QUANTIFYING THE BENEFIT OF REDUNDANT FLUORESCENT PENETRANT INSPECTION

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INTRODUCTION

Redundant fluorescent penetrant inspection (RFPI) is the practice of performing multiple inspections on a single part. The philosophy behind multiple inspections is to increase the probability of detecting a flaw which may exist. In reality, the multiple inspections may not increase inspection capability as much as expected because the inspections may be performed by the same operator or multiple operators, the procedure may be reset before performing the next inspection, and the part may be cleaned.

Historically, calculations expressing the benefits of redundant fluorescent penetrant inspection have been made assuming complete independence between inspections. For example, if the probability of detecting (POD) a flaw of a certain size is 0.9 then the probability of a single miss (POM) is 0.1, the probability of two (independent) misses is 0.1 * 0.1 = 0.01, and so the POD for two inspections is 1 - (0.01) = 0.99, assuming independence.

Unfortunately, fluorescent penetrant inspection has been found to be not independent inspection-to-inspection. Events which cause this dependency include inspection of the same crack twice (location, size, etc. correlation between inspections), or the same operator may inspect the crack two times, or the surface of the part, and the crack itself, may not be restored to its initial state between inspections.

Assume Correlation between Inspections

The probability of detection using redundant FPI, POD(A or B), and the difference between single and double inspections assuming inspection-to-inspection dependency is presented pictorially in Figures 1-7. For simplicity, assume multiple inspections of one flaw size, a*, and two inspectors, A and B. POD(A or B) is found by combining the probability that A will detect a* (see Figure 1) with the probability that B will detect a* (see Figure 2) resulting in the probability as shown in Figure 3 as POD(A) + POD(B). The POD(A) + POD(B) is however an overestimate of the POD(A or B), since the area of intersection, the redundancy, POD(A and B), has been added in twice, once for A and once for B. To eliminate this problem, the intersection needs to be subtracted out once. Upon subtraction the correct expression of POD for redundant FPI is:

$$\text{POD(A or B)} = \text{POD(A)} + \text{POD(B)} - \text{POD(A and B)}$$

This calculation is expressed in Figure 4.

Now the "difference" between single and double inspections can be considered in the following three cases for flaw size a*. If A would have done the single inspection, then the increase in POD due to RFPI is that of inspector B, or:

$$\begin{aligned}\text{Increase} &= \text{POD(A or B)} - \text{POD(A)} \\ &= [\text{POD(A)} + \text{POD(B)} - \text{POD(A and B)}] - \text{POD(A)} \\ &= \text{POD(B)} - \text{POD(A and B)}\end{aligned}$$

this is shown by the shaded area in Figure 5.

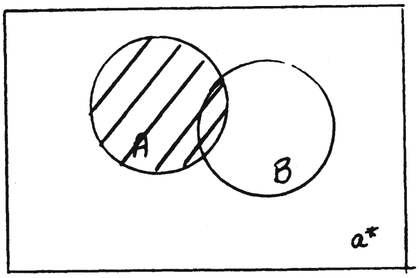

Figure 1. POD(A)

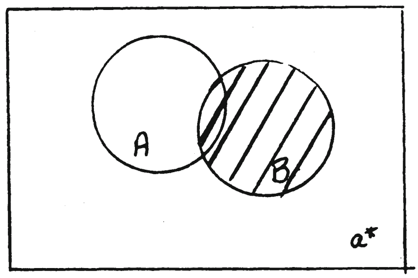

Figure 2. POD(B)

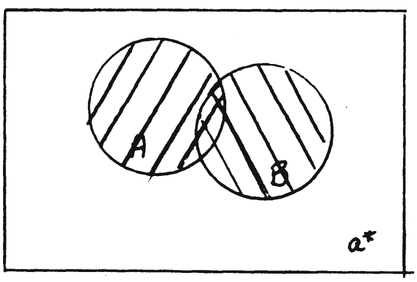

Figure 3. POD(A) + POD(B)

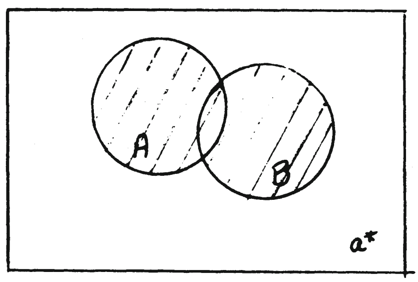

Figure 4. POD(A) + POD(B) - POD(A and B)

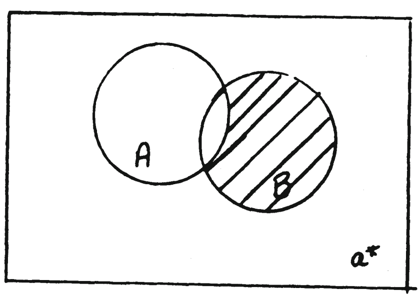

Figure 5. Difference due to RFPI
Case 1 - A Inspects Single FPI

Similarly the difference due to RFPI POD if inspector B would have done single FPI is the shaded area in Figure 6, this difference can be written as:

Increase = POD(A or B) - POD(B)
= [POD(A) + POD(B) - POD(A and B)] - POD(B)
= POD(A) - POD(A and B)

Usually, A and B would take an equal share in inspecting for a flaw, so that in the long run, A may inspect 50% of the time and B would inspect 50%. Under these conditions, the difference between single and redundant FPI is:

Increase = POD(A) + POD(B) -POD(A and B) -0.5POD(A) - 0.5POD(B)
= 0.5*POD(A) + 0.5*POD(B) - POD(A and B)

This difference is shown in Figure 7.

The preceding argument can be extended to RFPI with more than double inspections, or with more than two operators, or with the division of labor being other than 50/50.

Modeling Probability of Detection

FPI data are dichotomous: either a defect is found or it isn't. This type of data can be described by a binomial probability density function. The POD a flaw of size "a" for a range of crack sizes is determined using a statistical procedure called maximum likelihood estimation. Maximum likelihood estimation finds the most likely estimates of the parameters of the binomial distribution, given the data that have been observed.

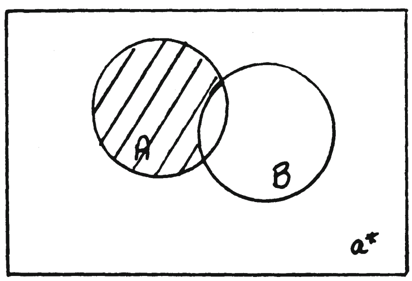

Figure 6. Difference due to RFPI
Case 2 - B Inspects Single FPI

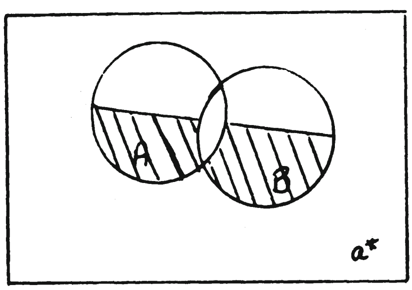

Figure 7. Difference due to RFPI
Case 3 - A Inspects 50%
Case 3 - B Inspects 50%

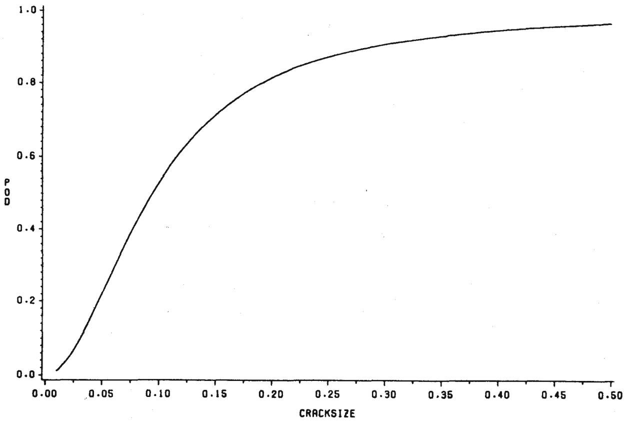

Figure 8. POD vs.a

It has been seen that there is a relationship between the probability of detection and flawsize - as the flaw size increases, POD is expected to increase as shown in Figure 8.

This pattern can be described by the log-logistic model:

$$POD(a_i) = p_i = \exp(\alpha + \beta \times \ln(a_i))/(1 + \exp(\alpha + \beta \times \ln(a_i))) \qquad (1)$$

where:

a_i = flaw size

α, β = constants to be estimated from the data

Given the data that are observed, the MLE of p is found by maximizing L to find the best values of alpha and beta above, or, maximize

$$L((\alpha,\beta),a_i) = \prod_{i=1}^{n} (p_i)^{(x_i)}(1-p_i)^{(1-x_i)} = \prod_{i=1}^{n} \left(\frac{\exp(\alpha + \beta \times \ln a_i)}{(1+\exp(\alpha + \beta \times \ln a_i))}\right)^{(x_i)} \left(\frac{1}{(1+\exp(\alpha + \beta \times \ln a_i))}\right)^{(1-x_i)} \qquad (2)$$

where:

x_i = 1 if i^{th} crack of size a_i is detected

= 0 if not detected

This is accomplished by forming the log likelihood and setting the first partial derivatives of lnL with respect to alpha and beta equal to 0 and solving for alpha and beta.

Example

The following is an analysis of the data in the tables in the appendix. The data of Table 1 resulted from a group of specimens inspected in the same FPI processing situation by both inspectors A and B. From this data, the "difference" due to RFPI can be expressed as:

Increase = 0.5*POD(A) + 0.5*POD(B) - POD(A and B)

where:

POD(A and B) = POD(A)*POD(B, given A found the crack) (3)

or, equivalently:

= POD(B)*POD(A, given B found the crack) (4)

The POD(A, given B found the crack) can be estimated by taking only those cracks found by B and modeling the hit/miss status of those cracks inspected by A. Similarly POD(B given A) is estimated from the hit/miss data for B considering only those cracks found by A. Data for these two situations is found in Tables 2 and 3.

Curves expressing the above calculations were used to estimate the RFPI curves of Figures 9 and 10. If the estimates of the model parameters could be determined exactly, then the two RFPI curves would be equal. Visual inspection of Figures 9 and 10 shows that although the difference vs. cracksize relationship is approximately the same, these two curves are not equal. How well the POD vs. a curve fits the data is a function of how accurately our sample of inspection data represents the population of total inspection data (how many data points are available and how well they cover the cracksize spectrum) and how accurately the model fits the data.

There is an alternative to the calculations of Equations (3) and (4) given above which is computationally easier and a more reliable estimate of POD(A or B). The POD(A or B) curve is fit as before to data which consists of cracks which are considered found if either A or B find the crack.

Similarly, cracks are considered missed if both A and B miss the crack. These data can be found in Table 4 with the corresponding POD(A or B) curve shown in Figure 11. Assuming 50/50 inspection for A and B, the benefit of redundant FPI can be calculated as:

Increase = POD(A or B) - 0.5*POD(A) - 0.5*POD(B)

and is shown in Figure 12.

A statistical goodness of fit test can be performed to assess how well the data are explained by the model. This test shows whether the maximum likelihood estimates of alpha and beta accurately describe the data, or if departures in the actual data values cause alpha and beta to be not well defined.

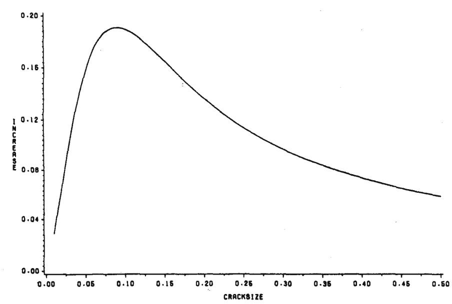

Figure 9. Increase in POD vs. Cracksize for Inspector B Given Inspector A Initially Found the Crack

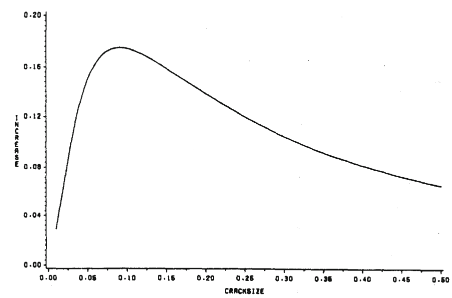

Figure 10. Increase in POD vs. Cracksize for Inspector A Given Inspector B Initially Found the Crack

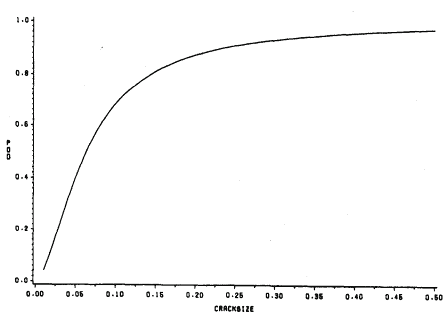

Figure 11. POD(A or B) vs. Cracksize Combined Data

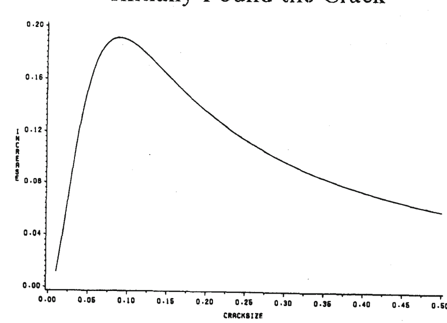

Figure 12. Increase in POD vs. Cracksize Combined Data

Appendix

Table 1

Crack	A	B
0.035	1	0
0.040	0	0
0.045	0	1
0.060	0	0
0.060	0	0
0.075	1	0
0.085	0	0
0.085	0	1
0.100	0	0
0.100	0	0
0.100	1	1
0.120	0	1
0.125	1	1
0.140	1	1
0.145	1	1
0.150	1	1
0.170	0	0
0.180	1	1
0.220	1	1
0.225	1	1
0.240	0	0
0.240	1	1
0.250	0	1
0.280	1	1
0.300	1	1
0.310	1	1
0.375	0	1
0.390	1	1
0.390	1	1
0.400	1	1
0.400	1	1
0.400	1	1
0.400	1	1
0.400	1	1
0.400	1	1
0.400	1	1
0.400	1	0
0.400	0	1
0.400	1	1
0.400	0	1
1.000	1	1

Table 2

Crack	A	B
0.045	0	1
0.085	0	1
0.100	1	1
0.120	0	1
0.125	1	1
0.140	1	1
0.145	1	1
0.150	1	1
0.180	1	1
0.220	1	1
0.225	1	1
0.240	1	1
0.250	0	1
0.280	1	1
0.300	1	1
0.310	1	1
0.375	0	1
0.390	1	1
0.390	1	1
0.400	1	1
0.400	1	1
0.400	1	1
0.400	1	1
0.400	1	1
0.400	1	1
0.400	1	1
0.400	0	1
0.400	1	1
0.400	0	1
1.000	1	1

Table 3

Crack	A	B
0.035	1	0
0.075	1	0
0.100	1	1
0.125	1	1
0.140	1	1
0.145	1	1
0.150	1	1
0.180	1	1
0.220	1	1
0.225	1	1
0.240	1	1
0.280	1	1
0.300	1	1
0.310	1	1
0.390	1	1
0.390	1	1
0.400	1	1
0.400	1	1
0.400	1	1
0.400	1	1
0.400	1	1
0.400	1	1
0.400	1	1
0.400	1	0
0.400	1	1
1.000	1	1

Table 4

Crack	A	B	AB
0.035	1	0	1
0.040	0	0	0
0.045	0	1	1
0.060	0	0	0
0.060	0	0	0
0.075	1	0	1
0.085	0	0	0
0.085	0	1	1
0.100	0	0	0
0.100	0	0	0
0.100	1	1	1
0.120	0	1	1
0.125	1	1	1
0.140	1	1	1
0.145	1	1	1
0.150	1	1	1
0.170	0	0	0
0.180	1	1	1
0.220	1	1	1
0.225	1	1	1
0.240	0	0	0
0.240	1	1	1
0.250	0	1	1
0.280	1	1	1
0.300	1	1	1
0.310	1	1	1
0.375	0	1	1
0.390	1	1	1
0.390	1	1	1
0.400	1	1	1
0.400	1	1	1
0.400	1	1	1
0.400	1	1	1
0.400	1	1	1
0.400	1	1	1
0.400	1	0	1
0.400	0	1	1
0.400	1	1	1
0.400	0	1	1
1.000	1	1	1

The following numbers are probability values, p*, describe the goodness of fit.

POD(A)	p* = 0.0063
POD(B)	p* = 0.0002
POD(A given B)	p* = 0.1179
POD(B given A)	p* = 0.0218
POD(A or B)	p* = 0.0022

The closer p* is to 0, the better the model is. This means that the curve fit of POD(A given B) is not as good as the other three curve's estimates. Usually a criterion of $p^* > 0.05$ or $p^* > 0.10$ is used to reject the hypothesis that the model explains the data well.

Figures 13 and 14 show a comparison between the RFPI curves calculated using the three methods described previously. The curve showing POD(B)*POD(A given B) (Figure 18) compares well with the combined method whereas a difference between POD(A)*POD(B given A) and the combined method exist (Figure 14).

Lack of Fit or Convergence Failure

The value of p* may become too large at times indicating that the adequacy of fit of the model is lacking. Lack of fit of a model indicates that even though estimates of alpha and beta can be obtained by maximum likelihood estimation, there are departures from the model such that the model doesn't describe the data well. At other times, the data may be unable to be fit; no estimate of alpha and beta can be found to satisfy Equation 1.

If either of these situations occur, it is more likely that they will occur in the case of POD(A given B) or POD(B given A). This lack of fit occurs due to the sensitivity of a model built with fewer data points, with increased lack of fit leading to convergence failure due to increased influence of spurious occurences which disrupt the model. When the data cause the model to suffer from lack of fit or cause the MLE to fail to converge, the range of cracksizes can be considered to be discrete instead of continuous, and an "n/N" (n = number found, N = total existing) analysis of POD can be performed to achieve a crude quantification of the difference between single and double fluorescent penetrant inspection.

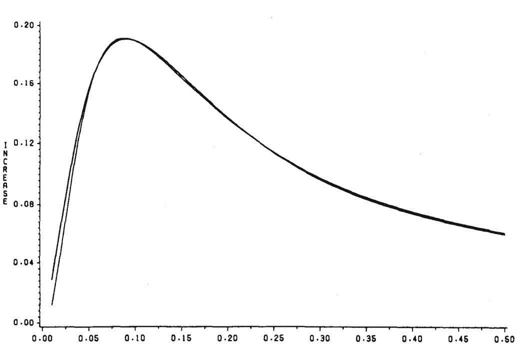

Figure 13. Increase in POD vs. Cracksize Combined Data vs. B Initially Finding Crack

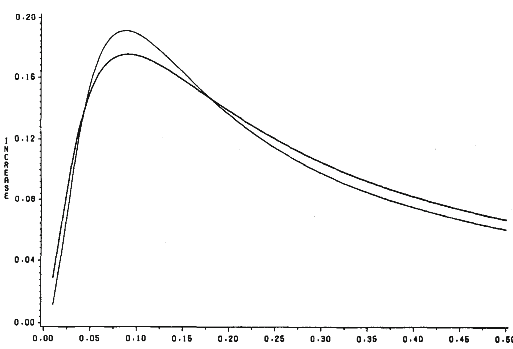

Figure 14. Increase in POD vs. Cracksize Combined Data vs. A Initially Finding Crack

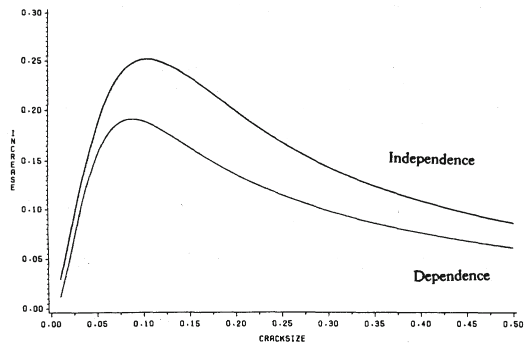

Figure 15. Increase in POD vs. Cracksize Independence vs. Dependence

Difference Assuming Independence

If independence between inspections could have been assumed, then the POD of redundant inspection would have been:

POD(A or B) = 1 - (1 - POD(A))*(1 - POD(B))

and the increase due to redundant FPI can be calculated as:

Increase = POD(A or B) - 0.5*POD(A) - 0.5*POD(B)

Figure 15 gives a visual comparison between the increase due to redundant FPI assuming independence and assuming correlation. The curve assuming independence over-estimates the benefit of redundant FPI.

Redundant FPI Assuming Differences in Inspector Capabilities

The above calculations apply more logically to the case where the skill levels of both inspectors A and B are the same. If differences occur, either between inspectors or within a single inspector, they must be corrected before any real benefit due to redundant FPI is considered. The reason for this correction is that part of the process of detection and elimination of known sources of variation is the elimination of a flaw such as the large dependence on operator ability from the system. The number of times a single part needs to be inspected is directly related to the variance in the system; that is, the more inspector to inspector differences exist, or the more the system is sensitive to these differences, the larger the number of needed inspections.

SUMMARY

The fluorescent penetrant inspection process is not independent inspection-to-inspection and therefore the probability of detection for redundant FPI cannot be obtained by a simple multiplication of probabilities. The correct POD in a multiple inspection system is determined assuming dependence. From this dependent POD, the benefit of redundant FPI is determined by comparing this POD to a single FPI POD and calculating a difference. The difference curve is an estimate of this benefit, and the goodness of this estimate is determined by a goodness of fit test to assess model fit.

REFERENCES

1. Cox, D.R. (1970), Analysis of Binary Data, Spottiswoode, Ballantyne & Co Ltd, London.