THIN ROD FLEXURAL ACOUSTIC WAVE DEVICES: A SENSOR CANDIDATE

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INTRODUCTION

In the past two decades there has been growing interest in the development of integrated acoustic sensors. Sensors based on bulk (BAW) [1,2] and surface (SAW) [3,4] were reported. Most of these sensors operate in a gaseous medium, although a few are used with liquids or solids, to determine concentrations of chemical and biological substances, as well as viscosity, acceleration, temperature, pressure, etc. Recently, Wenzel and White [5] have found out that flexural plate wave gravimetric sensors can have higher mass sensitivity at low operating frequencies (a few MHz) than the BAW or SAW counterparts. The lowest flexural (antisymmetric) Lamb mode, \( A_0 \), was used for thin isotropic plates [5]. The thickness of the plates is much smaller than an acoustic wavelength.

The mass sensitivity, \( S_m \), can be defined as [5]

\[
S_m = \lim_{\Delta m \to 0} \frac{\Delta V}{V_0 \Delta m}
\]  

(1)

where \( \Delta m \) is the uniformly distributed mass per unit area added to the surface of the device; \( \Delta V = (V - V_0) \), \( V_0 \) and \( V \) are the unloaded and loaded phase velocity respectively. The sensitivity constant \( S_m \) was found to be approximately \(-1/(4\rho h)\) for the \( A_0 \) mode. \( \rho \) is the density of the thin plate. The high achievable \( S_m \) relies on the fact that the thickness \( h \) can be made very small (2 \( \mu \)m in [5]).

In this paper we propose an alternative approach which mainly uses thin rods rather than thin plates. The analysis of the lowest flexural, \( F_{11} \), rod mode is emphasized because it is quite analogous to the \( A_0 \) plate mode.

FLEXURAL ACOUSTIC (F11) MODE OF THIN RODS

Let us consider an isotropic acoustic rod with a radius, \( a \), Young's modulus, \( E \), and material density, \( \rho \). In the low frequency limit the flexural mode, \( F_{11} \), can be correctly described by the elementary theory of flexure, which provides the equation of motion [6] and [7]:

\[
where $U$ is the transverse displacement of the $F_{11}$ mode, $z$ is the propagation distance along the rod, $m = \pi a^2 p, \rho$ is the mass per unit length, $T$ is the tension exerted on the thin rod and $I = \pi a^4 / 4$ is the moment of inertia of the cross section about the neutral plane [6].

Assume the $F_{11}$ mode travelling along the thin rod has the displacement $U$ of the form $U = A e^{i(\omega t - kz)}$ where $A$ is the amplitude, $\omega = 2\pi f$ and $f$ is the operating frequency. $k$ is the propagation constant. Equation (2) can be rewritten as

$$-m\omega^2 + EI\beta^4 + T\beta^2 = 0$$

(3)

The phase velocity, $V = \omega / k$, and the group velocity, $V_g = \partial \omega / \partial k$, of $F_{11}$ mode can then be expressed as follows:

$$V = \left[ \left( \frac{T}{2m} \right)^2 + \frac{EI}{m} \right]^{1/2} + \frac{T}{2m}$$

(4)

$$V_g = \frac{T}{mV} + \frac{2EI\omega}{m} - \frac{1}{V^3}$$

(5)

In this paper only the case of zero tension ($T = 0$) is considered. The corresponding phase and group velocities are given as follows:

$$V = \left( \frac{EI\omega}{m} \right)^{1/4} = (\omega)^{1/2} a \left( \frac{\pi E}{4m} \right)^{1/4} = (\omega a)^{1/2} \left( \frac{E}{4\rho} \right)^{1/4}$$

(6)

From equations (5) and (6) we can also obtain

$$V_g = 2V$$

(7)

It is interesting to note that equations (6) and (7) are very similar to the velocity expressions for the $A_0$ mode [8]. The nonzero tension cases have been reported elsewhere [9].

Since the sensitivity, $S_m^V$, for the gravimetric sensor geometry is of interest, it is convenient to introduce the concept of mass per unit area, $m_\alpha$, in a fashion similar to that adopted in Ref.[5].

$$m_\alpha = \frac{m}{2\pi a} = \frac{\rho a}{2}$$

(8)

Following the approach in [5] we assume that when the sensor is loaded, the mass per unit area, $m_\alpha$, is changed by an amount of $\Delta m_\alpha (= \Delta m / 2\pi a)$ with negligible change in the rod diameter and Young's modulus, where $\Delta m$ and $\Delta m_\alpha$ are the mass change per unit length and area respectively. Equation (6) can then be written as

$$V + \Delta V = (\omega)^{1/2} a \left( \frac{\pi E}{4(m + \Delta m)} \right)^{1/4}$$

(9)

Using equations (1), (8) and (9) we can obtain

$$S_m^V = -\frac{1}{2\rho_\alpha}$$

(10)
which is the mass sensitivity of our proposed thin rod gravimetric sensor with no tension exerted on the thin rod ($T = 0$).

The mass sensitivity of the gravimetric sensor can be alternatively expressed as [5]

$$S'_m = \frac{1}{f_0} \lim_{\Delta \omega \to 0} \frac{\Delta f}{\Delta m},$$

where $\Delta f = f - f_0$. Again, $f_0$ and $f$ are the resonant frequencies on unperturbed and mass loaded cases, respectively.

Using Rayleigh hypothesis [10] and equation (7) we can show that the mass sensitivity of the velocity measurement for the thin rod flexural acoustic gravimetric sensors can be alternatively expressed as:

$$S^V_m = S'_m \left( \frac{v}{V_g} \right) = \frac{1}{2} S'_m$$

where $V$ and $V_g$ are again the phase velocity and group velocity of the acoustic mode.

As expected equation (10) is very similar to the result for the mass sensitivity for a plate-mode, $A_0$, gravimetric sensor ($S^V_m = -1/(4\rho h)$ ) in [5]. We can visualize that if the rod diameter is reduced, the mass sensitivity, $S^V_m$ or $S'_m$, increases. Similar to the analysis and claims for plate-mode devices reported in [5], the proposed thin rod flexural acoustic wave gravimetric sensors also have higher sensitivity at low operating frequencies (a few MHz) than the BAW and SAW counterparts. For BAW and SAW gravimetric sensors, $S^V_m$ is directly proportional to acoustic frequency, $f$ [5]. It is noted that operation at lower frequency means lower acoustic absorption losses in the device. In addition, acoustic experiments with an operating frequency at a few MHz are routine.

**Velocity Dispersion Measurements**

Pure gold wires of 10.5 $\mu$m radius were used as thin rod samples. They were chosen because of their uniform diameter. Figure 1 shows the calculated velocity dispersion relation for the lowest order flexural mode, $F_{11}$ of the thin gold wire; the lower solid and dotted lines for $F_{11}$ mode are calculated phase velocities based on exact [11] and approximated (Equation 6) dispersion formulae respectively. The upper solid line represents the exact group velocity. It is clear that equation (6) is very accurate for small $f_a$ values. In addition, $F_{11}$ exhibits dispersion characteristics very similar to the zeroth antisymmetric plate mode [5].

![Figure 1](image-url)
The basic arrangement for exciting and receiving thin rod flexural acoustic waves is shown in Fig. 2. A 10.5 μm radius gold wire is fixed at two posts by adhesive tape. A piezoelectric longitudinal ultrasonic transducer is bonded at the end of a silica glass horn. The small displacements of the ultrasonic transducer are transformed into large displacements at the tip of the horn. The tip of the horn is adhesively bonded to the gold wire. When the longitudinal waves in the glass rod reach the thin tip, they excite the flexural acoustic F_11 mode in the wire. The same type of transducer is also used at the receiving end. A similar excitation geometry and mechanism has been reported in Ref. [11].

We have adopted the standard continuous wave (CW) technique to measure the phase velocity of the F_11 mode. The measured phase difference of the received CW acoustic signal due to the change in propagation distance of the F_11 mode is used to calculate the phase velocity. The measured phase velocities indicated as closed circles in Fig. 1 agree very well with the theoretical prediction which assumes zero tension. Radio-frequency (RF) pulses were used to measure the group velocity dispersion. The measured group velocities indicated as open circles in Fig. 1 also agree with theoretical calculations. For velocity measurements the operating frequency ranges from 0.4 to 3.0 MHz.

FLEXURAL ACOUSTIC (F_11) MODE OF THIN RODS IMMERSED IN LIQUIDS

Since the flexural wave velocity of the thin rod can be smaller than the compressional velocity, C, of the liquid surrounding the thin rod at a small value of f_a, the energy which leaks to the liquid is minimal. Thus the thin rod flexural acoustic wave device is a good sensor candidate even when immersed in liquids. When a thin rod is immersed in a liquid, its velocity V (< C) can be shown to be [12]

\[ V = (\omega \sigma)^{1/2} \left( \frac{E}{4 \rho_f} \right)^{1/4} \left[ 1 + \frac{\rho}{\rho_f} \right]^{-1/4} \] (13)

where

\[ \sigma = 1 + \omega^2 \alpha^2 \eta \ln(\gamma \omega \eta^{1/2}) \] (14)

\[ \eta = \frac{1}{v^2} - \frac{1}{c^2} \] (15)

where \( \rho \) is the density of the liquid and \( \gamma = 0.8905 \). Inside the bracket \( \{ \} \) of equation (11) we can consider the second term as a perturbation to the velocity of the F_11 mode of the thin rod in vacuum due to the liquid surrounding.

For mass sensitivity analysis, we can also obtain

\[ S_m = \frac{-1}{2 \rho^* \sigma} \] (16)

where

\[ \rho^* = \rho + \rho \left[ 1 - \frac{\omega^2 \alpha^2}{4v^2} \left[ 1 - 2 \left( 1 - \frac{2v^2}{c^2} \right) \ln(\gamma \omega \eta^{1/2}) \right] \right] \] (17)
If the thin rod device is in vacuum, the mass sensitivity, $S^V_m$, increases when $a$ and $\rho_s$ decrease [1]. The first term of the r.h.s. of equation (17) represents the $S^V_m$ for thin rods in vacuum and the second term is the perturbation on $S^V_m$ due to the water surrounding.

Figure 3 shows the calculated dispersion characteristics of the $F_{11}$ flexural acoustic mode propagating along a fused silica rod in vacuum (solid curve) and in water (dashed curve). Fused silica rod and water were arbitrarily chosen. The water which surrounds the thin rod slightly decreases the velocity, $V$, of $F_{11}$ mode. The difference, $\delta V/V$, between the above two velocities remains relatively constant in the interested $f_a$ range which is less than 40 m/s.

The mass sensitivity, $S^V_m$, for the thin fused silica rod gravimetric sensor in vacuum (solid curve) and in water (dashed curve) are shown in Figure 4. In this case, frequency ($f$) equal to 1 MHz was assumed. $|S^V_m|$ increases with a reduced rod radius, $a$, and/or density, $\rho_s$. The water surrounding decreases $|S^V_m|$ slightly. It is noted that for thin plate flexural acoustic wave devices the surrounding liquid reduces the velocity of $A_0$ mode [5] and its $|S^V_m|$ as well.

Fig. 3 The dispersion characteristics of the $F_{11}$ flexural acoustic mode propagating along a thin fused silica rod in vacuum (solid curve) and in water (dashed curve).

Fig. 4 The mass sensitivity, $|S^V_m|$, for the thin fused silica rod gravimetric sensor in vacuum (solid curve) and in water (dashed curve). $f = 1$ MHz.
The above analyses assumed that the mass change of the thin rod gravimetric sensor is due to a sorption process which means that only the density changes but not the dimension. If both sorption and deposition contribute to the mass change, then

\[ \Delta m_t \sim \frac{1}{2} [\Delta \rho a + (2\rho_d - \rho) \Delta a] \]  

(18)

where \( \Delta \rho \) is the density change due to the sorption and \( \Delta a \) is the radius change due to the deposition. \( \rho_d \) is the density of the deposition layer.

If \( \rho_s \sim \rho_d - \rho \) and \( \Delta \rho = 0 \) which refers a deposition process and the density of the deposited layer is nearly the same as that of the thin plate, then we can obtain

\[ S_m^V = \frac{1}{\rho_s a} \]  

(19)

If \( \rho_s \sim \rho_d - \rho \) and \( \Delta \rho \neq 0 \) which refers that both sorption and deposition exist, then we can obtain

\[ S_m^V = -\frac{1}{2\rho_s a}(1 - 3X) \]  

(20)

where

\[ \lim_{\Delta m_t \to 0} \frac{\Delta a}{\Delta m_t} = X \geq 0 \]  

(21)

The relation between \( S_m^V \) is shown in Figure 5. Around \( X = 1/3 \) the mass sensitivity is very low. Although equation (21) is the result of a special case (\( \rho_s \sim \rho_d - \rho \)), nevertheless, it demonstrates that it is essential to know the actual physical process of such flexural acoustic wave gravimetric sensors.

CONCLUSION

Because of their high mass sensitivity thin rod flexural acoustic devices are excellent candidates for gravimetric sensors. A smaller rod radius offers higher sensitivity. Because of the low velocity, flexural acoustic wave devices can be immersed in liquids with minimal leakage loss. However, the velocity and mass sensitivity are reduced when the device is immersed in liquids. It has also been found that for flexural acoustic wave devices \( S_m^V = 1/2 \) \( S_m^L \).

![Fig.5](image_url)  

Fig.5 Mass sensitivity of a thin rod as a function of X under the condition that both sorption and deposition contribute to the mass change.
ACKNOWLEDGEMENT

This work was partially supported by the Natural Sciences and Engineering Research Council of Canada. One coauthor (M.V.) wishes to thank F.C.A.R. foundation (Quebec) for a fellowship.

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