ONE SIDED INSPECTION FOR ELASTIC CONSTANT DETERMINATION OF ADVANCED MATERIALS

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INTRODUCTION

Knowledge of mechanical properties of a composite material is prerequisite to good engineering design. The problem of theoretically predicting the mechanical properties of a composite material as a function of the properties of its constituents has been thoroughly investigated by many authors [1-2]. In one general class of techniques, termed "effective modulus" theories, the composite is viewed as a homogeneous anisotropic material with "effective" elastic constants that are determined by the elastic constants of the constituent materials. All of these theories remove the microstructure of the composite from consideration and, as a result, cannot be expected to predict accurately the properties of the composite material over a wide range of deformation scales. One limitation which comes about from this "smearing" of the microstructure into a homogeneous continuum is that the effective modulus theories, and hence materials which are assumed to have "effective elastic constants", are incapable of predicting frequency dispersion of waves, which is sometimes very pronounced in composite materials [3].

The practical problem of experimentally determining the "effective" elastic constants of a composite material has a long history. One of the most common techniques is to cut cubes of the material, at specific angles with respect to the principal directions of the composite, and to measure the ultrasonic wave speed in those directions. Analytical formulas, derived from Christoffel's equation, are then used for "inverting" the collected wave speed information to determine the effective elastic constants of the composite. This technique can be very accurate, but is clearly destructive in nature, and requires rather careful specimen preparation. In recent years, our group, as well as several others [4,5,6] have been concerned with developing
Individual Lamina

- Fabricated from Hercules IM7-8551 Prepreg Tape.
- Prepreg 0° Modulus = 166 GPa
- Prepreg 90° Modulus = 8.3 GPa
- Density = 1.586 g/cm³
- Lamina Thickness: 0.127±0.013 mm

Fabricated Specimen

- Prepreg Layup:
  
  \[ [0, \pm 45, 0]_s \]

FIGURE 1 Specimen Configuration & Coordinate System

nondestructive experimental methods for determining the effective elastic constants.

In this study, we have calculated several of the effective elastic constants of a particular, assumed orthotropic, graphite epoxy layered composite material by two techniques, the cube cutting technique and a recently considered one sided, non-destructive inspection technique developed by our group [7]. We have also compared these results to those obtained from quasi-static compression tests performed on the same specimen. The specimen orientation, and the coordinate system to which we have referred the elastic constants are shown in Figure 1.

CUBE CUTTING TECHNIQUE

In the cube cutting technique, several cubes are cut from the specimen, at various orientations to the principal directions of the specimen, and phase velocity measurements are made in these directions.

For the assumed orthotropic structure of our specimen, there are nine independent “effective” elastic constants which necessitates making nine independent velocity measurements. These nine measurements come from using all three different modes, i.e., Quasi-longitudinal and both Quasi-shear, propagating in different directions. The large attenuation of fibrous composite materials, especially for the quasi-shear modes, requires the thickness of the cubes to be kept fairly small. In this study, the thicknesses were approximately 0.5 inch. Another major concern when dealing with fibrous composites is the frequency dispersion of the waves, that is, a frequency dependent propagation velocity. Both the (frequency dependent) attenuation and frequency dispersion effects produce changes in the pulse shape as it propagates through the material, making phase velocity measurements very difficult. In the cube cutting technique, these measurement error effects were minimized by making the cubes thin, and by using “tone burst” (narrow band) pulses of ultrasonic waves. The velocity measurement technique is described briefly below.
A pulse of a specific central frequency and number of cycles is generated, and then sent simultaneously through a sole buffer rod, and a buffer rod coupled to the specimen to be tested. The time delay caused by the propagation of one of the signals through the specimen is determined by shifting the delayed signal in time until the peaks of both signals overlap.

Once the velocities of the various modes are measured in the various directions, the elastic constants of the specimen can be determined by using the equations relating the phase velocity of a given mode to its propagation direction for generally anisotropic media which can be found, for instance, in [7].

**Cube Cutting Results**

The composite specimen used in this study exhibited dispersive behavior, in that the quasi-longitudinal wave mode velocity for all but the X3 direction was frequency dependent. The shear waves however, were not dispersive for any measured direction.

The elastic constants, measured for four different frequencies are tabulated in Table 1.

**ONE-SIDED TECHNIQUE**

In trying to determine the effective elastic constants of a material from one sided measurements, we have employed various ultrasonic wave modes including bulk quasi-longitudinal and quasi-shear, as well as guided surface and subsurface waves. With the assumption that the material can be considered as “effectively” orthotropic, there are, as mentioned above, nine independent elastic constants which must be determined from our wave velocity measurements. Closed form expressions can be obtained relating the phase velocity of a given wave mode (i.e., guided and/or bulk) to the direction of its wavevector and the material properties of the material. These expressions were then inverted to give closed form analytical formulas for the elastic constants of the material as functions of the nine measured wave velocities which can be found in [7].

<table>
<thead>
<tr>
<th>Table 1 Measured Elastic Constants [GPa]</th>
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<tr>
<td>0 MHz</td>
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</tr>
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<td>C11</td>
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<td>C23</td>
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The propagation characteristics of a surface (and subsurface) wave are only influenced by the properties of the specimen at the surface on which the wave propagates. This means that the data obtained from the surface and subsurface waves can only supply information about that portion of the surface of the specimen through which it propagates. We have however, in this study, assumed that the material is homogeneous with depth, i.e., that the effective elastic stiffness tensor of the material is independent of depth. This assumption seems plausible in this case, given the regular repeating structure, but it will clearly be violated if the structure is non-repeating or changing in some other manner with depth. See [9].

To determine the elastic constants of this specimen using the one-sided technique discussed earlier, we used three different wave modes. 0.5 and 1.0 MHz (nominal) transducers were used in a pulse echo arrangement to generate bulk longitudinal waves in the thickness direction of the specimen (which is a principal direction). The measurements were made at five different points on the surface of the specimen, and at each point, we recorded, with an eight bit A/D converter, the RF signal (which contained at least two successive back wall echoes). We then ran these signals through an envelope detection program, which calculated the time shift between the peaks of the envelopes of the two successive back wall echoes. We also used 1.0 MHz contact shear wave transducers to generate both pure shear waves, at five positions on the surface of the specimen, and the same recording and time measuring technique was used as described above.

The information obtained from the bulk wave data can only be used to calculate three of the nine effective elastic constants of the specimen, each of the three being one of the constants on the main diagonal of the stiffness tensor. To determine the other diagonal, as well as the off diagonal terms, we utilized surface waves and longitudinal subsurface waves in various directions.

Due to their low propagation speed on composites, surface waves cannot in general be generated using the commercially available perspex transducer shoes. We therefore used the wedge mediator technique discussed in [7], with 0.5 MHz sender and receiver, to generate a surface wave at five different areas on the surface of the specimen. To measure the velocity, we first kept the sender and receiver a given distance, say $L$, apart, and collected the received surface wave RF signal with an A/D converter. We then moved the receiver a distance, say $\Delta L$, and again collected the RF signal with an A/D recorder. The two waveforms were then sent through an envelope detection program, which accurately calculated the time between the peaks of the envelopes of the two signals, which, by knowing $\Delta L$, directly gives the velocity. This technique, and some typical waveforms which we received are shown in Figure 2a.

A similar technique was used to measure the subsurface longitudinal wave velocities, but the generation technique was different. To generate the subsurface longitudinal
waves, a sending transducer was mounted to a variable angle Plexiglas shoe, and the incident angle was varied until a wave propagating along the surface was detected. This wave is the fastest signal which appears on the c.r.t. screen. Figure 2b illustrates the technique, and shows typical subsurface wave RF signals received using a 0.5 Mhz source.

Results of One Sided Technique

The results obtained from the one sided technique are listed in the last column of table 1. Two of the constants, C12 and C66, when measured with this technique, gave results which are not possible for the matrix of elastic constants to be positive definite. The exact cause of this is being investigated. A similar occurrence was reported recently in \[8\].

STATIC COMPRESSION TEST

The cubes cut from the specimen were subjected to quasi-static compression up to a maximum strain of less than
0.4% to insure linearly elastic behavior and that no permanent damage was produced in the specimens. A chart recording of the stress vs. strain curve was plotted, and the Young’s Modulus, for the particular direction under test, was obtained as the slope of this curve. Young’s modulus was determined for the \(<100>\), \(<010>\), \(<001>\), \(<110>\), \(<011>\) and \(<101>\) directions.

The results of the compression tests were, in GPa, \(E_{<100>}=75.4\), \(E_{<010>}=26.9\), \(E_{<001>}=10.0\), \(E_{<110>}=50.3\), \(E_{<101>}=10.8\) and \(E_{<011>}=9.8\).

COMPARISONS AND CONCLUSIONS

Having the full set of elastic constants, one can determine the Young’s modulus in any direction, for a medium of orthotropic symmetry, by the formula [9]:

\[
\frac{1}{E_{4\alpha\beta}} = \frac{1}{E_1} + \frac{1}{E_2} + \frac{1}{E_3} + \frac{2G_{3j}}{E_3} + \frac{2G_{ij}}{E_3} + \frac{2G_{ij}}{E_3} + \frac{1}{C_{45}}
\]

Where,

\[
\frac{1}{E_1} = \frac{C_{11} - C_{13}}{A} \quad \frac{1}{F_1} = 2(C_{12} - C_{13}C_{45})A + \frac{1}{C_{45}}
\]

And

\[A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{12} & C_{22} & C_{23} \\ C_{13} & C_{23} & C_{33} \end{bmatrix}\]

Using this formula, along with the constants measured from the cube cutting technique, and listed in table 1, the Young’s moduli corresponding to the \(<100>\), \(<010>\), \(<001>\), \(<110>\), \(<011>\) and \(<101>\) directions were calculated for comparison with those obtained from compression tests. A typical comparison is presented in Figure 3. The maximum difference between the moduli calculated from static testing and the cube cutting technique was 15%, while the average was only 8.3%. Because \(C_{12}\) and \(C_{66}\) were not obtained from the one sided non-destructive tests, we could not calculate the Young’s moduli for that technique.

FUTURE POSSIBILITIES

Several researchers [4,5] are currently investigating the use of a non-linear curve fitting algorithms to determine the elastic constants of thin anisotropic substrates. One such algorithm, known as the SIMPLEX algorithm [10], seems to be a very efficient and accurate method for obtaining the constants. The SIMPLEX algorithm provides a highly efficient way of determining the set of constants which minimize the “error” (in the least square sense) between the experimentally and theoretically obtained data. The only requirement for use of the SIMPLEX algorithm is the ability to, given a set of elastic constants of the specimen, reproduce theoretically, the same information which is determined experimentally.
We now propose a measurement technique for use in conjunction with the SIMPLEX algorithm for determining the effective elastic constants of thick orthotropic composite materials. The first step is to measure the phase velocity of subsurface longitudinal waves, for "M" angles, $\Phi_i$, $i \in \{1, 2, \ldots M\}$ on the $X_1-X_2$ plane, and minimize the error,

$$SSR = \sum_{i=1}^{M} \left\{ f(p, C_{11}, C_{22}, C_{66}, C_{12}, \Phi_i) - V_{ph}(\Phi_i) \right\}^2$$

Where:

$$f = \sqrt{\xi + \xi^2 - 4C}$$

$$\xi = C_{66} + C_{11} \cos^2 \Phi_i + C_{22} \sin^2 \Phi_i$$

$$C = (C_{11} \cos^2 \Phi_i + C_{66} \sin^2 \Phi_i) (C_{66} \cos^2 \Phi_i + C_{22} \sin^2 \Phi_i) - (C_{12} + C_{66}) \sin \Phi_i \cos \Phi_i$$

The second step is to measure the group velocity of quasilongitudinal waves for "R" angles $\Psi_i$, $i \in \{1, 2, \ldots R\}$ in the $X_1-X_3$ plane, along with the group velocity of quasilongitudinal waves for "N" angles $\Theta_i$, $i \in \{1, 2, \ldots N\}$ in the $X_2-X_3$ plane and minimize the error,

$$SSR = \sum_{i=1}^{N} \left\{ f_1(p, C_{22}, C_{33}, C_{23}, C_{44}, \Theta_i) - V_{gr}(\Theta_i) \right\}^2 + \sum_{i=1}^{R} \left\{ f_2(p, C_{11}, C_{33}, C_{13}, C_{55}, \Psi_i) - V_{gr}(\Psi_i) \right\}^2$$

Where the numerical values of the functions $f_1$ and $f_2$ must, in general, be obtained from a computer program to calculate the group velocity as a function of its direction, that is, the Ray or Wave surface. It should be noted that the accuracy of reproducing the elastic constants can be greatly increased if, in the minimizations above, those constants whose determination depends only on bulk waves at normal incidence, are kept fixed. The density should also be calculated by other means and held fixed in the minimization procedure.
REFERENCES


