A FINITE ELEMENT FORMULATION FOR ULTRASONIC NDT MODELING

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INTRODUCTION

Numerical analysis techniques have been successfully applied to the modeling of electromagnetic field/defect interactions[1]. Studies of magnetostatic leakage field and eddy current NDT phenomena have clearly shown that finite element codes can be used effectively for probe design[2] and the simulation of test geometries difficult to replicate in the laboratory[3]. In extending these codes to three dimensional geometries[4] and pulsed eddy current phenomena[5], it was realized that the required computing capability should also be sufficient to model ultrasound/defect interactions directly in the time domain. Increasing availability of powerful vector computers[6] bodes well for the ultimate solution of the generic NDT problem in which it is desired to predict the probe response to any arbitrarily shaped defect. As a first step in this direction, the NDT research group at Colorado State University, following the pioneering numerical efforts of Bond[7] and Dewey[8], has developed a finite element code for direct time domain solution of the elastic wave equation (Figure 1 shows the relationship between numerical and analytical approaches). The following sections describe the finite element formulation and the application of the code to the prediction of 2-D displacements in a rectangular bar excited at one end by a step input of force.

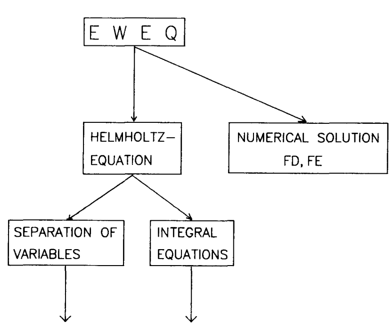

Fig. 1. Relationship between numerical (finite differences, finite elements) and analytical approaches

FORMULATION

The general equation of motion can be written in the form

$$\nabla \cdot \underline{\underline{T}} + \bar{F} = \rho \frac{\partial^2 \bar{u}}{\partial t^2} \tag{1}$$

where $\underline{\underline{T}}, \bar{F}, \bar{u}$ represent stress tensor, body force and displacement vectors respectively. ρ denotes the material density. Three restrictions are imposed:

1) no body forces

   $\bar{F} = 0$

2) no internal energy losses and small deformations such that Hook's law is applicable

   $\underline{\underline{T}} = \underline{\underline{\underline{\underline{C}}}} : \underline{\underline{S}}$

   with $\underline{\underline{\underline{\underline{C}}}}$ being the forth rank material tensor and $\underline{\underline{S}}$ representing the strain tensor

3) only a homogeneous isotropic solid is considered. Thus, the material tensor consists of only two independent coefficients λ and μ (Lame constants)

   $C_{ijk\ell} = \lambda \delta_{ij} \delta_{k\ell} + \mu(\delta_{ik}\delta_{j\ell} + \delta_{i\ell}\delta_{jk})$

Substitution of these three conditions into (1) yields the elastic wave equation in rectangular coordinates

$$(\mu+\lambda)\nabla(\nabla \cdot \bar{u}) + \mu \nabla^2 \bar{u} = \rho \frac{\partial^2 \bar{u}}{\partial t^2} \tag{2}$$

If $V_L^2 = (\lambda+2\mu)/\rho$ and $V_s^2 = \mu/\rho$ are introduced as longitudinal and shear velocities, (2) can be expressed for the two dimensional case as

$$V_L^2 \frac{\partial^2 u_x}{\partial x^2} + V_s^2 \frac{\partial^2 u_x}{\partial y^2} + (V_L^2 - V_s^2) \frac{\partial^2 u_y}{\partial x \partial y} = \frac{\partial^2 u_x}{\partial t^2} \tag{3a}$$

$$V_L^2 \frac{\partial^2 u_y}{\partial y^2} + V_s^2 \frac{\partial u^2 y}{\partial x^2} + (V_L^2 - V_s^2) \frac{\partial^2 u_x}{\partial y \partial x} = \frac{\partial^2 u_y}{\partial t^2} \tag{3b}$$

with the Neumann type boundary conditions given by

$$T_{xx/\rho} = V_L^2 \frac{\partial u_x}{\partial x} + (V_L^2 - 2V_s^2) \frac{\partial u_y}{\partial y} \tag{4a}$$

$$T_{xy/\rho} = V_s^2 \left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right) = T_{yx/\rho} \tag{4b}$$

$$T_{yy/\rho} = V_L^2 \frac{\partial u_y}{\partial y} + (V_L^2 - 2V_s^2)\frac{\partial u_x}{\partial x} \tag{4c}$$

FINITE ELEMENT IMPLEMENTATION

Instead of developing a direct discretization of (3) by means of collocation or Galerkin's method, we consider an energy related functional

$$\bar{F}(\bar{u}) = \int \underline{\underline{S}}:\underline{\underline{T}}dV + \int \bar{u}\,\frac{\partial^2 \bar{u}}{\partial t^2}\,\rho dV$$

or

$$\bar{F}(\bar{u}) = \frac{1}{2}\int\{V_L^2[(\frac{\partial u_x}{\partial x})^2 + (\frac{\partial u_y}{\partial y})^2] + 2(V_L^2 - 2V_s^2)\frac{\partial u_y}{\partial y} + V_s^2(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x})^2 + 2(u_x\frac{\partial^2 u_x}{\partial t^2} + u_y\frac{\partial^2 u_y}{\partial t^2})\}\rho dV \tag{5}$$

which, upon finding a stationary value with respect to the unknown displacements u_x, u_y, results in the same solution. An easy way to check the correctness of the above functional is to utilize variational calculus in order to arrive at the so called Euler equations which subsequently yield the original elastic wave equations (3a) and (3b). It can also be shown by the same derivation that the stress free boundary conditions are implicit in the energy related functional.

To solve (5) in terms of the unknown displacements, the following four steps have to be performed

a) discretize solution domain into a finite number of elements

b) find a stationary value for (5) with respect to u_x, u_y

c) replace u_x, u_y by the approximations

$$u_x = [N(x,y)]\{u_x\}_e \;,\; u_y = [N(x,y)]\{u_y\}_e$$

$$\frac{\partial u_x}{\partial x} = [\frac{\partial N(x,y)}{\partial x}]\{u_x\}_e \;,\; \frac{\partial u_y}{\partial y} = [\frac{\partial N(x,y)}{\partial y}]\{u_y\}_e$$ etc., where $[N(x,y)]$

denotes the shape functions as a row vector with $\{u_x\}_e$, $\{u_y\}_e$ being the unknown displacements at the nodal points of each element. The resulting elemental matrix equation takes on the form

$$[K]\{u\}_e + [M]\{\ddot{u}\}_e = \{\bar{F}\}$$

or

$$\begin{bmatrix} [K_{xx}] & [K_{xy}] \\ [K_{yx}] & [K_{yy}] \end{bmatrix} \begin{Bmatrix} \{u_x\}_e \\ \{u_y\}_e \end{Bmatrix} + \begin{bmatrix} [M_x] & 0 \\ 0 & [M_y] \end{bmatrix} \begin{Bmatrix} \{\ddot{u}_x\}_e \\ \{\ddot{u}_y\}_e \end{Bmatrix} = \begin{Bmatrix} \{\bar{F}_x\} \\ \{\bar{F}_y\} \end{Bmatrix} \tag{6}$$

with the coefficients of the submatrices given by

$$K_{xx}(I,J) = \int [V_L^2 \frac{\partial N_I}{\partial x} \frac{\partial N_J}{\partial x} + V_s^2 \frac{\partial N_I}{\partial y} \frac{\partial N_J}{\partial y}] \rho dV$$

$$K_{xy}(I,J) = K_{yx}(J,I)$$

$$K_{xy}(I,J) = \int [(V_L^2 - 2V_s^2) \frac{\partial N_I}{\partial x} \frac{\partial N_J}{\partial y} + V_s^2 \frac{\partial N_I}{\partial y} \frac{\partial N_J}{\partial x}] \rho dV$$

$$K_{yy}(I,J) = \int [V_L^2 \frac{\partial N_I}{\partial y} \frac{\partial N_J}{\partial y} + V_s^2 \frac{\partial N_I}{\partial x} \frac{\partial N_J}{\partial x}] \rho dV$$

$$M_x(I) = M_y(I) = \int N_I N_J \rho dV$$

$\bar{F}_x(I), \bar{F}_y(I)$ are external driving forces. The numerical integration is carried out by employing a 7 point Gaussian quadrature formula.

d) Assemble all the elemental matrices (6) into a global matrix which can be solved for $\{u_x\}_e$ and $\{u_y\}_e$ Before the assembly can be done however, the problem consists of integrating the second time derivative in (6). Possible integration schemes are termed as either explicit or implicit depending on whether a matrix inversion of [K] is involved. The central difference integration (explicit) as well as the Houbolt, Wilson and Newmark integration (implicit) have been implemented. For the purposes of this paper only the Newmark integration is given

$$(\frac{1}{\alpha \Delta t^2} [M] + [K])\{u\}_{t+\Delta t} = \{\bar{F}\}_{t+\Delta t} + \frac{1}{\alpha \Delta t} [M]\{u\}_t + \frac{1}{\alpha \Delta t} [M]\{\dot{u}\}_t + + (\frac{1}{2\alpha} - 1)[M]\{\ddot{u}\}_t$$

This scheme can be made unconditionally stable depending on the selection of α and δ.

APPLICATIONS

In order to validate the finite element code, a bar subject to a step tension T_0 was modeled as shown in Figure 2. In Figure 3 the u_x displacement is plotted at three different locations A,B,C within the bar. The results are in excellent agreement with the one dimensional displacement predictions by Dewey et al. who also shows the analytical series solution.

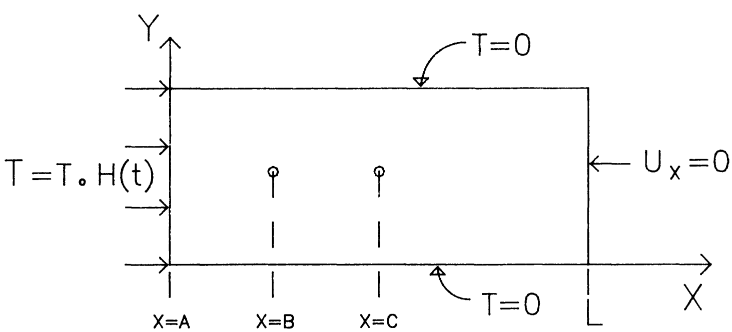

Fig. 2. Geometry and boundary conditions of rectangular bar

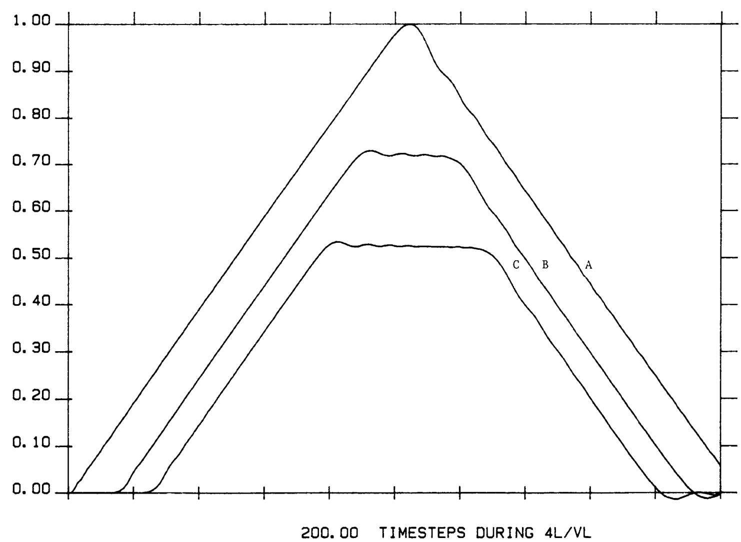

Fig. 3. Normalized u_x displacement at locations A,B,C

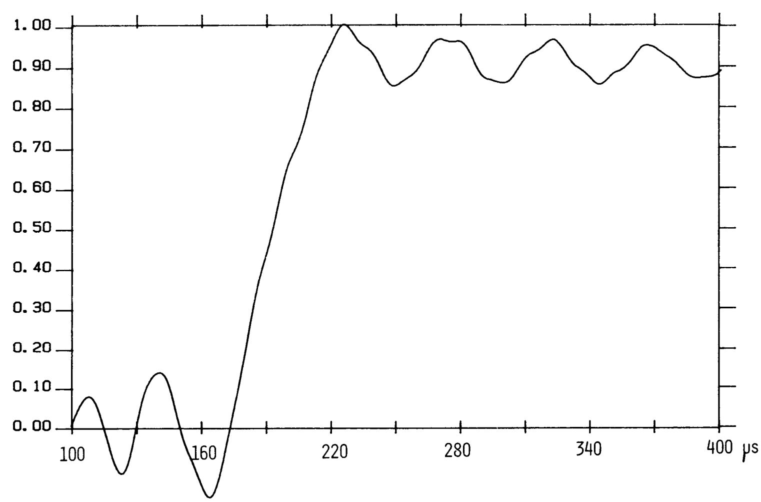

Fig. 4. Normalized u_y displacement at x-37.5 in.

A more crucial test results if one attempts to obtain the u_y displacements for a wide rectangular bar subject to a longitudinal step pressure loading. Jones and Ellis[9] compared the theoretical predictions of the plane-stress theory with their experimental observations. Their experimental results for a 130 inch long and 1.5 inch wide bar with V_s=1.248 x 10^5 in/s and a Poisson ratio of σ=0.335 also show good agreement with the finite element prediction shown in Figure 4. Typical solution times on a VAX 11/780 computer for two different mesh sizes are given in Table 1. These figures are somewhat misleading, however, since both mesh size and computer code are not yet optimized. In general, the computer time is a function of wave velocity, transducer frequency, sampling rate, distance of travel and mesh size. To illustrate this for a two dimensional case and show how a powerful computer like the CYBER 205 can significantly reduce the time requirement, consider the more sophisticated problem of pulse echo wave propagation as shown in Figure 5. Here a pulse created by a 1 MHz transducer propagates with a longitudinal velocity of V_L=5000 m/s through a specimen of 12.5 cm thickness. The total travel time for twice the thickness is, therefore, 50μs. Based on an assumed sampling rate of 32 MHz it follows that 1600 time steps solution time are required. If a solution domain requiring a mesh size of 3000 nodes is assumed, it will take a VAX 11/780 computer about 4.5 minutes to solve the resulting matrix equation at each time step. Based on our experience with eddy current calculations, the CYBER 205 reduces the solution time to 0.0056 min. For a total of 1600 time steps this would amount to 120 hours on the VAX versus 9 minutes on the CYBER 205.

Table 1. Solution time for transient bar analysis

CP–requirements on VAX 11/780

elements	nodes	CP–time(900 timesteps)
208	477	1h 51 min
624	1325	8h 20 min

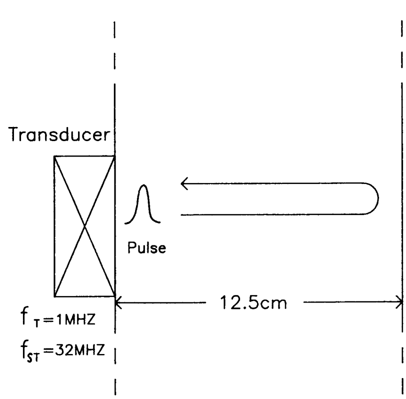

Fig. 5. Pulse echo time considerations

CONCLUSIONS

A considerable amount of work remains to be done before numerical code can be used as an engineering tool for the design and analysis of ultrasonic nondestructive tests. Early studies in this field show promise, however, and the increasing availability of supercomputers can provide the computational power needed to ultimately predict ultrasonic transducer responses from realistic defect geometries. Although the finite element formulation and applications described in this paper are 2-D in nature, only computational cost limits the extension to 3-D geometries.

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