ACOUSTIC NONLINEARITY IN METAL - MATRIX COMPOSITES

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INTRODUCTION

The elastic behavior of a solid consists of linear and nonlinear contributions. The linear part is represented by the well known Hooke's law which is given in tensorial notation as

$$\sigma_{ij} = C_{ijkl}\,\varepsilon_{kl} \qquad (1)$$

where σ_{ij} and ε_{kl} are the stress and strain tensors, respectively, and C_{ijkl} is the tensor of the second order elastic constants (SOEC). This relationship is sufficient for many engineering calculations since deviations from a purely linear elastic behavior are small. Hooke's law, however, is not sufficient for an advanced characterization of the elastic behavior of materials. This is due to the fact that many of the physical and mechanical properties of materials are of nonlinear nature. The nonlinear elastic behavior can be investigated using ultrasonic techniques because of their high sensitivity for small nonlinear effects. Among the nonlinear effects are the stress and the temperature dependences of ultrasonic velocities in the solid. These effects have gained considerable interest in the last decade, particularly for the nondestructive evaluation of applied and residual stresses [1], and also for the microstructural characterization of materials [2]. Another physical manifestation of the nonlinear elastic behavior of solids is the acoustic nonlinearity parameter. This parameter can be determined from measurements of the amplitudes of fundamental and second harmonic when an originally sinusoidal wave gets distorted while propagating through the solid. The nonlinearity parameter can also be calculated from a combination of second and third order elastic constants. In previous studies [3], the nonlinearity parameter was found to be sensitive to microstructural changes in aluminum alloys and in particular to the content of precipitates of the second phase.

Metal - matrix composites are a new class of materials wich contain a metallic matrix and a metallic or ceramic material as a reinforcement. The bulk properties of these composites can be tailored by changing the volume percentage, geometry, distribution, and orientation of the reinforcement. Due to the different coefficients of thermal expansion for the matrix and the reinforcement material in MMCs, the creation of thermal stresses during manufacturing is unavoidable. Therefore, the nondestructive characterization of these composites is necessary in order to monitor their mechanical properties and to guarantee their quality.

In the present study the effect of the volume content of reinforcement in metal - matrix composites (MMC) on the two nonlinear elastic quantities, namely acoustoelastic constants and nonlinearity parameter has been investigated. The nonlinearity parameter is determined using two different methods and the results are compared.

THEORETICAL

Basically, all nonlinear elastic effects are due to the anharmonicity of the interatomic potential. Thus, relationships between quantities describing the elastic nonlinearity are expected. In order to develop quantitative parameters for the description of elastic nonlinearity, it is convenient to use the thermodynamic derivation of the elastic constants starting from the elastic potential of the solid. If the lattice arrangement of a solid is disturbed by an infinitesimal strain ε due to the presence of an elastic wave, the energy of deformation per unit volume $\Phi(\varepsilon)$ can be expanded as a power series of strains such that

$$\Phi(\varepsilon) = \phi_0 + \frac{\partial\Phi}{\partial\varepsilon_{ij}}\varepsilon_{ij} + \frac{\partial^2\Phi}{\partial\varepsilon_{ij}\partial\varepsilon_{kl}}\varepsilon_{ij}\varepsilon_{kl} + \frac{\partial^3\Phi}{\partial\varepsilon_{ij}\partial\varepsilon_{kl}\partial\varepsilon_{mn}}\varepsilon_{ij}\varepsilon_{kl}\varepsilon_{mn} + \dots \tag{2}$$

According to Brugger [4], the elastic constants of the order n are defined as the n-th partial derivatives of the elastic potential with respect to strain as

$$C_{ij\dots} = \partial^{(n)}\Phi/\partial\varepsilon_{ij\dots} \tag{3}$$

If up to third order terms in ε are considered in eq.(2), the stress-strain relationship can be written as

$$\sigma_{ij} = C_{ijkl}\,\varepsilon_{kl} + C_{ijklmn}\,\varepsilon_{kl}\varepsilon_{mn} \tag{4}$$

where the C_{ijklmn} are the third order elastic constants (TOEC) which need to be added to Hooke's law (1) to allow for nonlinear deviations.

For isotropic materials, the tensor of the second order elastic constants reduces to two independent second order elastic constants, known as Lamé constants λ and μ. These constants can be determined directly from ultrasonic experiments by measuring the wave speeds of longitudinal and shear waves using the relationships :

$$\mu = \rho\, v_T^2 \qquad \text{and} \qquad \lambda + 2\mu = \rho\, v_L^2 \tag{5}$$

The tensor of the third order elastic constants reduces to three independent third order elastic constants in isotropic materials. These are called the Murnaghan constants l, m and n. In order to measure these constants using ultrasonic methods the propagation velocities of three different wave modes have to be determined as a function of an applied uniaxial strain. The relative change in the ultrasonic velocity of a given wave mode with the applied elastic strain normalized by the velocity of the strain-free specimen is called the acoustoelastic constant (AEC) which is a characteristic of the material. The Murnaghan constants can be evaluated from acoustoelastic constants using the relationships [6]

$$l = \frac{\lambda}{1-2\nu}\left[\frac{1-\nu}{\nu}\frac{\partial v_{22}/v_0}{\partial\varepsilon} + \frac{2}{1+\nu}\left(\frac{\partial v_{21}/v_0}{\partial\varepsilon} + \nu\frac{\partial v_{23}/v_0}{\partial\varepsilon}\right) + 2\nu\right]$$

$$m = 2(\lambda+\mu)\left[\frac{\nu}{1+\nu}\frac{\partial v_{23}/v_0}{\partial\varepsilon} + \frac{1}{1+\nu}\frac{\partial v_{21}/v_0}{\partial\varepsilon} + 2\nu - 1\right] \tag{6}$$

$$n = \frac{4\mu}{1+\nu}\left[\frac{\partial v_{21}/v_0}{\partial\varepsilon} - \frac{\partial v_{23}/v_0}{\partial\varepsilon} - 1 - \nu\right]$$

where ν is the Poisson's ratio, λ and μ are the Lamé constants, v_{ij} is the velocity of an ultrasonic wave with propagation direction i and polarization direction j, v_0 is the velocity in the unstrained specimen and ε is a uniaxial strain applied in a direction perpendicular to the propagation direction.

Another nonlinear quantity which can be obtained if eq.(4) is inserted into the equation of motion for particles in the solid's lattice, leads to the nonlinear wave equation

$$\frac{\partial^2 u_i}{\partial t^2} - v_i^2 \frac{\partial^2 u}{\partial a^2} = - \beta_i v_i^2 \frac{\partial u_i}{\partial a} \frac{\partial^2 u_i}{\partial a^2} \qquad (7)$$

where u is the particle displacement, i is a mode index depending on the polarization and the propagation direction of the wave, a is a coordinate along the direction of wave propagation and v_i is the wave velocity of the mode i. The quantity β_i is the modal acoustic nonlinearity parameter of the solid and can be expressed as a linear combination of the second and third order elastic constants [5]. The solution of the nonlinear wave equation can be given as

$$u_i = A_1 \sin (ka - \omega t) + A_2 \cos 2(ka - \omega t) \qquad (8)$$

where A_1 and A_2 are the amplitudes of the fundamental and second harmonic waves, respectively. The acoustic nonlinearity parameter is related to these amplitudes as

$$\beta = \frac{8}{k^2 a} \frac{A_2}{A_1^2} \qquad (9)$$

where ω is the fundamental frequency, k is the propagation constant and a is the distance measured from the generating transducer to the instantaneous position of the fundamental wave in the solid. Its magnitude determines the extend of the distortion of the fundamental wave. If one considers a longitudinal wave mode in an isotropic solid, the acoustic nonlinearity parameter is related to the Lamé and the Murnaghan constants by

$$-\beta = 3 + \frac{2l + 4m}{\lambda + 2\mu} \qquad (10)$$

Using this relationship the acoustic nonlinearity parameter can be calculated when the third order elastic constants are known.

EXPERIMENTAL

In this study, two different sets of specimens were used. The matrix was an Al - 8091 alloy in one case and in the other case an Al - 7064 alloy. The chemical compositions of both alloys are shown in table 1. As reinforcement material these composites contain SiC - particles. The particles had a more or less globular shape, ranging from 1 - 5 µm in size. The material has finally been extruded to rods. Micrographs taken for different cuts of the specimens revealed a planar random distribution of the reinforcement in the plane normal to the extrusion direction and a particle alignment along the extrusion direction.

A block diagram of the experimental set up for the determination of the nonlinearity parameter is shown in Fig.1 . A lithium niobate transducer attached to the specimen by a solid bond is used to generate ultrasonic pulses. These pulses have typically a center frequency of 10 MHz and a bandwidth of 200 kHz. The ultrasonic signal propagates along the extrusion direction of the specimen whose surfaces are made parallel to each other and are lapped optically flat. The 10 MHz fundamental as well as the 20 MHz harmonic signal cause a distortion of the free surface of the specimen. The displacement amplitudes carried by the two frequencies are measured in order to determine the nonlinearity parameter using eq.(9).

Table 1. Chemical compositions of the aluminum alloys in weight percent.

	Alloying Elements								
Alloy	Si	Fe	Cu	Mg	Zr	Li	Zn	Cr	Co	Al
8091	0.02	0.01	1.90	0.80	0.11	2.70	----	----	----	rem
7064	0.05	0.10	2.00	2.30	0.20	----	7.10	0.12	0.22	rem

For measurements of the absolute amplitudes, the capacitive detector technique described in [8] has been used. This technique allows the detection of displacement amplitudes of a free surface with a sensitivity of 10^{-3} Å.

In order to verify the behavior predicted by eq.(9), the amplitudes of the fundamental and second harmonic have been measured as a function of increasing source voltage. A plot of the harmonic amplitude A_2 vs. the square of the fundamental amplitude A_1 yields a linear relationship over the whole range of the driving voltages used as shown in Fig.2 . The slope of a linear fit through these data is used to calculate the nonlinearity parameter. In many of the engineering materials the attenuation of ultrasonic waves depends on their frequencies. Unless the attenuation of the second harmonic is twice that of the fundamental, a correction has to be made according to relationship [9]

$$\beta = \beta_{meas.} \frac{\alpha_2 - 2\alpha_1}{1 - \exp[(2\alpha_1 - \alpha_2)\, a]} \tag{11}$$

where α_1 and α_2 are the attenuation coefficients of the fundamental and the second harmonic waves respectively. The attenuation coefficients for 10 and 20 MHz longitudinal waves in the MMC-specimens have been used to perform this correction. Another correction due to diffraction effects has been neglected because of the large lateral dimensions of the specimens.

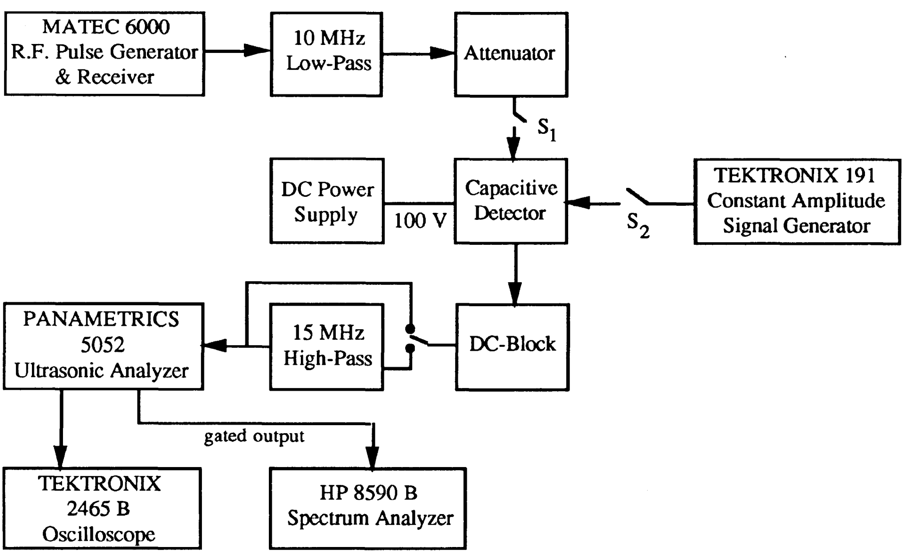

Fig. 1. Block diagram of the capacitive detector system.

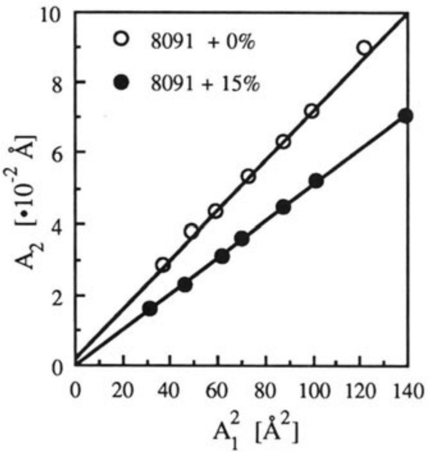

Fig. 2. Measured displacement amplitudes for fundamental and second harmonic signals.

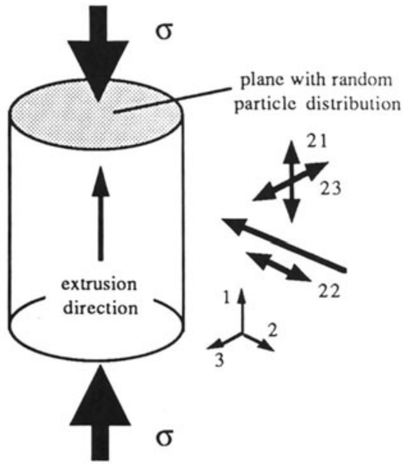

Fig. 3. Velocity designations and direction of applied stress

Acoustoelastic constants are determined by applying a uniaxial compressive stress while ultrasonic waves are propagating transversely to the load direction. The relative changes in velocity for three different wave modes (Fig.3) are measured using the pulse echo overlap technique. This technique is described in detail in [10] and can resolve velocity changes up to one part in 10^6. The changes caused by the acoustoelastic effect are typically in the order of one part in 10^4 and depend on the polarization of the wave. The slope of the velocity data vs. the elastic strain is used to determine the acoustoelastic constants and then, with eqs.(6), to calculate the third order elastic constants. The average values resulting from measurements in two different propagation directions within the quasi-isotropic plane are used to represent the material parameters.

RESULTS AND DISCUSSION

From measurements of densities and of ultrasonic velocities the second order elastic constants of the metal matrix composites have been evaluated by Lee et al. [13] using eq.(5). Their results are listed in Tab.3 . From this table it is seen that both, the Young's and shear moduli increase linearly with the SiC content. The Young's modulus follows closely the isostress condition represented by

$$E_{comp} = [E_{Al} \cdot E_{SiC}]/[E_{SiC} \cdot f_{Al} + E_{Al} \cdot f_{SiC}] \qquad (12)$$

where f is the volume fraction of the indicated phase.

The values of the acoustoelastic constants of the metal-matrix composites, determined from the relative changes in the ultrasonic velocity as a function of elastic strain, are the average values of the data obtained by Lee et al. [13] and are listed in Tab.2 . The AECs are reproducible within 2%. From the table, one can see that the values of the AECs of the 7064 alloy are larger than those of the 8091 alloy. Also the AECs for both sets of MMCs decrease as the content of SiC increases. Smaller absolute values of AECs indicate a smaller change in the ultrasonic velocity as a function of elastic strain which means a smaller deviation from the ideal Hookean behavior of the material.

Table 2 . Averaged acoustoelastic constants of the examined MMCs.

Material	Acoustoelastic Constants					
	AEC 22	Δ%	AEC 21	Δ%	AEC 23	Δ%
Al - 8091	1.12	0	-2.59	0	0.87	0
+10% SiC	1.08	-3.7	-2.17	-19.4	0.82	-5.7
+15% SiC	0.92	-17.9	-1.91	-35.6	0.74	-14.9
Al - 7064	1.44	0	-3.10	0	1.93	0
+15% SiC	1.20	-16.7	-2.35	-24.2	1.27	-34.2
+20% SiC	0.88	-38.9	-1.85	-40.3	0.92	-52.3

From the acoustoelastic constants and the second order elastic constants, the third order elastic constants were determined using eq.(6). From an analysis of error propagation the inaccuracy is found to be highest in the Murnaghan constant l and is equal to 15%. The inaccuracies of the constants m and n are 5% and 3% respectively. In general, the values of the Murnaghan constants m and n are not significantly influenced by the content of SiC within the experimental error. The constant m, however, has a tendency towards less negative values, whereas the constant n is showing no trend at all. The Murnaghan constant l changes considerably and even becomes positive when SiC is added. Compared to the elastic moduli μ and E, the third order elastic constants do not show a clear relationship with the content of reinforcement, whereas those of the elastic moduli increase in a predictable manner.

The Murnaghan constants l and m are used to calculate the nonlinearity parameter according to eq.(8) . The values of the calculated nonlinearity parameter are listed in Tab.4 as $\beta_{calc.}$ Also included in Tab.4 are the values of the directly measured nonlinearity parameter as $\beta_{meas.}$ The experimental error is 10% for the calculated and 15% for the measured nonlinearity parameter. For both $\beta_{calc.}$ as well as $\beta_{meas.}$ the values decrease considerably with increasing volume percentages of SiC.

As can be seen from Fig.4, the values of the calculated nonlinearity parameter for the 8091 as well as for the 7064 alloys change linearly as a function of second phase content. The values of the composites with the Al - 8091 matrix appear to be smaller than those of the composites with the Al - 7064 matrix.

Table 3 . Second and third order elastic constants of MMCs.

Material	SOEC [GPa]			TOEC [GPa]		
	λ	μ	E	l	m	n
Al - 8091	42.0	30.1	77.7	-34	-320	-438
+10% SiC	42.8	35.4	91.0	34	-313	-466
+15% SiC	42.4	37.6	95.7	33	-288	-454
Al - 7064	54.1	26.9	71.4	-33	-359	-515
+15% SiC	57.4	35.1	91.5	43	-343	-516
+20% SiC	54.3	38.0	99.2	24	-309	-486

Table 4 . Calculated and directly determined nonlinearity parameter.

Material	βcalc.	Δ%	βmeas.	Δ%
Al - 8091	10.2	0	10.6	0
+10% SiC	7.4	-27.5	8.5	-19.8
+15% SiC	6.2	-39.2	6.9	-34.9
Al - 7064	10.9	0	8.7	0
+15% SiC	7.1	-34.9	6.6	-24.1
+20% SiC	6.2	-43.1	5.8	-33.3

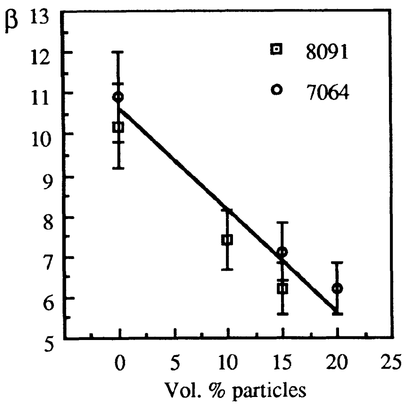

Fig. 4. Calculated nonlinearity parameter as a funtion of particle content

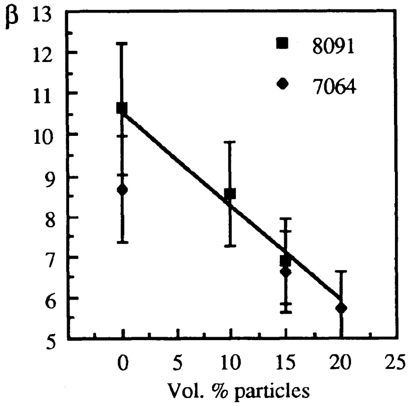

Fig. 5. Measured nonlinearity parameter as a funtion of particle content

A plot of $\beta_{meas.}$ vs. the volume fraction of reinforcement is displayed in Fig.5 . The values of both composites are close to a linear relationship between $\beta_{meas.}$ and the volume fraction of SiC. One value, namely that of the unreinforced 7064 specimen, is clearly deviating from this behavior. Interferences in the backwall echo sequence obtained on this specimen indicate a strong texture in the extrusion direction. Since the nonlinearity parameter varies significantly in different lattice directions of single crystalline materials, a texture is likely to change the value of the nonlinearity parameter.

The lower values for the nonlinearity parameter of SiC reinforced aluminum alloys can be understood from the fact, that ceramic materials with low nonlinearity will lower the nonlinearity parameter of aluminum alloys when a composite is formed. This does not mean that a law of mixture is applicable to model the bulk nonlinearity of a metal-matrix composite. Influences from the interfacial region between matrix and reinforcement are expected to contribute to the nonlinearity of the composite.

Because there is no significant difference between the calculated nonlinearity parameter determined in the isotropic plane and the one directly measured along the extrusion direction, it can be assumed that the nonlinearity parameter depends primarily on the overall content of reinforcement. Influences from the particle alignment in the extrusion direction could not be detected within the accuracy of the measurements.

Theoretical investigations by Cantrell [12] have shown that the nonlinearity parameter is related to the coefficient of thermal expansion (CTE) which is also a nonlinear quantity. In previous studies [11, 14], the coefficient of thermal expansion in MMCs has been found to decrease with increasing amounts of reinforcement. Measurements of the CTEs of the composites investigated in this paper are in progress.

CONCLUSIONS

The results of this study show that the acoustoelastic constants and the acoustic nonlinearity parameter are influenced by the amount of reinforcement in metal-matrix composites. Therefore, they are promising candidates to characterize the mechanical behavior of MMCs nondestructively. Also, the two quantities clearly indicate a decreasing elastic nonlinearity of the composite with the increasing content of SiC. The nonlinearity parameter changes linearly as a function of second phase content.

The absolute values of the calculated as well as of the directly measured acoustic nonlinearity parameter are in good agreement within the accuracy of the measurements. This shows that both techniques, measurements of absolute amplitudes using the capacitive gap receiver and measurements of the acoustoelastic effect, are suitable methods for the determination of the nonlinearity parameter.

The direct measurement of the nonlinearity parameter using the capacitive gap receiver requires a careful preparation of the specimen surfaces. The measurement of the acoustoelastic effect is restricted to simple specimen geometries since it requires the application of external stresses. The selection between the two methods depends on the geometry condition of the sample.

ACKNOWLEDGEMENT

This work is financially supported by the Army Research Office under contract No. DAAL03-88-K-0096.

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