality and its dependence on the anisotropy factor is unknown. A better understanding of this phenomenon is important, especially for improving ultrasonic testing methods of

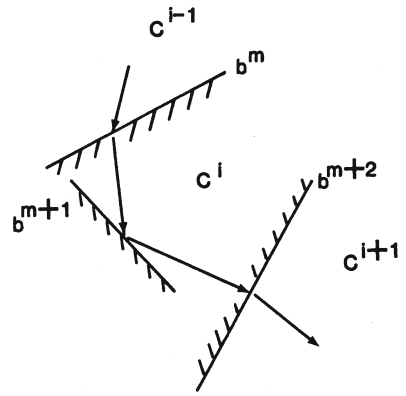
austenitic steels and nickel based alloys having very large grains.

In this paper we formulate a new approach for studying this problem by using ray tracing simulation in polycrystalline media formed by randomly oriented anisotropic grains. By finding the statistical characteristics for propagation through grain boundaries as functions of the anisotropy factor of the material, it is possible to characterize the problem generally. The model includes the texture characteristics of the material. While the method and algorithm are valid and the computer programs are written for general grain anisotropy, the results are demonstrated for cubic anisotropy of grains.

THEORETICAL APPROACH

We consider ultrasonic wave propagation in a polycrystalline material in the frame of the geometric acoustic approximation, $\lambda << \bar{D}$, where λ is an ultrasonic wavelength for either quasi-longitudinal or quasi-transverse waves in the given crystal and $\bar{D}$ is an average grain diameter. We consider polycrystalline materials both with and without texture. Untextured material has randomly oriented elastic grains, while the grains in textured material have a preferred orientation in a particular range of angles.

The grain boundary is also taken to be oriented randomly in three-dimensional space forming grains with average diameter $\bar{D}$. This is illustrated in Fig. 1 where c^{i-1}, c^{i}, and c^{i+1} are three neighboring randomly oriented grains; b^{m}, b^{m+1}, and b^{m+2} are the corresponding randomly oriented boundaries. Interfaces between grains are considered plane, as shown in the figure; therefore the ray interaction with the grain boundary is locally approximated as an elastic plane wave interaction with an arbitrary plane interface between two anisotropic media.



b^{m} - boundary oriented randomly in 3-D space

C_i - crystal oriented randomly in 3-D space

Fig. 1. Schematic explanation of ray tracing in polycrystalline media.

To treat elastic wave propagation through an interface arbitrarily oriented between two generally anisotropic media, we used a unified algorithm which we developed previously [10,11]. The ray tracing through the grains is organized as follows: The top surface of the medium forms a plane-grained surface with random orientation of each grain. The ultrasonic wave is incident on the top sample surface (for example from water) at a preselected angle. Next the transmitted wave is calculated in the first grain. The wave with the maximum energy-flow transmission coefficient is selected for further tracing. The angle of refraction and the energy transmission coefficients are calculated for this wave. This wave is now selected to be incident on the next grain boundary which is positioned at a distance equal to the average grain size. The Euler angles of the orientation of the next boundary relative to the global coordinate system are randomly generated. After its orientation is selected, the incident plane, which includes the incident ray and the normal to the grain boundary, is determined. The next step is the random generation of the three orientation angles of the crystallographic system of the neighboring grain relative to the global coordinate system.

The following step involves calculation of the amplitudes and angles of these reflected and transmitted waves. It may happen that the angle of incidence is close to the critical angle and, therefore, one of the reflected waves is stronger than the transmitted wave; in this case the reflected wave is traced further, as shown in Fig. 1 for boundary b^{m+1}. If one of the transmitted waves has a high energy transmission coefficient, it is selected for further tracing as shown for grain boundary b^{m} and b^{m+2}.

In our model we assume that the grain diameter is greater than a wavelength but significantly less than the diameter of the ultrasonic beam. Since the ultrasonic beam crosses different grains, it separates into different groups of rays (passing through different grains) which are not collinear with each other. Since the wave vector and ray direction change their orientation from grain to grain, strictly speaking, all of the energy of the ultrasonic wave is scattered away from the original beam direction even in the first layer of grains of the sample. But we can consider an average flow of elastic energy in a particular direction, for example from the transmitting to the receiving transducers. This is analogous to turbulent flow in a liquid, where each particle of liquid is random movement while an average flow may exist in some preferred direction. Of course, different rays will arrive at the receiving transducer mainly incoherently, and, therefore, a phase-insensitive transducer should preferably be selected as a receiver.

From this point of view, special care should be taken in the definition of the ultrasonic wave attenuation, which in this case is associated with transmission losses.

Let us consider as transmission loss of energy for a given ray when it propagates from grain to grain:

$$\prod_{m=1}^{N} T_m = \bar{T}^N(A) \qquad (1)$$

where T_m is the energy transmission (reflection) coefficient through crystalline boundary m; N is the number of crystalline boundaries through which the ray propagates; $\bar{T}(A)$ is an average transmission coefficient for a crystal of a given crystal anisotropy A. We can introduce an effective average acoustic path length by

$$L^{eff} = N\bar{D} \quad (2)$$

where $\bar{D}$ is an average grain size. Now introducing formally an average attenuation factor α for the ultrasonic ray on the acoustic path we can write

$$\bar{T}(L^{eff}/\bar{D}) = e^{-2\alpha L^{eff}} \quad (3)$$

Here absorption of ultrasonic energy in the crystal itself is neglected. By taking the logarithm of both sides of Eq. 3 we can see that the effective length L^{eff} cancels and we can write the average attenuation coefficient of this ultrasonic ray

$$\alpha = \ln[\bar{T}(A)]/2\bar{D} \quad (4)$$

DEPENDENCE OF ATTENUATION COEFFICIENT ON MATERIAL ANISOTROPY

The average attenuation factor given by Eq. 4 may serve as a useful material characteristic if it can be described in terms of material properties. This can be done if the parameters of the statistical distribution for transmission coefficient T_m through the grain boundary (m = 1, 2, 3, ...) can be determined.

Our calculations show that the value of T_m is not distributed normally for randomly oriented grain boundaries. But, according to the Central Limit Theorem, the mean of m identically distributed independent random variables will be distributed normally regardless of the distributions of the individual vairables. Using the fact that for several consecutive transmissions, the transmission coefficients multiply and the tangents of the deviation angles add algebraically, after several grains the logarithm of the overall transmission coefficients and the overall deviation tangents will be distributed normally. Normalized to a single boundary, the distributions of these values therefore will be characterized by only two numbers: the mean value and the variance. These values can be found as functions of the anisotropy coefficient. In this way the attenuation coefficient is found as a function of the anisotropy factor.

The parameters of the normal distribution for the transmission coefficient (average and standard deviation) were found as functions of the anisotropy factor for cubic crystals. The computer simulation algorithm includes:

1. Space averaging of the transmission coefficient for several grains since the diameter of the ultrasonic beam is larger than the grain size. At this step, wave propagation through the grain is simulated with random selection of the grain boundary. If the transmitted energy is less than the reflected energy, the energy reflection coefficient is taken instead of the transmission coefficient for averaging.

2. Calculation of the energy transmission coefficient through m (30 - 40) grains, taking as the transmission coefficient at each grain boundary the averaged value for the layer of grains found in Step 1.

3. Finding the distribution of the logarithms of the transmission factors obtained in Step 2.

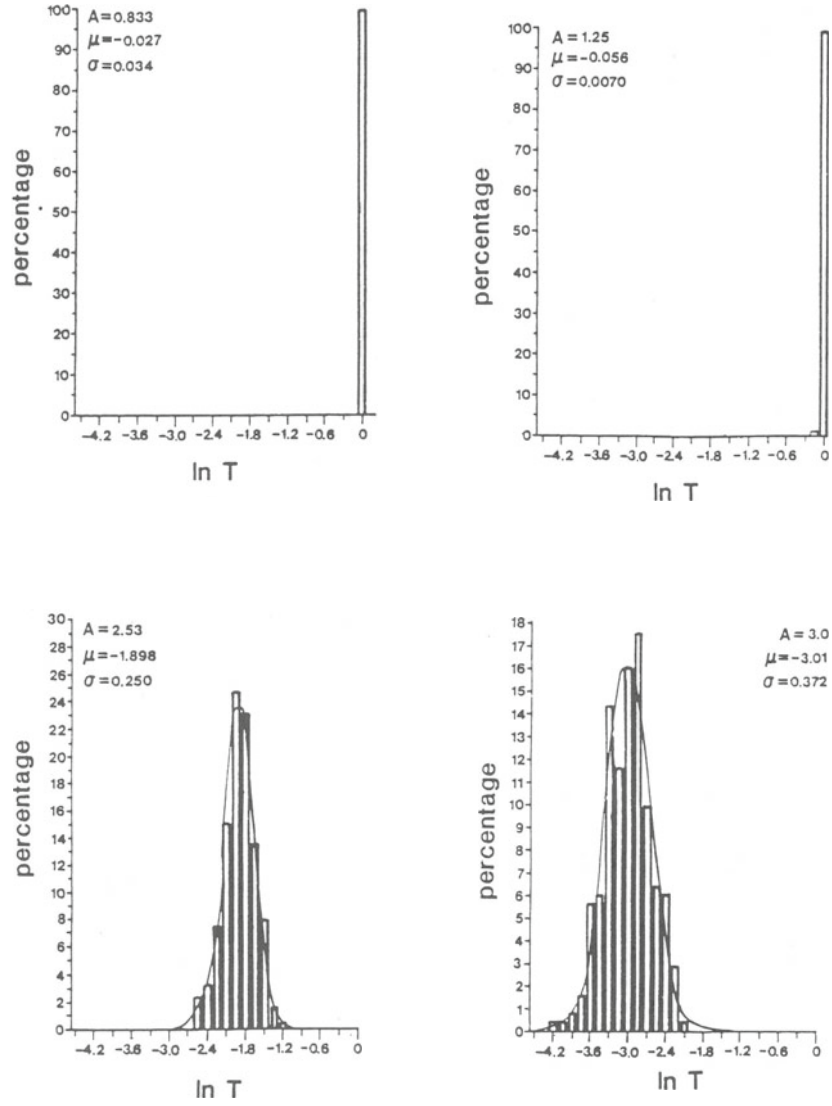


Fig. 2. Simulated distributions of transmission coefficients for different anisotropy factors. Mean values μ and variances σ are given on each figure.

4. By dividing the values found in Step 3 by the number of layers, the average transmission factor $\overline{\ln T}$ and the standard deviation σ for our grain boundary may be found for each value of the anisotropy factor.

Therefore, as can be seen from Eq. 4, the average attenuation factor for an ultrasonic ray will depend on the anisotropy factor and the average grain size. The elastic properties of a cubic material depend on three independent constants, one of which can be taken as the anisotropy factor $A = 2C_{44}/(C_{11} - C_{12})$, where C_{11}, C_{12}, and C_{44} are the elastic constants of the crystal. However, the scattering from the grain boundaries depends mainly on A, so we have chosen A as a parameter and fixed C_{11} and C_{12} to have the values found in nickel.

As an example, the distributions of the transmission coefficient found as discussed above are shown in Fig. 2, with the same horizontal scale for different anisotropy factors. Solid lines give theoretical normal distributions. Note the large changes of the width of the distribution and the shifts of the mean value.

The dependence of the attenuation coefficient times average grain size as a function of the anisotropy factor is shown in Fig. 3. The data are presented for two cases: (i) random orientation of grains with randomly oriented grain boundaries, and (ii) approximation of grains by a layered structure where the crystallographic axis of each layer is randomly oriented while the boundaries of the layers are parallel to each other. In the second case the wave is incident normally on the interface and, therefore, the wave preserves the character of a plane wave, while the ray vectors, which deviate from the wave normal, change their directions when the wave passes through the interfaces. Small anisotropy attenuation goes as the square of the anisotropy factor for layered structures (case 2) and forms a cusp when plotted against the anisotropy factor, for randomly oriented grain boundaries (case 1). The nature of this cusp formation is currently not clear and additional calculation is required to clarify this phenomenon.

CONCLUSION

A new approach for studying high frequency ultrasonic wave propagation in polycrystalline materials is introduced. Elastic wave propagation through crystalline grains is treated exactly, in the framework of geometrical acoustics. The treatment is based on a theory previously developed by the authors. The problem is addressed generally by giving the dependence of the attenuation coefficient on the crystalline anisotropy using the statistical characteristics of transmitted and scattered waves.

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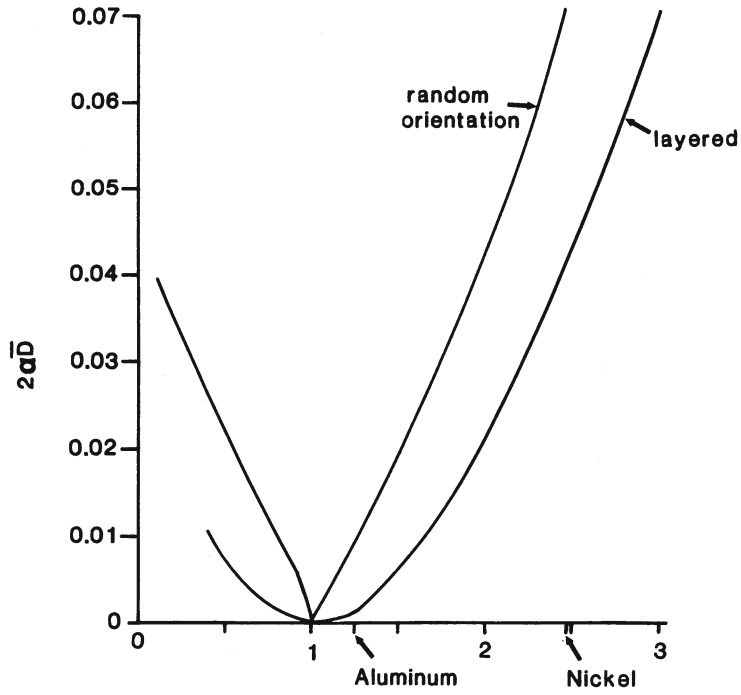

Fig. 3. Theoretical dependence of ultrasonic attenuation coefficient versus anisotropy factor.

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