SCATTERING OF ELASTIC WAVES BY MULTIPLE INCLUSIONS AND CRACKS

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INTRODUCTION

A new volume integral equation method (VIEM) is used to study problems involving wave scattering by multiple inclusions and cracks. The major objective of this study is to determine the characteristics of the scattered field in metal matrix composites with large fibers in presence of microcracks and other defects. Since composites consist of a large number of closely packed fibers, the elastic field in these materials is strongly affected by the interaction between microcracks and the fibers. Thus, to analyze the effects of microcracks on the scattered field in composites is of particular interest in ultrasonic NDE for crack detection and characterization. In this paper we consider the scattering of elastic waves by a distribution of long parallel fibers in presence or absence of interface debonding. We use the VIEM to calculate the scattering cross section from the fibers and discuss the possible application of the results in the nondestructive characterization of the fiber-matrix interface degradation in structural composites.

THE VOLUME INTEGRAL EQUATION

The geometry of the general problem is sketched in Fig. 1. Let ρ and C_{ijkl} denote the density and the elastic tensor of the solid. The matrix is assumed to be homogeneous, but the inclusions may, in general, be inhomogeneous. The interfaces between the inclusions and the matrix are assumed to be perfectly bonded insuring continuity of the displacement and stress vectors.

Let $e^{-i\omega t} u^o_m(\mathbf{x}, \omega)$ denote the mth component of the displacement vector due to the incident field at $\mathbf{x}$ in absence of the inclusions and let $e^{-i\omega t} u_m(\mathbf{x}, \omega)$ denote the same in presence of the inclusions, where ω is the circular frequency of the waves. In what follows the common time factor $e^{-i\omega t}$ and the explicit dependence on ω of all field quantities will be suppressed.

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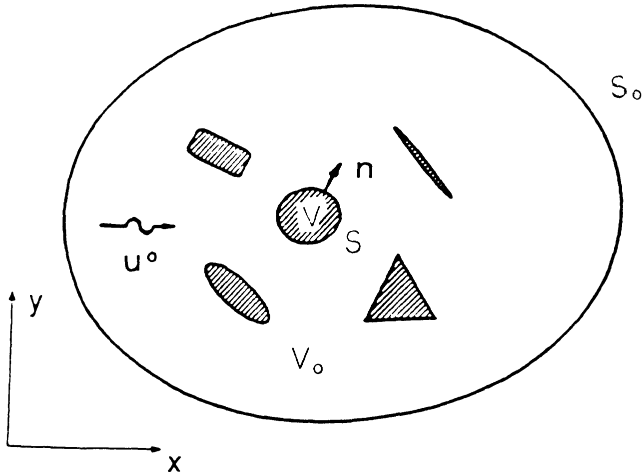


Fig. 1. Geometry of the general scattering problem.

It has been shown by Mal and Knopoff [1] that $u_m(x)$ satisfies the equation here

$$u_m(x) = u_m^o(x) + \int_V [\delta\rho\omega^2 G_i^m(\xi, x) u_i(\xi) - \delta C_{ijkl} G_{i,j}^m(\xi, x) u_{k,l}(\xi)] d\xi \qquad (1)$$

the integral is over the whole space, $\delta\rho$ and δC_{ijkl} are the contrasts in the density and elastic tensor of the inclusions and the matrix and $G_i^m(\xi, x)e^{-i\omega t}$ is the Green's function for the unbounded homogeneous matrix material; i.e., $G_i^m(\xi, x)e^{-i\omega t}$ represents the ith component of the displacement at ξ due to a concentrated force, $e_m e^{-i\omega t}$, at **x** in the mth direction. In (1) the summation convention and comma notation have been used and the differentiations are with respect to the integration variable, ξ_i. It should be noted that the integrand is nonzero within the inclusion only.

If **x** lies within the inclusions, then (1) is an integrodifferential equation for the unknown displacement vector **u**(x) within the inclusions. It is extremely difficult if not impossible to solve this coupled system of equations analytically even in the presence of a single inclusion of arbitrary shape. Mal and Knopoff [1] gave an approximate solution at low frequencies when the inclusions are well separated spheres. Mal and Yin [2] gave the solution of (1) in the one-dimensional case. For an arbitrary-shaped isolated inclusion the equation can be solved by successive approximations, (e.g., the Born series) provided the contrast in the properties of the matrix and the inclusion is "small" [3]. In this paper a numerical scheme is used to solve the equation for the general two-dimensional case. Once **u**(x) within the inclusions is determined, the scattered field outside the inclusion can be obtained from (1) by evaluating the integral.

In the next section we consider the scattering of P-SV (in-plane) waves in a metal matrix composite containing a distribution of parallel cylindrical fibers. In particular we calculate the scattering cross section form a distribution of partially or totally debonded fibers. It should be noted that the derivation of (1) requires the presence of elastic materials within the inclusions, i.e., it is not strictly valid if the inclusions become void or infinitely rigid. However, as has been shown in Lee and Mal [4] the void case can be obtained as the limiting case of vanishing material properties of the inclusions through careful evaluation of the limit.

SCATTERING OF P AND SV WAVES.

For the in-plane problem of the scattering of P and SV waves on the 1-2 plane, the volume integral equation (1) reduces to

$$\begin{aligned} u_1(x) = u_1^0(x) + \int_R \{ & \delta\rho\omega^2[G_1^1(\xi,x)u_1(\xi) + G_2^1(\xi,x)u_2(\xi)] \\ & - [\delta(\lambda + 2\mu)G_{1,1}^1 u_{1,1} + \delta\lambda G_{1,1}^1 u_{2,2} + \delta\mu G_{1,2}^1(u_{1,2} + u_{2,1})] \\ & - [\delta(\lambda + 2\mu)G_{2,2}^1 u_{2,2} + \delta\lambda G_{2,2}^1 u_{1,1} + \delta\mu G_{2,1}^1(u_{1,2} + u_{2,1})]\} d\xi_1 d\xi_2 \end{aligned} \tag{2}$$

and

$$\begin{aligned} u_2(x) = u_2^0(x) + \int_R \{ & \delta\rho\omega^2[G_1^2(\xi,x)u_1(\xi) + G_2^2(\xi,x)u_2(\xi)] \\ & - [\delta(\lambda + 2\mu)G_{1,1}^2 u_{1,1} + \delta\lambda G_{1,1}^2 u_{2,2} + \delta\mu G_{1,2}^2(u_{1,2} + u_{2,1})] \\ & - [\delta(\lambda + 2\mu)G_{2,2}^2 u_{2,2} + \delta\lambda G_{2,2}^2 u_{1,1} + \delta\mu G_{2,1}^2(u_{1,2} + u_{2,1})]\} d\xi_1 d\xi_2 \end{aligned} \tag{3}$$

where $u_1(x)$, $u_2(x)$ are the in-plane displacement components, $\delta\lambda = \lambda_1 - \lambda_2$, $\delta\mu = \mu_1 - \mu_2$, λ_1 and λ_2 are the Lame′ constants and μ_1 and μ_2 are the shear moduli of the inclusions and the matrix, respectively. The Green's function for the two-dimensional time-harmonic elastodynamic state is given by [6]

$$\begin{aligned} G_2^1 = \frac{i}{4\rho\omega^2} \Bigg\{ & \frac{(x_1 - \xi_1)(x_2 - \xi_2)}{r^2}[k_1^2 H_0(k_1 r) - k_2^2 H_0(k_2 r)] \\ & - \frac{2(x_1 - \xi_1)(x_2 - \xi_2)}{r^3}[k_1 H_1(k_1 r) - k_2 H_1(k_2 r)] \Bigg\} \end{aligned} \tag{4}$$

$$G_1^2 = G_2^1 \tag{5}$$

$$\begin{aligned} G_1^1 = \frac{i}{4\rho\omega^2} \Bigg\{ & \frac{(x_1 - \xi_1)^2}{r^2}[k_1^2 H_0(k_1 r) - k_2^2 H_0(k_2 r)] \\ & + \frac{1}{r}[1 - 2\frac{(x_1 - \xi_1)^2}{r^2}][k_1 H_1(k_1 r) - k_2 H_1(k_2 r)] + k_2^2 H_0(k_2 r) \Bigg\} \end{aligned} \tag{6}$$

$$\begin{aligned} G_2^2 = \frac{i}{4\rho\omega^2} \Bigg\{ & \frac{(x_2 - \xi_2)^2}{r^2}[k_1^2 H_0(k_1 r) - k_2^2 H_0(k_2 r)] \\ & + \frac{1}{r}[1 - 2\frac{(x_2 - \xi_2)^2}{r^2}][k_1 H_1(k_1 r) - k_2 H_1(k_2 r)] + k_2^2 H_0(k_2 r) \Bigg\} \end{aligned} \tag{7}$$

where H_0 and H_1 denote the Hankel functions of the first kind of the zeroth and first orders, respectively, $r = |\mathbf{x} - \xi|$, and k_1, k_2 are the P and S wavenumbers in the solid. As in the case of the SH waves [5], finite element discretization of the inclusions in (4) and (5) results in a system of two coupled linear algebraic equations for the unknown nodal displacements inside the inclusion.

The integrands in (2) and (3) contain singularities of different orders due to the singular nature of the Green's function at $\mathbf{x} = \xi$. In general, the singular behavior of the Green's function at $\mathbf{x} = \xi$ is given by

$$G = -\frac{1}{2\pi\mu}\ln r + O(1) \tag{8}$$

and

$$\frac{\partial G}{\partial \xi_1}, \frac{\partial G}{\partial \xi_2} = -\frac{2}{\pi k r} + O(1) \tag{9}$$

Although the singularities in the VIEM are integrable, special care must be taken in the numerical treatmentof the integrals in equations (2) and (3). We have used the direct integration scheme to handle both singularities; the details can be found in [4].

The Single Inclusion Problem

In order to check the accuracy of the numerical method the single cylindrical inclusion is considered first. Analytical solution based on standard techniques (e.g., Bose and Mal [7]) and the boundary integral equation method (BIEM) was also used to solve the problem for comparison with the VIEM solution. The standard 8-node quadrilateral and 6-node triangular finite elements were used in the VIEM while quadratic elements were used in the BIEM. The number of elements used in VIEM was 256 and in BIEM it was 100.

Two different normalized wavenumbers, $k_2a = 0.25$ (low frequency) and 2.5 (intermediate frequency), were used. Figs. 2 and 3 show the comparison between the analytical and the numerical solutions using BIEM and VIEM for two different frequencies. It can be seen that the three solutions are indistinguishable in the graphs. A number of other scattering problems involving inclusions as well as cracks with were solved by the methods and the results were compared with the available solutions. The agrement was found to be excellent in all cases. The details can be found in [4].

THE SCATTERING CROSS SECTION IN METAL MATRIX COMPOSITES

In this section, the effect of fiber debonding on the scattering cross section in metal matrix composites is examined. Since fiber debonding affects the overall strength of composites, it is important to detect and characterize the fiber-matrix interfacial damage [8-11]. Here, we assume that the carbon interphase layer fails during debonding. It should be noted that the derivation of (1) requires the presence of elastic materials within the inclusions, i.e., it is not strictly valid if the inclusions become void or infinitely rigid. However, the void case can be obtained as the limiting case of vanishing material properties of the inclusions through careful evaluation of the limit [4, 12].

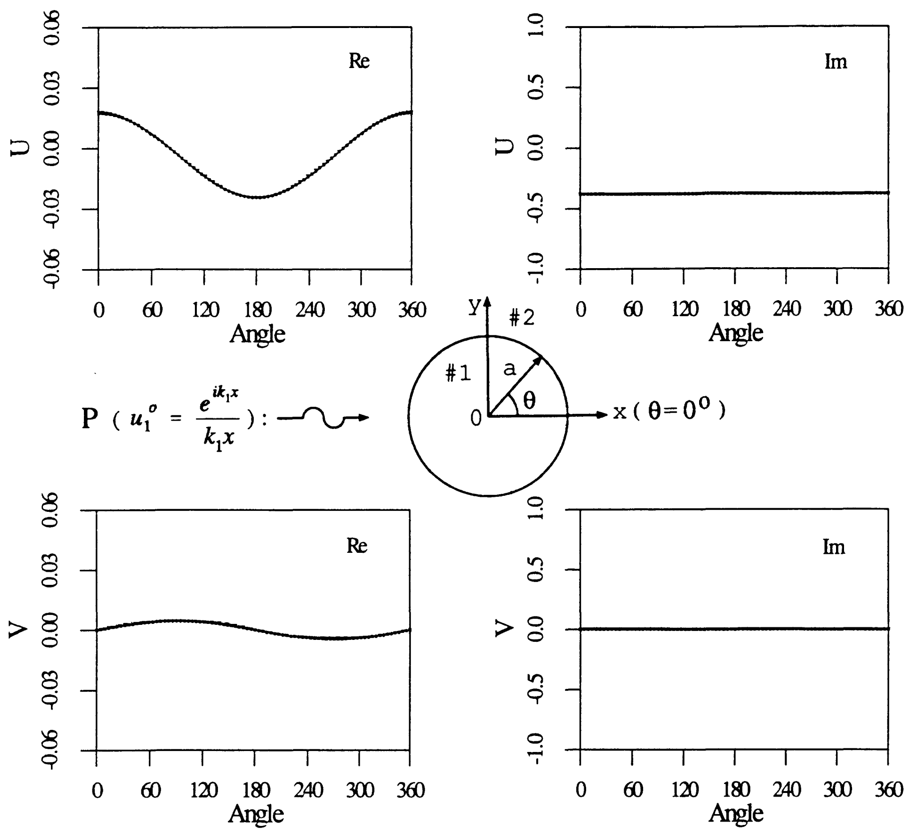


Fig. 2. The real and imaginary parts of the normalized displacements ($U = u_1$, $V = u_2$) at the fiber-matrix interface for $k_2a = 0.25$ for an incident P wave. k_2 is the S wavenumber in the matrix. Calculations are done using analytical method, VIEM and BIEM giving almost identical results.

For a given frequency ω, a measure of the fraction of the incident power scattered in a particular direction is the differential cross section. Using the asymptotic behavior of the Green's function as $r \rightarrow \infty$, the far-field differential cross section can be calculated from the volume integral equation. The influence of progressive fiber-matrix debonding on the differential cross section of scattered P waves due to an incident P wave is presented here.

The generalized self consistent model introduced by Yang and Mal [11] is used in the calculations. In this model, a SiC fiber with a carbon interphase and surrounded by an annulus of the matrix (i.e., Titanium) is assumed to be embedded in an unbounded effective medium. The model, sketched in Fig. 4, approximately incorporates the effect of microstrucure in the near field and of fiber distribution in the far field of each fiber. The properties of the SiC/Ti composite and of the effective medium are given in Table 1. The fiber core is omitted since the effect of a core has been found to be negligible [12].

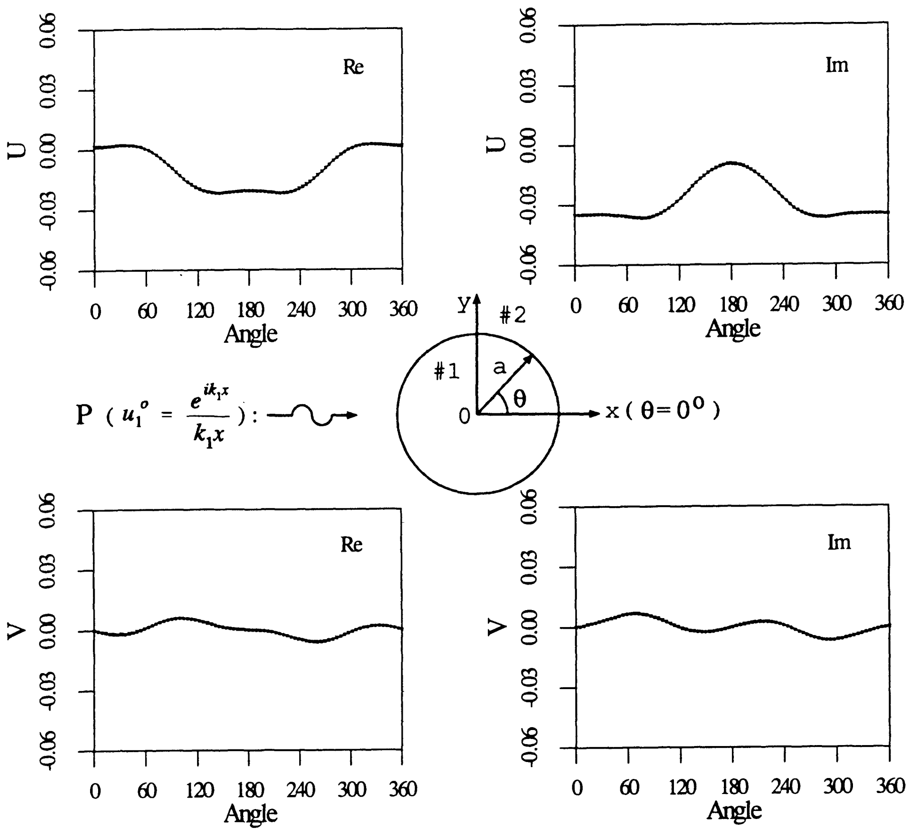
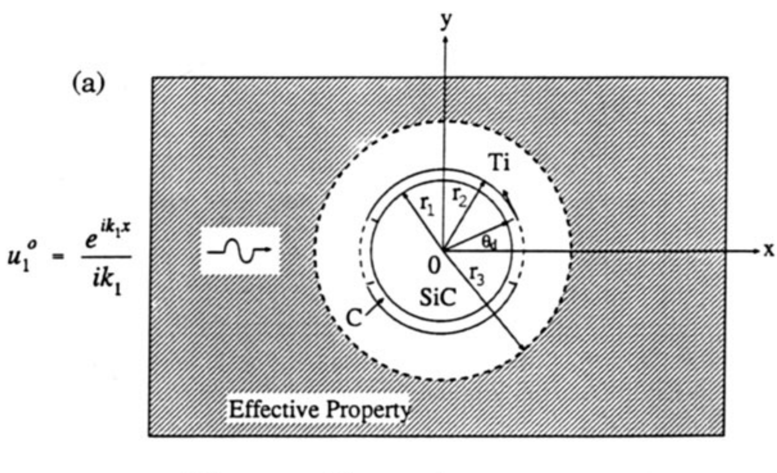


Fig. 3. Same as in Fig. 2 for $k_2a = 2.5$.



r_1 = 70 μm, r_2 = 77 μm and r_3 = 118.32 μm, C_f = 0.35

Fig. 4. A SiC fiber with a broken carbon coating (dashed segments) in the GSCM.

Table 1. Material properties of SiC/Ti composite with coated fibers

	ρ (g/cm^3)	E (GPa)	μ (GPa)
SiC fiber	3.18	431.0	172.0
Titanium matrix	4.54	96.5	37.1
Carbon coating	1.41	34.48	14.34
Effective medium	3.83	123.84	48.64

The normalized scattering cross section is calculated at different normalized frequencies, k_1a at a normalized distance $k_1r = 60$ from the center of the fiber, where k_1 is the P-wavenumber in the matrix. Typical results are presented in Fig. 5. Fig. 5a shows the discretized model of the fiber and Figs. 5b, c and d show the scattering cross sections for $k_1a = 0.1$, 1.0 and 3.0, respectively. Fiber debonding can be seen to have a strong influence on the scattered field at all frequencies considered.

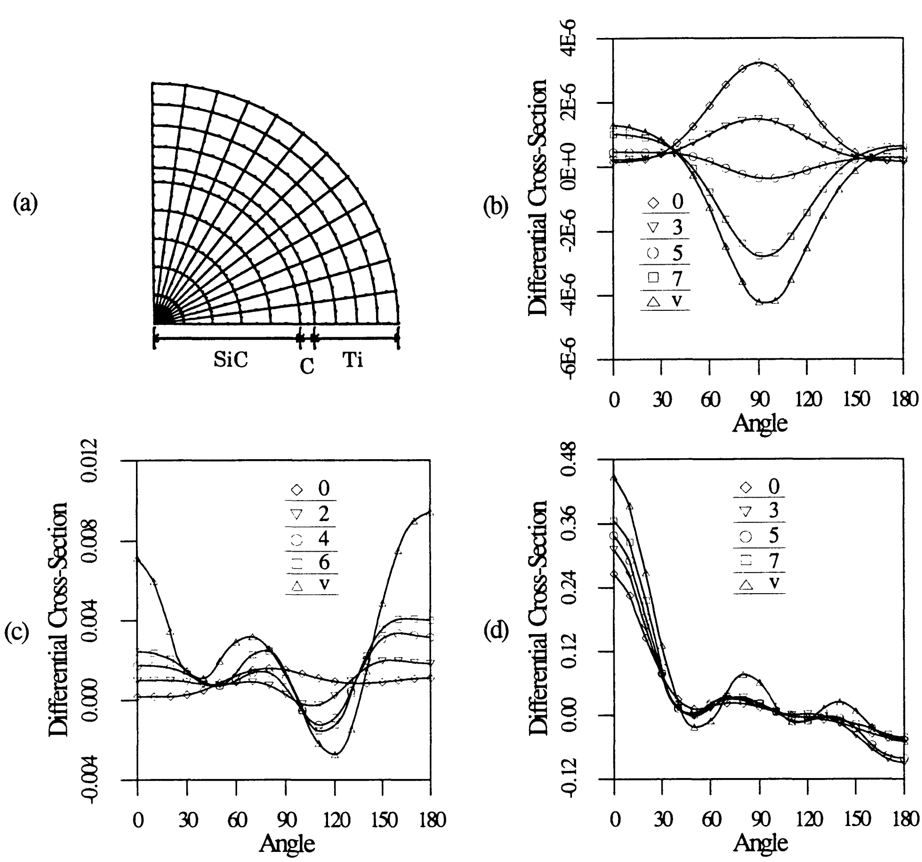


Fig. 5. Normalized differential cross section in SiC/Ti composites with different fiber debonding lengths for (b) $k_1a = 0.1$, (c) $k_1a = 1.0$, and (d) $k_1a = 3.0$. The numbers at the top right of each panel indicates the number of broken elements in carbon coating layer; 0 is for full bonding and v is for complete debonding.

CONCLUSIONS

The volume integral equation method is proposed as a new numerical solution scheme for two dimensional elastodynamic problems with arbitrary shapes and number of inclusions. The volume integral equation method is also applied to the calculation of scrattering cross section in metal matrix composites. The effect of fiber debonding on the cross section was investigated as the debonding length increases. Finally, a more convenient formulation, involving a combination of the VIE and BIE, was introduced and was applied to a simple illustrative problems. Further studies on the numerical implementation of this mixed formulation should lead to a highly efficient method for solving elastodynamic problems involving multiple inclusions and multiple cracks.

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