ULTRASONIC REFLECTIVITY OF DIFFUSION BONDS

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INTRODUCTION

Solid state welds that bond a piece of metal to a piece of the same metal are increasingly important in industrial applications. Consequently new inspection methods that use ultrasound are being developed based on two characteristics of the weld. One inspection method uses the bond-line's specular reflectivity to indicate microcracking or porosity [1-4]. A second method measures the attenuation (or velocity shift) to infer changes in the near-bond microstructure of the metal [4,5]. To date both methods give useful but primarily qualitative information on bond integrity.

This report of "work in progress" is focused on a quantitative method to infer the average defect size and the area fraction of defects from the frequency-dependent reflectivity of the bond plane. The report is split into two unequal parts. First a simple model is given for approximating the reflectivity in terms of the scattering amplitudes of the individual defects at the bond-plane. Second, the reflectivity model is then related to the bond-plane parameters. It is briefly shown how the defect size and area fraction can be determined once the broadband reflectivity is known.

Baik and Thompson [6] and Sotiropolous and Achenbach [3,7] have also developed theories of bond-plane reflectivity. Baik and Thompson developed an approximation suitable to low frequencies (the quasi-static approximation). This approximation is suitable for any area fraction of flaws and any angle of incidence. However it is only capable of giving limited information on the needed bond-plane parameters. In particular, as shown by Baik and Thompson, low frequency data is not sufficient to determine area fraction and defect size. Sotiropolous and Achenbach have proposed formally exact expressions (for microcracks) that are valid for all frequencies and all area fractions. Unfortunately the numerical evaluation of these expressions for randomly distributed defects appears to be intractable at present. Sotiropolous and Achenbach have also proposed a series of approximations that are based on their exact expressions. These approximations require one to know the crack opening displacement. This quantity is difficult to evaluate and is not generally available.

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Our calculation, on the other hand, relates the reflectivity to the average far-field scattering amplitude, a quantity that is readily available for many types of defects. Finally our expression is valid at all frequencies for a sufficiently dilute distribution of bond-line defects and for angles not too near grazing incidence.

FREQUENCY DEPENDENT REFLECTIVITY

The sample geometry is shown in Fig. (1). The sample is assumed to consist of two half-spaces of an otherwise homogeneous and isotropic material that has been imperfectly bonded at the plane labeled "BOND" in the figure.

Our basic assumption is that the specular reflection of the incident beam occurs due to isolated microdefects that are randomly distributed over the bond-plane. Typical microdefects are disbonds (cracks), inclusions or porosity. The reflectivity induced by a collection of such defects is modeled by using elastic wave scattering theory and by assuming that each microdefect scatters independently of the others. The assumption of independent scatterers is crucial to our analysis; it allows us to approximate the frequency-dependent reflected wave in terms of the scattering amplitude of single, completely isolated defects. Consequently it is simple to generate the (specular) reflection coefficient for a broad variety of experimental set-ups and defect distributions.
DEFINITION OF SPECULAR COEFFICIENTS

The specular transmission and reflection coefficients are defined as follows. Let a plane wave displacement field be incident on the bond-plane from the left side. This plane wave interacts with the defects causing them to radiate ultrasound. Part of the incident plane wave is specularly transmitted, part is specularly reflected. The rest scatters diffusely in all directions.

The specular transmission and reflection coefficients are actually averaged quantities, which we now define. We start by considering a longitudinally polarized, incident plane wave propagating in the direction \( \mathbf{e} \).

\[
\mathbf{U}^{\mathbf{IN}}(k, \mathbf{e}, \mathbf{r}) = U^\circ \exp(i k \mathbf{e} \cdot \mathbf{r}) \mathbf{e}.
\]  

(1)

Here \( k \) denotes the wavevector for propagation of ultrasound in the material, \( \mathbf{e} \) is a unit vector and \( \mathbf{r} \) denotes the spatial coordinate. This incident field gives rise to scattered waves. By convention \( z > 0 \) is to the right of the bond-plane, while \( z < 0 \) is to the left. The specular transmitted field is defined as the field averaged over planes perpendicular to \( \mathbf{e} \) and for \( z > 0 \). This averaged field, \( \langle \mathbf{u} \rangle \), then behaves as

\[
\langle \mathbf{u}(k, \mathbf{e}, \mathbf{r}) \rangle = T(k, \mathbf{e}) \exp(i k \mathbf{e} \cdot \mathbf{r}) \mathbf{e}.
\]  

(2)

Here the transmission coefficient \( T(k, \mathbf{e}) \) depends on the wavevector and direction of incidence. The specular reflection coefficient is defined analogously. In this case we consider waves reflected in the direction \( \mathbf{e}' \) defined by Snell's law. The scattered field is averaged over planes perpendicular to \( \mathbf{e}' \) for \( z < 0 \). The resulting averaged field can be written as

\[
\langle \mathbf{u}(k, \mathbf{e}, \mathbf{r}) \rangle = \mathbf{u}^{\mathbf{IN}}(k, \mathbf{e}, \mathbf{r}) + R(k, \mathbf{e}) \exp(i k \mathbf{e} \cdot \mathbf{r}) \mathbf{e}'.
\]  

(3)

The reflection coefficient \( R(k, \mathbf{e}) \) depends on the wavevector and the direction of incidence.

NORMAL INCIDENCE REFLECTION COEFFICIENT

The general strategy for computing specular reflection and transmission coefficients was schematized above. Here details are provided for calculating the normal incidence reflection coefficient, \( R(k, \mathbf{z}) \). We start by stating our basic assumptions: (1) Each defect scatters the incident wave independently; i.e. each scatters the incident field as if no other defect existed; (2) the far-field scattering amplitude adequately characterizes the scattering from each flaw; and (3) \( k|z|>>1 \), where \( |z| \) is the distance from the point of observation to the interface.

The geometry needed to carry out our calculation is shown in Fig. (2). The defects at the interface are labeled \( J-1, J, J+1, \ldots \). We start by computing the average field of an individual defect over the plane \( S \), which is parallel to and a distance \( |z| \) from the interface. The total average field is then obtained by summing the contributions of all the defects. Upon explicitly computing the sum one obtains \( R(k, \mathbf{z}) \) from Eq. (3).

The field scattered from the \( J^{th} \) defect is, according to our assumptions,

\[
\mathbf{u}_J(k, \mathbf{z}, \rho) = u^\circ \exp(i k z) \hat{z} + u^\circ A_J(k, f_J, \mathbf{z}) \sqrt[p^2 + z^2] \rho \, f_J.
\]  

(4)

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Here the first term on the right-hand-side of Eq. (4) represents the incident field, while the scattered field is represented by the second term. Also \( \mathbf{r} = (\rho, z) \) denotes a point on the plane \( S \). The \( j \)th defect is assumed to be centered about the origin of coordinates, and \( \mathbf{r}_j \) is a unit vector pointing from the \( j \)th defect to the observation point \( \mathbf{r} \). Finally \( A_j(k, \mathbf{r}_j, z) \) is the far-field longitudinal scattering amplitude for the \( j \)th defect. \( A_j \) is computed for an isolated defect in an otherwise uniform and isotropic elastic solid. The second argument of \( A_j \), \( \mathbf{r}_j \), denotes the direction of scattering, while the third argument, \( z \), denotes the direction of incidence.

\[
\text{Fig. 2. Shows the geometry for calculating the reflection coefficient. The point } \mathbf{r} = (z, \rho) \text{ lies on the plane labeled } "S" \text{ over which the scattered field is averaged.}
\]

The scattered field of the \( j \)th scatterer, averaged over the plane \( S \), is computed as follows. First we note that the scattered field is defined by

\[
\mathbf{u}_{j}^{sc} = \mathbf{u}_j - \mathbf{u}^\text{IN}.
\]

We plug Eq. (4) into Eq. (5) and average \( \mathbf{u}_{j}^{sc} \) over the plane \( S \) to obtain

\[
<\mathbf{u}_{j}^{sc}> = \frac{1}{\text{area}} \int d^2 \rho \left\{ u^a A_j(k, \mathbf{r}_j, z) \frac{a^k \sqrt{\rho^2 + z^2}}{\sqrt{\rho^2 + z^2}} \mathbf{r}_j \right\}.
\]

The integral is over the area of the plane \( S \) and is normalized by that area as indicated symbolically by the prefactor in Eq. (6). At first sight the integral in Eq. (6) appears to be quite formidable since the scattering amplitude \( A_j \) depends on \( \rho \) through \( \mathbf{r}_j \). However this difficulty is removed by our far-field assumption that \( k |z| \gg 1 \). This assumption allows us to evaluate Eq. (6) asymptotically for large \( kz \) to obtain
\[ <u_{sc}^{\infty}> = \frac{u^0}{\text{area}} \frac{2\pi}{ik} \exp(-ikz) A_j(k,-\hat{z},\hat{z}). \] 

The reflection coefficient is now obtained as follows. First we calculate the total averaged scattered field by summing Eq. (7) over all \( J \) to obtain

\[ <u_{tor}^{\infty}> = u^0 \frac{2\pi}{ik} \exp(-ikz) \frac{N}{\text{area}} \left[ \frac{1}{N} \sum_{j=1}^{N} A_j(k,-\hat{z},\hat{z}) \right](-\hat{z}). \]

Here \( N \) denotes the total number of defects. The ratio \( n = N/\text{area} \) denotes the number of defects per unit area. Comparing Eqs. (2) and (8) yields

\[ R(k,\hat{z}) = 2\pi n(ik)^{-1} \bar{A}(k,-\hat{z},\hat{z}). \]

Here \( \bar{A} \) is the average scattering amplitude of the defects and is defined by

\[ \bar{A}(k,-\hat{z},\hat{z}) = N^{-1} \sum_{j=1}^{N} A_j(k,-\hat{z},\hat{z}). \]

The formula for the reflection coefficient given in Eq. (9) is one of the major results of this report. When examined in detail for certain defect distribution Eq. (9) allows one to predict the average defect size as well as the area fraction of defects. Before turning to these results we note that Eq. (9) can be simplified and its physical content more clearly delineated if we assume (1) all the defects are the same (for example, all in-plane penny shaped cracks) and (2) they all have the same size. Upon noting that the scattering amplitude can be written in the following scaled form

\[ A(k,-\hat{z},\hat{z}) = \alpha \Phi(k\alpha,-\hat{z},\hat{z}). \]

we find

\[ R(k,\hat{z}) = b \Pi(k\alpha,-\hat{z},\hat{z}). \]

Here \( b \) is the area fraction of defects, \( n\alpha^2 \), and

\[ \Pi(k\alpha,-\hat{z},\hat{z}) = 2 \Phi(k\alpha,-\hat{z},\hat{z})/(ik\alpha). \]

Note that \( R \) now depends only on the product of the area fraction and the dimensionless (or reduced) reflectivity \( \Pi(k\alpha,-\hat{z},\hat{z}) \).

RESULTS

The frequency-dependent reduced reflectivity is presented for two types of interface defects: penny-shaped cracks and spherical pores. In each case we assume that the defects each have the same radius and that the elastic host material is characterized by a longitudinal-to-shear velocity ratio of 2. Figure (3A) shows \( |\Pi(k\alpha,-\hat{z},\hat{z})| \) for penny shaped cracks that lie in the plane of the interface. Key characteristics to note are: the linear behavior of \( |\Pi| \) for small \( k\alpha \),
Fig. 3. Shows the reduced reflectivity $|\Pi|$ as a function of $ka$ for (a) bond-plane microcracks and (b) bond-plane porosity.
the peak in $\pi$ for $ka \approx 0.9$ and the plateau in $|\pi|$ for large values of $ka$. The plot of $|\pi|$ versus $ka$ for spherical pores is shown in Fig. (3B). The low and intermediate features are similar to those of the penny-shaped crack. That is, there is a linear region for small $ka$ and a peak at approximately $ka \approx 0.7$. However the high frequency asymptotics differ, with $|\pi|$ for the pores falling off as $(ka)^{-1}$ for large $ka$.

AREA FRACTION AND DEFECT SIZE

The strategy for computing the defect size and area fraction are straightforward given the results shown in Figs. 3A and 3B. First an estimate for the average radius of microcracks is obtained by noting the frequency, $f_{\text{max}}$ at which the peak in the reflectivity occurs,

$$\alpha = 0.9(2\pi v f_{\text{max}}). \quad (14)$$

Here $v$ is the velocity of longitudinal sound. Once the radius has been estimated, Eq. (12) can be used to determine the area fraction. Crudely we can say that the radius depends on the characteristic frequency of the maximum, while the area fraction is measured by the overall strength of the reflected signal.

SUMMARY

A simple method has been proposed for computing the reflection and transmission coefficients of a defective diffusion bond. As an application the reflection coefficient for a normally incident plane wave was computed for bond-plane microcracking and bond-plane porosity. These results were shown imply a simple way to infer defect area fraction and average defect size.

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REFERENCES

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