REFLECTION BY DISTRIBUTED MICROFLAWS IN A DIFFUSION BOND

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INTRODUCTION

Ultrasonic methods have shown considerable promise for the evaluation of solid state welds. The interaction of an ultrasonic signal with the geometrical and material configuration in the bond plane produces a reflected signal which can, in principle, be used to separate a good from a poor bond. The details of such an evaluation would be greatly assisted by a physical model.

In this paper we consider a specific class of poor bonds, namely, bonds that are permeated with small cracks. A simple expression for the reflection coefficient, which has been derived in Ref.[1], is generalized to the case of transverse (shear) waves. The expression is valid for arbitrary distributions of microcracks in a bond plane. The reflection coefficient is expressed in terms of the number of cracks per unit area, and the average crack-opening volume. Considerable simplifications are achieved when the cracks are all approximately penny-shaped and do not interact with each other, and when the frequency is low. A generalization for the case of a statistical distribution of crack radii is also presented. Finally, a connection has been established between stress-intensity factors for a service loading condition (in tension), and a zero frequency limit (for shear waves) of the reflected ultrasonic signal.

The work in this paper is closely related to that of Refs.[1] and [2]. Reflection by spherical cavities, periodically distributed in a plane, has been studied by Kitahara and Achenbach [3].

REFLECTION COEFFICIENT

It is well known that at large distances from a crack, the scattered field generated by an incident wave is proportional to the crack-opening volume. When a plane ultrasonic wave is incident on a planar distribution of cracks, the scattered fields generated by the cracks superimpose, to generate reflected and transmitted plane waves. It has been shown by Sotiropoulos and Achenbach [1,2] that the reflection coefficient is proportional to the number of cracks per unit area and the average crack opening volume.

Let us consider a homogeneous, isotropic, linearly elastic solid which contains a plane with a large number of cracks. The plane is defined by $z = 0$, and the cracks are of arbitrary shape. Figure 1

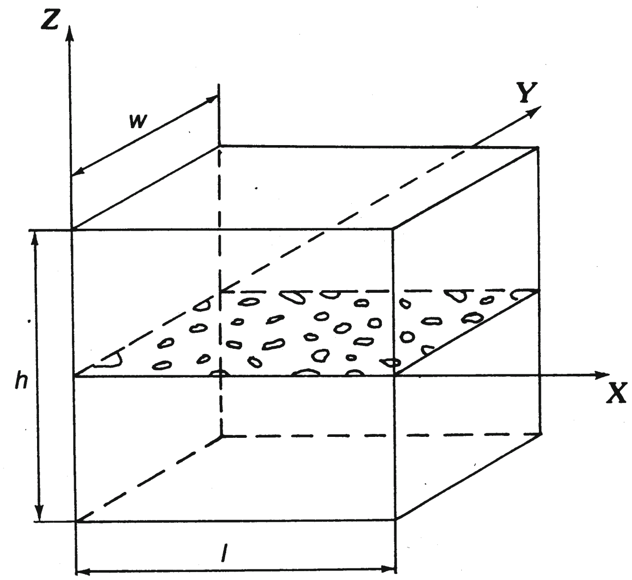

Fig.1 Distribution of cracks in the plane z = 0.

shows a cell of length ℓ, width w, and height h, that contains N cracks in the plane z = 0. A plane time-harmonic wave is incident on the cracked plane. The incident wave is a transverse (or shear) wave of the form $\underset{\sim}{u}^I(\underset{\sim}{x})\exp(-i\omega t)$, where

$$\underset{\sim}{u}^I(\underset{\sim}{x},t) = u_o\underset{\sim}{d}\exp(ik_T z) \tag{1}$$

and

$$\underset{\sim}{d} = (1,0,0). \tag{2}$$

$$k_T = \omega/c_T \quad , \quad c_T^2 = \mu/\rho \tag{3a,b}$$

The amplitude, u_o, and the angular frequency, ω, characterize the incident plane wave, while the shear modulus, μ, and the density, ρ, characterize the elastic solid. The multiplicative factor $\exp(-i\omega t)$, common to all field variables, will be omitted in the sequel. The incident field generates only shear stresses, σ_{zx}, in the plane z = 0. It is then concluded that the corresponding crack-opening displacements are also in the x-direction only.

Far from the plane of the cracks, and at sufficiently low frequency, the dominant parts of the displacement fields will be transverse waves only, and they may be written as

$$z < 0 \quad : \quad u = u_o\exp(ik_T z) - u_o R^T\exp(-ik_T z) \tag{4}$$

$$z > 0 \quad : \quad u = u_o T^T\exp(ik_T z) \tag{5}$$

In (4)-(5), R^T denotes the reflection coefficient for transverse wave incidence while T^T denotes the corresponding transmission coefficient. At higher frequencies we generally have mode conversion, but not at the frequencies which are being considered here.

As shown by Sotiropoulos and Achenbach [1,2] the reflection coefficient may be written as

$$R^T = \left[\frac{1}{2u_o}\right]\left[\frac{N}{\ell w}\right]\left[\frac{1}{N}\sum_{n=1}^{N} V_n\right] = \frac{N^* V^*}{2u_o} \tag{6}$$

where V_n is the crack opening volume of the n-th crack

$$V_n = \int_{A_n} \Delta^n u(\zeta_1,\zeta_2)\,dA_\zeta \tag{7}$$

In (7), A_n is the area of the n-th crack, ζ_1 and ζ_2 are local coordinates on the crack surface and

$$\Delta^n u(\zeta_1,\zeta_2) = u^n(\zeta_1,\zeta_2,0^+) - u^n(\zeta_1,\zeta_2,0^-) \ , \tag{8}$$

is the crack-opening displacement. The second expression in Eq.(6) gives the reflection coefficient for a transverse wave as the product of an amplitude factor, the number of cracks per unit area,

$$N^* = N/\ell w \quad , \tag{9}$$

and the average crack opening volume,

$$V^* = \frac{1}{N}\sum_{n=1}^{N} V_n \quad . \tag{10}$$

The corresponding expression for the transmission coefficient, T^T, is

$$T^T = 1 + R^T \tag{11}$$

in which R^T is given by (6).

Equation (6) can be rewritten in still another form by introducing

$$N^* = \frac{1}{a^* b^*} \tag{12}$$

where a^* and b^* are the average distances between crack centers in the x- and y- directions, respectively. Equation (6) then becomes

$$R^T = \frac{1}{2u_o}\,\frac{V^*}{a^* b^*} \quad , \tag{13}$$

Simple expressions are obtained when the cracks are of equal size and form a doubly periodic array. Suppose a and b are the distances between the geometrical centers of consecutive cracks in the x and x directions, respectively. Then

$$R^T = \frac{V}{2u_o ab} \tag{14}$$

where $V_n = V$ is the same for all cracks,

$$V = \int_A \Delta u(\zeta_1,\zeta_2)\,dA_\zeta \tag{15}$$

DISTRIBUTION OF MICROCRACKS

For a distribution of cracks in a diffusion bond plane, the cracks are generally of very small size (of the order of tens of microns), and it may be assumed that $k_T c$ (where c is the crack radius) and $k_T a$ (where a is a typical distance between crack centers) are much smaller than unity. It is then admissible to introduce two simplifications. The first one is that interactions between the cracks may be neglected, and the second one is that the crack-opening volumes may be represented by their quasi-static equivalents. Consequently the only information needed is the crack-opening volume for a crack in the xy-plane which is subjected to shear tractions

$$\sigma_{zx} = ik_T \mu u_o \quad . \tag{16}$$

The crack opening displacement Δu of a penny-shaped crack of radius c can be found in Ref.[4]. It is

$$\Delta u = \frac{8(1-\nu)}{\pi(2-\nu)} ik_T c \left(1 - \frac{x^2+y^2}{c^2}\right)^{\frac{1}{2}} u_o \tag{17}$$

The corresponding crack-opening volume follows by integration as

$$V^T = \frac{16(1-\nu)}{3(2-\nu)} ik_T c^3 u_o \tag{18}$$

If all the cracks are of the same radius, and when there is no interaction, all crack-opening volumes are the same, and the reflection coefficient follows from Eq.(13) as

$$R^T = \frac{8(1-\nu)}{3(2-\nu)} \frac{c^2}{a^* b^*} ik_T c \tag{19}$$

The discussion up to this point has been for deterministic crack-distributions. A statistical crack-distribution of penny-shaped cracks will now be considered. The radii $\tilde{c}_n$ of the cracks, and their separation distances $\tilde{a}_n$ and $\tilde{b}_n$ are random variables, indicated by a tilde. The probability densities of $\tilde{a}_n$, $\tilde{b}_n$ and $\tilde{c}_n$ are such that there is no interaction between cracks. For normal incidence of a transverse wave the cracks' opening volumes then follow from (18) as

$$\tilde{V}_n^T = \frac{16}{3} \frac{1-\nu}{2-\nu} ik_T \tilde{c}_n^3 u_o \tag{20}$$

Let us now assume that the $\tilde{c}_n$'s, which are independent random variables, are identically distributed, i.e., have identical probability density functions. Then $\tilde{c}_n$ can be replaced by $\tilde{c}$ in Eq.(20).

Substitution of the result in Eq.(13) yields

$$\tilde{R}^T = \frac{8i}{3} \frac{1-\nu}{2-\nu} \frac{\tilde{c}^2}{\tilde{a}\tilde{b}} k_T \tilde{c} \tag{21}$$

The expectation of $\tilde{R}^T$ may then be approximated by

$$<\tilde{R}^T> = \frac{8i}{3} \frac{1-\nu}{2-\nu} < \tilde{c}^2/\tilde{a}\tilde{b} > <k_T \tilde{c}> \tag{22}$$

We now compare the deterministic reflection coefficient for a doubly periodic array of equal penny-shaped cracks, with the reflection coefficient, Eq.(22), for a statistical distribution of cracks. For the statistical distribution, we consider the case that the ratios of crack radii and crack spacings are uniformly distributed over the range $0.1 \le \tilde{c}/\tilde{a}$, $\tilde{c}/\tilde{b} \le 0.3.$, i.e., their probability density functions are constants over that range. For the deterministic problem we take $c/a = c/b = 0.2$. The result may be shown versus $k_T <\tilde{c}>$. Since

$$<\tilde{c}^2/\tilde{a}\tilde{b}> = \frac{1}{0.08} \int_{0.01}^{0.09} \eta d\eta = 0.05 \quad , \tag{23}$$

it immediately follows that

$$\frac{R^T}{<\tilde{R}^T>} = \frac{0.04}{0.05} = 0.8 \tag{24}$$

Hence the statistical distribution has a slightly higher reflection coefficient.

STRESS INTENSITY FACTORS

A measurement of the reflection coefficient provides information that can be used, probably together with other a-priori information, to estimate the dimensions of the cracks. In the next step these estimates have to be used to calculate the strength of the crack-permeated bond. This might be done by a calculation of the maximum stress intensity factor. In this section the intermediate step of crack-site estimation is omitted, and stress-intensity factors are directly estimated from reflection data.

Let us consider a flat penny-shaped crack in the xy-plane, and let a plane transverse wave of the special form

$$u(z,t) = \delta(t-z/c_T) \tag{25}$$

be incident on the crack. Here $\delta(\cdot)$ is the Dirac delta function, and c_T is the speed of transverse waves. Let the corresponding crack-opening volume be denoted by $V^{T\delta}(t)$, where $V^{T\delta}(t)$ is defined analogously to Eq.(15). Now if the incident wave is of the more general form

$$u(z,t) = f(t-z/c_T)H(t-z/c_T) \quad , \tag{26}$$

it follows immediately by linear superposition that the corresponding crack-opening volume may be expressed as

$$V^T(t) = \int_o^t f(t-s)V^{T\delta}(s)ds \tag{27}$$

The asymptotic form of $V^T(t)$ as time increases, depends on the stress field corresponding to Eq.(26). If the stress component $\sigma_{zx}(z,t)$, which corresponds to Eq.(26), approaches a finite limit, i.e., as

$$\lim_{t\to\infty} (-\mu/c_T)f'(t-z/c_T) = \tau_o \quad , \tag{28}$$

then $V^T(t)$ also approaches a finite limit, which just equals the static crack-opening volume induced by the static shear stress τ_o.

Equation (27) is a convolution integral. It is well known that the Fourier transform (over time) of (27), which is indicated by a bar, is of the form

$$\bar{V}^T(\omega) = \bar{f}(\omega)\bar{V}^{T\delta}(\omega) \ . \tag{29}$$

It is also known that the long-time value of a quantity is related to the value at small ω of its Fourier transform. It can be shown that

$$(V^T)_{st} = \lim_{\omega\to o} i\omega\bar{V}^T(\omega) = \lim_{\omega\to o} i\omega\bar{f}(\omega)\bar{V}^{T\delta}(\omega) \tag{30}$$

Now, if it would be possible to obtain $V^{T\delta}(\omega)$ from an ultrasonic test, then $(V^T)_{st}$ could be obtained directly from Eq.(30).

The Fourier transform of the plane transverse wave which is incident on the crack in an ultrasonic test may be written as

$$\bar{u}(z,\omega) = \bar{A}(\omega)e^{ik_Tz} \tag{31}$$

By comparing the Fourier transform of Eq.(25) with (31), it follows that the crack-opening volume corresponding to Eq.(31) may be written as

$$\bar{V}(\omega) = \bar{A}(\omega)\bar{V}^{T\delta}(\omega) \tag{32}$$

Hence the static crack opening volume may be related to the crack-opening volume of the ultrasonic test by the relation

$$(V^T)_{st} = \lim_{\omega\to o} i\omega \frac{\bar{f}(\omega)}{\bar{A}(\omega)} \bar{V}(\omega) \tag{33}$$

The crack-opening volume may subsequently be eliminated in favor of the reflection coefficient. For a distribution of equal sized cracks we obtain by using Eq.(6) for an incident wave of the form (31):

$$(V^T)_{st} = \frac{2}{N^*} \lim_{\omega\to o} i\omega \ \bar{f}(\omega) \ R^T(\omega) \tag{34}$$

Under most practical service conditions, a diffusion bond will be statically loaded in tension. Let us consider a distant tension stress σ_o. For sufficiently wide-spaced penny-shaped cracks, the relevant static crack opening displacement may then be written as

$$V^L_{st} = \frac{8}{3}\frac{1-\nu}{\mu}\sigma_o c^3 \tag{35}$$

It is also known that [5]

$$K_I = \frac{2}{\pi}\sigma_o(\pi c)^{1/2} \tag{36}$$

By eliminating the crack radius, c, from Eqs.(35) and (36) we find

$$\frac{K_I}{\sigma_o} = \left[\frac{24\mu}{(1-\nu)\pi^3}\frac{V^L_{st}}{\sigma_o}\right]^{1/6} \tag{37}$$

The term V^L_{st}/σ_o is the crack opening volume per unit distant tensile stress. It is of interest to compare V^L_{st}/σ_o, which follows from Eq.(35), with the corresponding expression for a distant shear stress τ_o. The latter follows from Eq.(18) as

$$\frac{V^T_{st}}{\tau_o} = \frac{16(1-\nu)}{3(2-\nu)\mu} \; c^3 \qquad (38)$$

Evidently we have the following relation

$$V^L_{st}/\sigma_o = \frac{1}{2}(2-\nu)\,|V^T_{st}/\tau_o| \qquad (39)$$

By combining Eqs.(37), (39) and (34) it is then concluded that

$$\frac{K_I}{\sigma_o} = \left[\frac{12(2-\nu)}{(1-\nu)\pi^3} \frac{2c_T}{N^*} \lim_{\omega\to o} \frac{R^T(\omega)}{i\omega}\right]^{1/6} , \qquad (40)$$

where we have also used the relation

$$\tau_o = - \frac{\mu}{c_T} \lim_{\omega\to o} (i\omega)^2 \;\bar{f}(\omega) \qquad (41)$$

A measurement of the reflection coefficient and an estimate of the number of cracks per unit area, N^*, will then provide an estimate of the static Mode-I stress-intensity factor. It has been shown in Ref.[2] that errors in N^* as high as 50% give rise to errors of less than 10% in K_I/σ_o.

By interpreting the results for K_I/σ_o, it should be kept in mind that the allowable K_I for short cracks is generally much smaller than that of large cracks, the latter being the fracture toughness of the material. It is also conceivable that the allowable K_I for cracks in a diffusion bond is quite different from the fracture toughness of the bonded material(s).

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