ULTRASONIC SCATTERING IN COMPOSITES USING SPATIAL FOURIER TRANSFORM TECHNIQUES

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INTRODUCTION

The heterogeneous nature of composite materials often makes their inspection using ultrasonics difficult unless the flaws are sufficiently large so that common B- or C- scans can be employed. Thus, flaw scattering models are essential in order to interpret the measured ultrasonic responses. However, even at low frequencies where the composite may be able to be replaced by an equivalent homogeneous, anisotropic material, conventional direct scattering methods such as the T-Matrix and Boundary Element techniques are not effective. This is because both methods rely on the superposition of exact solutions to the governing equations of elastodynamics and, except for very special anisotropics, such exact solutions are not available in closed form. One way around this difficulty is to pose the scattering problem in a spatial Fourier frequency domain where exact fundamental solutions for elastodynamics are available, even for general anisotropic materials (1). Employing these solutions in a conventional volume or surface integral equation for the scattering wavefields then yields a spatial frequency domain formulation to the direct scattering problem. Because the boundary conditions are given in the real spatial domain, it is necessary to iteratively satisfy these conditions via fast Fourier transforms (2). This approach is called the Spectral-Iteration Technique and has been applied successfully for a variety of electromagnetic scattering problems (3), (4). Here, we will obtain the equivalent elastic wave scattering formulations for cracks and volumetric flaws in a general anisotropic medium. Modifications of the standard Spectral-Iteration technique needed to ensure its convergence at low frequencies will also be discussed.

PLANAR CRACK MODEL

Consider first a plane harmonic wave incident on an open crack which occupies the surface S in the plane $z = 0$ (Fig. 1).

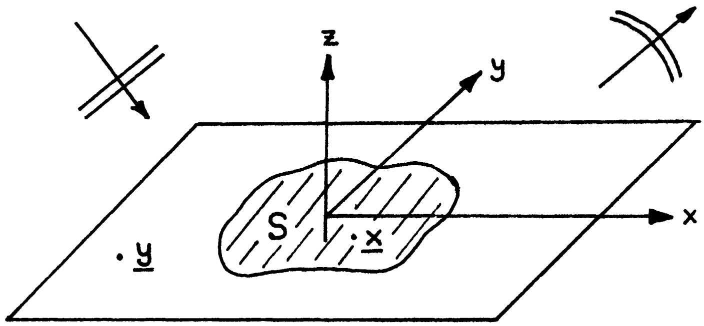

Figure One. Crack Scattering Geometry

The integral relation which governs this scattering problem is (5):

$$t_r^{scatt}(\underline{y}) = -C_{rskm}\, C_{ijqp}\, n_s\, n_j \int_S \partial^2 G_{qk}(\underline{x}-\underline{y})/\partial x_m \partial x_p \Delta u_i(\underline{x}) dS(\underline{x}) \tag{1}$$

where t_r^{scatt} are the tractions at point $\underline{y}$ due to the scattered waves, Δu_i are the components of the displacement discontinuity on S, C_{ijkl} is the fourth order elastic constant tensor, and G_{qk} is the fundamental solution tensor for the equations of elastodynamics. If the crack orientation is as shown in Fig. 1, $n_s = \delta_{s3}$ and $n_j = \delta_{j3}$ where δ_{ij} is the Kronecker delta symbol.

On the surface of the crack the total tractions must vanish. Thus, for $\underline{y}$ in S in Eq. (1) we have

$$t_r^{scatt}(\underline{y}) = -t_r^{inc}(\underline{y}) \tag{2}$$

where t_r^{inc} are the tractions due to the incident waves. Now, consider taking spatial Fourier transforms on x and y of both sides of Eq. (1) where the Fourier transform is defined as

$$\tilde{f}(\xi_1,\xi_2,z) = \iint_{-\infty}^{\infty} f(x,y,z)\, e^{i\xi_1 x}\, e^{i\xi_2 y}\, dx\, dy \tag{3}$$

Since the integral relationship is of the convolution type, we obtain on the plane z = 0:

$$\tilde{t}_r^{scatt} = -C_{rskm}\, C_{ijqp}\, n_s\, n_j\, \partial^2 \tilde{G}_{qk}/\partial x_m \partial x_p \Delta\tilde{u}_i \tag{4}$$

Equation (4) is a purely algebraic matrix equation relating these Fourier-transformed quantities which can be written symbolically as

$$\underset{[3x1]}{\tilde{t}_r^{scatt}} = \underset{[3x3]}{\tilde{K}_{ri}}\ \underset{[3x1]}{\Delta\tilde{u}_i} \tag{5}$$

Equation (4) shows that in order to determine the values of the coefficient matrix K_{ri} in Eq. (5), the 2-D spatial Fourier transforms of the derivatives of the fundamental solution, G_{qk}, must be known. For an isotropic medium these are given simply as

$$\partial^2\tilde{G}_{qk}/\partial x_m \partial x_p = -i/2G \left\{ \xi m \xi p(\delta_{qk} + \xi_q \xi_k/K_2^2)/\xi_3 \Big|_{\xi_3 = i(\xi_1^2 + \xi_2^2 - K_2^2)^{\frac{1}{2}}} \right.$$
$$\left. \xi_m \xi_p \xi_q \xi_k / K_2^2 \xi_3 \Big|_{\xi_3 = i(\xi_1^2 + \xi_2^2 - K_1^2)} \right\} \quad (6)$$

where G is the shear modulus, $K_\alpha = \omega/c_\alpha$ ($\alpha = 1,2$) are the wavenumbers for dilational and shear waves, respectively, with c_α ($\alpha = 1,2$) being the dilatational and shear wavespeeds and ω the frequency. Even in the general anisotropic medium case, however, we have

$$\partial^2\tilde{G}_{qk}/\partial x_m \partial x_p = \sum_\alpha [\xi_m \xi_p f_{qk}(\xi_1,\xi_2,\xi_3)/S_{,3}]_{\xi_3} = G_\alpha(\xi_1,\xi_2,\omega) \quad (7)$$

where is an algebraic form that can be evaluated explicitly for special anisotropics and numerically in the general case (1). Thus, composite materials having many types of equivalent anisotropics can be handled effectively by this approach.

Although Eq. (5) is a purely algebraic equation, it cannot be used directly to solve for the unknown displacement discontinuities Δu_i because t_r^{scatt} is unknown outside the crack surface S, and so $\tilde{t}_r^{scatt}$ is also unknown. Equation (5) can, however, be used as part of the iterative method described by Kastner and Mittra (3).

THE SPECTRAL-ITERATION TECHNIQUE

The Spectral-Iteration technique relys on the ability to go rapidly between spatially transformed quantities and real quantities using the fast Fourier transform (FFT) algorithm. This capability is essential because Eq. (5) is a relationship between transformed quantities that guarantees that the equations of elastodynamics are satisfied while the boundary conditions on the crack surface, Eq. (2), must be enforced on actual (untransformed) variables. To simultaneously satisfy both the equations of motion and boundary conditions we can follow the following sequence of steps:

1. Begin with an initial guess for Δu_i on the crack face. This could simply be $\Delta u_i = 0$ or the values of Δu_i obtained from the Kirchoff approximation, for example.
2. Use a 2-D FFT to compute $\Delta\tilde{u}_i$.
3. Form the matrix product $\tilde{K}_{ri}\ \Delta\tilde{u}_i$.
4. Compute the inverse 2-D FFT to obtain t_r^{scatt} on the plane z = 0.

5. Replace t_r^{scatt} values on S by $-t_r^{inc}$ (i.e., satisfy the boundary conditions) leaving the remaining values unchanged. Note that a convergence check on how well the boundary conditions are being satisfied is available at this step.
6. Compute the 2-D FFT of the results of step 5, $\tilde{t}_r^{scatt}$.
7. Form the matrix product $\Delta\tilde{u}_m = \tilde{K}_{mr}^{-1}\, \tilde{t}_r^{scatt}$.
8. Compute the inverse 2-D FFT to obtain a new Δu_m.
9. Outside S, set $\Delta u_m = 0$, keeping the remaining values inside S unchanged. Since the exact Δu_m should automatically vanish outside S, a convergence check is also available at this step.
10. Compute the 2-D FFT of this updated Δu_m, $\Delta\tilde{u}_m$.
11. Go to step 3.

The above steps are analogous to those presented by Kastner and Mittra for electromagnetic problems (3). As they point out, the method is very storage efficient since no large matrices are ever inverted and the same storage can be used over and over during the iteration. Thus, very large scatterers can be treated. In fact, results up to non-dimensional frequencies of ka = 50 have been obtained for certain electromagnetic problems (3). This capability greatly exceeds standard matrix scattering methods such as the T-Matrix or Boundary Elements.

Very often, in scattering problems the end result desired is the far-field values of the scattered waves. Following the same procedures outlined by Gubernatis et. al. (5), these far-field quantities for our crack problem can be shown to be directly obtained from the f-vectors defined as

$$f_i(\underline{\xi}) = -i\xi^3\, C_{ijkl}\, \hat{r}_j\, n_k/4\pi\rho\omega^2 \int_S \Delta u_l\, e^{-i\underline{\xi}\cdot\underline{r}'}\, dS' \qquad (8)$$

where $\underline{\xi} = K_{1,2}\hat{\underline{r}}$

Equation (8) shows that the f-vectors can be obtained from the spatial Fourier transform of Δu_e which is bandlimited to the "visible" region $|\underline{\xi}| \leq K_{1,2}$. This suggests that we need seek only to solve for a filtered version of Δu_e which has finite support in the spatial Fourier transform domain. Naturally, this can lead to greatly reduced sampling requirements.

INCLUSION MODEL

The previous section illustrated the application of the Spectral-Iteration technique for the special case of a planar crack. More general volumetric scatterers, such as inclusions, can, however, also be handled using a "stacked" version of the proceeding method. Briefly, this method consists of breaking up an inclusion into N parallel planes, each separated by a distance Δ (Fig. 2).

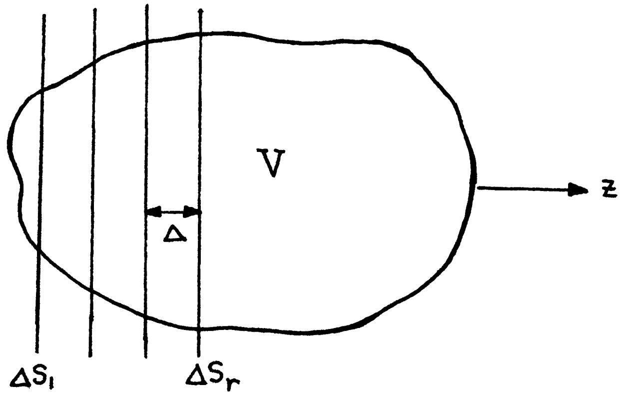

Figure Two. Stacked plane inclusion model.

Here, the governing equations are

$$u_i^{scatt}(\underline{y}) = \int_V \delta\rho\omega^2 G_{im}(\underline{x}-\underline{y})\, u_m(\underline{x})\, dV(\underline{x}) + \int_V \delta C_{jklm}\, \partial G_{ij}(\underline{x}-\underline{y})/\partial y_k\, \varepsilon_{lm}(\underline{x})\, dV(\underline{x}) \tag{9}$$

and

$$\varepsilon_{ip}^{scatt}(\underline{y}) = \int_V \delta\rho\omega^2\, \partial G_{im}(\underline{x}-\underline{y})/\partial y_p\, u_m(\underline{x})\, dV(\underline{x}) + \int_V \delta C_{jklm}\, \partial^2 G_{ij}(\underline{x}-\underline{y})/\partial y_k \partial y_p\, \varepsilon_{lm}(\underline{x})\, dV(\underline{x}) \tag{10}$$

where u_i, ε_{ip} are the displacement and strain components, respectively, and δ_ρ, δC_{jklm} are the density and elastic constant differences between the inclusion and the host material. If these equations sampled at N planes, we obtain

$$u_i^{scatt}(y_1, y_2, r\Delta) = \sum_{s=1}^{N} \Delta \left[\int_{\Delta S_S} \{\delta\rho\omega^2\, \partial G_{im}(u, v, (r-s)\Delta) \cdot u_m(x_1,x_2,s\Delta) + \delta C_{jklm}\, \partial G_{ij}(u, v, (r-s)\Delta)/\partial y_k \cdot \varepsilon_{lm}(x_1,x_2,s\Delta)\}\, dx_1, dx_2 \right. \tag{11}$$

where $u = x_1 - y_1$, $v = x_2 - y_2$.

Taking the spatial Fourier transform on y_1, y_2 of both sides of Eqs. (11) and (12) then yields

$$\tilde{u}_i^{scatt}(\xi_1,\xi_2,r\Delta) = \sum_{S=1}^{N} [\delta\rho\omega^2 \tilde{G}_{im}(\xi_1, \xi_2, (r-s)\Delta) \tilde{u}_m(\xi_1,\xi_2,s\Delta) + \delta C_{jklm} \partial\tilde{G}_{ij}(\xi_1, \xi_2, (r-s)\Delta)/\partial y_k \tilde{\varepsilon}_{lm}(\xi_1, \xi_2, s\Delta)] \quad (13)$$

and

$$\tilde{\varepsilon}_{ip}^{scatt}(\xi_1,\xi_2,r\Delta) = \sum_{S=1}^{N} [\delta\rho\omega^2 \partial\tilde{G}_{im}(\xi_1, \xi_2, (r-s)\Delta)/\partial y_p \tilde{u}_m(\xi_1,\xi_2,s\Delta) + \partial C_{jklm} \partial^2\tilde{G}_{ij}(\xi_1, \xi_2, (r-s)\Delta)/\partial y_k \partial y_p \tilde{\varepsilon}_{lm}(\xi_1,\xi_2,s\Delta)] \quad (14)$$

Equations (13) and (14) are again purely algebraic equations that allow us to use the Spectral-Iteration technique one plane at a time, sweeping through the inclusion in the same fashion as was done for electromagnetic problems (3). At each plane inside V the "constitutive" equation for the scatterer must be applied. For example, for a void these "constitutive" equations would be in V:

$$u_i^{scatt} = - u_i^{inc}$$
$$\varepsilon_{ip}^{inc} = - \varepsilon_{ip}^{inc} \quad (15)$$

Since the details of the iterative procedure again follow those of the electromagnetic case (3), we do not present any more explicit results here.

CONCLUDING REMARKS

Although the Spectral-Iteration technique is an effective tool for handling many previously intractable scattering problems, there are some difficulties in obtaining convergent solutions via the standard Spectral-Iteration method as described above at low frequencies. However, by imbedding this technique within an iterative procedure which minimizes the error in the solution at each step in an integrated square error sense, Van den Berg (6) has shown how such convergence difficulties can be avoided.

We are currently in the process of applying these iterative procedures to elastodynamic crack and inclusion scattering problems. Initially, software is being developed for the special cases of isotropic and transversely isotropic media. The latter case is particularly important since many fiber-reinforced composites can be modelled by such an equivalent anisotropy. Later, more general anisotropic cases will be considered and the results integrated into a layered geometry model (Fig. 3) for application to laminated composite structures.

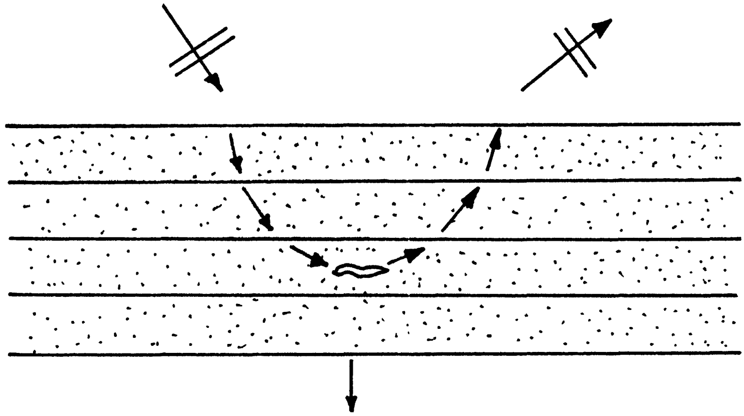

Figure Three. Layered composite geometry.

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